Econ 103 – Statistics for Economists

Chapter 3: Group Work

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Dice

“Odd Question” # 4

To throw a total of 7 with a pair of dice, you have to get a 1 and a 6, or a 2 and a 5, or a 3 and a 4. To throw a total of 6 with a pair of dice, you have to get a 1 and a 5, or a 2 and a 4, or a 3 and another 3. With two fair dice, you would expect:

(a) To throw 7 more frequently than 6.

(b) To throw six more frequently than 7.

(c) To throw 6 and 7 equally often.

Coin Flipping

“Odd Question” # 5

“Imitate” a coin. That is, write down a sequence of 100 H (for heads) and T (for tails) without tossing a coin–but a sequence that you think will fool everyone into thinking it is the reporting of tossing a fair coin.

Which of these is a real sequence of coin flips?

Exhibit A

H H H H T H H T T T H H T T T H T T T H T T H H T
T T T H T T T H T H T T H H H H T T T T T H H H H
H T T H T T T H H T T H H H H H T H H T H T H T H T
T H H T H H T T T H T T T T T T T T T H H T T T T

Exhibit B

H H T H T T T H H T H H H T H T T T H T H H T T T
T H H T T T H H H T H T T T H T T H T H H T H T
T T H H H T H T T H T H H T T H H H T H T T H H H
T T H H H H T H T T H H T T T H H T H H H T T H T

How could we tell which are the real coin flips?

Hardly anyone making up a sequence of 10 tosses puts in a run of 7 heads in a row. It is true that the chance of getting 7 heads in a row with a fair coin is only 1/64. But in tossing a coin 100 times, you have at least 93 chances to start tossing 7 heads in a row, because each of the first 93 tosses could begin a run of 7. It is more probable than not, in 100 tosses, that you will get 7 heads in a row. It is certainly more probable than not, that you will get at least 6 heads in a row. Yet almost no one writes down a pretend sequence, in which there are even 6 heads in a row.

Cards

Poker – Deal 5 Cards, Order Doesn't Matter

Basic Outcomes

$\binom{52}{5}$ possible hands

How Many Hands have Four Aces?

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Basic Outcomes

$\binom{52}{5}$ possible hands

How Many Hands have Four Aces?

48 (# of ways to choose the single card that is not an ace)

Probability of Getting Four Aces

$48/\binom{52}{5} \approx 0.00002$

Poker – Deal 5 Cards, Order Doesn't Matter

What is the probability of getting 4 of a kind?

Poker – Deal 5 Cards, Order Doesn't Matter

What is the probability of getting 4 of a kind?

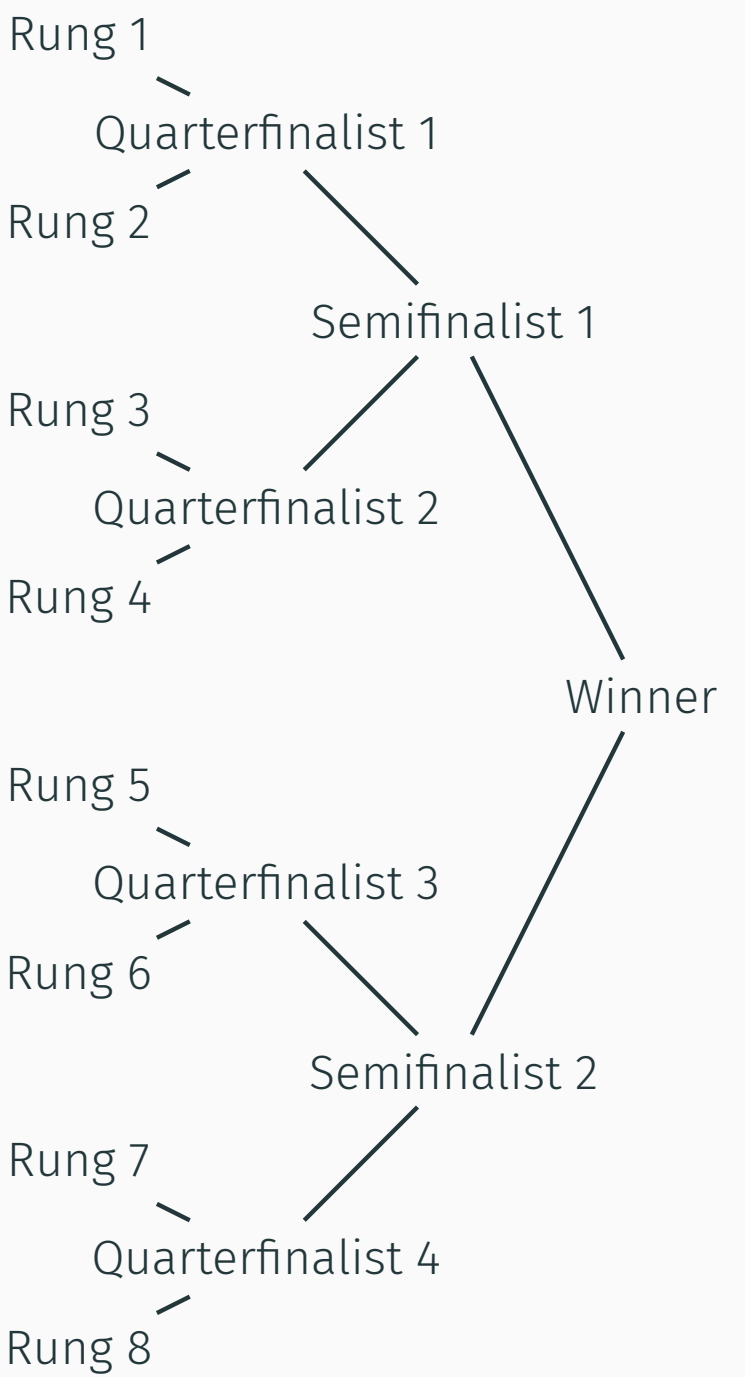
- 13 ways to choose *which* card we have four of
- 48 ways to choose the last card in the hand
- $13 \times 48 = 624$

$$624 / \binom{52}{5} \approx 0.00024$$

Tennis Tournament

A Fairly Ridiculous Example

Roger Federer and Novak Djokovic have agreed to play in a tennis tournament against six Penn professors. Each player in the tournament is randomly allocated to one of the eight rungs in the ladder (next slide). Federer always beats Djokovic and, naturally, either of the two pros always beats any of the professors. What is the probability that Djokovic gets second place in the tournament?


Rung 1
Quarterfinalist 1
Rung 2
Semifinalist 1
Rung 3
Quarterfinalist 2
Rung 4
Winner
Rung 5
Quarterfinalist 3
Rung 6
Semifinalist 2
Rung 7
Quarterfinalist 4
Rung 8

Solution: Order Matters!

Denominator

8! basic outcomes – ways to arrange players on tournament ladder.

Numerator

Sequence of three decisions:

1. Which rung to put Federer on? (8 possibilities)
2. Which rung to put Djokovic on?
   - For any given rung that Federer is on, only 4 rungs prevent Djokovic from meeting him until the final.
3. How to arrange the professors? (6! ways)

$$\frac{8 \times 4 \times 6!}{8!} = \frac{8 \times 4}{7 \times 8} = 4/7 \approx 0.57$$

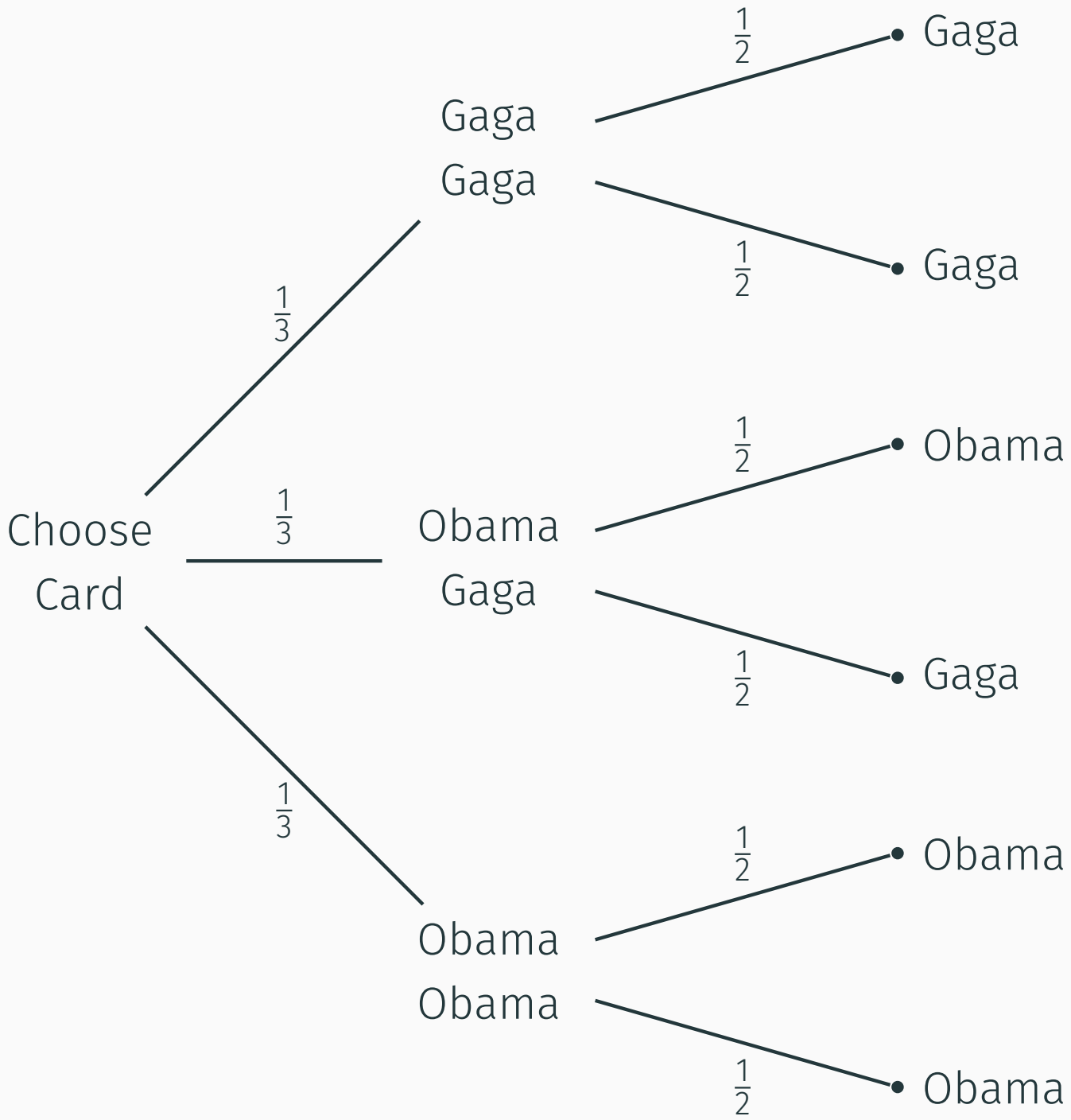
The Card Game

Three Cards, Each with a Face on the Front and Back

1. Gaga/Gaga
2. Obama/Gaga
3. Obama/Obama

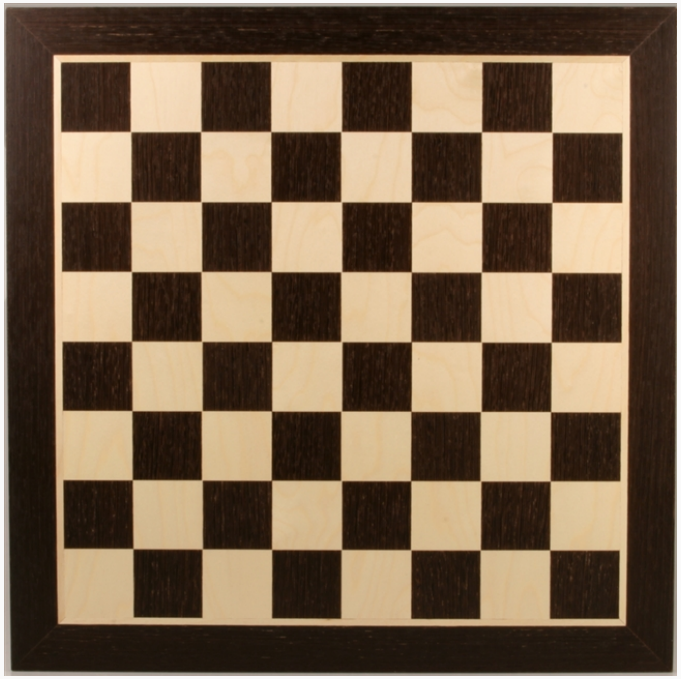
I draw a card at random and look at one side: it's Obama.

What is the probability that the other side is also Obama?


Choose Card
1/3
Gaga Gaga
1/2
Gaga
1/2
Gaga
1/3
Obama Gaga
1/2
Obama
1/2
Gaga
1/3
Obama Obama
1/2
Obama
1/2
Obama

Actual Interview Question

- Imagine a chess board
- You start in the top left corner of the board
- Your goal is to get to the bottom right
- You can only move down or right
- How many possible ways are there to get to the bottom right?



The Secretary Problem (aka the Marriage Problem)

- You have n applicants for a secretary position
- There is a ranking of the candidates (but you cannot observe it)
- You can only determine relative ranks (this person was better than that person)
- Your goal is to choose the best candidate
- You interview them one at a time, but at the end of the interview, you have to either reject them or give them an offer
- Once you've rejected someone, you cannot get them back
- What strategy will give you the highest chance of success?

The Secretary Problem (aka the Marriage Problem)

- **Strategy:** Let some number k candidates go by and then pick the next candidate that is better than anyone you've seen so far.
- **Question:** What should $k - 1$ be?