sampling to Section 6-5. Everywhere else, we will assume a VSRS, and often refer to it simply as a *random sample*.

E—HOW RELIABLE IS THE SAMPLE?

The purpose of random sampling, of course, is to make an inference about the underlying population. As a familiar example, we hope the sample mean $\overline{X}$ is a close estimate of the population mean μ. There are two ways we can study just how close $\overline{X}$ comes to μ:

1. Recall how we sampled the blue dots from the gray population in Figure 6-2, and then calculated the sample mean $\overline{X}$. We could repeat this and get a new $\overline{X}$, over and over. By recording how $\overline{X}$ varies from sample to sample, we would build up the *sampling distribution of* $\overline{X}$, denoted $p(\overline{x})$.

   Rather than *actually* sampling a physical population, we can *simulate* this sampling (just as we simulated the roll of a die earlier). This is normally done on a computer, where even complicated sampling can be repeated hundreds of times every second. Since this process—described in detail in Section 6-6—may be viewed as something like "rolling dice," it is called *Monte Carlo* sampling. (The relationship between gambling games and statistics has a long history; in fact, it was gamblers who provided the impetus for probability theory about 300 years ago.)
2. A more precise and useful (but often more difficult) alternative is to derive mathematical formulas for the sampling distribution of $\overline{X}$. Once we have derived such formulas (as we do in the next section) they can be applied broadly to a whole multitude of sampling problems.

PROBLEMS

6-1 A firm selling stationery in California found it could get a tax-exemption on past orders that were for resale (rather than personal use). Since its records did not state which orders were for resale, however, the firm had to estimate this from a survey.

From the 552,000 customers who purchased from it in the past, the firm randomly sampled 5000 by mail questionnaire. Of the 5000, there were 2970 replies, broken down as follows (simplified version of Freedman, 1986):

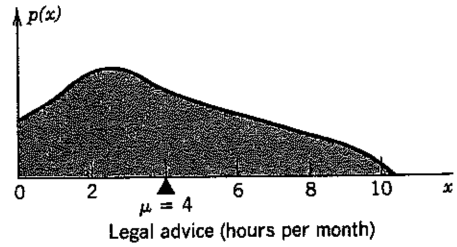
Sample Replies	*Total Dollar Value*
280 for resale	\$ 8,030
2690 for personal use	\$30,130
2970 total	\$38,160

a. The value of all 552,000 orders was $7,010,000. Estimate the value of just the *resale* orders.

b. What assumptions did you make? Why might they be questionable? Do you think the true figure would therefore be higher or lower than your answer in **a**?

c. If the tax rate was $6\frac{1}{2}\%$, use your answer in **a** to estimate the size of the tax refund.

6-2 Because of possible product liability suits, suppose that the hundreds of small U.S. toy manufacturers require varying amounts of legal advice, as follows:



a. Suppose a questionnaire is sent to a random sample of 15 firms. It is returned, however, by only those 5 that have been sued in the past and are therefore most concerned with this issue. Sketch on the graph where the sample of 15 might typically be located, and the 5 replies. Where would the average reply be? How close to the target μ?

b. Repeat part **a** for a random sample of just 5 questionnaires, carefully followed up to obtain a 100% response rate. Is $\overline{X}$ likely to be closer or further from μ?

6-3 In a small and hypothetical midwestern city, the 10,000 adults watch football on TV in widely varying amounts. The weekly amount X varies from 0, 1, 2, . . . , 9 hours, and for each of these levels of X, there are 1000 adults.

a. Graph the population distribution $p(x)$.

b. To simulate drawing one adult (a value of X) at random, take the first digit from Table I. Similarly, to simulate a small sample survey of 5 adults, take the first 5 digits from Table I. Mark these 5 observations in the graph in part **a**.

c. Calculate the population mean μ. (Or, by symmetry, what must it be?)

d. Calculate the sample mean $\overline{X}$. Is it closer to μ than most of the individual observations? Does this illustrate equation (6-2)?

*6-4 **a.** Repeat the simulation of Problem 6-3, a dozen times in all (each time starting at a different place in Table I, of course).

b. In Figure 6-2, the one sample shown in color had $\overline{X} = 70$. Is this a fairly typical sample, or was it a lucky one, particularly close to μ?

c. Suppose the sample size quadrupled to $n = 16$. Repeat part **a** above.

d. Answer True or False. If False, correct it: A doubling of sample size quadruples the accuracy of $\overline{X}$ in estimating μ. The reason is the square root divisor in (6-7).

SOLUTION

a.

$$E(\overline{X}) = \mu = 69 \qquad \text{(6-5) repeated}$$

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.2}{\sqrt{4}} = 1.6 \qquad \text{(6-7) repeated}$$

Thus, the many possible values of the sample mean $\overline{X}$ would fluctuate around the target of 69 inches, with a standard error of 1.6 inches.

b. This particular sample mean $\overline{X} = 70$ deviates from its target $\mu = 69$ by 1 inch. Since 1 inch is not far from the standard error of 1.6 inches, this sample is fairly typical.

c. For a larger sample size $n = 16$, $\overline{X}$ would still fluctuate around its target $\mu = 69$, but with a reduced standard error:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{3.2}{\sqrt{16}} = 0.8$$

d. False. A *quadrupling* of sample size *doubles* the accuracy . . .

PROBLEMS

6-5 Suppose that 10 men were sampled randomly (from the population such as Table 6-1) and their average height $\overline{X}$ was calculated. Then imagine the experiment repeated many times to build up the sampling distribution of $\overline{X}$. Answer True or False; if False, correct it.

a. The tall and short men in the sample tend to average out, making $\overline{X}$ fluctuate less than a single observation.

b. To be specific, the sample mean $\overline{X}$ fluctuates around its target μ with a standard error of only σ/n.

6-6 **a.** The population of American men in 1975 had incomes that averaged $\mu = 10$ thousand, with a standard deviation of 8 thousand, approximately. If a random sample of $n = 100$ men was drawn to estimate μ, what would be the standard error of $\overline{X}$?

b. The population of men in California is about 1/10 as large, but suppose it had the same mean and standard deviation. If a random sample of $n = 100$ was drawn, what would be the standard error of $\overline{X}$ now?

6-7 **a.** Continuing Problem 6-6, the population size was 78 million. If a 1% sample was taken (i.e., n = 1% of 78 million = 780 thousand), what would be the standard error of $\overline{X}$?

b. If a 1% sample of men in California was drawn, what would be the standard error of $\overline{X}$?

*6-8 In 1985, U.S. household income had the following distribution, crudely grouped:

Income x	Proportion $p(x)$
$10,000	.40
$30,000	.40
$50,000	.20

(*Stat. Abst. of the U.S.*, 1987, p. 431.)

a. Calculate the mean μ and standard deviation σ, and show them on a graph of the population distribution.

b. Now suppose a random sample of $n = 2$ incomes is drawn, X_1 and X_2, say. Since each observation has the population distribution, it is easy to tabulate the joint distribution of X_1 and X_2. Then calculate the sampling distribution of $\overline{X}$.

c. From the distribution of $\overline{X}$, calculate the expected value and standard error, and show them on a graph of the sampling distribution of $\overline{X}$.

d. From formulas (6-5) and (6-7), calculate the expected value and standard error of the sampling distribution:

i. When $n = 2$. Does this agree with part **c**?

ii. When $n = 5$.

iii. When $n = 20$.

6-3 THE SHAPE OF THE SAMPLING DISTRIBUTION

In Section 6-2 we found the expected value and standard error of $\overline{X}$. The remaining issue is the *shape* of the sampling distribution.

Across the top of Figure 6-3, we show three different populations. In the column below each, successive graphs show how the sampling distribu-

The next example will show how a problem involving a *total* can be solved by simply rephrasing it in terms of a mean.

EXAMPLE 6-4

A ski lift is designed with a total load limit of 10,000 pounds. It claims a capacity of 50 persons. Suppose the weights of all the people using the lift have a mean of 190 pounds and a standard deviation of 25 pounds. What is the probability that a random group of 50 persons will total more than the load limit of 10,000 pounds?

SOLUTION

First, we rephrase the question: "A random sample of 50 persons will total more than 10,000 pounds" is exactly the same as "A random sample of 50 persons will *average* more than 10,000/50 = 200 pounds each."

From the Normal Approximation Rule, the sample average $\overline{X}$ has an approximately normal distribution with expected value $= \mu = 190$, and a standard error $= \sigma/\sqrt{n} = 25/\sqrt{50} = 3.54$. We use these to standardize the critical average of 200 pounds:

$$Z = \frac{\overline{X} - \mu}{\text{SE}} = \frac{200 - 190}{3.54} = 2.83$$

Thus,

$$\Pr(\overline{X} > 200) = \Pr(Z > 2.83)$$
$$= .002$$

Thus the chance of an overload is only .2%.

The knowledge that the sample mean is normally distributed is very important—not only in making statements about a sample *mean*, but also in making statements about a sample *total* (such as the load limit in the last example).

PROBLEMS

6-9 **a.** The workers in a large meat packing plant earn annual incomes with mean $\mu = \$30{,}000$ and standard deviation $\sigma = \$9000$. A labor lawyer plans to randomly sample 25 incomes from this population. Her sample mean $\overline{X}$ will be a random variable that will only imperfectly reflect the population mean μ. In fact, the possible values of $\overline{X}$ will fluctuate around an expected value of ____ with a standard error of ____, and with a distribution shape that is ____.

b. The lawyer is worried that her sample mean will be misleadingly high. A statistician assures her that it is unlikely that $\overline{X}$

will exceed μ by more than 10%. Calculate just how unlikely this is.

6-10 The millions of SAT math scores of the population of U.S. college-bound seniors in 1978 were approximately normally distributed—around a mean of 470, with a standard deviation of 120.

a. For a student drawn at random, what is the chance of a score above 500? Show this on a graph of the distribution.

b. The registrar of Elora College does not know the mean score of the population, so she estimates it with the mean $\overline{X}$ of a random sample of 250 scores. She hopes $\overline{X}$ will be no more than 10 points off. What are the chances of this?

Show this on a graph of the distribution that shows how $\overline{X}$ fluctuates from sample to sample.

6-11 "One pound" packages filled by a well-worn machine have weights that vary normally around a mean of 16.2 oz, with a standard deviation of .12 oz. An inspector randomly samples a few packages from the production line to see whether their average weight is at least 16 oz. If not, the firm faces a $500 fine. What is the chance of such a fine if the sample size is:

a. $n = 1$?

b. $n = 4$?

c. $n = 16$?

6-12 Suppose that the population of weights of passengers on Flyways Airline has a mean of 150 pounds and standard deviation of 25 pounds. Flyways' commuter plane has a capacity of 7800 pounds, and Flyways is considering equipping it with a configuration of 50 passenger seats.

a. If it does so, what are the chances the aircraft would be overloaded on a fully booked flight?

b. What other information would you want to know before advising Flyways on whether or not it should use this seat configuration?

6-13 The managers of Mercury Mufflers find that the time t (in minutes) required for a worker to replace a muffler varies. Over a period of a year, they collected the following data:

t	*Relative Frequency*
20	10%
30	50%
40	30%
50	10%