

# Problem Set #6

Econ 103

## Lecture Progress

We made it to the end of the Chapter 5 slides.

## Homework Checklist

- ☐ **Book Problems (Chapter 4):** 19, 21, 25, 27, 29 (*When necessary, use  $R$  instead of the Normal tables in the textbook*)
- ☐ **Additional Problems:** See below
- ☐ **R Tutorial:** None this week so you can focus on discrete and continuous RVs
- ☐ **Ask questions on Piazza**
- ☐ **Review slides**
- ☐ **Work on R Project!**

## Additional Problems

1. Suppose that  $X$  is a random variable with the following PDF

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Graph the PDF of  $X$ .
- (b) Show that  $\int_{-\infty}^{\infty} f(x) dx = 1$ .
- (c) What is  $P(0.5 < X < 1.5)$ ?

2. Let  $X$  be a random variable with the following PDF

$$f(x) = \begin{cases} cx^2 & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) What is  $c$ ?
  - (b) Find  $E[X]$  and  $Var(X)$
  - (c) Find  $P(X \geq \frac{1}{2})$
3. A random variable is said to follow a  $\text{Uniform}(a, b)$  distribution if it is equally likely to take on any value in the range  $[a, b]$  and never takes a value outside this range. Suppose that  $X$  is such a random variable, i.e.  $X \sim \text{Uniform}(a, b)$ .
- (a) What is the support of  $X$ ?
  - (b) Explain why the PDF of  $X$  is  $f(x) = 1/(b - a)$  for  $a \leq x \leq b$ , zero elsewhere.
  - (c) Using the PDF from part (b), calculate the CDF of  $X$ .
  - (d) Verify that  $f(x) = F'(x)$  for the present example.
  - (e) Calculate  $E[X]$ .
  - (f) Calculate  $E[X^2]$ . *Hint:* recall that  $b^3 - a^3$  can be factorized as  $(b - a)(b^2 + a^2 + ab)$ .
  - (g) Using the shortcut formula and parts (e) and (f), show that  $Var(X) = (b - a)^2/12$ .
4. Suppose that  $X \sim N(0, 16)$  independent of  $Y \sim N(2, 4)$ . Recall that our convention is to express the normal distribution in terms of its mean and variance, i.e.  $N(\mu, \sigma^2)$ . Hence,  $X$  has a mean of zero and variance of 16, while  $Y$  has a mean of 2 and a variance of 4. In completing some parts of this question you will need to use the R function `pnorm` described in class. In this case, please write down the command you used as well as the numeric result.

For each of the following, use the transformation to a standard normal in your computations, but also include the R commands you would use for an arbitrary normal variable.

- (a) Calculate  $P(-8 \leq X \leq 8)$ .
- (b) Calculate  $P(0 \leq Y \leq 4)$ .
- (c) Calculate  $P(-1 \leq Y \leq 6)$ .
- (d) Calculate  $P(X \geq 10)$ .

**Note:** In the following five questions  $X_1, X_2 \sim iid N(\mu, \sigma^2)$ ,  $Y = (X_1 - \mu)/\sigma$ ,  $Z = (X_2 - \mu)/\sigma$ .

5. (a) What is the distribution of  $X_1 + X_2$ ?  
(b) Use R to calculate  $P(X_1 + X_2 > 5)$  if  $\mu = 5$  and  $\sigma^2 = 50$ .  
(c) Use R to calculate the 10th percentile of the distribution of  $X_1 + X_2$ .
6. (a) What is the distribution of  $Y^2$ ?  
(b) Use R to calculate  $P(Y^2 \geq 1)$ .
7. (a) What is the distribution of  $Y^2 + Z^2$ ?  
(b) Use R to calculate the 95th percentile of the distribution of  $Y^2 + Z^2$ .
8. (a) What is the distribution of  $Z/\sqrt{Y^2}$ ?  
(b) What value of  $c$  satisfies  $P(-c \leq Z/\sqrt{Y^2} \leq c) = 0.95$ ?  
(c) How does the interval in part (b) compare to the corresponding interval for  $Z$ ?
9. (a) What is the distribution of  $Y^2/Z^2$ ?  
(b) Use R to calculate the 95th percentile of the distribution of  $Y^2/Z^2$ .