COMP30026 Models of Computation Assignment 1

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Challenge 1

a.

P	Q	R	$((\neg P \wedge Q) \Rightarrow \neg R) \Rightarrow R$	R
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	0	0
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

$\therefore ((\neg P \wedge Q) \Rightarrow \neg R) \Rightarrow R \equiv R$

b.

P	Q	R	$(P \Rightarrow (Q \vee R)) \wedge (P \Leftrightarrow Q)$	$P \Leftrightarrow Q$
0	0	0	1	1
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

$\therefore (P \Rightarrow (Q \vee R)) \wedge (P \Leftrightarrow Q) \equiv P \Leftrightarrow Q$

c.

P	Q	$P \Rightarrow (P \Leftrightarrow Q)$	$P \Rightarrow Q$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

$\therefore P \Rightarrow (P \Leftrightarrow Q) \equiv P \Rightarrow Q$

d.

P	Q	R	$P \vee (R \Rightarrow Q)$	$(Q \Rightarrow P) \Rightarrow (R \Rightarrow P)$
0	0	0	1	1
0	0	1	0	0
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

$\therefore P \vee (R \Rightarrow Q) \equiv (Q \Rightarrow P) \Rightarrow (R \Rightarrow P)$

Challenge 2

Firstly, let us define both inhabitant's statements in propositional logic. Inhabitant P's statements are equivalent to $\neg Q$ and Q', respectively. Inhabitant Q's statements are equivalent to P and $\neg P'$, respectively.

We are given information that the statements given by the inhabitants P and Q respectively must be either true or false, but *cannot* be a mixture of both. In other words, if an inhabitant lies, then every statement they say is guaranteed to be a false statement.

This means that if inhabitant P is telling the truth, then $\neg Q \wedge Q'$ is true, otherwise $Q \wedge \neg Q'$ must be true. Similarly, if inhabitant Q is telling the truth, then $P \wedge \neg P'$ must be true, otherwise $\neg P \wedge P'$ must be true.

Let A mean P *is telling the truth*, and B mean Q is *telling the truth*. We can surmise when P and Q are lying or telling the truth via truth tables.

P	P'	A
0	0	1
0	1	0
1	0	0
1	1	1

Q	Q'	B
0	0	1
0	1	0
1	0	0
1	1	1

It can be seen that P is truthful when $P \Leftrightarrow P'$ is true, and lies when $P \oplus P'$ is true. The same notions hold for inhabitant Q, with respect to literals Q and Q'.

Now, we can extract the maximal amount of information provided by the two inhabitants, in the form of propositional formulas (1) and (2).

$$((P \Leftrightarrow P') \wedge (\neg Q \wedge Q')) \oplus ((P \oplus P') \wedge (Q \wedge \neg Q')) \tag{1}$$

$$((Q \Leftrightarrow Q') \wedge (P \wedge \neg P')) \oplus ((Q \oplus Q') \wedge (\neg P \wedge P')) \tag{2}$$

Using a truth table, we can determine for each of P and Q whether they are a knight or a knave, and whether they are healthy or sick.

P	P'	Q	Q'	$((P \Leftrightarrow P') \wedge (\neg Q \wedge Q')) \oplus ((P \oplus P') \wedge (Q \wedge \neg Q'))$	$((Q \Leftrightarrow Q') \wedge (P \wedge \neg P')) \oplus ((Q \oplus Q') \wedge (\neg P \wedge P'))$
0	0	0	0	0	0
0	0	0	1	1	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	0	1
0	1	1	0	1	1
0	1	1	1	0	0
1	0	0	0	0	1
1	0	0	1	0	0
1	0	1	0	1	0
1	0	1	1	0	1
1	1	0	0	0	0
1	1	0	1	1	0
1	1	1	0	0	0
1	1	1	1	0	0

We know that both formulas must be true. Therefore, there is only *one* possible scenario. That is, P is a *healthy knave* and Q is a *sick knight*.

Challenge 3

a. The two formulas are not equivalent. For example, consider the interpretation I with domain $D = \{0,1\}$ and $P(0)$. Additionally, let Q always be *false*.

   For this interpretation, it can be seen that $I \vDash \forall x(P(x)) \Rightarrow Q$, but $I \nvDash \forall x(P(x) \Rightarrow Q)$. Hence, $\forall x(P(x)) \Rightarrow Q \nvDash \forall x(P(x) \Rightarrow Q)$.

   Therefore, $\forall x(P(x)) \Rightarrow Q$ is *not* equivalent to $\forall x(P(x) \Rightarrow Q)$.

Challenge 4

a. $\forall x \forall y(M(x, y) \Rightarrow F(x))$

b. $\forall x \forall y((U(x) \wedge M(y, x)) \Rightarrow U(y))$

c. $\forall x((F(x) \wedge U(x) \wedge B(x)) \Rightarrow D(x))$

d. $\forall x \forall y((U(x) \wedge M(x, y) \wedge D(x)) \Rightarrow D(y))$

e. $\forall x((S(x) \wedge U(x)) \Rightarrow B(x))$

The *Horn clauses* generated are as follows:

a. $\{\neg M(x, y), F(x)\}$

b. $\{\neg U(x), \neg M(y, x), U(y)\}$

c. $\{\neg F(x), \neg U(x), \neg B(x), D(x)\}$

d. $\{\neg U(x), \neg M(x), \neg D(x), D(y)\}$

e. $\{\neg S(x), \neg U(x), B(x)\}$

Challenge 5

a. (a1) $\forall x \forall y (N(x,y) \Rightarrow N(y,x))$

(a2) $\forall x \forall y \exists u ((M(u,x) \wedge \neg M(u,y)) \Rightarrow N(x,y))$

(a3) $\forall x (\neg \exists u (M(u,x)) \Rightarrow E(x))$

(a4) $\forall x \forall y \forall u ((D(x,y) \wedge M(u,x)) \Rightarrow \neg M(u,y))$

Challenge 6

See Grok.