Stochastic Optimization for Stocks

Stock Allocation Under Economic Uncertainty Using Mixed Integer Programming

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GB 730 Prescriptive Modeling and Optimization for Business Analytics

December 16, 2024

Optimization Model and Purpose

- The purpose of this model is to help investors optimize portfolios for different economic scenarios.

- This for financial advisors, investment firms, and independent individuals to improve their decision-making under stress conditions.

- The value of the model is to reduce risk exposure while maximizing returns under varying market conditions.

- Mixed integer optimization allows optimization of a function subject to constraints, handling both continuous and integer decision variables. This increases the model flexibility.

- It is suitable for this problem because it allows for discrete decision-making of investments, incorporates complex constraints, and handles risk factors effectively. It adapts to various economic scenarios and scales well to large problems, all of which are crucial for making robust financial decisions in dynamic market conditions.

Model Overview: Objective

The **objective** is the model output.

This model **minimizes the mean absolute deviation or the risk of the portfolio** across all the assets and **maximizes the mean annual return**.

There is equal weight given for to the risk and return. This promotes selecting assets with lower volatility and minimizing losses incurred during financial downturn events, while still yielding high returns over a year.

Model Overview: Decision Variables

The **decision variables** are the choices about the model input.

Whether or not we invest in a stock, which is binary and then the **proportion** of investment in each stock, which is a real number.

Having the binary decision variable allows for model flexibility.

Input data is historical data on stocks from Yahoo! Finance 2005 to 2023

The stocks: Apple, Google, Microsoft, Amazon, Meta, Tesla, Netflix, NVIDIA, Advanced Micro Devices, and Intel.

Model Overview: Constraints

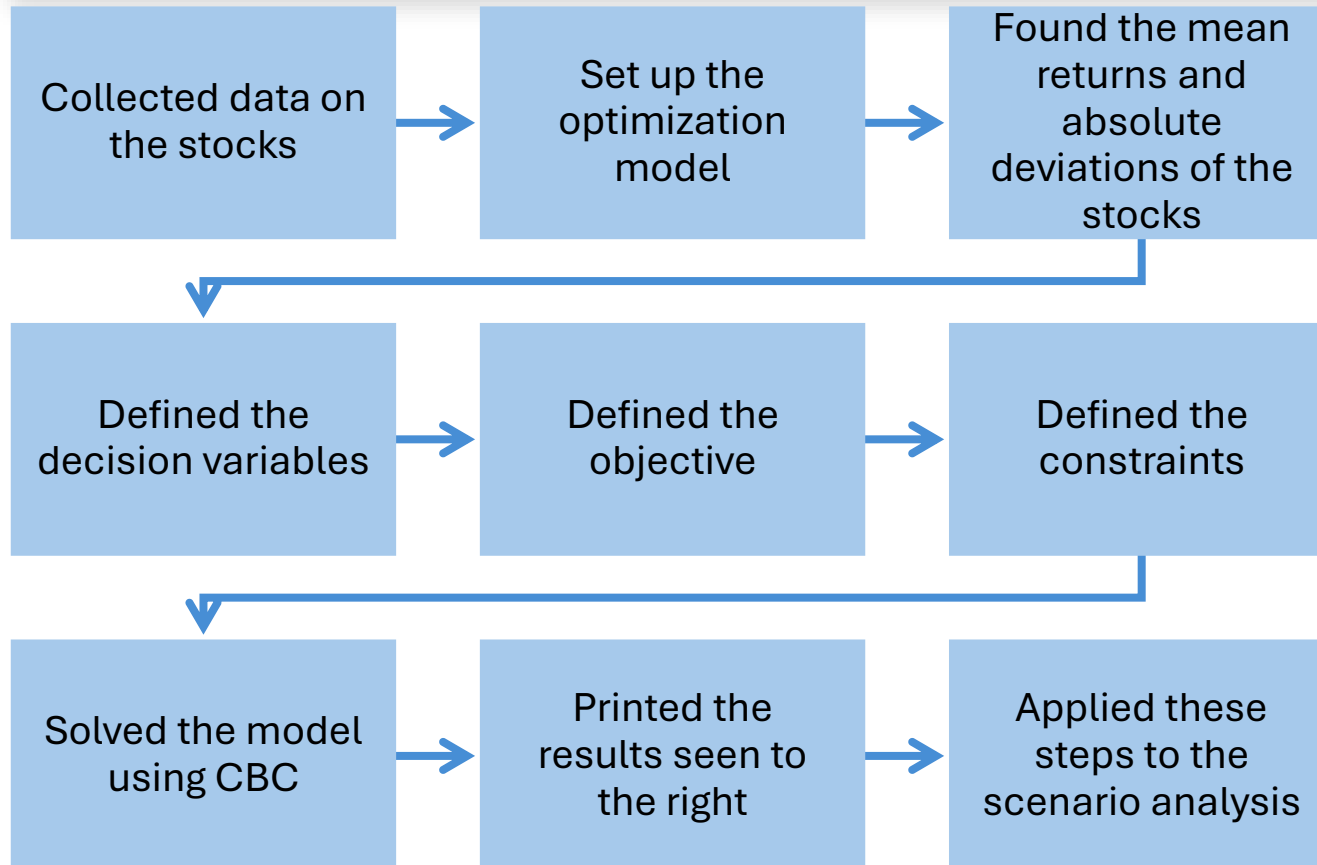The **constraints** are restrictions about an input, calculation, or output.

The **budget** is set to $1M. The total **proportion of money invested in selected stocks must equal 100%**

Maximum allowable exposure to high-risk stocks. The **expected return of the portfolio must be above 0.1%**

Diversification constraint, **no more than 40% of the budget can be invested in any single stock** to spread the risk

Hedging Effectiveness: For each asset included, the percent allocated needs to be at least 3%. This ensures there are meaningful investments

Analysis and Results from Model under typical conditions

Collected data on the stocks → Set up the optimization model → Found the mean returns and absolute deviations of the stocks

Defined the decision variables → Defined the objective → Defined the constraints

Solved the model using CBC → Printed the results seen to the right → Applied these steps to the scenario analysis

Stock allocation under typical conditions

TESLA
= 40%
$400K

NETFLIX
= 20%
$200K

NVIDIA
= 40%
$400K

The **mean absolute deviation** (**Risk** associated with the portfolio): 0.0213
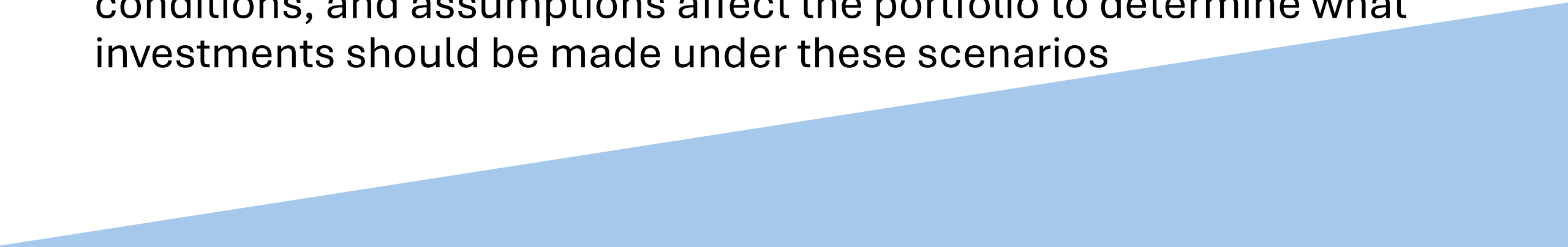
The returns on average, **fluctuate up and down by 2.13 percentage points** from its **mean annual return of 71.82%.**

Optimization Model compared to Baseline (User Selection of Stocks)

- Baseline model used Apple, Microsoft, Meta, and NVIDIA stocks
- 25% of the $1M budget was allocated to each
- Risk from this selection was 0.0148 and mean return was 41.02%
  - The returns on average, **fluctuate up and down by 1.48 percentage points** from its **mean return of 41.02%**

Scenario Analysis

- Scenario analysis is a process used to evaluate and understand potential outcomes of different hypothetical situations

- This will be used to asses how different economic factors, market conditions, and assumptions affect the portfolio to determine what investments should be made under these scenarios

Scenario Factors

- The market drops show declines in stock prices, reflecting the decreased economic activity and investor confidence

- The interest rate changes refer to the cost of borrowing money or return on savings

- Inflation is the rate at which prices for goods and services rise

- Regulation factor is the measure of the regulatory impact or pressure on markets

  - Lower numbers indicate greater regulation and stricter rules or policies.

Scenario Analysis – Economic Conditions

Scenario	Key Risks	Portfolio Strategy
Recession	Decline in consumer demands, moderate inflation, and moderate regulation	Choosing stocks that are typically more stable and less risky
Economic slowdown	Sluggish growth with moderate inflation to stimulate the market and no regulation	Balanced portfolio diversified across different assets, focusing on stable stocks
Inflation	Similar to Recession, higher inflation	Inflation-hedged assets – maintain their value
Deflation	Similar to Economic slowdown, declining prices, consumer spending drops	Cash, high-quality bonds, and stable stocks that are less sensitive to volatility

Scenario Analysis – Economic Conditions Continued

Scenario	Key Factors	Portfolio Strategy
Market crash	Severe market volatility , tight regulation with minimal interest and inflation	Safe-haven assets (bonds, cash), avoiding high-risk equities
Regulatory changes	Higher compliance costs, sector specific risks that affect market access	Diversified portfolio, focus on sectors resilient to change and regulations
Change to High Interest Rates	Significant decline in stock prices with high borrowing costs, moderate inflation and high regulation pressure	Defensive with less risk associated, favoring bonds and defensive sectors that typically aren't affected
Change to Low Interest Rates	Modest decline in stock prices with cheap borrowing costs to stimulate economic growth or counteract deflation and low regulation pressures	Aggressive with higher risk aiming for higher returns, growth stocks for long-term periods and riskier assets like emerging markets

Scenario Analysis – Economic Conditions Continued

Scenario	Key Factors	Portfolio Strategy
Change to risk averse appetite (more weight given to absolute deviation)	Higher interest rates with small decline in stock market value and low regulation	Safe-haven assets (bonds, cash), avoiding high-risk equities
Change to risk seeking appetite (more weight given to annual return)	Higher compliance costs with larger decline in the market. Much more volatile conditions	Growth stocks (technology or high-growth startups), aggressive and accepting higher volatility

Economic Conditions Risk and Allocations

Recession:

Risk: 2.11%

Return: 60.85%

- Tesla = 34%
  $340K
- Netflix = 32%
  $320K
- NVIDIA = 34%
  $340K

Economic Slowdown:

Risk: 2.12%

Return: 65.85%

- Tesla = 36%
  $360K
- Netflix = 28%
  $280K
- NVIDIA = 36%
  $360K

Inflation:

Risk: 2.12%

Return: 60.16%

- Tesla = 32%
  $320K
- Netflix = 32%
  $320K
- NVIDIA = 32%
  $320K
- AMD = 4%
  $40K

Deflation:

Risk: 2.13%

Return: 69.05%

- Tesla = 40%
  $400K
- Netflix = 20%
  $200K
- NVIDIA = 40%
  $400K

Economic Conditions Risk and Allocations

Market Crash:

Risk: 2.16%

Return: 50.75%

- Tesla = 28% $280K
- Netflix = 28% $280K
- NVIDIA = 28% $280K
- AMD = 16% $160K

High Interest Rate:

Risk: 2.12%

Return: 60.16%

- Tesla = 32% $320K
- Netflix = 32% $320K
- NVIDIA = 32% $320K
- AMD = 4% $40K

Low Interest Rate:

Risk: 2.13%

Return: 68.34%

- Tesla = 38% $380K
- Netflix = 24% $240K
- NVIDIA = 38% $380K

Regulatory changes:

Risk: 2.14%

Return: 63.62%

- Tesla = 30% $300K
- Netflix = 30% $300K
- NVIDIA = 30% $300K
- AMD = 10% $100K

Economic Conditions Risk and Allocations

Risk Averse:

Risk: 1.99%

Return: 56.74%

- Microsoft = 20% $200K
- Tesla = 20% $200K
- Netflix = 20% $200K
- NVIDIA = 20% $200K
- AMD = 20% $200K

Risk Seeking:

Risk: 2.17%

Return: 60.75%

- Tesla = 48% $480K
- Netflix = 4% $40K
- NVIDIA = 48% $480K

Discussion of Model under Economic Uncertainty

- All the scenarios prioritize **significant allocations to high growth potential and resilient stocks** (Netflix, Tesla, and NVIDIA)
  - These stocks stay relatively stable with predictable returns and are relevant in future focused industries
- For other assets (AMD, Microsoft), **allocations vary depending on scenario**
- Some assets receive no allocations across scenarios
- **Risk and return associated** is generally **similar** across scenarios. This reflects **robust portfolio design** tailored to each situation

- **Recession, High Interest Rate, Inflation, Slowdown:** allocations are toward growth stocks with potential for robust (faster) performance. Diversification with AMD and Netflix
- **Market Crash, Risk Averse:** Spreading risk among high growth assets and stable stocks, reflecting heightened uncertainty and need for safety
- **Regulation Changes :** Balancing portfolio with diversification to AMD
- **Deflation:** Growth stocks as they maintain profitability
- **Risk Seeking:** Aggressive investments, high risk, high reward stocks

Economic Conditions Risk and Returns – Baseline Model

Recession:

Risk: 1.56%

Return: 31.38%

Market Crash:

Risk: 1.63%

Return: 22.97%

Economic Slowdown:

Risk: 1.52%

Return: 35.07%

High Interest Rate:

Risk: 1.56%

Return: 29.53%

Low Interest Rate:

Risk: 1.51%

Return: 37.80%

Inflation:

Risk: 1.56%

Return: 29.53%

Deflation:

Risk: 1.51%

Return: 39.79%

Regulatory Changes:

Risk: 1.52%

Return: 29.22%

Risk Averse:

Risk: 1.51%

Return: 19.89%

Risk Seeking:

Risk: 1.60%

Return: 41.84%

Optimization Model compared to Baseline Under Economic Uncertainty

- The baseline model prioritizes stability over potential gains
  - **Lower risk (1.5 – 1.6%) and lower returns**
  - Conservative, one-size-fits-all stance
- Optimized model embraces higher risk assets to achieve higher returns
  - **Higher risk than baseline (2 – 2.2%) and higher returns (2x baseline)**
  - Risk associated is still relatively low, stability present
  - **More tailored to exploit opportunities within each specific scenario**

Discussion

| **Potential improvements** | **Further research areas** | **Key obstacles** | **Limitations** |

Potential improvements

- Using more sophisticated risk management techniques like Value at Risk or Conditional Value at Risk
- Using more advanced hedging strategies
- Including other stocks (startups, defensive, dividend)

Further research areas

- Applying a machine learning model to enhance predictive accuracy for stock returns and to forecast returns
- Implementing a company's policies and seeing how well they hold under stress
- Testing more volatile stocks

Key obstacles

- Computation resources
- Regulatory changes that can affect the constraints and strategies used in the model
- Market volatility can impact the predictions and effectiveness of the calculations used in the model

Limitations

- The current model may not be able to adjust dynamically to market changes
- The hedging effectiveness may vary in practice
- The current stocks included are mainly growth stocks with more risk associated with them