Problem Set 11.1, page 485

3. $2\pi/n$, $2\pi/n$, k, k, k/n, k/n

13. $\frac{1}{2} + \frac{2}{\pi}\left(\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - + \cdots\right)$

15. $\frac{\pi}{2} + \frac{4}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right)$

17. $\frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right)$
$+ \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - + \cdots$

19. $-\frac{4}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right)$
$+ 2\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right)$

21. $\frac{1}{3}\pi^2 - 4\left(\cos x - \frac{1}{4}\cos 2x + \frac{1}{9}\cos 3x - + \cdots\right)$

23. $\frac{1}{6}\pi^2 - \frac{4}{\pi}\cos x - \frac{1}{2}\cos 2x + \frac{4}{27\pi}\cos 3x + \frac{1}{8}\cos 4x - \cdots$

29. $f' = 2x$, $f'' = 2$, $j_1 = 0$, $j_1' = -4\pi$, $j_1'' = 0$, $a_n = \frac{1}{n\pi}\left(-\frac{1}{n}\right)(-4\pi)\cos n\pi$, etc.

Problem Set 11.2, page 490

1. $\frac{4}{\pi}\left(\sin\frac{\pi x}{2} + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{5}\sin\frac{5\pi x}{2} + \cdots\right)$

3. $\frac{1}{3} - \frac{4}{\pi^2}\left(\cos\pi x - \frac{1}{4}\cos 2\pi x + \frac{1}{9}\cos 3\pi x - + \cdots\right)$

5. Rectifier, $\frac{2}{\pi} - \frac{4}{\pi}\left(\frac{1}{1\cdot 3}\cos 2\pi x + \frac{1}{3\cdot 5}\cos 4\pi x + \frac{1}{5\cdot 7}\cos 6\pi x + \cdots\right)$

7. Rectifier, $\frac{1}{2} - \frac{4}{\pi^2}\left(\cos\pi x + \frac{1}{9}\cos 3\pi x + \frac{1}{25}\cos 5\pi x + \cdots\right)$

9. $\frac{2}{3} + \frac{4}{\pi^2}\left(\cos\pi x - \frac{1}{4}\cos 2\pi x + \frac{1}{9}\cos 3\pi x - \frac{1}{16}\cos 4\pi x + - \cdots\right)$

11. $\frac{3}{4} - \frac{4}{\pi^2}\left(\cos\frac{\pi x}{2} + \frac{1}{2}\cos\pi x + \frac{1}{9}\cos\frac{3\pi x}{2} + \frac{1}{25}\cos\frac{5\pi x}{2} + \frac{1}{18}\cos 3\pi x\right.$
$\left. + \cdots\right)$

13. $\frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

15. Translate by $\frac{1}{2}$.

17. Set $x = 0$.

Problem Set 11.3, page 496

1. Even, odd, neither, even, neither, odd

3. Odd

5. Neither

7. Odd

9. Odd

11. $\frac{\pi}{2} + \frac{4}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right)$

13. $\frac{4}{\pi}\left(\sin x - \frac{1}{9}\sin 3x + \frac{1}{25}\sin 5x - + \cdots\right)$

15. $1 - \frac{4}{\pi}\left(\sin\frac{\pi x}{2} + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{5}\sin\frac{5\pi x}{2} + \cdots\right)$

17. (a) 1, (b) $\frac{4}{\pi}\left(\sin\frac{\pi x}{2} + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{5}\sin\frac{5\pi x}{2} + \cdots\right)$

19. (a) $1 + \frac{8}{\pi^2}\left(\cos\frac{\pi x}{2} + \frac{1}{9}\cos\frac{3\pi x}{2} + \frac{1}{25}\cos\frac{5\pi x}{2} + \cdots\right)$

(b) $\frac{4}{\pi}\left(\sin\frac{\pi x}{2} + \frac{1}{2}\sin \pi x + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{4}\sin 2\pi x + \cdots\right)$

21. (a) $\frac{3}{2} - \frac{2}{\pi}\left(\cos\frac{\pi x}{2} - \frac{1}{3}\cos\frac{3\pi x}{2} + \frac{1}{5}\cos\frac{5\pi x}{2} - \frac{1}{7}\cos\frac{7\pi x}{2} + - \cdots\right)$

(b) $\frac{6}{\pi}\left(\sin\frac{\pi x}{2} - \frac{1}{3}\sin \pi x + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{5}\sin\frac{5\pi x}{2} - \frac{1}{9}\sin 3\pi x + \cdots\right)$

23. (a) $\frac{L}{2} - \frac{4L}{\pi^2}\left(\cos\frac{\pi x}{L} + \frac{1}{9}\cos\frac{3\pi x}{L} + \frac{1}{25}\cos\frac{5\pi x}{L} + \cdots\right)$

(b) $\frac{2L}{\pi}\left(\sin\frac{\pi x}{L} - \frac{1}{2}\sin\frac{2\pi x}{L} + \frac{1}{3}\sin\frac{3\pi x}{L} - + \cdots\right)$

25. (a) $\frac{\pi}{2} + \frac{4}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right)$

(b) $2\left(\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right)$

Problem Set 11.4, page 499

3. Use (5).

7. $-\frac{2i}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{2n+1}e^{(2n+1)ix}$

9. $i\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{(-1)^n}{n}e^{inx}$

11. $\frac{\pi^2}{3} + 2\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{(-1)^n}{n^2}e^{inx}$

13. $\pi + i\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{1}{n}e^{inx}$

Problem Set 11.5, page 501

3. $(0.05n)^2$ in D_n changes to $(0.02n)^2$, which gives $C_5 = 0.5100$, leaving the other coefficients almost unaffected.

5. $y = c_1 \cos \omega t + c_2 \sin \omega t + A(\omega) \cos t$, $A(\omega) = 1/(\omega^2 - 1) < 0$ if $\omega^2 < 1$ (phase shift!) and > 0 if $\omega^2 > 1$

7. $y = c_1 \cos \omega t + c_2 \sin \omega t + \sum_{n=1}^{N} \dfrac{a_n}{\omega^2 - n^2} \cos nt$

9. $y = c_1 \cos \omega t + c_2 \sin \omega t + \dfrac{\pi}{2\omega^2} + \dfrac{4}{\pi}\left(\dfrac{1}{\omega^2 - 1} \cos t + \dfrac{1/9}{\omega^2 - 9} \cos 3t + \cdots\right)$

11. $y = c_1 \cos \omega t + c_2 \sin \omega t + \dfrac{1}{2\omega^2} - \dfrac{1}{1 \cdot 3(\omega^2 - 4)} \cos 2t - \dfrac{1}{3 \cdot 5(\omega^2 - 16)} \cos 4t - \cdots$

13. The situation is the same as in Fig. 53 in Sec. 2.8.

15. $y = -\dfrac{3c}{64 + 9c^2} \cos 3t - \dfrac{8}{64 + 9c^2} \sin 3t$

17. $y = \sum_{n=1}^{N} \left(-\dfrac{ncb_n}{D_n} \cos nt + \dfrac{(1 - n^2)b_n}{D_n} \sin nt\right)$, $D_n = (1 - n^2)^2 + n^2c^2$

19. $I(t) = \sum_{n=1}^{\infty} (A_n \cos nt + B_n \sin nt)$, $A_n = (-1)^{n+1} \dfrac{240(10 - n^2)}{n^2 D_n}$,

$B_n = (-1)^{n+1} \dfrac{2400}{nD_n}$, $D_n = (10 - n^2)^2 + 100n^2$

Problem Set 11.6, page 505

1. $F = 2\left(\sin x - \dfrac{1}{2} \sin 2x + \cdots + \dfrac{(-1)^{N+1}}{N} \sin Nx\right)$, $E^* = 8.1, 5.0, 3.6, 2.8, 2.3$

3. $F = \dfrac{\pi}{2} - \dfrac{4}{\pi}\left(\cos x + \dfrac{1}{9} \cos 3x + \dfrac{1}{25} \cos 5x + \cdots\right)$, $E^* = 0.0748, 0.0748, 0.0119, 0.0119, 0.0037$

5. $F = \dfrac{2}{\pi} - \dfrac{4}{\pi}\left(\dfrac{1}{1 \cdot 3} \cos 2x + \dfrac{1}{3 \cdot 5} \cos 4x + \dfrac{1}{5 \cdot 7} \cos 6x + \cdots\right)$, $E^* = 0.5951, 0.0292, 0.0292, 0.0066, 0.0066$

7. $F = \dfrac{4}{\pi}\left(\sin x + \dfrac{1}{3} \sin 3x + \dfrac{1}{5} \sin 5x + \cdots\right)$, $E^* = 1.1902, 1.1902, 0.6243, 0.6243, 0.4206$ $(0.1272$ when $N = 20)$

9. $\dfrac{8}{\pi}\left(\sin x + \dfrac{1}{27} \sin 3x + \dfrac{1}{125} \sin 5x + \cdots\right)$, $E^* = 0.0295, 0.0295, 0.0015, 0.0015, 0.00023$

Problem Set 11.7, page 512

1. $f(x) = \pi e^{-x}$ $(x > 0)$ gives $A = \int_0^\infty e^{-v} \cos wv \, dv = \dfrac{1}{1 + w^2}$, $B = \dfrac{w}{1 + w^2}$ (see Example 3), etc.

3. $f(x) = \frac{1}{2}\pi e^{-x}$ gives $A = 1/(1 + w^2)$.

5. Use $f = (\pi/2) \cos v$ and (11) in App. 3.1 to get $A = (\cos (\pi w/2))/(1 - w^2)$.

7. $\dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\sin aw \cos xw}{w} \, dw$

9. $\dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\cos w + w \sin w - 1}{w^2} \cos xw \, dw$

11. $\dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\cos \pi w + 1}{1 - w^2} \cos xw \, dw$

15. $\dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\sin \pi w}{1 - w^2} \sin xw \, dw$

17. $\dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\pi w - \sin \pi w}{w^2} \sin xw \, dw$

19. $\dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{wa - \sin wa}{w^2} \sin xw \, dw$

Problem Set 11.8, page 517

1. $\sqrt{\dfrac{2}{\pi}}\left(\dfrac{\sin 2w - 2 \sin w}{w}\right)$

5. $\sqrt{\pi/2}\, e^{-x}$ $(x > 0)$

7. $\sqrt{\pi/2} \cos w$ if $0 < w < \pi/2$ and 0 if $w > \pi/2$

9. Yes, no

11. $\sqrt{(2/\pi)}\, w/(w^2 + \pi^2)$

13. $\mathcal{F}_s(xe^{-x^2/2}) = \mathcal{F}_s(-(e^{-x^2/2})') = w\mathcal{F}_c(e^{-x^2/2}) = we^{-w^2/2}$

17. $\mathcal{F}_c(f') = \mathcal{F}_c(-af) = -a\mathcal{F}_c(f) = -\sqrt{\dfrac{2}{\pi}}\dfrac{a^2}{a^2 + w^2} = w\mathcal{F}_s(f) - \sqrt{\dfrac{2}{\pi}} \cdot 1$,

$\mathcal{F}_s(f) = \sqrt{\dfrac{2}{\pi}}\dfrac{w}{a^2 + w^2}$

19. In (5) for $f(ax)$ set $ax = v$.

Problem Set 11.9, page 528

3. $ik(e^{-ibw} - 1)/(\sqrt{2\pi}w)$

5. $\sqrt{(2/\pi)}k (\sin w)/w$

7. $[(1 + iw)e^{-iw} - 1]/(\sqrt{2\pi}w^2)$

9. $\sqrt{(2/\pi)}i(\cos w - 1)/w$

11. $\frac{1}{2}e^{-w^2/2}$

13. $(e^{ibw} - e^{-ibw})/(iw\sqrt{2\pi}) = \sqrt{2/\pi}(\sin bw)/w$

Chapter 11 Review Questions and Problems, page 532

11. $\dfrac{4k}{\pi}\left(\sin \pi x + \dfrac{1}{3}\sin 3\pi x + \dfrac{1}{5}\sin 5\pi x + \cdots\right)$

13. $4\left(\sin\dfrac{x}{2} - \dfrac{1}{2}\sin x + \dfrac{1}{3}\sin\dfrac{3x}{2} - \dfrac{1}{4}\sin 4x + \dfrac{1}{5}\sin\dfrac{5x}{2} - + \cdots\right)$

15. $\dfrac{8}{\pi^2}\left(\sin\dfrac{\pi x}{2} - \dfrac{1}{9}\sin\dfrac{3\pi x}{2} + \dfrac{1}{25}\sin\dfrac{5\pi x}{2} - + \cdots\right)$

17. $\dfrac{2}{\pi} - \dfrac{4}{\pi}\left(\dfrac{1}{1\cdot 3}\cos 16\pi x + \dfrac{1}{3\cdot 5}\cos 32\pi x + \dfrac{1}{5\cdot 7}\cos 48\pi x + \cdots\right)$

19. $\dfrac{\pi^2}{12} - \cos 2x + \dfrac{1}{4}\cos 4x - \dfrac{1}{9}\cos 6x + \dfrac{1}{16}\cos 8x - + \cdots$

21. $\pi/4$ by Prob. 11 **23.** $\pi^2/8$ by Prob. 15

25. $\dfrac{1}{2}[f(x) + f(-x)],\ \dfrac{1}{2}[f(x) - f(-x)]$

27. $\pi - \dfrac{8}{\pi}\left(\cos\dfrac{x}{2} + \dfrac{1}{9}\cos\dfrac{3x}{2} + \dfrac{1}{25}\cos\dfrac{5x}{2} + \cdots\right)$

29. 8.105, 4.963, 3.567, 2.781, 2.279, 1.929, 1.673, 1.477

31. $y = C_1\cos\omega t + C_2\sin\omega t + \dfrac{\pi^2}{3\omega^2} - 4\left(\dfrac{\cos t}{\omega^2 - 1} - \dfrac{1}{4}\cdot\dfrac{\cos 2t}{\omega^2 - 4} + \dfrac{1}{9}\cdot\dfrac{\cos 3t}{\omega^2 - 9}\right.$
$\left. - \dfrac{1}{16}\cdot\dfrac{\cos 4t}{\omega^2 - 16} + - \cdots\right)$

33. $\dfrac{1}{\pi}\displaystyle\int_0^\infty \dfrac{(\cos w + w\sin w - 1)\cos wx + (\sin w - w\cos w)\sin wx}{w^2}\,dw$

35. $\dfrac{2}{\pi}\displaystyle\int_0^\infty \dfrac{w - \sin w\cos w}{w^2}\sin wx\,dw$

37. $\dfrac{4}{\pi}\displaystyle\int_0^\infty \dfrac{\sin 2w - 2w\cos 2w}{w^3}\cos wx\,dw$ **39.** $\sqrt{\dfrac{8}{\pi}}\cdot\dfrac{1}{w^2 + 4}$

Problem Set 12.1, page 537

1. $u = c_1(x)\cos 4y + c_2(x)\sin 4y$
3. $u = c_1(x) + c_2(x)y$
5. $u = c(x)e^{-y} + e^{xy}/(x + 1)$
7. $u = c(x)\exp(\frac{1}{2}y^2\cosh x)$
9. $u = c_1(x)y + c_2(x)y^{-2}$
11. $u = c(x)e^y + h(y)$
15. $c = 1/4$
17. Any c
19. $\pi/4$
21. Any c and ω
27. $u = 110 - (110/\ln 100)\ln(x^2 + y^2)$
29. $u = c_1x + c_2(y)$

Problem Set 12.3, page 546

1. $k\cos 2\pi t\sin 2\pi x$

3. $\dfrac{8k}{\pi^3}\left(\cos\pi t\sin\pi x + \dfrac{1}{27}\cos 3\pi t\sin 3\pi x + \dfrac{1}{125}\cos 5\pi t\sin 5\pi x + \cdots\right)$

5. $\dfrac{4}{5\pi^2}\left(\cos\pi t\sin\pi x - \dfrac{1}{9}\cos 3\pi t\sin 3\pi x + \dfrac{1}{25}\cos 5\pi t\sin 5\pi x - + \cdots\right)$

7. $\dfrac{2}{\pi^2}\left((\sqrt{2} - 1)\cos\pi t\sin\pi x + \dfrac{1}{2}\cos 2\pi t\sin 2\pi x\right.$
$\left. + \dfrac{1}{9}(\sqrt{2} + 1)\cos 3\pi t\sin 3\pi x - \cdots\right)$

9. $\frac{2}{\pi^2}\left((2-\sqrt{2})\cos \pi t \sin \pi x - \frac{1}{9}(2+\sqrt{2})\cos 3\pi t \sin 3\pi x + \frac{1}{25}(2+\sqrt{2})\cos 5\pi t \sin 5\pi x + \cdots\right)$

17. $u = \frac{8L^2}{\pi^3}\left(\cos\left[c\left(\frac{\pi}{L}\right)^2 t\right]\sin\frac{\pi x}{L} + \frac{1}{3^3}\cos\left[c\left(\frac{3\pi}{L}\right)^2 t\right]\sin\frac{3\pi x}{L} + \cdots\right)$

19. (a) $u(0, t) = 0$, (b) $u(L, t) = 0$, (c) $u_x(0, t) = 0$, (d) $u_x(L, t) = 0$. $C = -A$, $D = -B$ from (a), (b). Insert this. The coefficient determinant resulting from (c), (d) must be zero to have a nontrivial solution. This gives (22).

Problem Set 12.4, page 552

3. $c^2 = 300/[0.9/(2 \cdot 9.80)] = 80.83^2$ [$\mathrm{m^2/sec^2}$]

11. Hyperbolic, $u = f_1(x) + f_2(x + y)$

13. Elliptic, $u = f_1(y + 3ix) + f_2(y - 3ix)$

15. Parabolic, $u = xf_1(x - y) + f_2(x - y)$

17. Parabolic, $u = xf_1(2x + y) + f_2(2x + y)$

19. Hyperbolic, $u = (1/y)f_1(xy) + f_2(y)$

Problem Set 12.5, page 560

5. $u = \sin 0.4\pi x\, e^{-1.752 \cdot 16\pi^2 t/100}$

7. $u = \frac{2}{\pi}\left(\frac{2}{\pi}\sin 0.1\pi x\, e^{-0.01752\pi^2 t} + \frac{1}{2}\sin 0.2\pi x\, e^{-0.01752(2\pi)^2 t} - \cdots\right)$

9. $u = \frac{20\sqrt{2}}{\pi^2}\left(\sin 0.1\pi x\, e^{-0.01752\pi^2 t} + \frac{1}{9}\sin 0.3\pi x\, e^{-0.01752(3\pi)^2 t} - - \cdots\right)$

11. $u = u_{\mathrm{I}} + u_{\mathrm{II}}$, where $u_{\mathrm{II}} = u - u_{\mathrm{I}}$ satisfies the boundary conditions of the text, so that $u_{\mathrm{II}} = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L}\, e^{-(cn\pi/L)^2 t}$, $B_n = \frac{2}{L}\int_0^L [f(x) - u_{\mathrm{I}}(x)]\sin\frac{n\pi x}{L}\,dx$

13. $F = A\cos px + B\sin px$, $F'(0) = Bp = 0$, $B = 0$, $F'(L) = -Ap\sin pL = 0$, $p = n\pi/L$, etc.

15. $u = 1$

17. $u = \frac{2\pi^2}{3} + 4\left(\cos x\, e^{-t} - \frac{1}{4}\cos 2x\, e^{-4t} + \frac{1}{9}\cos 3x\, e^{-9t} - + \cdots\right)$

19. $u = \frac{\pi^2}{12} + \cos 2x\, e^{-4t} + \frac{1}{4}\cos 4x\, e^{-16t} + \frac{1}{9}\cos 6x\, e^{-36t} + \cdots$

23. $-\frac{K\pi}{L}\sum_{n=1}^{\infty} nB_n e^{-\lambda_n^2 t}$

25. $w = e^{-\beta t}$

27. $v_t - c^2 v_{xx} = 0$, $w'' = -Ne^{-\alpha x}/c^2$, $w = \frac{N}{c^2\alpha^2}\left[-e^{-\alpha x} - \frac{1}{L}(1 - e^{-\alpha L})x + 1\right]$, so that $w(0) = w(L) = 0$.

29. $u = (\sin \frac{1}{2}\pi x \sinh \frac{1}{2}\pi y)/\sinh \pi$

31. $u = \frac{80}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)\sinh(2n-1)\pi}\sin\frac{(2n-1)\pi x}{24}\sinh\frac{(2n-1)\pi y}{24}$

33. $u = A_0 x + \sum_{n=1}^{\infty} A_n \dfrac{\sinh (n\pi x/24)}{\sinh n\pi} \cos \dfrac{n\pi y}{24}$,

$A_0 = \dfrac{1}{24^2} \int_0^{24} f(y)\, dy, \quad A_n = \dfrac{1}{12} \int_0^{24} f(y) \cos \dfrac{n\pi y}{24}\, dy$

35. $\sum_{n=1}^{\infty} A_n \sin \dfrac{n\pi x}{a} \sinh \dfrac{n\pi(b-y)}{a}, \ A_n = \dfrac{2}{a \sinh (n\pi b/a)} \int_0^a f(x) \sin \dfrac{n\pi x}{a}\, dx$

Problem Set 12.6, page 568

1. $A = \dfrac{2 \sin ap}{\pi p}, \ B = 0, \ u = \dfrac{2}{\pi} \int_0^{\infty} \dfrac{\sin ap}{p} \cos px\, e^{-c^2p^2t}\, dp$

3. $A = e^{-p}, \ B = 0, \ u = \int_0^{\infty} \cos px\, e^{-p-c^2p^2t}\, dp$

5. Set $\pi v = s$. $A = 1$ if $0 < p/\pi < 1$, $B = 0$, $u = \int_0^{\pi} \cos px\, e^{-c^2p^2t}\, dp$

7. $A = 2[\cos p + p \sin p - 1)/(\pi p^2)]$, $B = 0$, $u = \int_0^{\infty} A \cos px\, e^{-c^2p^2t}\, dp$

Problem Set 12.8, page 578

1. (a), (b) It is multiplied by $\sqrt{2}$. (c) Half
3. $B_{mn} = 16/(mn\pi^2)$ if m, n odd, 0 otherwise
5. $B_{mn} = (-1)^{n+1}8/(mn\pi^2)$ if m odd, 0 if m even
7. $B_{mn} = (-1)^{m+n}4/(mn\pi^2)$
11. $k \cos \sqrt{29}\, t \sin 2x \sin 5y$
13. $\dfrac{6.4}{\pi^2} \sum_{\substack{m=1 \\ m,\, n \text{ odd}}}^{\infty} \sum_{n=1}^{\infty} \dfrac{1}{m^3n^3} \cos (t\sqrt{m^2+n^2}) \sin mx \sin ny$

17. $c\pi\sqrt{260}$ (corresponding eigenfunctions $F_{4,16}$ and $F_{16,14}$), etc.
19. $B_{mn} = 0$ (m or n even), $B_{mn} = 16k/(mn\pi^2)$ (m, n odd)
21. $B_{mn} = (-1)^{m+n}144a^3b^3/(m^3n^3\pi^6)$
23. $\cos \left(\pi t \sqrt{\dfrac{9}{a^2} + \dfrac{16}{b^2}}\right) \sin \dfrac{3\pi x}{a} \sin \dfrac{4\pi y}{b}$

Problem Set 12.9, page 585

7. $30r \cos \theta + 10r^3 \cos 3\theta$

9. $55 + \dfrac{220}{\pi}\left(r \cos \theta - \dfrac{1}{3} r^3 \cos 3\theta + \dfrac{1}{5} r^5 \cos 5\theta - + \cdots\right)$

11. $\dfrac{\pi}{2} - \dfrac{4}{\pi}\left(r \cos \theta + \dfrac{1}{9} r^3 \cos 3\theta + \dfrac{1}{25} r^5 \cos 5\theta + \cdots\right)$

15. Solve the problem in the disk $r < a$ subject to u_0 (given) on the upper semicircle and $-u_0$ on the lower semicircle.

$u = \dfrac{4u_0}{\pi}\left(\dfrac{r}{a} \sin \theta + \dfrac{1}{3a^3} r^3 \sin 3\theta + \dfrac{1}{5a^5} r^5 \sin 5\theta + \cdots\right)$

17. Increase by a factor $\sqrt{2}$

19. $T = 6.826\rho R^2 f_1^{\,2}$

21. No

23. Differentiation brings in a factor $1/\lambda_m = R/(c\alpha_m)$.

Problem Set 12.10, page 593

11. $v = F(r)G(t)$, $F'' + k^2F = 0$, $\dot{G} + c^2k^2G = 0$, $F_n = \sin(n\pi r/R)$,

$$G_n = B_n \exp(-c^2n^2\pi^2 t/R^2),\ B_n = \frac{2}{R}\int_0^R rf(r)\sin\frac{n\pi r}{R}\,dr$$

13. $u = 100$

15. $u = \frac{8}{5}r^3P_3(\cos\phi) - \frac{3}{5}rP_1(\cos\phi)$

17. $64r^4P_4(\cos\phi)$

21. Analog of Example 1 in the text with 55 replaced by 50

23. $v = r(\cos\theta)/r^2 = x/(x^2 + y^2)$, $v = xy/(x^2 + y^2)^2$

Problem Set 12.11, page 596

5. $W = \dfrac{c(s)}{x^s} + \dfrac{x}{s^2(s+1)}$, $W(0, s) = 0$, $c(s) = 0$, $w(x, t) = x(t - 1 + e^{-t})$

7. $w = f(x)g(t)$, $xf'g + f\dot{g} = xt$, take $f(x) = x$ to get $g = ce^{-t} + t - 1$ and $c = 1$ from $w(x, 0) = x(c - 1) = 0$.

9. Set $x^2/(4c^2\tau) = z^2$. Use z as a new variable of integration. Use $\text{erf}(\infty) = 1$.

Chapter 12 Review Questions and Problems, page 597

19. $u = c_1(y)e^{x} + c_2(y)e^{-2x}$

21. $u = g(x)(1 - e^{-y}) + f(x)$

23. $u = \cos t \sin x - \frac{1}{2}\cos 2t \sin 2x$

25. $u = \frac{3}{4}\cos t \sin x - \frac{1}{4}\cos 3t \sin 3x$

27. $u = \sin(0.02\pi x)\, e^{-0.004572t}$

29. $u = \dfrac{200}{\pi^2}\left(\sin\dfrac{\pi x}{50}\, e^{-0.004572t} - \dfrac{1}{9}\sin\dfrac{3\pi x}{50}\, e^{-0.04115t} + \cdots\right)$

31. $u = 100\cos 4x\, e^{-16t}$

33. $u = \dfrac{\pi}{2} - \dfrac{16}{\pi}\left(\dfrac{1}{4}\cos 2x\, e^{-4t} + \dfrac{1}{36}\cos 6x\, e^{-36t} + \dfrac{1}{100}\cos 10x\, e^{-100t} + \cdots\right)$

37. $u = f_1(y) + f_2(x + y)$ **39.** $u = f_1(y - 2ix) + f_2(y + 2ix)$

41. $u = xf_1(y - x) + f_2(y - x)$

49. $u = (u_1 - u_0)(\ln r)/\ln(r_1/r_0) + (u_0\ln r_1 - u_1\ln r_0)/\ln(r_1/r_0)$

Problem Set 13.1, page 606

5. $x - iy = -(x + iy)$, $x = 0$

7. 484

9. $-5/169$

11. $-7/13 - (22/13)i$

13. $-273 + 136i$

15. $-7/17 - (11/17)i$

17. $x/(x^2 + y^2)$

19. $(x^2 - y^2)/(x^2 + y^2)^2$

Problem Set 13.2, page 611

1. $3\sqrt{2}(\cos(-\frac{1}{4}\pi) + i\sin(-\frac{1}{4}\pi))$

3. $5(\cos\pi + i\sin\pi) = 5\cos\pi$

5. $\cos\frac{1}{2}\pi + i\sin\frac{1}{2}\pi$

7. $\frac{1}{3}\sqrt{61}\,(\cos \arctan \frac{6}{5} + i \sin \arctan \frac{6}{5})$ **9.** $-3\pi/4$
11. $\arctan(\pm 3/4)$ **13.** $\pm\pi/4$ **15.** $3\pi/4$
17. $2.94020 + 0.59601i$ **19.** $0.54030 - 0.84147i$
21. $\cos(-\frac{1}{4}\pi) + i \sin(-\frac{1}{4}\pi)$, $\cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi$
23. $\pm(1 \pm i)/\sqrt{2}$ **25.** -1, $\cos \frac{1}{5}\pi \pm i \sin \frac{1}{5}\pi$, $\cos \frac{3}{5}\pi \pm i \sin \frac{3}{5}\pi$
27. $4 + 3i$, $4 - 8i$ **29.** $\frac{5}{2} - i$, $2 + \frac{1}{4}i$
35. $|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)} = (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$. Multiply out and use $\text{Re}\, z_1\bar{z}_2 \leqq |z_1\bar{z}_2|$ (Prob. 32):
$z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2 = |z_1|^2 + 2\,\text{Re}\, z_1\bar{z}_2 + |z_2|^2 \leqq |z_1|^2 + 2|z_1||z_2| + |z_2|^2$
$= (|z_1| + |z_2|)^2$.
Take the square root to get (6).

Problem Set 13.3, page 617

1. Circle of radius $\frac{4}{3}$, center $3 + 2i$
3. Set obtained from an open disk of radius 1 by omitting its center $z = 1$
5. Hyperbola $xy = 1$ **7.** y-axis
9. The region above $y = x$
13. $f = 1 - 1/(z + 1) = 1 - (x + 1 - iy)/[(x + 1)^2 + y^2]$; $0.9 - 0.1i$
15. $(x^2 - y^2 - 2ixy)/(x^2 + y^2)^2$, $-i/2$ **17.** Yes since $r^2(\sin 2\theta)/r \to 0$
19. Yes **21.** $6z^2(z^3 + i)$
23. $2i(1 - z)^{-3}$

Problem Set 13.4, page 623

1. Yes **3.** No **5.** Yes
7. No **9.** Yes for $z \neq 0$
11. $r_x = x/r = \cos\theta$, $r_y = \sin\theta$, $\theta_x = -(\sin\theta)/r$, $\theta_y = (\cos\theta)/r$,
(a) $0 = u_x - v_y = u_r \cos\theta + u_\theta(-\sin\theta)/r - v_r \sin\theta - v_\theta(\cos\theta)/r$.
(b) $0 = u_y + v_x = u_r \sin\theta + u_\theta(\cos\theta)/r + v_r \cos\theta + v_\theta(-\sin\theta)/r$.
Multiply (a) by $\cos\theta$, (b) by $\sin\theta$, and add. Etc.
13. $z^2/2$ **15.** $\ln|z| + i\,\text{Arg}\, z$ **17.** z^3
19. No **21.** No **23.** $c = 1$, $\cos x \sinh y$
27. Use (4), (5), and (1).

Problem Set 13.5, page 626

3. $-1.13120 + 2.47173i$, $e = 2.71828$ **5.** $-i$, 1
7. $e^{0.8}(\cos 5 - i \sin 5)$, 2.22554 **9.** $e^{-2x}\cos 2y$, $-e^{-2x}\sin 2y$
11. $\exp(x^2 - y^2)\cos 2xy$, $\exp(x^2 - y^2)\sin 2xy$
13. $e^{i\pi/4}$, $e^{5\pi i/4}$
15. $\sqrt[n]{r}\,\exp[i(\theta + 2k\pi)/n]$, $k = 0, \cdots, n - 1$
17. $9e^{\pi i}$ **19.** $z = \ln 2 + \pi i + 2n\pi i$ $(n = 0, \pm 1, \cdots)$
21. $z = \ln 5 - \arctan \frac{3}{4}i \pm 2n\pi i$ $(n = 0, 1, \cdots)$

Problem Set 13.6, page 629

3. Use (11), then (5) for e^{iy}, and simplify. **5.** Use (11) and simplify.
7. $\cos 1 \cosh 1 - i \sin 1 \sinh 1 = 0.83373 - 0.98890i$