Nonlinear Control Design for a Jet-Powered Wingsuit

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***Abstract*—For thousands of years, humans have dreamed of taking to the sky as birds do. Wingsuits, now readily available to the experienced skydiver, provide one means to accomplish this. However they are unstable in the air, and flying remains particularly dangerous and limited. In this paper we begin to address this issue through the design of a nonlinear controller for thrusters strapped to the ankles of a wingsuit flyer. A number of techniques are attempted in the design, and a sliding mode controller for velocity is shown to be superior because of its immunity to deviations in parameters of the system. A method for using this same controller to control glide angle is proposed.**

***Index Terms*—Nonlinear control, sliding mode, wingsuit, awesome.**

I. INTRODUCTION

WINGSUITS are suits that incooperate membranes between the wearer's arms and legs so as to allow the wearer to experience some of the joys of gliding flight. However they are relatively inefficient, which causes significant loss of altitude during all but the most gentle of maneuvers. In order to address this Visa Parviainen has experimented with strapping small RC airplane thrusters to his ankles, and nearly attained level flight. However, during this flight he experienced uncontrollable oscillations in velocity [1]. In addition, he would not be able to fully address the loss of altitude during rapid maneuvers.

We begin to address these problems through design of controllers to track slow changes in velocity or glide angle through use of thrust magnitude as input.

II. CONTROL DESIGN

The longitudinal equations of motion for the system shown in figure 1 are given in [1]. These are derived with the following simplifying assumptions:

1) The glide angle does not change much with time.
2) Drag does not affect the pitching moment M.
3) The lift of the wingsuit behaves as though it comes from 3 lifting surfaces, rigidly positioned at constant distance from the centre of mass.

We choose our state vector to be given by $\vec{x} = \begin{bmatrix} \dot{\beta} \\ \beta \\ V \\ \vartheta \end{bmatrix}$, with variables defined as in figure 1. Our equations of motion are then:

$$\dot{x} = f(\vec{x}) + g(\vec{x})T$$

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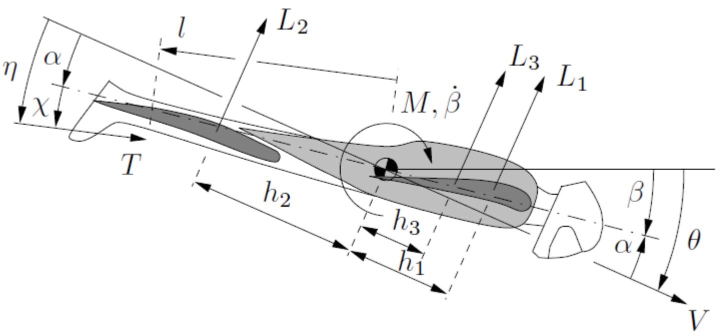

Figure 1. Free body diagram of the system considered in this paper. L_1, L_2 and L_3 are the lifts acting on the wingsuit, at distances h_1, h_2 and h_3 respectively. V is the velocity, ϑ is the glide angle, β is the body pitch angle - a measure of the angle of the body to the direction of travel. T is the thrust, which acts at distance l through the centre of mass at constant angle χ to the body axis. M is the net moment about the body. [Image from [1]]

Where T is our input thrust and

$$f(\vec{x}) = \begin{bmatrix} \frac{\rho V^2}{J}\left(c_m(\vartheta - \beta) + \frac{c_{md}\dot{\beta}}{V}\right) \\ \dot{\beta} \\ g\sin(\vartheta) - \frac{drag(\vartheta,\beta,V)}{m} \\ \frac{g\cos(\vartheta)}{V} - \frac{lift(\vartheta,\beta,V)}{mV} \end{bmatrix}$$

and

$$g(\vec{x}) = \begin{bmatrix} \frac{1}{J}(1-r)(\beta-\beta_0) \\ 0 \\ \frac{1}{m}\cos(\vartheta + \chi - r(\beta-\beta_0) - \beta_0) \\ \frac{1}{mV}\sin(\vartheta + \chi - r(\beta-\beta_0) - \beta_0) \end{bmatrix}$$

Here drag and lift are calculated as:

$$drag(\vartheta,\beta,V) = \rho V^2 (c_p + \frac{c_1(\vartheta - \beta) + c_2}{c_i})$$

and

$$lift(\vartheta,\beta,V) = \rho V^2 (c_1(\vartheta - \beta) + c_2)$$

We have parameters: the mass of the wingsuit-man system $m = 83\,\mathrm{kg}$, the density of air $\rho = 1\,\mathrm{kgm}^{-3}$, the moment of inertia of the wingsuit-man system: $J = 16\,\mathrm{kgm}^2$, the distance from the centre of gravity to the thrusters: $l = 1\,\mathrm{m}$, $g = 9.81\,\mathrm{ms}^{-1}$, $x = 25°$ and aerodynamic parameters which were determined experimentally in [1] $c_1 = 1.17$, $c_2 = 0.39$, $c_m = 0.2\,\mathrm{m}^3$, $c_{md} = 0.28\,\mathrm{m}^4$, $c_p = 0.056$, $c_i = 1.67$. The parameters β_0 and r are used to model non-rigid effects on the thrust angle. This was neglected for most of my study by setting the equilibrium body pitch angle $\beta_0 = 0$ and the rigidity factor $r = 1$. These values were used except when explicitly stated otherwise.

It is of use to note that this is a naturally stable system, and reaches an equilibrium of ${}^1x_{eq} = \begin{bmatrix} 0 \\ 36.6^\circ \\ 40.9\,\mathrm{ms}^{-1} \\ 36.6^\circ \end{bmatrix}$.

A. Feedback linearisability

As a first attempt, the system was checked for full-state feedback linearisability through evaluating the conditions of a theorem given in [2] determining the rank of the matrix:

$$\begin{bmatrix} g(x) & ad_{f(x)}g(x) & ad^2_{f(x)}g(x) & ad^3_{f(x)}g(x) \end{bmatrix}$$

Where $ad_{f(x)}g(x) = L_f(g) - L_g(f)$, is the lie bracket, $L_f(g)$ being the Lie derivative of f with respect to g. This was determined to have full rank, however, the distribution $D = span\{g, ad_f g, ad_f^2 g\} := span\{f_1, f_2, f_3\}$ was not involutive. This was checked in matlab through use of the fact that a distribution is involutive if the lie bracket of every combination of generating vector fields is in the distribution. Explicitly, the matrix $[f_1, f_2, f_3, ad_{f_i}(, f_j)]$ had rank 4 for each $1 < i, j < 3$.

B. Input-output linearisation

The system could still be input-output linearised, however. Such an approach was attempted for the control of the glide angle ϑ. The equation for ϑ alone can be written in the form:

$$\dot{\vartheta} = f_\vartheta(\vec{x}) + g_\vartheta(\vec{x})T$$

Thus we may select $T = \frac{-k(\vartheta - \vartheta_d)}{g_{\vartheta(x)}} - f_\vartheta(\vec{x})$, for some gain k, and desired angle ϑ_d. This was found to be very unstable, however, and only worked for short periods of time. In addition, over the periods for which it was stable, the thrust required was exceedingly large. This can be seen from the fact that $g_\vartheta(\vec{x})$ can become small if $\vartheta + x - r(\beta - \beta_0) - \beta_0$ is small. In addition, the controller required negative thrust, which is not physically possible.

C. Sliding mode for velocity control

Given the significant uncertainty in the dynamics of the system, sliding mode control presents itself as an ideal choice. For the model given, there is uncertainty in the aerodynamic parameters of the system, as well as uncertainty from the non-rigidity of the system. From the studies in [1], the following ranges were determined for the aerodynamic parameters $0.05 < c_p < 0.08$, $1.4 < c_i < 1.9$, $0.6 < c_1 < 1.8$, $0.3 < c_2 < 0.5$. Given this, a sliding-mode controller could be designed as follows. Take the sliding manifold to be $s = (V - V_d)$ for some desired velocity V_d. We can guarantee convergence to this manifold through ensuring the time derivative of the Lyapunov function $\frac{d}{dt}\frac{1}{2}s^2 \leq 0$. Denote by f_V, g_V the actual, possibly unknown, dynamics of the system in the equation for $\dot{V}$. We require

$$s\dot{s} = s(\dot{V} - \dot{V}_d) = s(f_V + g_V T - \dot{V}_d) \leq 0$$

Taking $T = -(|\frac{f_V^* - \dot{V}_d}{g_V^*}| + b_0)sgn(s)$ ensures this is the case. Here $f^*$and $g^*$are formed with parameters from the known ranges so as to maximise the term $|\frac{f_V^* - \dot{V}_d}{g_V^*}|$, and $b_0 > 0$. This implies:

$$\begin{aligned} s\dot{s} &= s(f_V + g_V T - \dot{V}_d) \\ &\leq |s|(f_V - V_d) - (|\frac{f_V^* - \dot{\dot{V}}_d}{g_v^*}| + b_0)sgn(s)s \\ &= |s|(f_V - V_d) - (g_V|\frac{f_V^* - \dot{V}_d}{g_v^*}| + g_V b_0)|s| \\ &\leq -g_V b_0 |s| \end{aligned}$$

This controller was then modified be replacing $sgn(s)$ with $\tanh(s)$ to reduce chatter in the thrust, which would be detrimental to the engines, and lower bounding it by zero, since our engines cannot produce negative thrust. The final form was:

$$T_v(V_d, \vec{x}) \quad :=$$

$$max(-(mg + drag(\vartheta, \beta, V, c_p^*, c_i^*, c_p^*, c_1^*, c_2^*) + b_0)\tanh(V - V_d), 0)$$

Where the aerodynamic parameters have been selected in order to maximise the drag force, and[2] $b_0 = 150$. This was tested in simulation, and found to accurately track velocities, however only within velocities of[3] $40.9 - 60\,\mathrm{ms}^{-1}$ were the body-pitch angle, and angle of attack sensible. The wingsuit-flyer invariably ended up spinning longitudinally outside this range. Since the terminal velocity of a man is $\sim 56\,ms^{-1}$, this was taken to be an acceptable range of control. The controller reached steady state for a large variety of initial conditions.

Of significant interest is the performance of the controller subject to the afore mentioned uncertainty in system parameters. The aerodynamic parameters were uniformly randomly sampled 200 times within their estimated range, and the controller made to track a slowly varying ramp in velocity over the controllable range of velocities. The least squares error was recorded for each perturbation and compared with the reference least squares error, taken from tracking the same trajectory with no perturbations. The least squares error rarely differed by more than 80% from the reference. In fact, with some perturbed dynamics, the tracking improves. Figure 2 shows a sample of 20 trajectories with perturbed dynamics tracking the ramp shown in green. This shows the controller is encouragingly robust to perturbation of parameters.

This was not the case for perturbations in the form of unmodeled dynamics, however. When the approximation for non-rigid attachment of the thrusters is included, the controller becomes unstable, as is shown in figure 3. The period of the oscillations observed was roughly $\sim 1.25\,s$. This corresponds exactly to the period of oscillation of one of the two stable modes of the linearised system, which have period $1.25\,s$, and $24\,s$, for the parameter values used.

[1]Note that all angles in equations are measured in radians, but displayed degrees for intuition.

[2]It was necessary to tune this value for the controller to function.

[3]Clearly without negative thrust, we cannot track velocities below the equilibrium value.

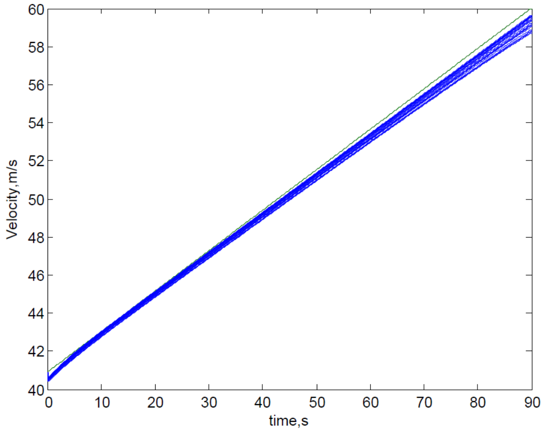

Figure 2. A sample of 20 trajectories with randomised parameters tracking the line shown in green. This demonstrates the controller is robust to parameter variation.

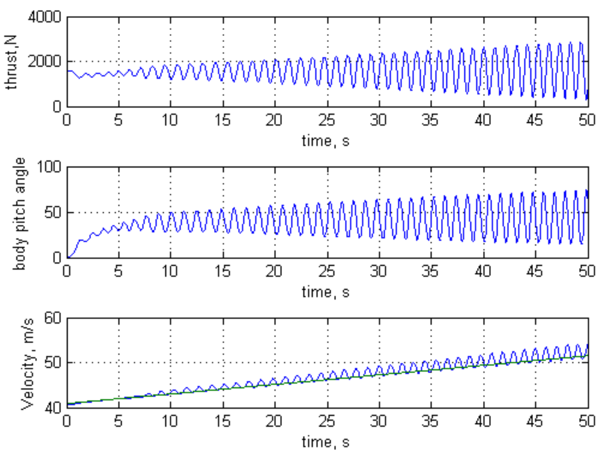

Figure 3. Dynamics observed for sliding mode controller tracking a ramp with rigidity factor $r = 0.5$, other parameters as before.

D. Glide-angle control

Sliding mode was also attempted for control of the gliding angle, with the sliding manifold $s = \vartheta - \vartheta_d$, and control $T(\vartheta_d, \vec{x}) := -(|\frac{f_\vartheta(\vec{x})}{g_\vartheta(\vec{x})}| + b_0)$. This did not work well, however because it drove the velocity to zero, and didn't work at all without negative thrust. It was noted, however, that when velocity was controlled, the glide-angle reached a steady state. Thus it is possible to control glide angle indirectly, through control of velocity, and makes sense to do so, as our velocity controller performs well. The mapping between steady-state glide angle and desired velocity was found empirically in simulation while running the velocity controller. This produced a mapping $q : V_d \to \vartheta_s$ which could be inverted so as to map desired glide-angles to desired velocities. The control law was then taken to be:

$$T(\vartheta_d, \vec{x}) := T_v(q^{-1}(\vartheta_d), \vec{x})$$

This controller was able to track glide angle well, as shown in figure 4. As the angle is only reached after the velocity has reached steady state, there is some lag. For the sinusoid shown in the figure, the lag was $6.4\,s$.

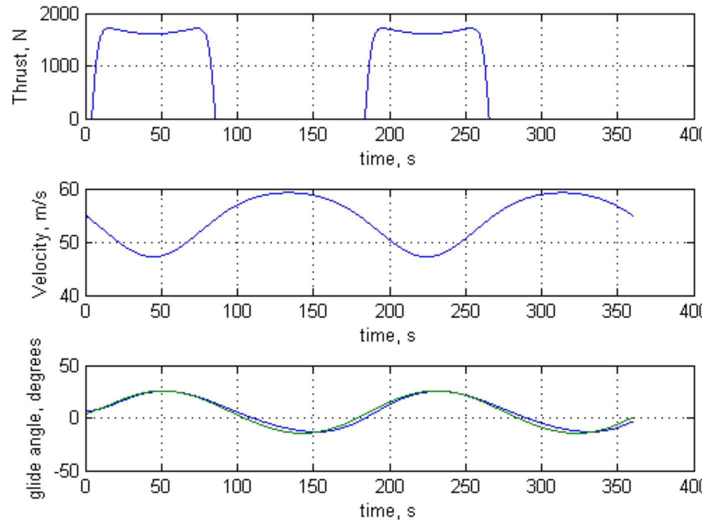

Figure 4. Inputs and state values for indirect control of glide angle through velocity. The glide angle is tracking a sinusoidal trajectory with a period of $180\,s$ shown in green. Note that this trajectory has been phase-shifted by $6.4\,s$, to show the similarity in the trajectories.

III. Conclusions

Attempts to stabilise slow changes in the velocity and angle of a Jet-powered wingsuit with thrust as the only input were simulated with the techniques of feedback linearisation, input-output linearisation, and sliding mode considered. The most successful of these controllers was found to be the sliding mode control for velocity stabilisation, as it was found to be robust to perturbations in parameters in the model, given appropriate bounds on the variation. It which was also found to suffice for attitude control, since the attitude reached an equilibrium for desired velocities, thus allowing indirect control of the attitude.

There are still many issues to resolve before this could be implemented on a real system, however. The velocity controller was not robust to changes in the rigidity of the direction of thrust, as modeled by the rigidity parameter, r. Inclusion of these dynamics resulted in excitation of a stable mode of the linearised system. The thrust required for the controller was also too high for realistic engines, which are only capable of around $\sim 300\,\mathrm{N}$ each. Further, the use of thrust magnitude as a control input is not ideal for the reliability of the engines. A logical extension would hence be an analysis of thrust-vectoring control as an additional, or replacement input. This should be coupled with input-output stability analysis to resolve the resonance issues.

References

[1] G. Robson and R. D. Andrea, "Longitudinal Stability Analysis of a Jet-Powered Wingsuit," *American Institute of Aeronautics and Astronautics*, no. August, 2010. [Online]. Available: http://www.idsc.ethz.ch/Research_DAndrea/Wingsuit/AIAA-237998-291.pdf

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