YALE
INTERNATIONAL
CENTER
FOR FINANCE

ILLIQUID ALTERNATIVE ASSET FUND MODELING

Dean Takahashi
Yale University Investments Office

Seth Alexander
Yale University Investments office

January 2001

Yale SCHOOL *of* MANAGEMENT

Illiquid Alternative Asset Fund Modeling

Dean Takahashi
Senior Director, Yale University Investments Office

Seth Alexander
Associate Director, Yale University Investments Office

January 2001

Abstract:

This paper describes a financial model that enables institutional investors to project future asset values and cash flows for funds in illiquid alternative asset classes such as venture capital, leveraged buyouts, real estate, and natural resources. The model incorporates actual historical information to provide a base for forecasts. Importantly, the model allows investors with a portfolio of alternative assets to assess the impact of changing fund commitment levels and of varying assumptions regarding contributions, distributions and underlying net returns.

Illiquid Alternative Assets

Illiquid alternative assets have become an increasingly significant portion of institutional portfolios as investors, most notably endowments and foundations, seek high returns and diversification relative to traditional stock and bond investments. Allocations to non-publicly traded investments, including venture capital, leveraged buyouts, real estate and natural resource investments have grown from an average of 5.1% of university endowments in June, 1990 to 13.3% as of June, 1999.[i] Investment returns from venture capital, in particular, have been spectacular, often differentiating top performing institutional funds from the rest of the pack.

Yale University has been an early and leading investor in non-publicly traded, alternative asset classes with initial commitments to natural resources in 1950, leveraged buyouts in 1973, venture capital in 1976, and real estate in 1978. As of June 30, 2000, Yale's allocation to these investments totaled more than 40% of its $10.1 billion Endowment.

Illiquid Alternative Asset Fund Modeling

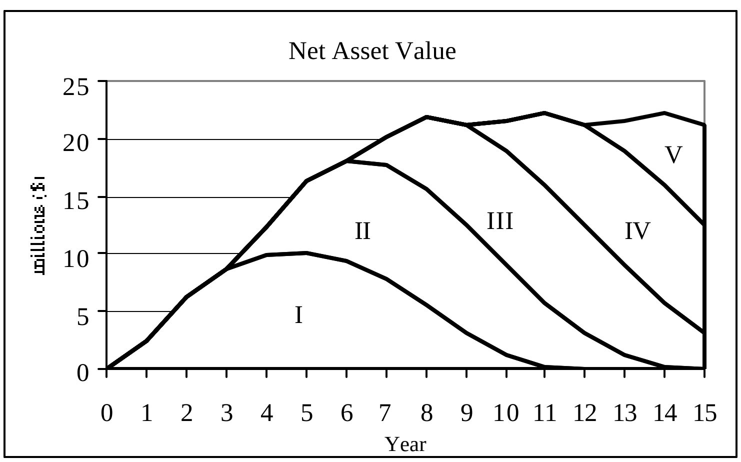
Funds that invest in illiquid assets present particular challenges for portfolio managers. Asset class allocations depend not only on the level of commitment to individual funds, but also on the rate and timing of managers' drawdowns and distributions. Effective management of alternative assets requires reasonably sophisticated modeling of projected manager behavior.

---

[i] National Association of College and University Business Officers dollar-weighted universe as cited by Cambridge Associates.

Institutions invest in alternative assets predominantly through commingled limited partnership funds. The funds are raised every few years on a blind pool basis by general partners who actively invest, manage and harvest portfolio investments. At the onset of the partnership, investors commit capital that gets drawn down over several years by the general partner. The uncertain schedule of drawdowns, unknowable changes in the valuation of the partnership's investments, and unpredictable distributions of cash or securities to the limited partners combine to make it difficult to predict accurately the future value of partnership interests. Nonetheless, because investors target a percentage allocation for illiquid alternatives that translates into an asset value, some form of projection must be employed. In addition, it is often helpful for the investor to estimate expected future cash needed to meet capital commitments, as well as projected distributions that would generate liquidity.

Prior to the mid 1990s, most institutional investors were trying to increase their relatively small and growing allocations to alternative assets. The U.S. stock market was exceptionally strong and investors were focused on reaching their target allocations to alternative assets. At that time, simple rules of thumb relating commitment levels to expected asset values of funds were good enough. One common method for venture capital was to invest roughly fifty cents in each of a series of funds to achieve a dollar of ongoing value aggregated across funds. This method works in certain model circumstances, as illustrated below, in which a series of $10 million commitments every three years results in an approximate aggregate exposure of $20 million.



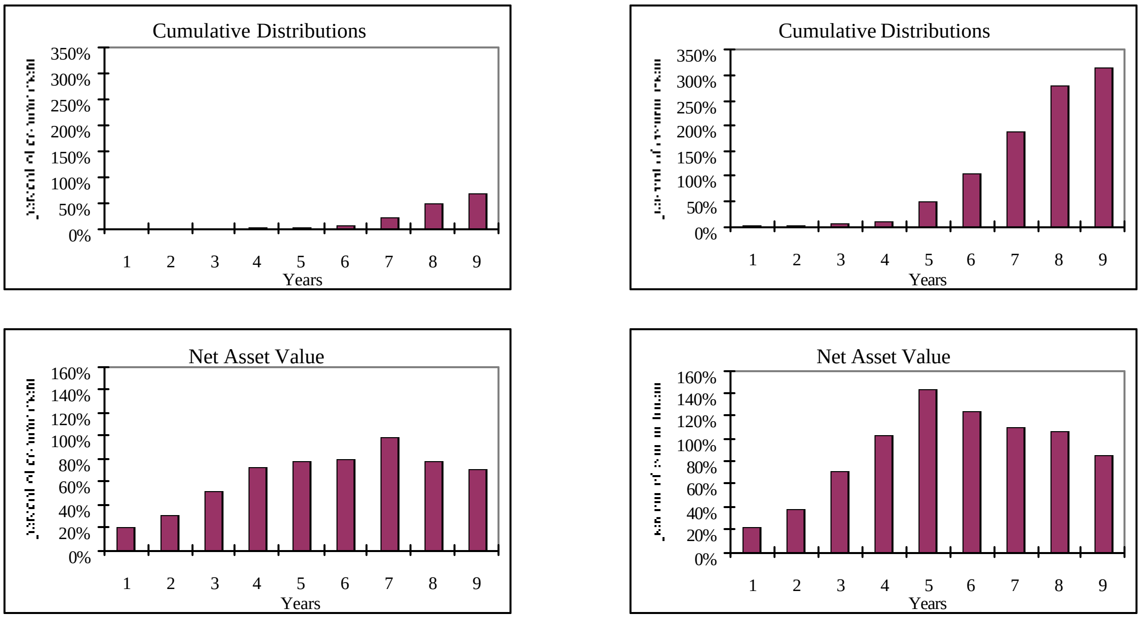
Commitments

Fund	Year	$$
I	0	10 million
II	3	10 million
III	6	10 million
IV	9	10 million
V	12	10 million

As allocations to alternative assets grew, however, the inability of these models to make reasonable future projections became apparent. Seeking more accurate projections, we created a model in the mid 1990s which assumed future funds would have the same pattern of capital contributions, distributions, and asset values as historical averages. This type of model, while superior to simple rules of thumb, still had a number of problems. For example, the "average" fund experience varied greatly depending on the time sample, raising the question of which historical period upon which to base the model. Many funds raised in the early and mid 1980s had dismal returns with little growth in asset values and delayed distributions. On the other hand, venture capital funds in the late 1980s and early 1990s exhibited strong performance and asset values significantly above capital contributed. Yale's venture capital funds exhibited marked differences in returns between time periods, as shown below.

Venture Capital: Vintage Years 1984 - 1986*

Venture Capital: Vintage Years 1990 - 1992**



* sample size equals 6 funds
** sample size equals 8 funds

Prospective investment environments do not necessarily resemble any historical period. For example, in the late 1990s venture capitalists drew down capital at unprecedented rates with unprecedented returns. Unlike earlier funds which drew down capital over four to five years, funds raised in the late 1990s frequently were fully invested within two years. Performance soared as well with many funds earning triple digit rates of return. Historical data from any time period provided a poor template from which to model these funds.

New Model

In 1998, faced with a need for better estimates of future values in making private equity and real asset capital commitments, we set out to develop an alternative asset fund projection model which met several criteria. First, the model should be simple and sensible on a theoretical basis. Second, the model should be able to incorporate and respond to actual capital commitment experience and current partnership asset values. Third, the model should be able to analyze the portfolio impact of varying return scenarios and changing rates of investment and distribution. Fourth, the model should be useful for a variety of asset types.

Recognizing that the fund experience varied greatly depending on the time sample and the investment environment, we initially fit the sample model to historical data, but then allowed it to be adjusted as needed. The model uses six inputs to project capital contributions, distributions, and net asset value.

Inputs	Description
RC	Rate of contribution
CC	Capital commitment ($)
L	Life of the fund (years)
B	Factor describing changes in the rate of distribution over time
G	Annual growth rate (%)
Y	Yield (%)

Outputs	Description
C	Capital contributions ($)
D	Distributions ($)
NAV	Net asset value ($)

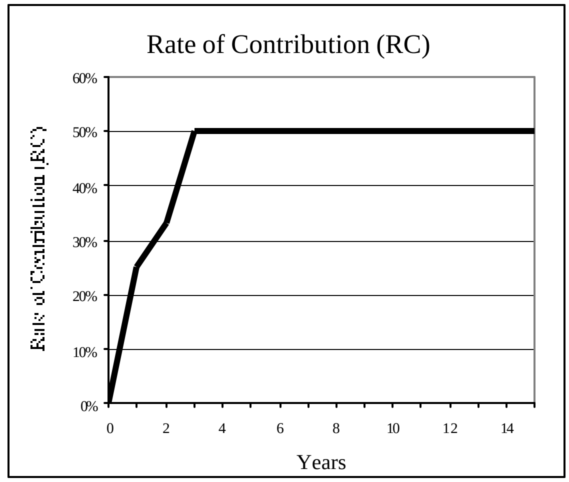
Capital Contributions

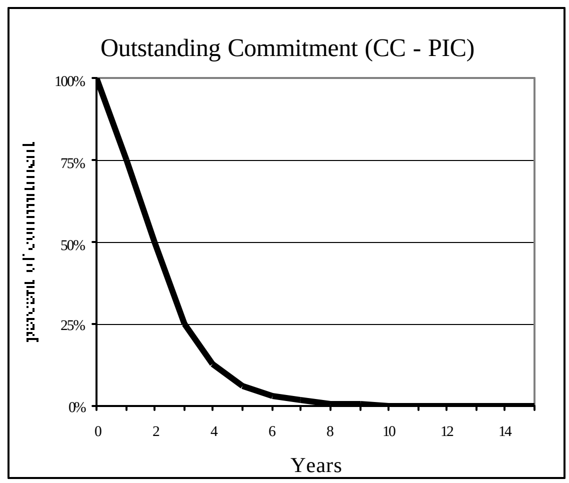
In most cases, capital contributions are heavily concentrated in the first few years of a fund's life. After the initial burst of investment activity, contributions to a fund are made at a declining rate as fewer new investments are made and follow-on investments and fees are spread out over a number of years. As shown below, capital contributions are calculated by multiplying the rate of contribution by the remaining capital commitment, or the initial capital commitment minus the sum of the paid in capital (PIC)[ii].

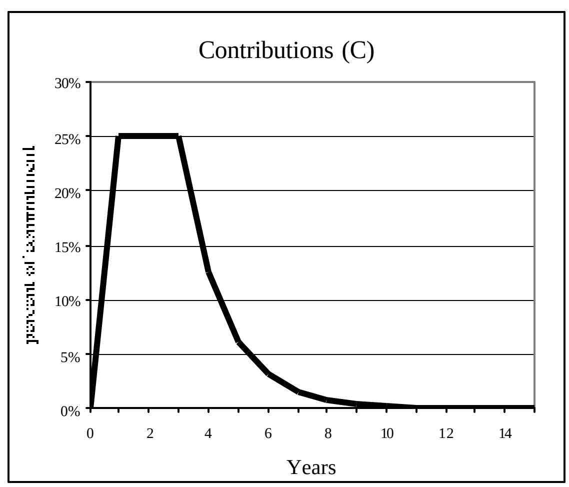
$$C_{(t)} = RC_{(t)} \left(CC - PIC_{(t)}\right)^{\text{iii}}$$

The graphs shown to the right use the following inputs: 25% contribution rate in year one, 33.3% contribution rate in year two, and 50% contribution rate in subsequent years.

In the model, the sum of the capital contributions never quite equals the capital commitment, a fair forecast since many funds never completely draw down all capital.[iv]







ii $PIC_{(t)} = \sum_{0}^{t-1} C_{(t)}$

iii Rather than specify a different rate of contribution every year, we simplified the model by separating the first two years of contributions from subsequent years. Typically, we would assume two similarly sized large contributions in years one and two, and geometrically declining contributions in subsequent years.

iv In fact, the model projects capital contributions will persist ad infinitum, an obviously incorrect expectation. However, the dollar amounts involved are insignificant as long as RC is set at a reasonable level. For example, if RC = 25% in year one, 33.3% in year two, and 50% in subsequent years, the projected drawdown in year 15 is less than one basis point of the capital commitment. Once a fund actually draws down the total capital commitment, the model adjusts and assumes no further drawdowns.

Distributions

Distributions vary with the different stages of a fund's life. In the early years of a fund, distributions tend to be minimal as investments have not had time to be harvested. The middle years of a fund tend to have the most distributions as investments come to fruition. Later years are marked by a steady decline in distributions as fewer investments are left to be harvested.

Recognizing that the size and timing of distributions are based on the performance of the fund, we established a distribution formula based on a rate of distribution (RD) and the net asset value (NAV), as shown below.

$$D_{(t)} = RD * [NAV_{(t-1)} * (1 + G)]$$

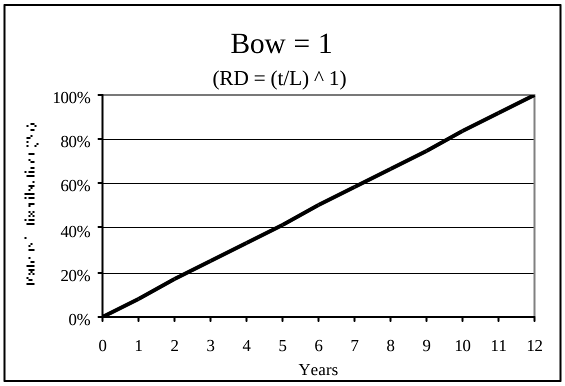
The distribution rate (RD) has two components: yield (Y) and realizations [(t/L)^B], which occur as assets are sold or investments are harvested.

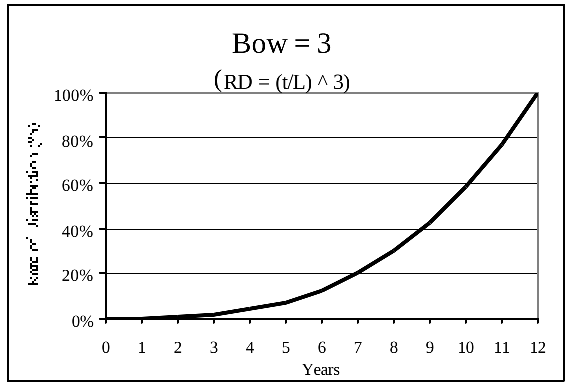
$$RD = \text{Max}\ [Y, (t/L)^B]$$

For income generating asset types such as real estate, the yield sets a minimum distribution level. For other asset types such as venture capital, the yield term often is irrelevant and set at zero.

As time (t) passes, realizations will increase and overshadow the yield term (Y). In the last year of the fund, when t equals the Life (L), the distribution rate reaches a peak as 100% of the remaining asset value gets distributed. Despite the high distribution rate, the dollar amount of the final distribution typically will be fairly small as the net asset value has declined significantly in the later years of a fund.

The bow (B) controls the rate at which the distribution rate changes over time. As shown in the charts below, the higher the bow, the slower the initial increase of the distribution rate and the faster the later acceleration. The examples shown below assume a life of 12 and a yield of 0.



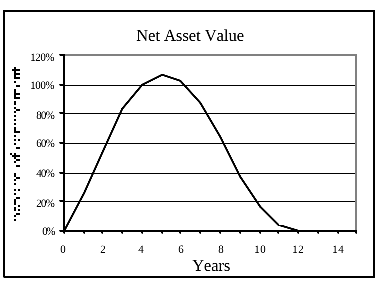


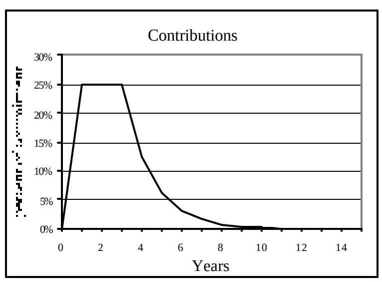
Net Asset Value

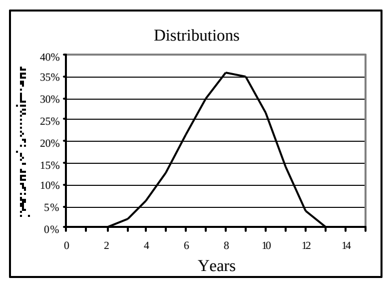
Partnership values are affected by three variables: investment performance, capital inflows, and capital outflows. In the model, as shown below, the net asset value (NAV) is calculated by multiplying the previous year's net asset value by the projected growth rate (G), adding contributions (C), and subtracting distributions (D).

$$NAV_{(t)} = [NAV_{(t-1)} * (1 + G)] + C_{(t)} - D_{(t)}$$

A sample model is shown below, using the following inputs: 13% growth rate, 12 year life, 25% contribution rate in year one, 33.3% contribution rate in year two, 50% contribution rate in subsequent years, a bow of 2.5, and a yield of 0%.



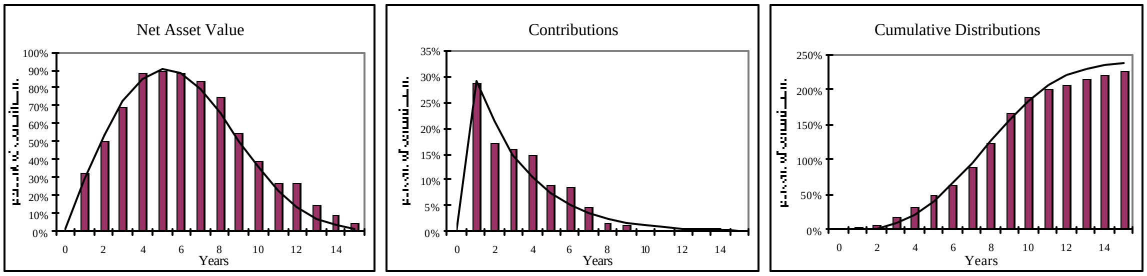




<u>Testing the Model</u>

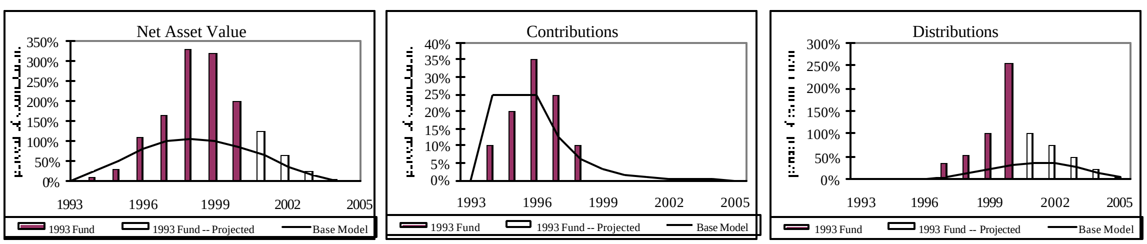
To test the model, we examined our four initial criteria: (1) the model should be simple and sensible on a theoretical basis; (2) the model should be able to incorporate and respond to actual capital commitment experience and current partnership asset values; (3) the model should be able to analyze the portfolio impact of varying return scenarios and varying rates of investment and distribution; (4) the model should be useful for multiple asset types.

The model is simple and fundamental. The net asset value is increased by contributions and a growth rate (which equals the total return net of management fees) and decreased by distributions. The basic patterns of the model graphs of net asset value, contributions and distributions conform to reasonable expectations of fund behavior. To verify our intuition, we compared the model to historical data. Shown below is the model (thick black line) graphed against a sample of Yale's historical venture capital fund data (shaded columns).[v] The inputs used for the model are as follows: 20% growth rate, 20 year life, 29% contribution rate in year one, 30% contribution rate in subsequent years, a bow of 1.2, and a yield of 0%.[vi]



In this example, using the model's predicted cash flows to calculate an internal rate of return results in a 20% return, equal to the assumed growth rate of 20%.

The model incorporates and responds to actual capital commitment experience and current partnership asset values. It can be updated each year with actual data, allowing it to adjust to current events. For example, the capital commitment in any given year is based on the outstanding commitment to a fund, meaning that unusually large early capital calls diminish the amount of remaining capital. Likewise, estimates of net asset values and distributions are based on prior years' data, allowing actual data to influence future projections. An example of this adjustment process is shown below.



In this exhibit, the base model (unadjusted for actual data) is graphed against a 1993 vintage year venture capital fund in which Yale participated.[vii,viii] The 1993 fund data through the year 2000 are actual data (shaded columns); future data (un-shaded columns) are projected using the model. Though the venture capital fund performed well above the

---

[v] This data sample includes all 33 venture capital funds in which the University participated in the 1980s.

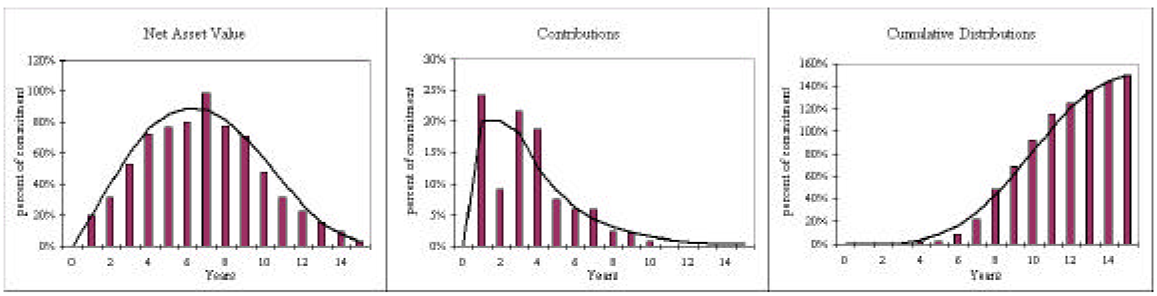
[vi] Although we might have defined the rate of contribution to match perfectly the actual contributions each year to establish a closer fit to observed capital drawdowns, we used the standard format of the model since it is more representative of what we want to use on an on-going basis.

[vii] The assumptions used for the model are as follows: 13% growth rate, 12 year life, 25% contribution rate in year one, 33.3% contribution rate in year two, 50% contribution rate in subsequent years, a bow of 2.5, and a yield of 0%. These assumptions, as is described in the next section of the paper, are the base model assumptions used by Yale for venture capital funds.

[viii] Vintage year refers to the year in which the fund was formed.

expectations of the base model, the use of actual data allowed the model to continue to make reasonable future projections for the 1993 fund.

The model can be modified as investment environments change. For example, recent assumptions in Yale's model include an accelerated pace of capital contributions to venture capital funds, reflecting the faster pace at which funds have been calling money. This flexibility is helpful in analyzing the impact of varying market conditions. For example, a forecast of an unfavorable exit environment for venture capital could be reflected by extending the life of funds or by slowing down the distribution rate. To demonstrate the model's ability to depict below average return environments, we examined Yale's venture capital funds with vintage years between 1984 and 1986. The inputs used for the model are as follows: 7% growth rate, 18 year life, 20% contribution rate in year one, 25% contribution rate in year two, 30% contribution rate in subsequent years, a bow of 2.2, and a yield of 0%. As is shown below, the model fits historical data nicely.



Though we have focused on venture capital throughout the paper, the model can be used for a variety of asset types. For example, by altering the inputs, the model can be used to represent assets with cash yields such as real estate, depleting assets such as oil and gas, and more traditional private equity assets such leveraged buyouts. Exhibit 1 displays how the model's inputs can be modified to depict a variety of asset classes.

<u>Employing The Model</u>

Once the model was created, we established a set of assumptions for projecting the cash flows and performance of existing and future funds. Despite the excellent performance in recent years of many private asset classes, we used as inputs the conservative return assumptions built into our asset allocation modeling. For example, the assumptions for a new venture capital fund were as follows: 13% growth rate, 12 year life, 25% contribution rate in year one, 33.3% contribution rate in year two, 50% contribution rate in subsequent years, a bow of 2.5, and a yield of 0%. The projected 13% rate of return is significantly below the 48% annual rate of return Yale has earned on venture capital investments over the past ten years.

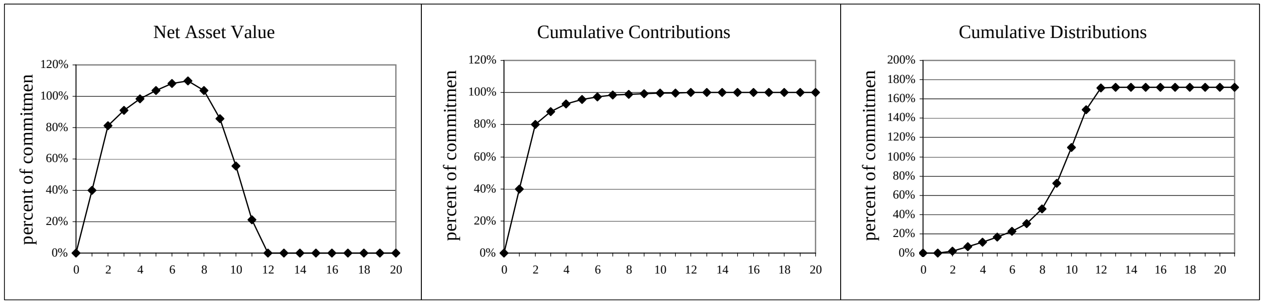
Once individual funds were modeled and reasonable assumptions made about commitments to new funds, the results were aggregated to examine predicted future exposures to, contributions to, and distributions from entire asset classes. Exhibit 2 displays an analysis we use to examine our asset class exposures. By varying the commitments to various managers, we can assess the effect on projected asset class exposures and cash flows. Also, we can examine the effect of different investment environments by changing the inputs for all the managers. For example, we might, decrease the growth rate of each manager by several percent to understand the impact of lower-than-expected returns on our asset allocation.

The model can be used to size commitments to individual managers, providing insight into exposure to a single organization or investment strategy. Recently, Yale considered its options in making a commitment to a new fund. The University had committed to five previous funds with excellent results, but wanted to examine the effect of an increased commitment size. As shown in Exhibit 3, we set up an analysis which depicted the effect of making a new commitment of $150 million. Despite the seemingly large size of the commitment, the model was helpful in showing that the growth in the Endowment and future projected sales from older funds justified a significant step-up in the new fund.

Conclusion

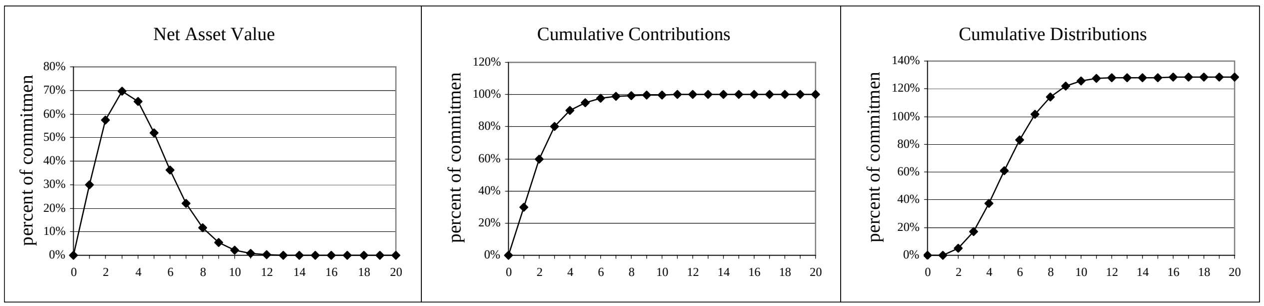
With the growth in allocation to illiquid alternative asset classes such as venture capital, leveraged buyouts, real estate, and natural resources, institutional investors are compelled to find ways of projecting future asset values and cash flows for funds. Older tools such as simple rules of thumb or models based solely on historical data are flawed by a lack of flexibility and adaptability, and a failure to utilize actual data. Our model provides a simple, sensible way to estimate future exposures and cash flows and provides a flexible management tool to assess the impact of changing fund commitment levels and of varying assumptions regarding contributions, distributions and underlying net returns.

Real Estate **Exhibit 1**



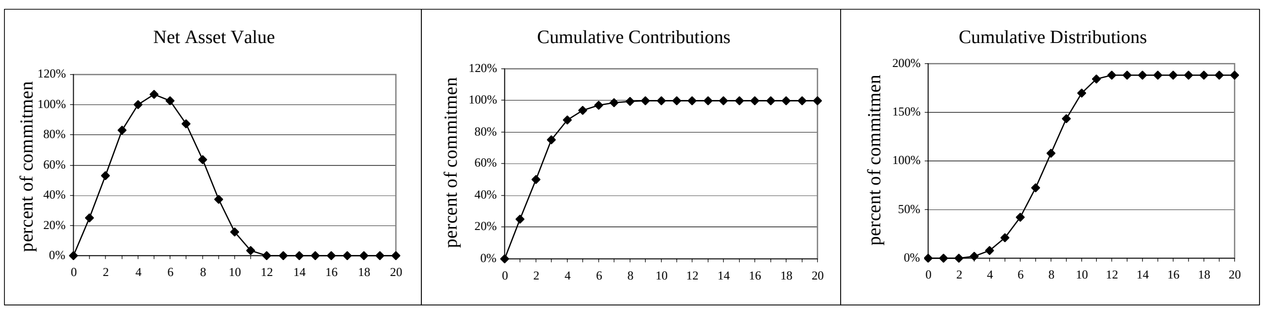
Growth	8.0	%
Life	12	years
Initial Contribution Rate	40.0	%
Contribution Rate	40.0	%
Bow	5.0	
Yield	5.0	%

Oil & Gas



Growth	8.0	%
Life*	15	years
Initial Contribution Rate	30.0	%
Contribution Rate	50.0	%
Bow	1.0	
Yield	15.0	%

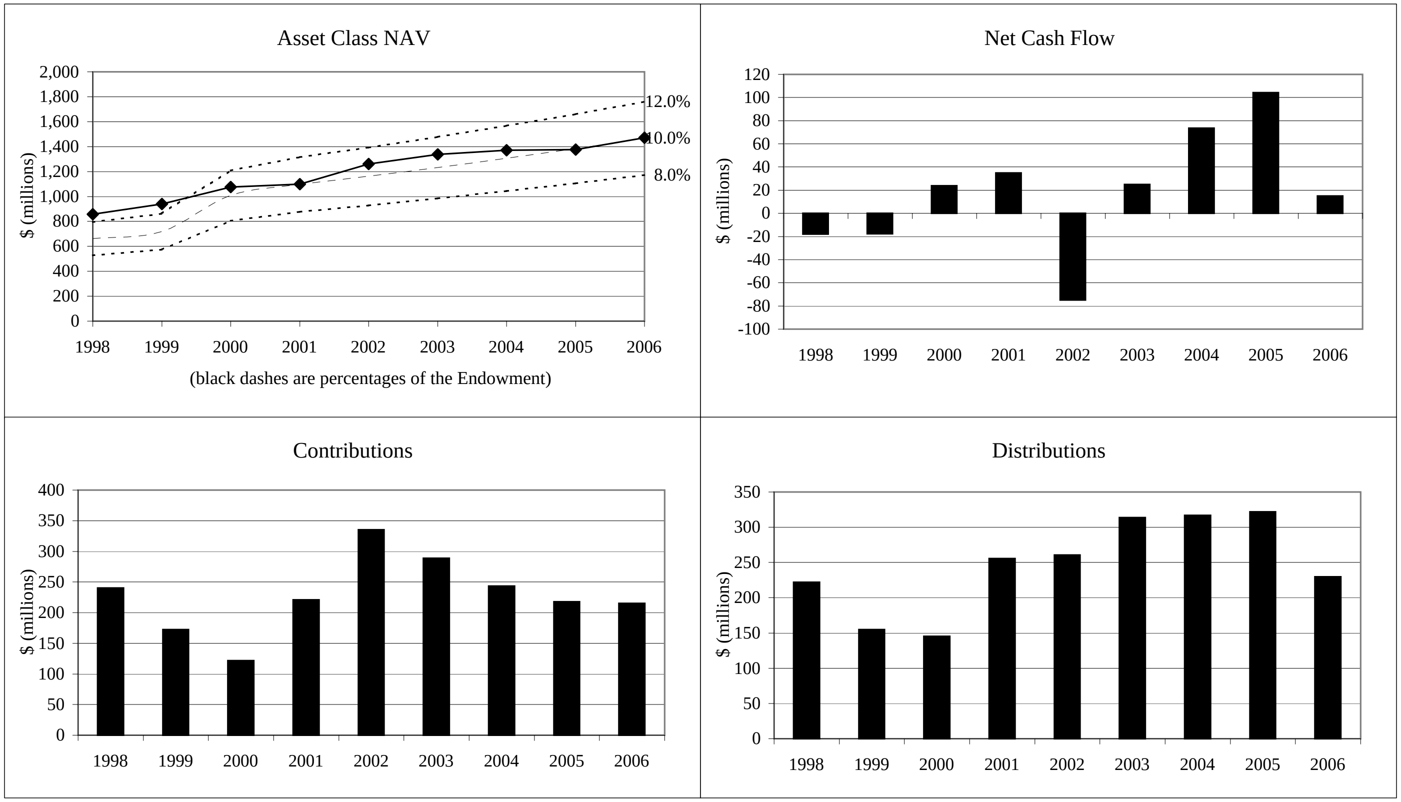
Leveraged Buyouts



Growth	13.0	%
Life	12	years
Initial Contribution Rate	25.0	%
Contribution Rate	50.0	%
Bow	2.5	
Yield	0.0	%

* Because of the fast distribution rate of many oil and gas investments, the stated life doesn't equal the actual life of fund. The model is representative; input factors are used as mathematical inputs and should not be taken literally.

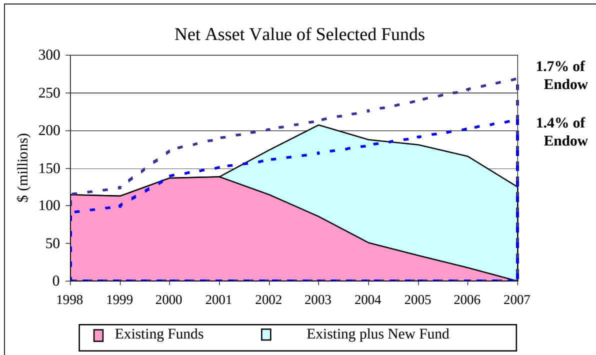
Exhibit 2

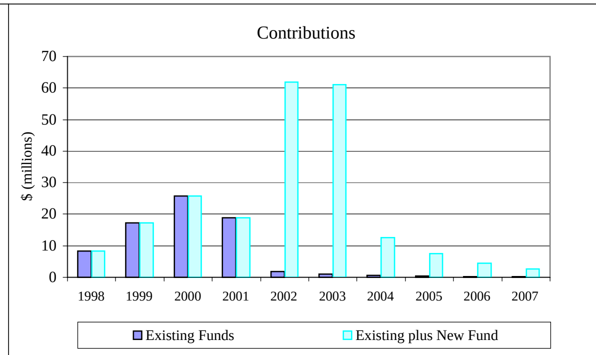


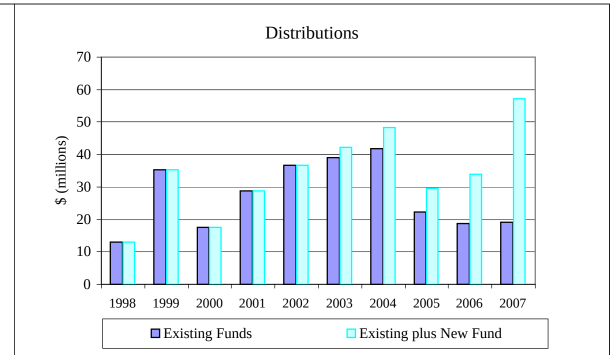
Future Commitments

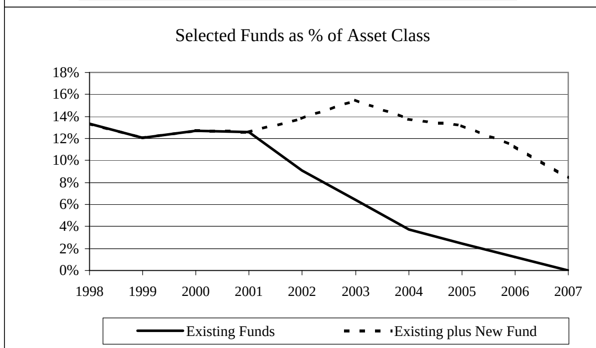
Fund	Year	$$
Manager A	2001	25
Manager B	2001	50
Manager C	2001	75
Manager D	2001	150
Manager E	2002	25
Manager F	2002	75
Manager G	2002	75
Manager H	2002	100
Manager I	2003	25
Manager J	2003	50
Manager K	2003	50
Manager L	2003	60
Manager M	2003	100
Manager N	2003	150
Manager A	2004	25
Manager B	2004	75
Manager C	2004	100
Manager D	2005	60
Manager E	2005	150
Manager F	2005	75
Manager G	2005	50
Manager H	2006	25
Manager I	2006	75
Manager J	2006	50

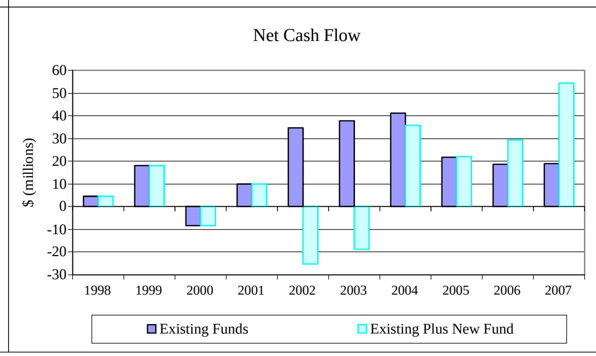
Exhibit 3

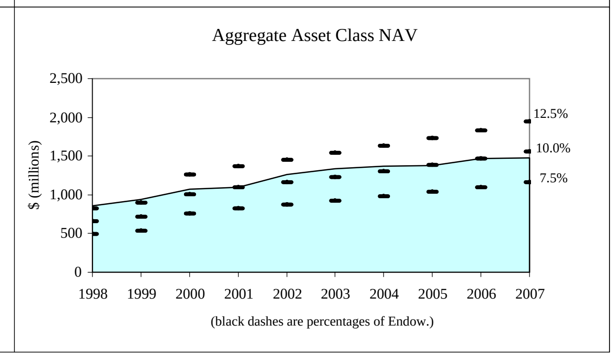












Existing Funds:	Inception Date	Yale Commitment	Unfunded Commitment	Approximate Size of Fund	Yale as % of Fund	NAV (Yale)	Multiple	Distributions (as a % of Contrib.)
Fund I	1993	25.0	0.0	150.0	16.7%	19.5	1.6	88.6%
Fund II	1995	47.0	0.0	200.0	23.5%	47.8	1.8	71.6%
Fund III	1996	25.0	0.0	100.0	25.0%	26.6	1.5	62.3%
Fund IV	1997	25.0	6.4	100.0	25.0%	20.3	1.5	45.8%
Fund V	1999	40.0	15.6	300.0	13.3%	23.9	1.0	0.4%
TOTALS		162.0	22.0	850.0	19.1%	138.2	1.5	57.3%

New Funds:	Inception Date	Yale Commitment
Fund VI	2002	150.0
TOTALS		150.0