

Exercise 1 Let $g(x) = -x^2$ and $h(x) = 2|x - 1| - 1$.

(a) Calculate the following limits.

(i) $\lim_{x \rightarrow 1} g(x) = \boxed{-1}$

(ii) $\lim_{x \rightarrow 1} h(x) = \boxed{-1}$

(iii) $\lim_{x \rightarrow 1} \frac{g(x)}{h(x)} = \boxed{1}$

(iv) $\lim_{x \rightarrow 1} \frac{h(x) - h(4)}{x - 4} = \boxed{2}$

(b) Find the expression for $h(x)$, if $x < 1$.

$$h(x) = \boxed{-2}x + \boxed{1}, \text{ if } x < 1.$$

(c) Find the expression for $h(x)$, if $x \geq 1$.

$$h(x) = \boxed{2}x - 3, \text{ if } x \geq 1.$$

(d) Make the correct choice.

The given inequality, $g(x) \leq h(x)$, is (true ✓/ not true) for all values of x .

(e) Let f be a function such that $g(x) \leq f(x) \leq h(x)$ for all $0 < x < 2$.

Then $\lim_{x \rightarrow 1} f(x) = \boxed{-1}$, by (limit laws/ continuity/ difference law/ quotient law/ the Squeeze Theorem ✓).
