Solving Optimization Problems

Problem1

Find the value of x that gives minimum value for the following function:

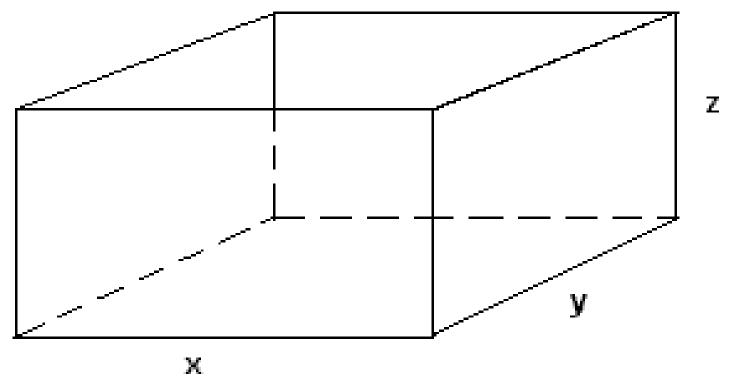
$$f(x) = 1.2(x-2)^2 + 3.2$$

Problem2

You decide to construct a rectangle of perimeter 400 mm with maximum area. Find the length and the width of the rectangle.

Problem3

You decide to build a box that has the shape of a rectangular prism with a volume of 1000 cubic centimeters. Find the dimensions x, y and z of the box so that the total surface area of all 6 faces of the box is minimum.



Approaches to Solve Unconstrained Optimization Problems

- Try all possible solutions and pick the optimal one.

- Find the optimal solution(min/max) using slopes and equation solving.

- Guess the solution and refine it until we get close to optimal solution.

Optimization with constraints

- What if I want to constrain the parameters of the model.

- Two functions:
  - An objective function to maximize
  - An equality/inequality that must be satisfied

Lagrange Multiplier Solution for General constrained problem

- Maximizing: $f(x, y)$
- Subject to: $g(x, y) = c$

- Introduce a new variable, and find a maxima.

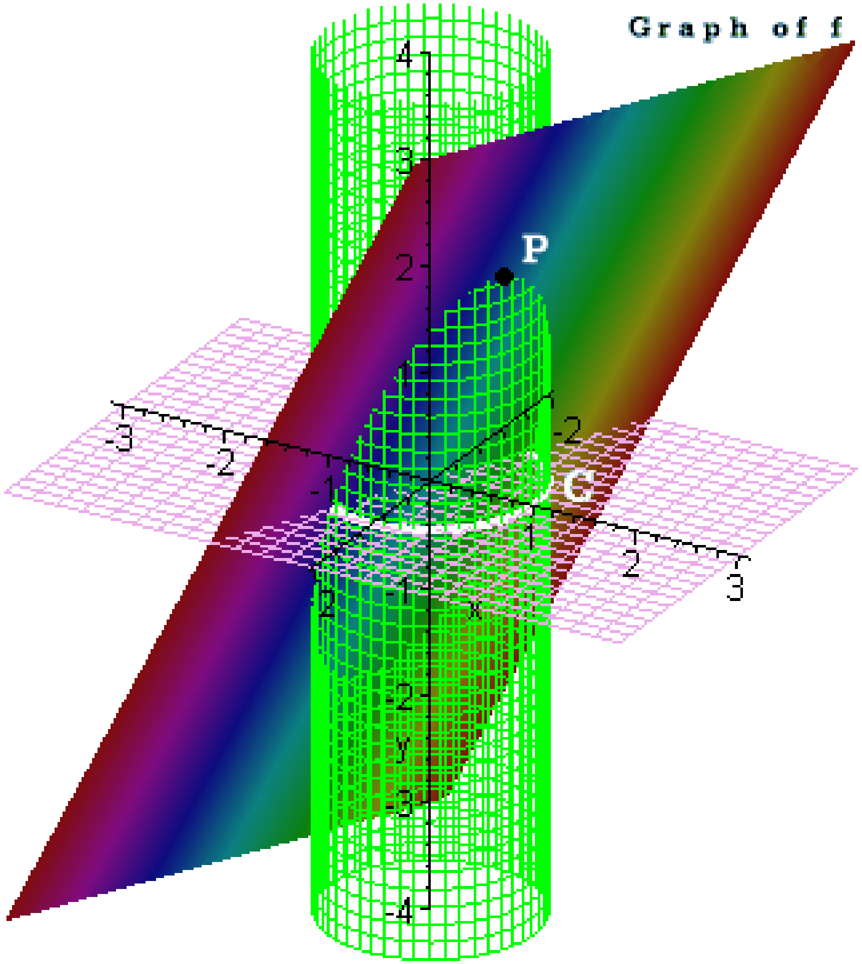
$$\Lambda(x, y, \lambda) = f(x, y) + \lambda(g(x, y) - c)$$

Lagrange Multipliers

- Find maxima of f(x,y) subject to a constraint.

$$f(x, y) = x + 2y$$

$$x^2 + y^2 = 1$$



Example

- Maximizing: $f(x, y) = x + 2y$
- Subject to: $x^2 + y^2 = 1$

- Introduce a new variable, and find a maxima.

$$\Lambda(x, y, \lambda) = x + 2y + \lambda(x^2 + y^2 - 1)$$

Example

$$\frac{\partial \Lambda(x, y, \lambda)}{\partial x} = 1 + 2\lambda x = 0$$

$$\frac{\partial \Lambda(x, y, \lambda)}{\partial y} = 2 + 2\lambda y = 0$$

$$\frac{\partial \Lambda(x, y, \lambda)}{\partial \lambda} = (x^2 + y^2 - 1) = 0$$

Now have 3 equations with 3 unknowns.

Example

Eliminate Lambda

$$1 = 2\lambda x$$

$$2 = 2\lambda y$$

$$\frac{1}{x} = 2\lambda = \frac{2}{y}$$

$$y = 2x$$

Substitute and Solve

$$x^2 + y^2 = 1$$

$$x^2 + (2x)^2 = 1$$

$$5x^2 = 1$$

$$x = \pm\frac{1}{\sqrt{5}}$$

$$y = \pm\frac{2}{\sqrt{5}}$$