Econ 103 – Statistics for Economists

Chapter 2

Mallick Hossain

University of Pennsylvania

Survey Results

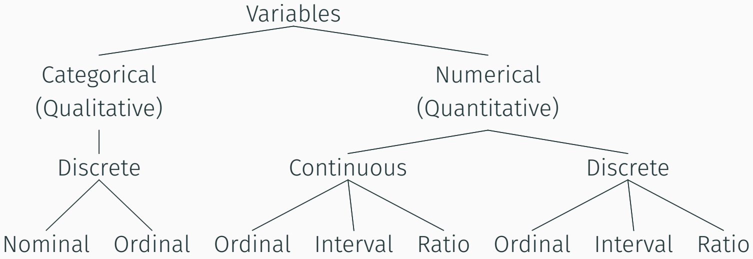
To be added soon.

Types of Variables

Discussion!

- What are the differences between the following variables?
  - “Age” and “gender”
  - “Gender” and “class standing”
  - “SAT score” and “job creation”

A Few Definitions: A Taxonomy of Variables



Definitions

- **Discrete:** Can be a countable number of values
- **Continuous:** Can take on any value

Discussion!

Can you order the following from weakest to strongest?

Interval, Nominal, Ordinal, Ratio.

From Weakest to Strongest

- **Nominal:** no order to the categories

From Weakest to Strongest

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From Weakest to Strongest

- **Nominal:** no order to the categories
- **Ordinal:** categories with natural order
- **Interval:** only differences meaningful, no natural zero
- **Ratio:** differences and ratios meaningful, natural zero

Summary Statistics

Definitions

1. Measures of Central Tendency
   - **Mean:** the average ("balance point")

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

   - **Median:** the middle observation (if data has even number of observations, take the mean of the middle two observations)

Definitions

2. Percentiles

P^{th} Percentile = Value in $(P/100) \cdot (n+1)^{th}$ Ordered Position

An Example: n = 12

60 63 65 67 70 72 75 75 80 82 84 85

$$
\begin{aligned}
Q_1 &= \text{value in the } 0.25(n+1)^{th} \text{ ordered position} \\
&= \text{value in the } 3.25^{th} \text{ ordered position} \\
&= 0.75 * 65 + 0.25 * 67 \\
&= 65.5
\end{aligned}
$$

Definitions

3. Measures of Spread
    - **Variance:** the spread from the mean

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

    - **Standard Deviation:** another way to measure the spread

$$s = \sqrt{s^2}$$

    - **Range:** the distance between the highest and lowest value

$$Range = |x_{max} - x_{min}|$$

    - **Interquartile Range (IQR):** the distance between the upper and lower quartiles

$$IQR = |x_{75\%} - x_{25\%}|$$

Why Squares?

$$\boxed{s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Why Squares?

$$\boxed{s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

What's Wrong With This?

$$\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x}) = \frac{1}{n-1}\left[\sum_{i=1}^{n}x_i - \sum_{i=1}^{n}\bar{x}\right] = \frac{1}{n-1}\left[\sum_{i=1}^{n}x_i - n\bar{x}\right]$$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n}x_i - n\cdot\frac{1}{n}\sum_{i=1}^{n}x_i\right]$$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n}x_i - \sum_{i=1}^{n}x_i\right] = 0$$

4. Measure of Symmetry
    - **Skewness:** a measure of symmetry, positive values means the right tail is longer and vice versa

$$Skewness = \frac{1}{n}\frac{\sum_{i=1}^{n}(x_i - \bar{x})^3}{s^3}$$

Skewness – A Measure of Symmetry

$$\boxed{\text{Skewness} = \frac{1}{n}\frac{\sum_{i=1}^{n}(x_i - \bar{x})^3}{s^3}}$$

What do the values indicate?

Zero $\Rightarrow$ symmetry, positive right-skewed, negative left-skewed.

Why cubed?

To get the desired sign.

Why divide by s^3?

So that skewness is unitless

Rule of Thumb

Typically (but not always), right-skewed $\Rightarrow$ mean $>$ median
left-skewed $\Rightarrow$ mean $<$ median

Definitions

5. Relationship between variables (to be covered later)
    - **Covariance:** how two variables vary together
    - **Correlation:** normalized version of covariance (can only range between -1 and +1)
    - **Regression:** an estimation of how two variables are related
    - We'll cover these in-depth later in the semester

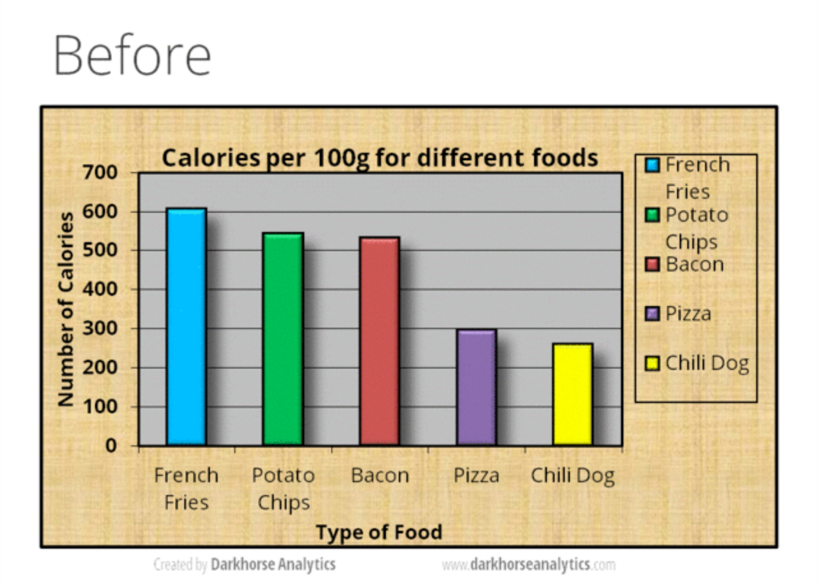
Charts

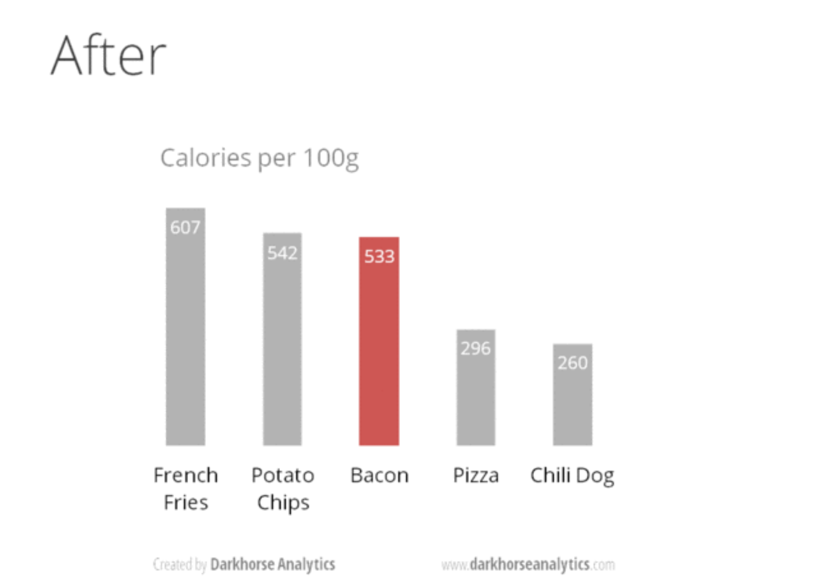
Some Data Visualization Quotes

1. “Overload, clutter, and confusion are not attributes of information, they are failures of design” –Edward Tufte
2. “...few people will appreciate the music if I just show them the notes. Most of us need to listen to the music to understand how beautiful it is. But often, that’s how we present statistics; we just show the notes we don’t play the music.” –Hans Rosling

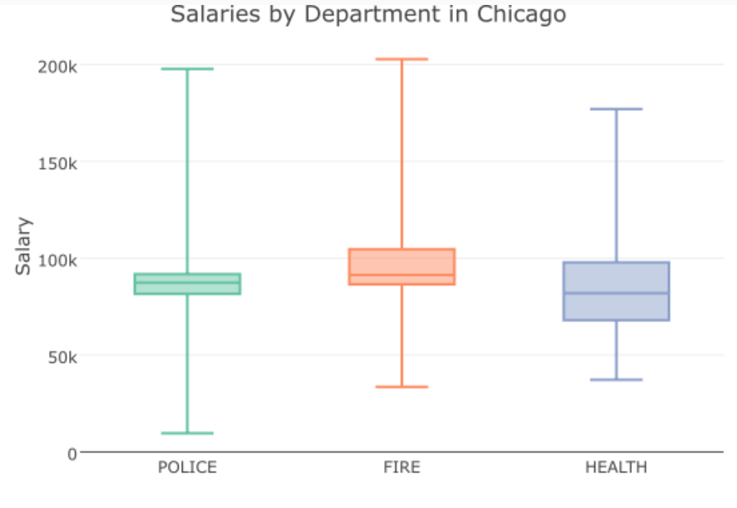
An Illustration

`https://i.imgur.com/W4BKCVU.gif`

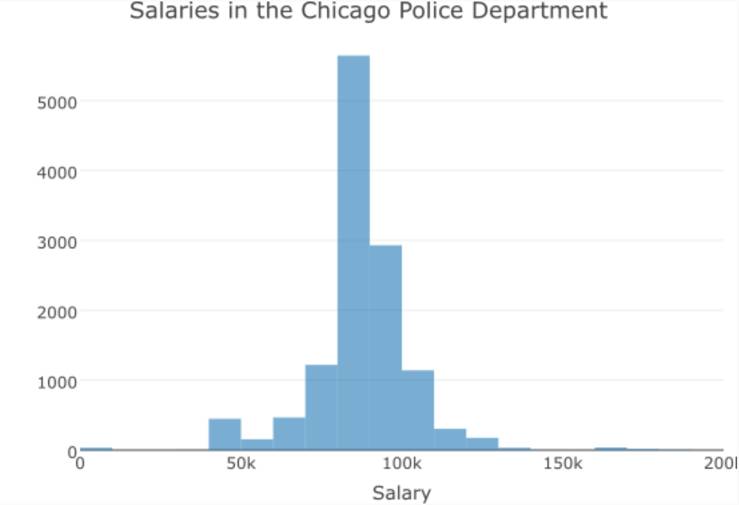

Before
Calories per 100g for different foods
Number of Calories
700
600
500
400
300
200
100
0
French Fries
Potato Chips
Bacon
Pizza
Chili Dog
Type of Food
French Fries
Potato Chips
Bacon
Pizza
Chili Dog
Created by Darkhorse Analytics
www.darkhorseanalytics.com


After
Calories per 100g
607
542
533
296
260
French Fries
Potato Chips
Bacon
Pizza
Chili Dog
Created by Darkhorse Analytics
www.darkhorseanalytics.com

Illustrating with Charts (Box and Whisker Chart)



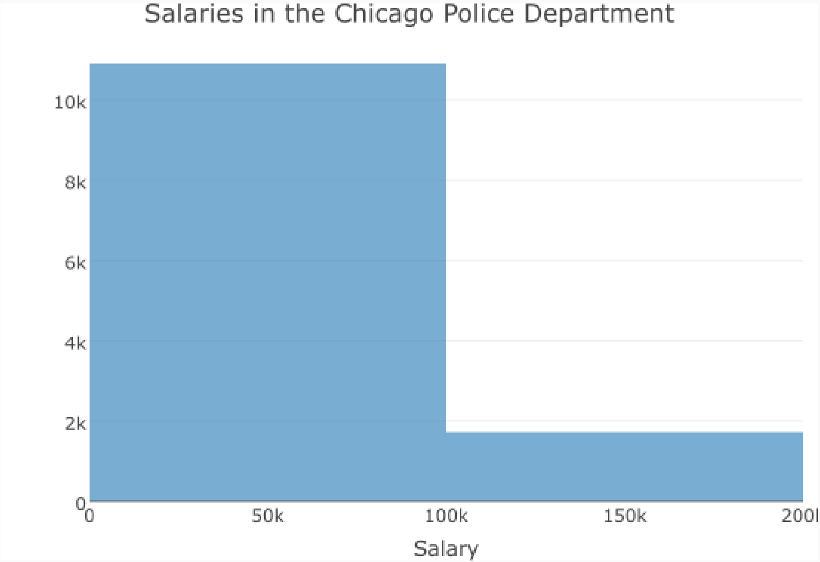
Illustrating with Charts (Histogram)



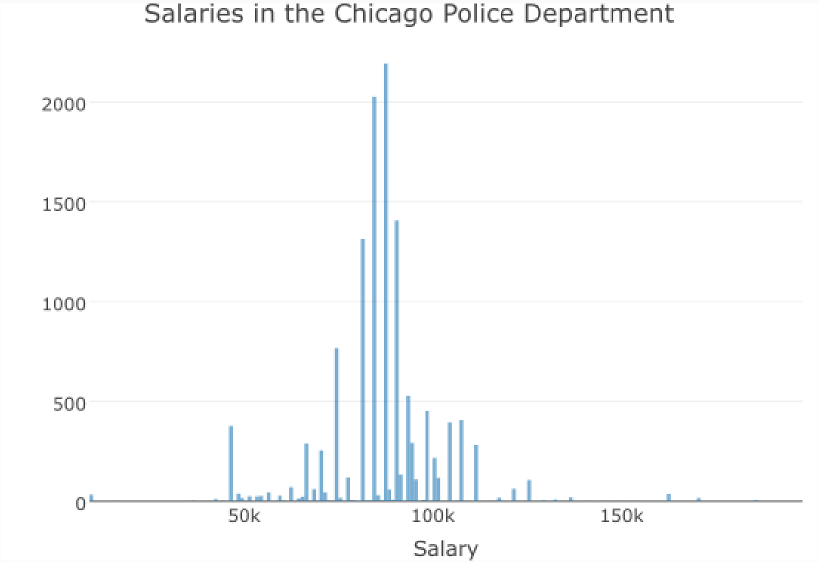
Histograms are *Really* Important

1. Histograms show the frequency of different observations
2. Important Choice: How many bins?

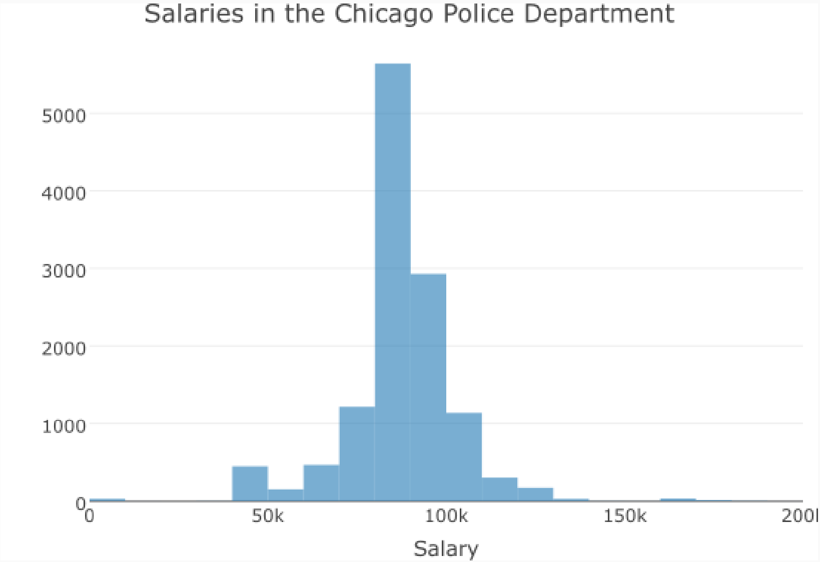
Too Few Bins (Oversmoothing)



Too Many Bins (Undersmoothing)



Just Right! (Usually around 20 bins or so)



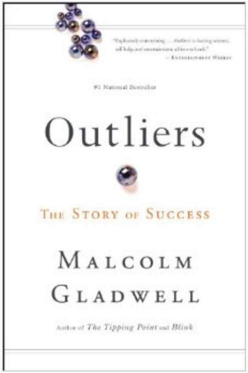
Questions to Ask Yourself about Each Summary Statistic

1. What does it measure?
2. What are its units compared to those of the data?
3. How do its units change if those of the data change?
4. What are the benefits and drawbacks of this statistic?

Some of the information regarding items 2 and 3 is on the homework rather than in the slides because working it out for yourself is a good way to check your understanding.

Outliers

What is an Outlier?



Outlier: A very unusual observation relative to the other observations in the dataset (i.e. very small or very big).

Which Summary Stats are Sensitive to Outliers?

- Assume our data is 1, 2, 3, 4, 5. What are our summary stats (mean, median, variance, range, IQR)
- What will be affected if the data includes an outlier and becomes 1, 2, 3, 4, 4990?

Which Summary Stats are Sensitive to Outliers?

- Mean changes from 3 to 1000

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- When Does the Median Change? IQR?

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- Mean changes from 3 to 1000
- Median remains at 3
- Variance changes from 2.5 to 4,975,032
- Range changes from 4 to 4889
- IQR remains at 2
- When Does the Median Change? IQR?
    - Ranks would have to change.

Summary of Sensitivity

Variance

Essentially the average squared distance from the mean. Sensitive to both skewness and outliers.

Standard Deviation

$\sqrt{\text{Variance}}$, but more convenient since same units as data

Range

Difference between larges and smallest observations. *Very* sensitive to outliers.

Interquartile Range

Range of middle 50% of the data. Insensitive to outliers, skewness.

Sample vs. Population

Essential Distinction: Sample vs. Population

For now, you can think of the population as a list of N objects:

Population: $x_1, x_2, \ldots, x_N$

from which we draw a sample of size $n < N$ objects:

Sample: $x_1, x_2, \ldots, x_n$

Important Point:

Later in the course we'll be more formal by considering probability models that represent the *act of sampling* from a population rather than thinking of a population as a list of objects. Once we do this we will no longer use the notation N as the population will be *conceptually infinite*.

Essential Distinction: Parameter vs. Statistic

N individuals in the Population, n individuals in the Sample:

	Parameter (Population)	**Statistic** (Sample)
Mean	$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
Var.	$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$	$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$
S.D.	$\sigma = \sqrt{\sigma^2}$	$s = \sqrt{s^2}$

Key Point

We use a sample $x_1, \ldots, x_n$ to calculate statistics (e.g. $\bar{x}$, s^2, s) that serve as estimates of the corresponding population parameters (e.g. μ, σ^2, σ).

Why Do Sample Variance and Std. Dev. Divide by $n - 1$?

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

There is an important reason for this, but explaining it requires some concepts we haven't learned yet.

Some Intuition

- **Intuition 1:** If we only had one data point, what would be the sample variance? Would it even be defined?
- **Intuition 2:** We know that the deviations from the sample mean sum to zero (see discussion of why variance is squared). Hence, we only need to know $n-1$ of the deviations since the last one will be whatever it takes to make the sum of them equal to 0. Hence, it would be proper to divide by $n-1$ instead of n

Z Scores

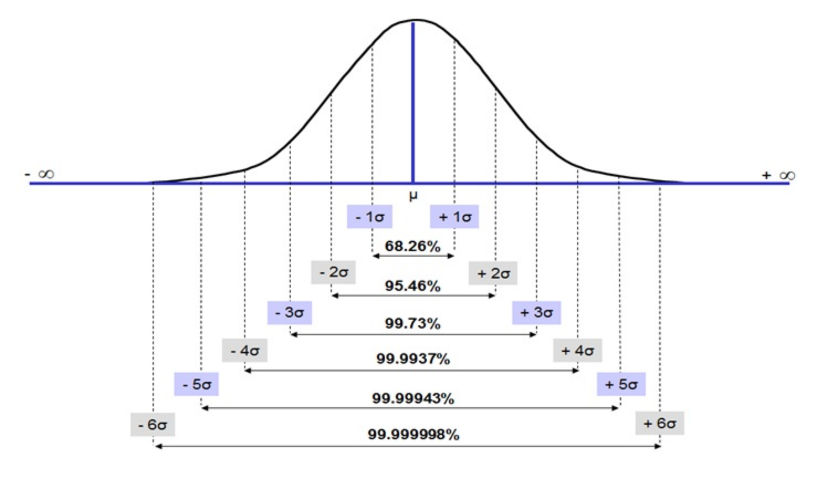
Why Mean and Variance (and Std. Dev.)?

Empirical Rule

For large populations that are approximately bell-shaped, standard deviation tells where most observations will be relative to the mean:

- $\approx$ 68% of observations are in the interval $\mu \pm \sigma$
- $\approx$ 95% of observations are in the interval $\mu \pm 2\sigma$
- Almost all of observations are in the interval $\mu \pm 3\sigma$

Standard Deviations



Z-scores: How many standard deviations from the mean?

$$Z_i = \frac{X_i - \bar{X}}{S}$$

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Unitless

Allows comparison of variables with different units.

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Detecting Outliers

Measures how "extreme" one observation is relative to the others.

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Linear Transformation

What is the sample mean of the z-scores?

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$$\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i = \frac{1}{n}\sum_{i=1}^{n}\frac{x_i - \bar{x}}{s} = \frac{1}{n \cdot s}\left[\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\bar{x}\right]$$

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$$= \frac{1}{n \cdot s}\left[\sum_{i=1}^{n} x_i - n\bar{x}\right] = \frac{1}{n \cdot s}\left[\sum_{i=1}^{n} x_i - n \cdot \frac{1}{n}\sum_{i=1}^{n} x_i\right]$$

What is the sample mean of the z-scores?

$$
\begin{aligned}
\bar{z} &= \frac{1}{n}\sum_{i=1}^{n} z_i = \frac{1}{n}\sum_{i=1}^{n}\frac{x_i-\bar{x}}{s} = \frac{1}{n\cdot s}\left[\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\bar{x}\right] \\
&= \frac{1}{n\cdot s}\left[\sum_{i=1}^{n} x_i - n\bar{x}\right] = \frac{1}{n\cdot s}\left[\sum_{i=1}^{n} x_i - n\cdot\frac{1}{n}\sum_{i=1}^{n} x_i\right] \\
&= \frac{1}{n\cdot s}\left[\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i\right] = 0
\end{aligned}
$$

What is the variance of the z-scores?

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$$s_z^2 = \frac{1}{n-1}\sum_{i=1}^{n}(z_i - \bar{z})^2 = \frac{1}{n-1}\sum_{i=1}^{n} z_i^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s_x}\right)^2$$

What is the variance of the z-scores?

$$
\begin{aligned}
s_z^2 &= \frac{1}{n-1}\sum_{i=1}^{n}(z_i-\bar{z})^2 = \frac{1}{n-1}\sum_{i=1}^{n} z_i^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{s_x}\right)^2 \\
&= \frac{1}{s_x^2}\left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2\right] = \frac{s_x^2}{s_x^2} = 1
\end{aligned}
$$

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$$= \frac{1}{s_x^2}\left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right] = \frac{s_x^2}{s_x^2} = 1$$

So what is the *standard deviation* of the z-scores?

Population Z-scores and the Empirical Rule: $\mu \pm 2\sigma$

If we knew the population mean μ and standard deviation σ we could create a *population version* of a z-score. This leads to an important way of rewriting the Empirical Rule: Bell-shaped

population $\Rightarrow$ approx. 95% of observations x_i satisfy

$$\mu - 2\sigma \leq x_i \leq \mu + 2\sigma$$

$$-2\sigma \leq x_i - \mu \leq 2\sigma$$

$$-2 \leq \frac{x_i - \mu}{\sigma} \leq 2$$

Covariance and Correlation

Covariance and Correlation: Linear Dependence Measures

Two Samples of Numeric Data

$x_1, \dots, x_n$ and $y_1, \dots, y_n$

Dependence

Do x and y both tend to be large (or small) at the same time?

Key Point

Use the idea of centering and standardizing to decide what "big" or "small" means in this context.

Are These Related?

Engine Cylinders and Horsepower?

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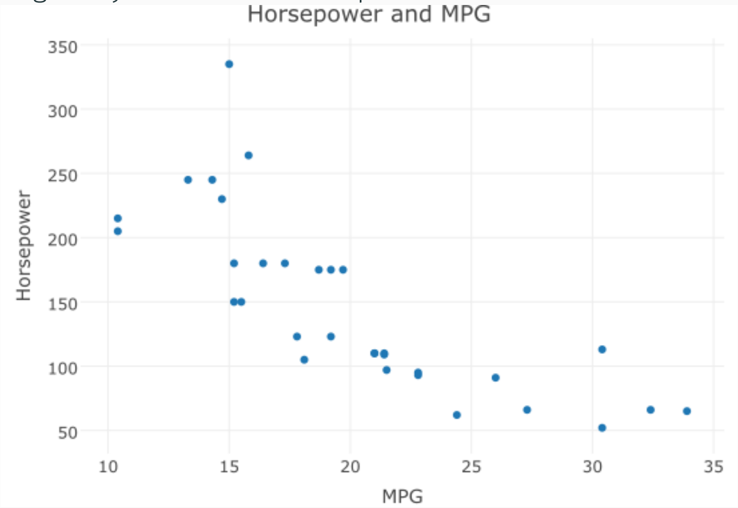

Recall Formulas

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$s_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$s_y = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

Covariance

$$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

- Centers each observation around its mean and multiplies.
- Zero $\Rightarrow$ no linear dependence
- Positive $\Rightarrow$ positive linear dependence
- Negative $\Rightarrow$ negative linear dependence
- Population parameter: σ_{xy}
- Units?

Correlation

$$r_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right) = \frac{s_{xy}}{s_x s_y}$$

- Centers *and* standardizes each observation
- Bounded between -1 and 1
- Zero $\Rightarrow$ no linear dependence
- Positive $\Rightarrow$ positive linear dependence
- Negative $\Rightarrow$ negative linear dependence
- Population parameter: ρ_{xy}
- Unitless

Game!

`guessthecorrelation.com`

Review

Essential Distinction: Parameter vs. Statistic

N individuals in the Population, n individuals in the Sample:

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Mean	$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i$	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
Var.	$\sigma_x^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$	$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$
S.D.	$\sigma_x = \sqrt{\sigma_x^2}$	$s_x = \sqrt{s^2}$
Cov.	$\sigma_{xy} = \frac{\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)}{N}$	$s_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}$
Corr.	$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$	$r = \frac{s_{xy}}{s_x s_y}$

Related Reading

- Wonnacott: Chapter 2, Appendix 2-2, and Appendix 2-5
- How to Lie with Statistics: Chapters 2, 5, and 6