

**Course Title:** Advanced Mathematics **Major:** Petroleum Eng.(drilling) **Exam Date:** Bahman 2, 1389**Exam Time:** 180 Min.**OPEN BOOK****Level:** B.Sc ☐ M.Sc ☒ Ph.D. ☐**Student Full Name:****Student Number:**1. A clamped cubic Spline for a function $f(x)$ is defined by

$$f(x) = \begin{cases} 1 + Bx + 2x^2 - 2x^3 & 0 \leq x \leq 1 \\ 1 + b(x-1) - 4(x-1)^2 + 7(x-1)^3 & 1 \leq x \leq 2 \end{cases}$$

Find $f'(0)$ and $f'(2)$.

2. Calculate the smallest Eigen value of the following system.

$$\frac{d}{dx} \left(x \frac{dy}{dx} \right) + \frac{\lambda^2}{x} y = 0 \quad 1 < x < 2$$
$$y(2) = 0 = y'(1)$$

3. Calculate y''' at $x = 0.2$ with accuracy $O(h^3)$.

$$y''' + 2y'' - y' - 2y = e^x \quad x \geq 0$$
$$y(0) = 1, \quad y'(0) = 2 \quad \text{and} \quad y''(0) = 5$$

4. Solve the system for (x, y) in the first quadrant.

$$\begin{cases} x^3 - 2y^2 + 5x - 1 = 0 \\ y^3 - 2x^2 + 5y - 1 = 0 \end{cases}$$

5. Use Shooting Method to calculate $y\left(\frac{\pi}{4}\right)$ given,

$$y'' = y' + 2y + \cos x \quad 0 \leq x \leq \frac{\pi}{2}$$

$$y(0) = -0.3, \quad y\left(\frac{\pi}{2}\right) = -0.1$$

6. Calculate $u(5,5)$ and $u(5,10)$ given

$$e^{-y} \frac{\partial^2 u}{\partial x^2} + e^{-x} \frac{\partial^2 u}{\partial y^2} = x + y \quad 0 < x < 10, \quad 0 < y < 15$$

$$u = \frac{1}{x^2 + y^2 + 1} \quad \text{on } B$$

7. In laminar flow the function coefficient f can be related to the Reynolds number Re by the relation,

$$f = a(Re)^b$$

Use the following measured data to determine a and b .Then predict f for $Re = 2200$.

Re	500	1000	1500	2000
f	0.0320	0.0160	0.0107	0.0080

Good luck...