Petroleum University of Technology

Petroleum Faculty of Ahwaz

Final Exam of 1st Term of 89-90

Course Title: Advanced Mathematics	**Major:** Petroleum Eng.(drilling)	**Exam Date:** Bahman 2, 1389
Exam Time: 180 Min.	**OPEN BOOK**	**Level:** B.Sc☐ M.Sc☑ Ph.D.☐
Student Full Name:	**Student Number:**	

1. A clamped cubic Spline for a function $f(x)$ is defined by

$$f(x)=\begin{cases}1+Bx+2x^2-2x^3 & 0\le x\le 1\\ 1+b(x-1)-4(x-1)^2+7(x-1)^3 & 1\le x\le 2\end{cases}$$

Find $f'(0)$ and $f'(2)$.

2. Calculate the smallest Eigen value of the following system.

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right)+\frac{\lambda^2}{x}y=0 \qquad 1<x<2$$

$$y(2)=0=y'(1)$$

3. Calculate y''' at $x=0.2$ with accuracy $O(h^3)$.

$$y'''+2y''-y'-2y=e^x \qquad x\ge 0$$

$$y(0)=1\ ,\quad y'(0)=2 \qquad and \qquad y''(0)=5$$

4. Solve the system for (x,y) in the first quadrant.

$$\begin{cases}x^3-2y^2+5x-1=0\\ y^3-2x^2+5y-1=0\end{cases}$$

5. Use Shooting Method to calculate $y\left(\frac{\pi}{4}\right)$ given,

$$y''=y'+2y+\cos x \qquad 0\le x\le \frac{\pi}{2}$$

$$y(0)=-0.3 \quad , \quad y\left(\frac{\pi}{2}\right)=-0.1$$

6. Calculate $u(5,5)$ and $u(5,10)$ given

$$e^{-y}\frac{\partial^2 u}{\partial x^2}+e^{-x}\frac{\partial^2 u}{\partial y^2}=x+y \qquad 0<x<10 \quad , \quad 0<y<15$$

$$u=\frac{1}{x^2+y^2+1} \qquad on\ B$$

7. In laminar flow the function coefficient f can be related to the Reynolds number Re by the relation,

$$f=a(\mathrm{Re})^b$$

Use the following measured data to determine a and b.

Then predict f for $\mathrm{Re}=2200$.

Re	500	1000	1500	2000
f	0.0320	0.0160	0.0107	0.0080

Good luck…