SECTION 9.11, page 453

1. ± 1 3. ± 1 7. 0 11. 21 13. 0 15. 0

SECTION 9.12, page 461

1. Exact
3. Exact
5. Not exact
7. $u = x + y + z + c$
9. $u = \sin x - 2 \cos y$
11. $u = -1/(x^2 + y^2)$
13. $u = \sinh (y^2 + z^2)$
15. 5
17. -2
19. $-e^{-2} - 1$

SECTION 10.1, page 464

1. $2\pi, 2\pi, \pi, \pi, 2, 2, 1, 1$
17. 0 (n even), $2/n$ (n odd)
19. $0\ (n = 0),\ 2\pi/n\ (n = 1, 3, \cdots),\ -2\pi/n\ (n = 2, 4, \cdots)$
21. 0
23. $n[(-1)^n e^{-\pi} - 1]/(1 + n^2)$
25. $2\pi^3/3\ (n = 0),\ (-1)^n 4\pi/n^2\ (n = 1, 2, \cdots)$

SECTION 10.2, page 471

1. $\frac{1}{2} + \frac{2}{\pi}\left(\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - + \cdots\right)$
3. $\frac{1}{2} + \frac{2}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right)$
5. $\frac{4}{\pi}\left(\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - + \cdots\right)$
7. $2\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x + - \cdots\right)$
9. $\frac{\pi^2}{3} - 4\left(\cos x - \frac{1}{4}\cos 2x + \frac{1}{9}\cos 3x - \frac{1}{16}\cos 4x + - \cdots\right)$
11. $\frac{\pi}{2} + \frac{4}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right)$
13. $\frac{4}{\pi}\left(\sin x - \frac{1}{9}\sin 3x + \frac{1}{25}\sin 5x - + \cdots\right)$

SECTION 10.3, page 474

3. Use (1b).
5. $\frac{4}{\pi}\left(\sin \pi t + \frac{1}{3}\sin 3\pi t + \frac{1}{5}\sin 5\pi t + \cdots\right)$
7. $\frac{1}{2} + \frac{2}{\pi}\left(\sin\frac{\pi\tau}{2} + \frac{1}{3}\sin\frac{3\pi\tau}{2} + \frac{1}{5}\sin\frac{5\pi\tau}{2} + \cdots\right)$
9. $\frac{1}{4} - \frac{2}{\pi^2}\left(\cos \pi t + \frac{1}{9}\cos 3\pi t + \cdots\right) + \frac{1}{\pi}\left(\sin \pi t - \frac{1}{2}\sin 2\pi t + - \cdots\right)$
11. $\frac{2}{3} + \frac{4}{\pi^2}\left(\cos \pi t - \frac{1}{4}\cos 2\pi t + \frac{1}{9}\cos 3\pi t - + \cdots\right)$
13. $-\frac{4}{\pi^2}\left(\cos \pi t + \frac{1}{9}\cos 3\pi t + \cdots\right) + \frac{2}{\pi}\left(2\sin \pi t - \frac{1}{2}\sin 2\pi t + - \cdots\right)$

SECTION 10.4, page 478

1. Neither odd nor even, even, odd, even, odd, neither odd nor even, even, even.
3. Odd
5. Odd
7. Neither odd nor even
9. Odd
15. $(1 + x^2)/(1 - x^2)^2 + 2x/(1 - x^2)^2$
17. $-x^2/(1 - x^2) + x/(1 - x^2)$
23. $\frac{1}{2} + \frac{2}{\pi}\left(\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - + \cdots\right)$
25. $\frac{4}{\pi}\left(\sin x - \frac{1}{9}\sin 3x + \frac{1}{25}\sin 5x - + \cdots\right)$
27. $\frac{\pi}{2} - \frac{4}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right)$
29. $\frac{\pi^2}{12} - \cos x + \frac{1}{4}\cos 2x - \frac{1}{9}\cos 3x + \frac{1}{16}\cos 4x - + \cdots$

SECTION 10.5, page 482

1. $2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t + - \cdots\right)$
3. $\frac{2}{\pi}\left(\sin \pi t - \frac{1}{2}\sin 2\pi t + \frac{1}{3}\sin 3\pi t - + \cdots\right)$
5. $\left(1 + \frac{2}{\pi}\right)\sin t - \frac{1}{2}\sin 2t + \left(\frac{1}{3} - \frac{2}{9\pi}\right)\sin 3t - \frac{1}{4}\sin 4t + \cdots$
7. $\frac{1}{\pi}\left(\sin 4t - \frac{1}{9}\sin 12t + \frac{1}{25}\sin 20t - \frac{1}{49}\sin 28t + - \cdots\right)$
9. $f(t) = 1$
11. $\frac{l^2}{3} - \frac{4l^2}{\pi^2}\left(\cos\frac{\pi t}{l} - \frac{1}{4}\cos\frac{2\pi t}{l} + \frac{1}{9}\cos\frac{3\pi t}{l} - \frac{1}{16}\cos\frac{4\pi t}{l} + - \cdots\right)$
13. $\frac{2}{\pi} - \frac{4}{\pi}\left(\frac{1}{1\cdot 3}\cos\frac{2\pi t}{l} + \frac{1}{3\cdot 5}\cos\frac{4\pi t}{l} + \frac{1}{5\cdot 7}\cos\frac{6\pi t}{l} + \cdots\right)$
17. $i\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{(-1)^n}{n}e^{inx}$
19. $\frac{\sinh \pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n\frac{1 + in}{1 + n^2}e^{inx}$

SECTION 10.6, page 488

11. $12\left(\sin x - \frac{1}{2^3}\sin 2x + \frac{1}{3^3}\sin 3x - + \cdots\right)$
13. $\frac{l^3}{4} + \frac{6l^3}{\pi^2}\left[\left(\frac{4}{\pi^2} - 1\right)\cos\frac{\pi x}{l} + \frac{1}{2^2}\cos\frac{2\pi x}{l} + \left(\frac{4}{3^4\pi^2} - \frac{1}{3^2}\right)\cos\frac{3\pi x}{l} + \cdots\right]$
15. $a_0 = \pi^4/5$, $a_n = (-1)^n 8(\pi^2 n^2 - 6)/n^4$, $b_n = 0$ $(n = 1, 2, \cdots)$

SECTION 10.7, page 491

1. $y = C_1\cos\omega t + C_2\sin\omega t + A(\omega)\sin t$, $A(\omega) = 1/(\omega^2 - 1)$, $A(0.5) = -1.33$, $A(0.7) = -0.20$, $A(0.9) = -5.3$, $A(1.1) = 4.8$, $A(1.5) = 0.8$, $A(2) = 0.33$, $A(10) = 0.01$

3. $y = C_1 \cos \omega t + C_2 \sin \omega t + B_1 \sin t + B_3 \sin 3t + B_5 \sin 5t$ where

ω	0.5	0.9	1.1	2.0	2.9	3.1	4.0	4.9	5.1	6.0	8.0
$B_1 = 1/(\omega^2 - 1)$	−1.33	−5.3	4.8	0.33	0.13	0.12	0.07	0.04	0.04	0.03	0.02
$B_3 = 1/9(\omega^2 - 9)$	−0.013	−0.014	−0.014	−0.02	−0.19	0.18	0.02	0.01	0.01	0.004	0.002
$B_5 = 1/25(\omega^2 - 25)$	−0.002	−0.002	−0.002	−0.002	−0.002	−0.003	−0.004	−0.04	0.04	0.004	0.001

5. $y = C_1 \cos \omega t + C_2 \sin \omega t + \sum_{n=1}^{N} \dfrac{b_n}{\omega^2 - n^2} \sin nt$

7. $y = C_1 \cos \omega t + C_2 \sin \omega t + \dfrac{\pi^2}{12\omega^2} - \dfrac{1}{\omega^2 - 1} \cos t + \dfrac{1}{4(\omega^2 - 4)} \cos 2t - \cdots$

9. $y = -\dfrac{K}{c} \cos t$

11. $y = \dfrac{1 - n^2}{D} a_n \cos nt + \dfrac{nc}{D} a_n \sin nt, \quad D = (1 - n^2)^2 + n^2c^2$

13. $y = A_1 \cos t + B_1 \sin t + A_3 \cos 3t + B_3 \sin 3t + \cdots$
where $A_n = -ncb_n/D, \quad B_n = (1 - n^2)b_n/D, \quad D = (1 - n^2)^2 + n^2c^2,$
$b_1 = 1,\ b_2 = 0,\ b_3 = -\frac{1}{9},\ b_4 = 0,\ b_5 = \frac{1}{25}, \cdots$

15. $I = \sum_{n=1}^{\infty} (A_n \cos nt + B_n \sin nt),\ A_n = \dfrac{80(10 - n^2)}{\pi n^2 D_n},\ B_n = \dfrac{800}{n\pi D_n}$ (n odd),
$A_n = 0,\ B_n = 0$ (n even), $\quad D_n = (10 - n^2)^2 + 100\, n^2,$
$I = 1.266 \cos t + 1.406 \sin t + 0.003 \cos 3t + 0.094 \sin 3t + \cdots$

SECTION 10.8, page 494

1. $F = \dfrac{4}{\pi}\left[\sin x + \dfrac{1}{3} \sin 3x + \cdots + \dfrac{1}{N} \sin Nx\right]$ (N odd)

5. $F = \dfrac{\pi^2}{3} - 4\left(\cos x - \dfrac{1}{4} \cos 2x + \dfrac{1}{9} \cos 3x - \cdots + \dfrac{(-1)^{N+1}}{N^2} \cos Nx\right),$

$E^* = \dfrac{2\pi^5}{5} - \pi\left(\dfrac{2\pi^4}{9} + 16 + 1 + \dfrac{16}{81} + \dfrac{1}{16} + \cdots\right)$

SECTION 10.9, page 501

9. $\dfrac{2}{\pi} \int_0^{\infty} \left[\left(1 - \dfrac{2}{w^2}\right) \sin w + \dfrac{2}{w} \cos w\right] \dfrac{\cos wx}{w}\, dw$

11. $\dfrac{2}{\pi} \int_0^{\infty} \left[\dfrac{a \sin aw}{w} + \dfrac{\cos aw - 1}{w^2}\right] \cos xw\, dw$

13. $\dfrac{6}{\pi} \int_0^{\infty} \dfrac{2 + w^2}{4 + 5w^2 + w^4} \cos xw\, dw$

15. Differentiating (10) we have $\dfrac{d^2A}{dw^2} = -2 \int_0^{\infty} f^*(v) \cos wv\, dv,$
$f^*(v) = v^2 f(v)$, and the result follows.

SECTION 11.1, page 505

19. $u = c(x)e^{-y^2}$

21. $u = v(x) + w(y)$

23. $u = v(x)e^{-y} + w(y)$

25. $u = axy + bx + cy + k$

27. $u = cx + g(y)$

SECTION 11.3, page 514

1. $u = 0.02 \cos t \sin x$

3. $u = k(\cos t \sin x - \cos 2t \sin 2x)$

5. $u = \dfrac{4}{5\pi}\left(\dfrac{1}{4}\cos 2t \sin 2x - \dfrac{1}{36}\cos 6t \sin 6x + \dfrac{1}{100}\cos 10t \sin 10x - + \cdots\right)$

7. $u = \dfrac{8k}{\pi}\left(\cos t \sin x + \dfrac{1}{3^3}\cos 3t \sin 3x + \dfrac{1}{5^3}\cos 5t \sin 5x + \cdots\right)$

9. $u = 12k\left(\cos t \sin x - \dfrac{1}{2^3}\cos 2t \sin 2x + \dfrac{1}{3^3}\cos 3t \sin 3x - + \cdots\right)$

11. $u = \sum_{n=1}^{\infty} B_n^* \sin nx \sin nt, \quad B_n^* = \dfrac{0.04}{\pi n^3} \sin \dfrac{n\pi}{2}$

15. $27, 960/\pi^6 \approx 0.9986$

17. $u = ke^{c(x-y)}$

19. $u = ky^c e^{cx}$

21. $u = k \exp(x^2 + y^2 + c(x - y))$

23. $u = k \exp(cx + y/c)$

SECTION 11.4, page 517

9. $F_n = \sin(n\pi x/l)$, $G_n = a_n \cos(cn^2\pi^2 t/l^2)$

11. $u(0, t) = 0$, $u(l, t) = 0$, $u_x(0, t) = 0$, $u_x(l, t) = 0$

13. $\beta l \approx \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi, \cdots$ (more exactly 4.730, 7.853, 10.996, $\cdots$)

15. $\beta l \approx \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi, \cdots$ (more exactly 1.875, 4.694, 7.855, $\cdots$)

17. $u = f_1(x) + f_2(x + y)$

19. $u = xf_1(x + y) + f_2(x + y)$

21. $u = f_1(x + y) + f_2(2x - y)$

25. $y'^2 - 2y' + 1 = (y' - 1)^2 = 0, \quad y = x + c, \quad \Psi(x, y) = x - y, \quad v = x,$
$z = x - y$

SECTION 11.5, page 523

3. The solutions of the wave equation are periodic in t.

5. $u = \sin 0.1\pi x \, e^{-1.752\pi^2 t/100}$

7. $u = \dfrac{40}{\pi^2}\left(\sin 0.1\pi x \, e^{-0.01752\pi^2 t} - \dfrac{1}{9}\sin 0.3\pi x \, e^{-0.01752(3\pi)^2 t} + - \cdots\right)$

9. $u = \dfrac{800}{\pi^3}\left(\sin 0.1\pi x \, e^{-0.01752\pi^2 t} + \dfrac{1}{3^3}\sin 0.3\pi x \, e^{-0.01752(3\pi)^2 t} + \cdots\right)$

11. Since the temperatures at the ends are kept constant, the temperature will approach a steady-state (time-independent) distribution $u_I(x)$ as $t \to \infty$, and $u_I = U_1 + (U_2 - U_1)x/l$, the solution of (1) with $\partial u/\partial t = 0$, satisfying the boundary conditions.

15. $u = 1$

17. $u = 0.5 \cos 2x \, e^{-4t}$

19. $u = \frac{\pi}{4} - \frac{8}{\pi}\left(\frac{1}{4}\cos 2t\, e^{-4t} + \frac{1}{36}\cos 6t\, e^{-36t} + \cdots\right)$

21. $u = \frac{\pi}{8} + \left(1 - \frac{2}{\pi}\right)\cos x\, e^{-t} - \frac{1}{\pi}\cos 2x\, e^{-4t} - \left(\frac{1}{3} + \frac{2}{9\pi}\right)\cos 3x\, e^{-9t} + \cdots$

25. $w = e^{-\beta t}$

SECTION 11.6, page 528

7. $\frac{1}{\sqrt{\pi}}\int_{(a-x)/\tau}^{(b-x)/\tau} e^{-w^2}\, dw - \frac{1}{\sqrt{\pi}}\int_{(a+x)/\tau}^{(b+x)/\tau} e^{-w^2}\, dw$

SECTION 11.8, page 537

3. c increases and so does the frequency.

9. $B_{mn} = (-1)^{m+n}\frac{4ab}{mn\pi^2}$

11. $B_{mn} = 0$ (m or n even), $B_{mn} = \frac{64a^2b^2}{\pi^6 m^3 n^3}$ (m, n both odd)

13. $B_{mn} = (-1)^{m+n}\frac{144a^3b^3}{m^3n^3\pi^6}$

15. $u = k\cos \pi\sqrt{5}\,t \sin \pi x \sin 2\pi y$

17. $u = \frac{144k}{\pi^6}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{m+n}}{m^3n^3}\cos(\pi t\sqrt{m^2+n^2})\sin m\pi x \sin n\pi y$

21. $u = \frac{440}{\pi}\sum_{n=0}^{\infty}\frac{\sin \frac{1}{20}(2n+1)\pi x \sinh \frac{1}{40}(2n+1)\pi y}{(2n+1)\sinh(2n+1)\pi}$

23. $u_y(x, 0, t) = u_y(x, \pi, t) = 0$, $u = \sin mx \cos ny \exp(-c^2(m^2+n^2)t)$

25. $u = \frac{64}{\pi^2}\sum_{\substack{m=1 \\ m,n \text{ odd}}}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m^3n^3}\sin mx \sin ny\, e^{-c^2(m^2+n^2)t}$

SECTION 11.9, page 540

9. $5 + 5r^2\cos 2\theta$

11. $u = \frac{200}{\pi}\left(r\sin\theta + \frac{1}{3}r^3\sin 3\theta + \frac{1}{5}r^5\sin 5\theta + \cdots\right)$

13. $u = \frac{4u_0}{\pi}\left(\frac{r}{a}\sin\theta + \frac{1}{3a^3}r^3\sin 3\theta + \frac{1}{5a^5}r^5\sin 5\theta + \cdots\right)$

SECTION 11.10, page 545

9. $u = 4k\sum_{m=1}^{\infty}\frac{J_2(\alpha_m)}{\alpha_m{}^2 J_1{}^2(\alpha_m)}\cos\alpha_m t\, J_0(\alpha_m r)$

SECTION 11.11, page 549

3. $u = 160/r + 30$

5. $u = -40\ln r/(\ln 2) + 150$

17. $[(u_1 - u_0)\ln r + u_0\ln r_1 - u_1\ln r_0]/\ln(r_1/r_0)$

SECTION 11.12, page 553

3. $u = 1$

5. $u = -\frac{2}{3}r^2P_2(\cos\phi) + \frac{2}{3} = r^2(-\cos^2\phi + \frac{1}{3}) + \frac{2}{3}$

7. $\cos 2\phi = 2\cos^2\phi - 1$, $2x^2 - 1 = \frac{4}{3}P_2(x) - \frac{1}{3}$, $u = \frac{4}{3}r^2P_2(\cos\phi) - \frac{1}{3}$

15. $i_{xx} = LCi_{tt} + (RC + GL)i_t + RGi$

SECTION 11.13, page 557

5. $U(x, s) = \dfrac{c(s)}{x^s} + \dfrac{x}{s^2(s+1)}$, $U(0, s) = 0$, $c(s) = 0$,

$u(x, t) = x(t - 1 + e^{-t})$

9. Set $x^2/4c^2\tau = z^2$. Use z as a new variable of integration. Use $\operatorname{erf}(\infty) = 1$.

SECTION 12.1, page 564

3. $32 - 24i$

5. $-\frac{7}{41} + \frac{22}{41}i$

7. $-\frac{46}{13}$

9. $2xy/(x^2 + y^2)$

SECTION 12.2, page 567

1. 1

3. $\sqrt{(x+1)^2 + y^2}/\sqrt{(x-1)^2 + y^2}$

5. $(x^2 + y^2)^2$

7. $\frac{1}{25}$

9. $\sqrt{2}(\cos\pi/4 + i\sin\pi/4)$

11. $8(\cos\pi + i\sin\pi)$

13. π

15. $\pi/3$

19. (10) holds when $z_1 + z_2 = 0$. Let $z_1 + z_2 \neq 0$ and $c = a + ib = z_1/(z_1 + z_2)$. By (12), $|a| \leqq |c|$, $|a - 1| \leqq |c - 1|$. Thus $|a| + |a - 1| \leqq |c| + |c - 1|$. Clearly $|a| + |a - 1| \geqq 1$. Together we have the inequality below; multiply it by $|z_1 + z_2|$ to get (10).

$$1 \leqq |c| + |c - 1| = \left|\frac{z_1}{z_1 + z_2}\right| + \left|\frac{z_2}{z_1 + z_2}\right|$$

SECTION 12.3, page 570

3. The region between the two branches of the hyperbola $x^2 - y^2 = 1$.

5. Horizontal strip of width 2π

7. $(x - \frac{17}{15})^2 + y^2 = (\frac{8}{15})^2$

9. The left half-plane without the closed disk of radius $\frac{1}{2}$ and center at $-\frac{1}{2}$.

SECTION 12.4, page 574

1. $2 + 4i$, $-1 + 2i$, $20 - 20i$

3. $(9 - 13i)/500$, $-i$, $(-2 - 11i)/1000$

5. $2(x^3 - 3xy^2) - 3x$, $2(3x^2y - y^3) - 3y$

7. $|\arg w| \leqq 3\pi/4$

9. $|w| > 9$

11. $\operatorname{Re}(z^2)/|z^2| = (x^2 - y^2)/(x^2 + y^2) = 1$ if $y = 0$ and -1 if $x = 0$. Ans. No.

13. $6z(z^2 - 4)^2$

15. $2/(1 - z)^3$

17. $-\frac{1}{27}$

19. Use $\operatorname{Re} f(z) = [f(z) + \overline{f(z)}]/2$, $\operatorname{Im} f(z) = [f(z) - \overline{f(z)}]/2i$.