Homework 3

Econ 103

Lecture Progress

We made it to slide 48 of the Chapter 3 lecture.

Homework Checklist

- ☐ **Book Problems (Chapter 3):** 1, 3, 5, 9, 11, 13, 15, 17ab, 35
- ☐ **Additional Problems:** See below
- ☐ **R Tutorial 3**
- ☐ **Ask questions on Piazza**
- ☐ **Review slides**

Additional Problems

1. Suppose you flip a fair coin twice.
   (a) List all the basic outcomes in the sample space.
   (b) Let A be the event that you get at least one head. List all the basic outcomes in A.
   (c) What is the probability of A?
   (d) List all the basic outcomes in A^c.
   (e) What is the probability of A^c?
2. Suppose I deal two cards at random from a well-shuffled deck of 52 playing cards. What is the probability that I get a pair of aces?
3. In poker, a flush is when you have 5 cards of the same suit. What is the probability of getting a flush? For poker aficionados, we are including straight flushes and royal flushes, so don't exclude those.

4. Suppose every Econ 103 student (assume there are 100 of you) set up a private trading firm. Every day, each student has a 50% chance of beating the market.
   (a) What is the probability that John Smith, a particular student in the class, beats the market five days in a row (exchanges are only open Monday-Friday)?
   (b) What is the probability that at least one person beats the market five days in a row?
   (c) What is the longest streak that someone in the class would be expected to pull off (with a greater than 50% probability)? We assume that each student's outcome is independent.
   (d) Based on the above, should we be surprised when some hedge funds pull off above-market annual returns (remember hedge funds don't normally tout their daily returns)? Why or why not? We'll assume hedge fund outcomes are independent of each other.
   (e) **Bonus (Hard):** Can you generalize the formula you found above to apply to any situation where we have k participants, each with a probability p of success and you can compute the probability of someone having a streak of length n?

5. (Adapted from Mosteller, 1965) A jury has three members: the first flips a coin for each decision, and each of the remaining two independently has probability p of reaching the correct decision. Call these two the "serious" jurors and the other the "flippant" juror (pun intended).
   (a) What is the probability that the serious jurors both reach the same decision?
   (b) What is the probability that the serious jurors each reach different decisions?
   (c) What is the probability that the jury reaches the correct decision? Majority rules.

6. Imagine that you're trying to find a suitable person to date. However, you're just too busy to spend all your time meeting people (and you prefer not to use Tinder). Hence, you've decided to save yourself some time and just hold interviews for those interested in being your boyfriend/girlfriend. On interview day, all of your suitors (suitresses?) gather in a room, hoping that they'll be the one you choose. You've made the following rules for the interviews:

   - You don't know the suitors beforehand (you're too busy and you didn't peek in the room where they gathered)
   - You interview one person at a time (in a random order)
   - At the end of the interview, you either choose them or reject them
   - If you reject them, you can't get them back

- While your suitors could be ranked (there is a "best" suitor), you can only determine their relative rank (i.e. the second person you interviewed was better/smarter/hotter than the first).

(a) Assume you have 3 suitors. If you've decided to accept the first person you interview, what's the probability you pick the best one?

(b) What if you decide to pick the second person (no matter what)?

(c) You decide to be more sophisticated with your choosing since just picking someone based on the pure order they walk in seems like a suboptimal strategy. You refine your strategy as follows, you'll straight out reject the first person no matter what (poor guy/lady) and then pick the next person that is better than that (i.e. you'll pick person 2 only if he/she is better than person 1. Otherwise, you'll pick person 3). What is the chance of picking the best person using this strategy?

(d) Would your chances of picking the best suitor improve if you rejected the first 2 people instead?

(e) What about if you had 4 suitors? How many should you outright reject to get the highest chance of finding the best one?

Part III – Challenge Problem

This is optional. If you're up for a challenge you'll learn quite a bit from this one.

7. Formally prove the Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ using the basic notions of set theory we learned in class and the axioms of probability. [Hint: Try to translate the intution from the Venn diagram into an equation.]