

SECOND MIDTERM EXAMINATION
ECON 103, STATISTICS FOR ECONOMISTS

NOVEMBER 2ND, 2016

You will have 90 minutes to complete this exam. Graphing calculators, notes, and textbooks are not permitted.

I pledge that, in taking and preparing for this exam, I have abided by the University of Pennsylvania's Code of Academic Integrity. I am aware that any violations of the code will result in a failing grade for this course.

Please sign the back of your blue book.

Question:	1	2	3	4	5	6	7	Total
Points:	7	15	22	12	30	14	0	100
Bonus Points:	0	0	0	0	0	0	5	5
Score:								

Instructions: Answer all questions in your blue book. Show your work for full credit but be aware that writing down irrelevant information will not gain you points. Be sure to sign the academic integrity statement in the back of your blue book. Make sure that you have all pages of the exam before starting.

Warning: If you continue writing after I call time, even if this is only to fill in your name, twenty-five points will be deducted from your final score. In addition, ten points will be deducted for not signing the back of your blue book.

CHECKLIST BEFORE CONTINUING:

- ☐ My name and Penn ID number are on my blue book
- ☐ I have signed the academic integrity statement in the back of the blue book

1. **Short Answer** Be sure to fully explain your answer.
 - (a) [2 points] What is a random variable? Be precise in your definition.
 - (b) [5 points] Prove that $E[aX + b] = aE[X] + b$ when X is a continuous random variable.
2. **Joint Distributions** For the following question, X and Y have the following joint distribution:

		Y		
		1	2	3
X	1	$\frac{1}{6}$	$\frac{1}{6}$	0
	2	0	$\frac{1}{6}$	$\frac{1}{6}$
	3	$\frac{1}{6}$	0	$\frac{1}{6}$

- (a) [6 points] What is the marginal pmf of X ? What is the marginal pmf of Y ?
 - (b) [4 points] What is $E[E[Y|X]]$?
 - (c) [5 points] Are X and Y independent? Why or why not?
3. **Continuous Random Variables:** Let X be a random variable with pdf $f(x) = (x + 2)/4$ and support set $x \in [-2, 0]$.
 - (a) [5 points] Calculate the CDF $F(x_0)$ of X .
 - (b) [5 points] Calculate $P(X > -1)$.
 - (c) [5 points] Calculate $E[X]$.
 - (d) [7 points] Calculate $Var(X)$.
4. **Distributions** Be sure to fully specify the following distributions. Let X_1, X_2 , and $X_3 \sim iidN(0, 1)$.
 - (a) [3 points] What is the distribution of $X_1 + X_2$?
 - (b) [3 points] What is the distribution of $X_1^2 + X_2^2 + X_3^2$?
 - (c) [3 points] What is the distribution of

$$\frac{(X_1^2 + X_2^2)/2}{(X_2^2 + X_3^2)/2} ?$$

- (d) [3 points] What is the distribution of

$$\frac{X_1}{\sqrt{(X_1^2 + X_2^2 + X_3^2)/3}} \quad ?$$

5. **Estimators** Let X_1, \dots, X_{10} be iid draws from a distribution with mean μ and variance σ^2 . Joe is trying to estimate μ and he proposed two estimators. His first estimator is $\hat{\mu}_1 = \frac{1}{n} \sum_i X_i$. His second estimator is $\hat{\mu}_2 = X_7$, where he reports the 7th observation as his estimate of the sample mean.

- (a) [3 points] What does it mean for an estimator to be unbiased? Be precise in your definition.
- (b) [3 points] What does efficiency mean? Be precise in your definition.
- (c) [2 points] What is mean-squared error (MSE) of an estimator? Be precise in your definition.
- (d) [3 points] What does MSE consistency mean? Be precise in your definition. (**Note:** this is the only kind of consistency we have talked about in class)
- (e) [5 points] Is $\hat{\mu}_1$ unbiased? Show your work
- (f) [3 points] Is $\hat{\mu}_2$ unbiased? Show your work.
- (g) [8 points] Which estimator is more efficient? Clearly explain how you reached your conclusion. Show your work.
- (h) [3 points] Which estimator (if any) is consistent?

6. **Confidence Intervals** Jane is trying to estimate the average height of American men. She has collected data on the heights of a random sample of 100 men. The average height of her sample is 70 inches. For this question, assume heights are normally distributed with mean μ and variance σ^2 .

- (a) [3 points] Jane knows the true population variance, σ^2 , is 25 inches. What is the 95% confidence interval for the population mean μ ?
- (b) [3 points] Jane actually wants to find the 98% confidence interval for the population mean μ . Once again, she knows the true population variance, σ^2 , is 25 inches. What is the confidence interval? Use R commands where necessary.
- (c) [3 points] Jane actually does not know the population variance σ^2 , but she knows the sample variance S^2 is 25 inches and she wants to construct the 95% confidence

interval for the population mean μ . What is the 95% confidence interval? Use R commands where necessary.

- (d) [5 points] Why was Jane's confidence interval different between parts (a) and (c)? There is no need to re-derive any results from the lecture slides, but you should write out the relevant expressions for confidence intervals and discuss what changed and how that changed the result. (**Note:** If you got the same answer for (a) and (c), you might want to double check your work)

7. [5 points (bonus)] **Bonus Question: PDFs and PMFs** Compare and contrast a probability mass function (pmf) and a probability density function (pdf). Be sure to reference their particular properties where possible. Remember, "compare" means to discuss similarities while "contrast" means to discuss differences.