Exercise 1 *Complete the following table. Use* ***exact*** *values.*

x	$\frac{1}{3}$	$\frac{-1}{30}$	$\frac{1}{300}$	$\frac{-1}{301}$	$\frac{1}{1000}$	$\frac{1}{1001}$	$\frac{1}{50000}$	$\frac{-1}{100004}$
$\frac{\pi}{x}$	3π	-30π	300π	-301π	1000π	1001π	50000π	-100004π
$\cos\left(\frac{\pi}{x}\right)$	$\boxed{-1}$	$\boxed{1}$	$\boxed{1}$	$\boxed{-1}$	$\boxed{1}$	$\boxed{-1}$	$\boxed{1}$	$\boxed{1}$

Question 1.1 *Based on the table above, make a conjecture about* $\lim_{x\to 0}\cos\left(\frac{\pi}{x}\right)$. *Does it exist? Explain.*

Free Response: *It seems like the limit does not exist.* $\cos\left(\frac{\pi}{x}\right)$ *doesn't seem to approach 1, -1, or any value in between even when* x *is very close to 0. It seems to oscillate in between.*

Exercise 1.2 *Complete the following table using* ***exact*** *values.*

x	$\frac{1}{3}$	$\frac{-1}{301}$	$\frac{1}{1000}$	$\frac{1}{1001}$	$\frac{1}{50000}$	$\frac{-1}{100004}$
$\lvert x\rvert\cos\left(\frac{\pi}{x}\right)$	$\boxed{\frac{-1}{3}}$	$\boxed{\frac{-1}{301}}$	$\boxed{\frac{1}{1000}}$	$\boxed{\frac{-1}{1001}}$	$\boxed{\frac{1}{50000}}$	$\boxed{\frac{1}{100004}}$

Question 1.2.1 *Based on the table above, make a conjecture about* $\lim_{x\to 0}\lvert x\rvert\cos\left(\frac{\pi}{x}\right)$. *Does it exist? Explain.*

Free Response: *We see that as* x *gets small the value of our function gets very close to 0. We conjecture then that the value of the limit is 0.*

Exercise 1.2.2 *Can you use the product limit law in order to evaluate* $\lim_{x\to 0}\lvert x\rvert\cos\left(\frac{\pi}{x}\right)$*?*

Multiple Choice:

(a) *Yes*

(b) *No* ✓

Question 1.2.2.1 *Explain why you can or cannot use the product limit law to evaluate the above limit.*

Free Response: *In order to use the product limit law we need for both* $\lim_{x\to 0}|x|$ *and* $\lim_{x\to 0}\cos\left(\frac{\pi}{x}\right)$ *to exist. However we can conjecture based on the first part of the problem that the second of these two limits does not exist, therefore we cannot use the product limit law.*

Exercise 1.2.2.2 *Use the Squeeze Theorem to determine* $\lim_{x\to 0}|x|\cos\left(\frac{\pi}{x}\right)$ *by performing the following steps.*

The following inequalities are true for all x.

$$-1 \leq \cos(x) \leq 1.$$

(a) *Multiple all sides by* $|x|$.

$$\boxed{-|x|} \leq |x|\cos\left(\frac{\pi}{x}\right) \leq \boxed{|x|}.$$

(b) *Since* $\lim_{x\to 0}|x| = \boxed{0} = \lim_{x\to 0}(-|x|)$, *it follows by the Squeeze Theorem that* $\lim_{x\to 0}|x|\cos\left(\frac{\pi}{x}\right) = \boxed{0}$.