PROBLEMS

3-1 Fill in the blanks:

a. We define probability or chance as ____.

b. To estimate the chance that a loaded die comes up 1, we could ____.

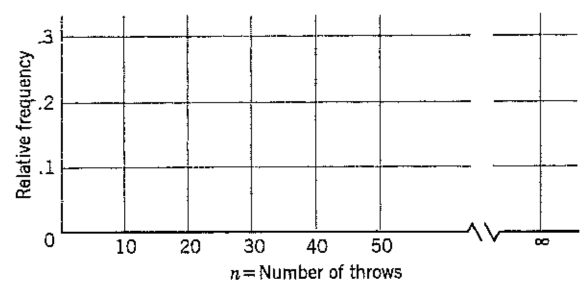
c. To estimate the chance that it will snow in Boston next Christmas day, we could ____.

d. Referring to the data in Problem 2-1, suppose a coil was randomly sampled from the same production run. Then the chance that its hardness would be 50 or less is approximately ____.

3-2 To see how random fluctuation settles down consistently in the long run, repeat Example 3-1. That is, throw a die 50 times (or simulate with random digits, starting at a random spot in Appendix Table I).

a. Graph the relative frequency distribution after 10 throws, after 50 throws, and finally after zillions of throws (guess).

b. These graphs can alternatively be condensed onto a time axis. That is, in the following graph, above $n = 10$ plot all six relative frequencies. Similarly, show the relative frequencies for $n = 50$ and $n = \infty$.



To make the graph more complete, also graph the relative frequencies at $n = 5$ and $n = 20$. Then note how the relative frequencies converge. (In Chapter 6 we will get a precise formula for this convergence.)

3-3 a. Toss a coin 50 times, and record how often it comes up heads. What is your best guess for the *probability* of a head?

b. Toss a thumb tack 50 times, and record how often it comes point up. What is your best guess for the *probability* of this?

c. Roll a pair of dice 50 times (or simulate by drawing pairs of digits from Table I), and record how often you get a "total of 7 or 11" (an outright win in "shooting craps"). What is your best guess for the *probability*?

TABLE 3-2 Several Events in Planning 3 Children

Three Alternative Ways of Naming an Event			
(1) *Arbitrary Symbol for Event*	(2) *Verbal Description*	(3) *Outcome List*	(4) *Probability*
E	At least 2 girls	$\{e_4, e_6, e_7, e_8\}$	4/8
F	Last two are girls	$\{e_4, e_8\}$	2/8
G	Less than 2 girls	$\{e_1, e_2, e_3, e_5\}$	4/8
H	All the same sex	$\{e_1, e_8\}$	2/8
K	Less than 2 boys	$\{e_4, e_6, e_7, e_8\}$	4/8
I	No girls	$\{e_1\}$	1/8
I_1	Exactly 1 girl	$\{e_2, e_3, e_5\}$	3/8
I_2	Exactly 2 girls	$\{e_4, e_6, e_7\}$	3/8
I_3	Exactly 3 girls	$\{e_8\}$	1/8

The first three columns of Table 3-2 show three different ways to name or specify an event. The value of specifying an event the third way—by its outcome list—is evident when we look at the event K (less than 2 boys). The list for K is the same as the list for the first event E (at least 2 girls). That is, $K = E$, an equality that is not so evident from the verbal description.

PROBLEMS

3-4 Using Figure 3-3, find the chance that in a family of three children, there will be:

a. Exactly 2 girls?

b. At least 2 girls?

c. At least one child of each sex?

d. The middle child opposite in sex to the other two?

3-5 **a.** In a learning experiment, a subject attempts a certain task twice in a row. Each time his chance of failure is .40. Draw the probability tree, and then calculate the chance of exactly one failure.

b. Repeat **a** if the subject attempts his task once more, for a total of three tries.

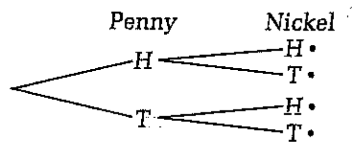
3-6 Repeat Problem 3-5**b**, if the subject makes three tries and he learns from his previous trials, especially his previous successes, as follows: His chance of failure is still .40 at the first trial. However, for later trials his chance of failure drops to .30 if his previous trial was a failure, and drops way down to .20 if his previous trial was a success.

3-7 **a.** (Acceptance sampling) The manager of a small hardware store buys electric clocks in cartons of 12 clocks each. To see whether each carton is acceptable, 3 clocks are randomly selected and thoroughly tested. If all 3 are of acceptable quality, then the carton of 12 is accepted.

Suppose in a certain carton, unknown to the hardware manager, only 8 of the 12 are of acceptable quality. What is the chance that the sampling scheme will inadvertently accept the carton?

b. Repeat part **a**, if the given carton of 12 clocks has only 6 of acceptable quality.

3-8 When a penny and a nickel are tossed, the outcome set could be written as a tree or as a rectangular array:



Penny \ Nickel	*H*	*T*
H	• (*H H*)	• (*H T*)
T	• (*T H*)	• (*T T*)

Draw up the same two versions of the outcome set when a pair of dice are thrown—one red, one white. Then, using whichever version is more convenient, calculate the probability of:

a. A total of 4 dots.

b. A total of 7 dots.

c. A total of 7 or 11 dots (as in Problem 3-3**c**).

d. A double (the same value on both dice).

e. A total of at least 8 dots.

f. A 1 on one die, 5 on the other.

g. A 1 on both dice ("snake eyes").

h. Would you get the same answers if both dice were painted white? In particular, would you get the same answers as before, for parts **f** and **g**?

3-3 COMPOUND EVENTS

A—DEFINITIONS

In planning their three children, suppose that the couple would be disappointed if there were fewer than two girls, or if all were the same sex. Referring to Table 3-2, you can see that this is the event "*G or H*." From

PROBLEMS

3-9 The chance that a factory's sprinkler system will fail is 20%; the chance that its alarm system will fail is 10%; and the chance that both will fail is 4%. What is the chance that:

a. At least one will work?

b. Both will work?

3-10 Of the U.S. population in 1980,

10% were from California
6% were of Spanish origin
2% were from California and of Spanish origin

If an American was drawn at random, what is the chance she would be:

a. From California or of Spanish origin?

b. Neither from California nor of Spanish origin?

c. Of Spanish origin, but not from California?

3-11 In a family of 10 children (assuming boys and girls are equally likely), what is the chance there will be:

a. At least one boy?

b. At least one boy and one girl?

3-12 Suppose that a class of 100 students consists of four subgroups, in the following proportions:

	Men	*Women*
Taking economics	17%	38%
Not taking economics	23%	22%

If a student is chosen by lot to be class president, what is the chance that the student will be:

a. A man?

b. A woman?

c. Taking economics?

d. A man, or taking economics?

e. A man, and taking economics?

f. If the class president in fact turns out to be a man, what is the chance that he is taking economics? not taking economics?

3-13 The men in a certain college engage in various sports in the following proportions:

Football, 30% of all men	Both football and basketball, 5%
Basketball, 20%	Both football and soccer, 10%
Soccer, 20%	Both basketball and soccer, 5%

All three sports, 2%

If a man is chosen by lot for an interview, use a Venn diagram to calculate the chance that he will be:

a. An athlete (someone who plays at least one sport).

b. A football player only.

c. A football player or a soccer player.

If an *athlete* is chosen by lot, what is the chance that he will be:

d. A football player only?

e. A football player or a soccer player?

3-14 A salesman makes 12 calls per day, and on each call has a 20% chance of making a sale.

a. What is the chance he will make no sales at all on a given day?

b. What is the chance he will make at least one sale?

c. If he sells for 200 days of the year, about how many of these days will he make at least one sale?

3-4 CONDITIONAL PROBABILITY

A—DEFINITION

Conditional probability is just the familiar concept of limiting relative frequency, but with a slight twist—the set of relevant outcomes is restricted by a condition. An example will illustrate.

EXAMPLE 3-7
PROBABILITY AFTER A CONDITION (EVENT) IS KNOWN

In a family of 3 children, suppose it is known that G (fewer than two girls) has occurred. What is the probability that H (all the same sex) has occurred? That is, if we imagine many repetitions of this experiment and consider just those cases in which G has occurred, how often will H occur? This is called the *conditional probability of H*, given G, and is denoted $\Pr(H|G)$.

good bulbs together would be:

$$\frac{5}{8} \text{ of } \frac{6}{9} = \frac{5}{12} \qquad \text{(3-19) confirmed}$$

Thus, the product formula (3-18) has a strong intuitive basis.

PROBLEMS

3-15 The table below shows the percentages of the 1985 U.S. labor force, classified by sex and employment status (*Stat. Abst. of U.S.*, 1987, p. 378):

	Sex		
	M (male)	*F (female)*	*Totals*
E (employed)	51.9%	40.9%	92.8%
U (unemployed)	3.9%	3.3%	7.2%
Totals	55.8%	44.2%	100.0%

a. What is the unemployment rate? That is, what is $\Pr(U)$, the chance that a worker drawn at random will be unemployed?

b. What is $\Pr(U|M)$? What is this called?

c. What is $\Pr(U|F)$? What is this called?

In the U.S. in 1974, the population was classified as male or female, and as favoring or opposing abortion.[1] The proportions in each category were approximately as follows (note that all the proportions add up to 1.00 = 100% of the population):

	Favor	*Opposed*
Male	.27	.21
Female	.24	.28

What is the probability that an individual drawn at random would be:

a. In favor of abortion?

b. In favor of abortion, if male?

c. In favor of abortion, if female?

[1] The *Gallup Opinion Index*, April 1974, p. 24. The exact question was, "The U.S. Supreme Court has ruled that a woman may go to a doctor to end pregnancy at any time during the first 3 months of pregnancy. Do you favor or oppose this ruling?" The 10% who had no opinion are not included.

3-17 In a family of three children, what is the chance of:

a. At least one girl?

b. At least two girls?

c. At least two girls, given at least one girl?

d. At least two girls, given that the eldest child is a girl?

3-18 Suppose that 4 defective light bulbs inadvertently have been mixed up with 6 good ones.

a. If 2 bulbs are chosen at random, what is the chance that they both are good?

b. If the first 2 are good, what is the chance that the next 3 are good?

c. If we started all over again and chose 5 bulbs, what is the chance they all would be good?

3-19 A student of statistics, so we're told, once heard that there was one chance in a million of a bomb being on an aircraft. He calculated there would be only one chance in a million million (trillion) of there being two bombs on an aircraft. In order to enjoy these longer odds, therefore, he always carried a bomb on with him (carefully defused, of course—he was no fool.)

Comment.

3-20 Two dice are thrown and we are interested in the following events:

E: first die is 5
F: total is 7
G: total is 10

By calculating the probabilities using Venn diagrams, show that:

a. $\Pr(F|E) = \Pr(F)$.

b. $\Pr(G|E) \neq \Pr(G)$.

c. Is the following a correct verbal conclusion? If not, correct it.
If I'm going to bet on whether the dice show 10, it will help (change the chances) to peek at the first die to see whether it is a 5. But if I'm going to bet on whether the dice show 7, a peek won't help.

3-5 INDEPENDENCE

A—DEFINITION

Independence is a very precise concept that we define in terms of certain probabilities. An example will illustrate.

TABLE 3-3 Review of Probability Formulas

	Pr(E or F)	Pr(E and F)
General Theorem	$= \Pr(E) + \Pr(F) - \Pr(E \text{ and } F)$	$= \Pr(E)\Pr(F \mid E)$
Special Case	$= \Pr(E) + \Pr(F)$ if E and F are mutually exclusive; i.e., if $\Pr(E \text{ and } F) = 0$	$= \Pr(E)\Pr(F)$ if E and F are independent; i.e., if $\Pr(F \mid E) = \Pr(F)$

C—CONCLUSION

Now we have completed our development of the most important formulas of probability. To review them, Table 3-3 sets out our basic conclusions for Pr(E or F) and Pr(E and F).

PROBLEMS

3-21 The 1980 U.S. population, broken down by region and attitude to legalization of marijuana, roughly turned out as follows (note that all proportions add up to 100%):

	In Favor (F)	*Opposed* ($\bar{F}$)
East (E)	7.8%	22.2%
All except East ($\bar{E}$)	18.2%	51.8%

a. What is $\Pr(F)$ (the probability that an individual drawn at random will be in favor of legalization?

b. What is $\Pr(F \mid E)$?

c. Is F independent of E?

3-22 In Problem 3-21, we found that F was independent of E.

a. Can you guess, or better still, state for certain on the basis of theoretical reasoning:

i. Whether E will be independent of F?

ii. Whether E will be independent of $\bar{F}$?

b. Calculate the appropriate probabilities to verify your answers in part (a).

3-23 The table below classifies the 115.5 million civilians in the 1985 U.S. labor force by age and employment status (*Stat. Abst. of U.S.*, 1987, p. 378):

	Age		
	Y (young, under 25)	O (older, 25 and over)	Totals
E (employed)	20.4	86.8	107.2
U (unemployed)	3.2	5.1	8.3
Totals	23.6	91.9	115.5 million

a. What is $\Pr(U)$, the probability that a worker drawn at random will be unemployed? That is, find the unemployment rate.

b. What is $\Pr(U|Y)$?

c. Is unemployment independent of age?

3-24 If E and F are two mutually exclusive events, what can be said about their independence? [*Hint:* What is $\Pr(E \text{ and } F)$? Then, using (3-17), what is $\Pr(E|F)$? Does it equal $\Pr(E)$?]

3-6 BAYES THEOREM: TREE REVERSAL

An important branch of applied statistics called *Bayes Analysis* can be developed out of conditional probability and trees. An example will illustrate.

EXAMPLE 3-10
HOW TO BUY A USED CAR

I am thinking of buying a used Q-car at Honest Ed's. In order to make an informed decision, I look up the records of Q-cars in an auto magazine, and find that, unfortunately, 30% have faulty transmissions.

To get more information on the particular Q-car at Honest Ed's, I hire a mechanic who can make a shrewd guess on the basis of a quick drive around the block. Of course, he isn't always right; but he does have an excellent record: Of all the faulty cars he has examined in the past he correctly pronounced 90% "faulty"; in other words, he wrongly pronounced only 10% "OK." He has almost as good a record in judging good cars: He has correctly pronounced 80% "OK," while he wrongly pronounced only 20% "faulty." (We emphasize that "*faulty*" in quotation marks describes the mechanic's opinion, while *faulty* with no quotation marks describes the actual state of the car.)

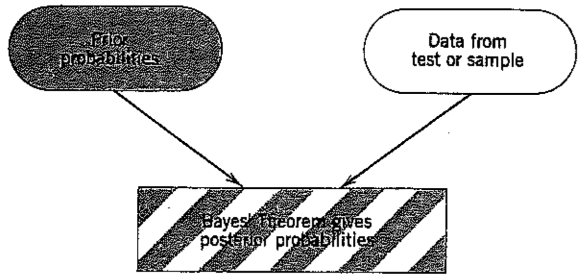


FIGURE 3-11
The logic of Bayes Theorem.

The point of Bayes Theorem may be stated more generally: Prior probabilities, combined with some sort of information such as a test or sample, yield posterior probabilities (the relevant betting odds). Figure 3-11 shows this schematically.

PROBLEMS

3-25 An incipient form of cancer occurs in three out of every 1000 Americans. To provide early detection, a screening test has been developed that rarely errs. Among healthy patients, only 5% get a + reaction (false alarm). Among patients with this incipient cancer, only 2% get a − reaction (missed alarm).

If this test is used to screen the American public, all those who get a + reaction will be hospitalized for exploratory surgery. What proportion of these people who are thought to have cancer, will *actually* have cancer?

3-26 A barometer manufacturer, in testing a very simple model, found that it sometimes erred: on rainy days it erroneously predicted "no rain" 10% of the time; and on the days when it didn't rain, it erroneously predicted "rain" 30% of the time.

In a small town on the Oregon coast, it rains 40% of the days in September. This is roughly the chance it will rain next Labor Day, for example.

As Labor Day approaches, its barometer prediction turns out to be "rain." Now what is the chance it will actually rain?

3-27 The chain saws produced by a large manufacturer for the first three months of 1990 were of poor quality. In each of the 300 shipments, 40% of the saws were defective. After tightening up quality control, however, this figure was reduced to 10% defective in each of the 900 shipments produced in the last nine months.

The manager of a large hardware store was sent a shipment of the 1990 model, and wanted to know whether it was one of the poor shipments produced early in 1990. (If so, he wanted to return it, or at least check it out, saw by saw.)

a. Before opening the shipment, what was the chance it was one of the poor ones?

b. He opened it, randomly drew out a saw, and tested it, and found it was defective. Now what is the chance it was one of the poor shipments?

***[3]3-28** Continuing Problem 3-27, calculate the chance it is one of the poor shipments if he:

a. Draws a second defective saw?

b. Draws a third defective saw?

***3-29** A small plane has gone down, and the search is organized into three regions. Starting with the likeliest, they are:

Region	*Initial Chance Plane is There*	*Chance of Being Overlooked in the Search*
Mountains	.50	.30
Prairie	.30	.20
Sea	.20	.90

The last column gives the chance that if the plane is there, it will not be found. For example, if it went down at sea, there is 90% chance it will have disappeared, or otherwise not be found.

Since the pilot is not equipped to long survive a crash in the mountains, it is particularly important to determine the chance that the plane went down in the mountains.

a. Before any search is started, what is this chance?

b. The initial search was in the mountains, and the plane was not found. Now what is the chance the plane is nevertheless in the mountains?

c. The search was continued over the other two regions, and unfortunately the plane was not found anywhere. Finally now what is the chance that the plane is in the mountains?

d. Write a few lines describing how and why the chances changed from **a** to **b** to **c**.

[3] As mentioned in the Preface, a star indicates a problem is more challenging, or requires optional starred material.

Note that this does indeed give us the odds in our example, that is,

$$\text{the odds of getting an ace, } d = \frac{1/6}{1 - 1/6} = \frac{1}{5}$$

Of course, we can algebraically solve (3-27) to express p in terms of d:

$$p = \frac{d}{d + 1} \tag{3-28}$$

In our example, knowledge that the odds for an ace are $d = 1/5$ would allow us to solve (3-28) for the probability of an ace, that is,

$$p = \frac{1/5}{1 + 1/5} = \frac{1}{6}$$

PROBLEMS

3-30 **a.** If you had your choice today between the following two bets, which would you take?
Election Bet: If the Democratic candidate wins the next presidential election, you will then win a $100 prize (and win nothing otherwise).
Jar Bet: A chip will be drawn at random from a jar containing 1 black chip and 999 white chips. If the chip turns out black, you will then win the $100 prize (and win nothing otherwise).

b. Repeat choice **a**, with a change—make the composition of the jar the opposite extreme: 999 black chips and 1 white chip.

c. Obviously, your answers in parts **a** and **b** depended on your subjective estimate of American politics; there were no objective "right answers." However, the number of chips in the urn were so lopsided that there undoubtedly is widespread agreement in part **a** to prefer the election bet, and in part **b** to prefer the jar bet. The question is: as you gradually increase the black chips from 1 to 999, at what point do you become indifferent between the two bets? Is it reasonable to call this your personal probability of a Democratic win?

3-31 Using the "calibrating jar" of Problem 3-30, roughly evaluate your personal probability that:

a. The Dow Jones average (of certain stock market prices) will advance at least 10% in the next twelve months.

b. U.S. population will increase at least 1% next year.

c. U.S. inflation will be at least 5% next year.

d. The next vice president of the U.S. will be a female.

e. The next president of your student council will be a female.

f. The next president of your student council will be a female, if the president is a senior student chosen at random.

g. At the next Superbowl game, the coin that is flipped (to determine the kickoff) will turn up heads.

h. Suppose you go to the largest Ford dealer in town, pick out a three-year-old used car, and offer 90% of the sticker price—cash, take it or leave it. What is your subjective probability that after a certain amount of huffing and bluffing, the dealer will finally accept your offer?

3-32 In Problem 3-31, for which answers do you think there will be least agreement among the students in your class? Most agreement? Which questions are amenable to a brief investigation that would result in everyone agreeing? (If you have time in class, check out your answers.)

3-33 Do you think the following conclusions are valid? If not, correct them:

i. Certain probabilities (as in part g of Problem 3-31) are agreed upon by practically everybody; we may call them "objective probabilities."

ii. Other probabilities (as in parts **a** or **c**) are disagreed upon, even by experts; we may call them "subjective probabilities."

iii. But in between, there is a continuous range of probabilities that are subjective to a greater or lesser degree.

3-34 From a deck of 52 cards you are dealt one face down. What is **(i)** the chance and **(ii)** the odds that the card will turn out to be:

a. A club?

b. A black card?

c. An ace?

d. An honor card (A, K, Q, or J)?

e. The same denomination as the next card to be dealt (for example, a Queen followed by another Queen)?

*f. A higher denomination than the next card to be dealt?

CHAPTER 3 SUMMARY

3-1 Probability is just the proportion that emerges in the long run, as the experiment is endlessly repeated (the sample size n grows larger and larger). The abbreviation Pr can therefore be thought of as representing *proportion* or *percentage*, as well as *probability*.

3-2 Probability trees break a complex experiment into small manageable stages. At each stage, the various outcomes are represented by branches, and so the tree eventually represents the whole outcome set.

3-3 An event is just a collection of individual outcomes, and its probability is just the sum of the individual probabilities. From this first principle, we can easily find the probability of G or H by addition:

$$\Pr(G \text{ or } H) = \Pr(G) + \Pr(H) - \Pr(G \text{ and } H)$$

3-4 The probability of G and H is obtained by multiplication:

$$\Pr(G \text{ and } H) = \Pr(G)\,\Pr(H|G)$$

This multiplication principle is so natural, in fact, that it was used in probability trees without formal introduction.

3-5 An event F is called independent of another event E, if the probability of F remains ~~the same~~ after E ~~has occurred~~:

$$\Pr(F|E) = \Pr(F)$$

3-6 Bayes Theorem is simply a clever use of conditional probability to reverse probability trees. It combines prior probabilities with sample information (in the original tree) to obtain the posterior probabilities (in the reversed tree).

3-7 As well as the relative-frequency view, other views of probability are of interest: symmetric probability in fair games; axiomatic probability in mathematics; and subjective probability, which is increasingly important whenever human judgment is required—as in business or social science.

REVIEW PROBLEMS

3-35 Suppose that A and B are independent events, with $\Pr(A) = .6$ and $\Pr(B) = .2$. What is:

a. $\Pr(A|B)$?

b. $\Pr(A \text{ and } B)$?

c. $\Pr(A \text{ or } B)$?

3-36 Repeat Problem 3-35 if A and B are mutually exclusive instead of independent.

3-37 Kevin Stern, marketing VP for a corporation manufacturing small pleasure craft, must decide whether or not to enter the sailboard market with an inexpensive new model. The high initial cost of its development and promotion can only be justified if it captures 10% or more of the market.

To estimate the potential market share, he contacts a random survey of 200 potential buyers, who are given a hypothetical choice between buying this model at $950, or an alternative. Their responses are classified in the first column of the table below.

Response	*Frequency of Response*	*Estimated Probability of Buying*
"Would definitely buy it"	24	40%
"Would probably buy it"	46	20%
"Would maybe buy it"	34	8%
"Would not buy it"	96	1%
	$n = 200$	

From bitter experience with similar surveys, Kevin has learned that of all those who say they will "definitely buy it," only 40% actually do. This estimated probability of buying is listed in the last column, varying from 40% at the top to 1% at the bottom. (For similar examples, see Schwarz 1978.)

a. Estimate the market share (i.e., the proportion who would actually buy the model). Does it surpass the 10% target?

b. Of those who buy it, what proportion had earlier felt they would "definitely buy it?"

3-38 To reduce theft, suppose a company proposes to screen its workers with a lie-detector test that has been proven correct 90% of the time (for guilty subjects, and also for innocent subjects). The company would fire all the workers who fail the test. Suppose also that 5% of the workers steal from time to time.

a. Of the fired workers, what proportion would actually be innocent?

b. Of the remaining workers not fired, what proportion would actually be guilty?

3-39 A national survey of couples showed that 30% of the wives watched a certain TV program, and 50% of the husbands. Also, if the wife watched, the probability that the husband watched increased to 60%. For a couple drawn at random, what is the probability that:

a. The couple both watch?

b. At least one watches?

c. Neither watches?

d. If the husband watches, the wife watches?

e. If the husband does not watch, the wife watches?

f. Answer True or False; if false correct it: The unconditional probability that the wife watches is somewhere between the two conditional probabilities given in parts **d** and **e.**

3-40 For various forms of transportation, the 1975–78 U.S. death rates were approximately as follows (deaths per billion passenger miles):

Car	16
Train	0.84
Scheduled airline	0.35

a. Graph these rates.

b. Suppose you travel about 20,000 miles per year. Over a remaining lifetime of 60 years, what is your approximate chance of being killed in an accident if you traveled:

i. Always by car?
ii. Always by plane?
iii. By car or plane, 50-50?

c. What assumptions did you make in **b**?

3-41 Yesterday I bought a packet of inexpensive videotapes and wondered whether they might be defective. I learned that 20% of the packets in the store came from a now-bankrupt wholesaler, with an appalling record of supplying 15% defective tapes. The remaining packets came from a new wholesaler, with a much improved record of only 1% defective tapes.

a. What is the chance that the first videotape I draw from the packet is defective?

b. As luck would have it, the first tape was indeed defective. Now what is the chance that the second tape I draw from the packet is defective?

3-42 Suppose that the last three men out of a restaurant all lose their hat checks, so that the hostess hands back their three hats in random order. What is the probability:

a. That no man will get the right hat?

b. That exactly one man will?

c. That exactly two men will?

d. That all three men will?

3-43 Find (without bothering to multiply out the final answer) the probability that:

a. A group of 3 people (picked at random) all have different birthdays?

b. A group of 30 people all have different birthdays?

c. In a group of 30 people there are at least two people with the same birthday?

d. What assumptions did you make above?

3-44 On November 24, 1968, two hijackings occurred on the same day, and made the front page of the *New York Times*. How unusual and newsworthy is such a coincidence? (Glick, 1970) To answer this question, the following data are relevant: During the 4 winter months November–February, there are 120 days, and it turns out that there were 22 hijackings. If we assume that the hijackings are independent and apt to occur equally likely on any day, what is the probability that in a 120-day period with 22 hijackings, there will be some day when two or more hijackings occur? (Hint: do Problem 3-43 first.)

3-45 True or False? If false, correct it:

a. When two events are independent, the occurrence of one event will not change the probability of the second event.

b. Two events are mutually exclusive if they have no outcomes in common.

c. A and B are mutually exclusive if $\Pr(A \text{ and } B) = \Pr(A)\Pr(B)$.

d. If a fair coin has been fairly tossed 5 times and has come up tails each time, on the sixth toss the conditional probability of tails will be $1/64$.

3-46 ***A Final Challenge: How to Guarantee Questionnaire Security***

In Chapter 1, we saw how random selection guaranteed fairness. Now let us look at yet another ingenious use of randomization—to guarantee anonymity in a sensitive area of survey sampling.

Suppose, for example, that we are taking a survey to determine what proportion of executives have ever engaged in "inside trading" (using privileged information about a company's financial position to gain an illegal advantage in trading its stock). What executive would truthfully admit to this without an absolute guarantee of being untraceable (anonymous)?

Let us therefore phrase our survey with airtight protection. Each executive interviewed could be asked to privately flip a coin, and then:

i. If it turns up tails, answer the question "Have you ever engaged in inside trading?"

ii. If it turns up heads, flip it again and now answer "Did the coin come up heads this second time?"

Suppose the executive returns with the answer "Yes." There is no possible way for the interviewer to know whether this came from inside trading, or merely a second head. Thus anonymity is guaran-