Exercise 1 *Evaluate the limit and justify your answer.*

(a)

$$\lim_{x\to\frac{\pi}{2}} e^{\sin x} = \boxed{e}$$

Justification:

Notice that $e^{\sin x} = f(g(x))$, *where*

$$f(x) = \boxed{e^x} \quad \textit{and} \quad g(x) = \boxed{\sin x}$$

and both functions, f *and* g, *are (not continuous / continuous ✓) on* $(-\infty, \infty)$.
So, we can apply (Composition ✓/ Product) limit law

$$\lim_{x\to\frac{\pi}{2}} e^{\sin x} = e^{\lim\limits_{x\to\frac{\pi}{2}} \boxed{\sin x}}$$

Since g *is (not continuous/ continuous ✓) at* $\frac{\pi}{2}$, *we have*

$$\lim_{x\to\frac{\pi}{2}} \sin x = \sin\left(\boxed{\frac{\pi}{2}}\right) = \boxed{1}$$

and, therefore

$$\lim_{x\to\frac{\pi}{2}} e^{\sin x} = e^{\boxed{1}} = \boxed{e}$$

(b)

$$\lim_{x\to 2} \cos\left(\pi\frac{3x^2-4x+1}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$$

Justification:
Notice that $\cos\left(\pi\frac{3x^2-4x+1}{4}\right) = f(g(x))$, *where*

$$f(x) = \boxed{\cos x} \quad \textit{and} \quad g(x) = \boxed{\pi\frac{3x^2-4x+1}{4}}$$

and both functions, f *and* g, *are (not continuous / continuous ✓) on* $(-\infty, \infty)$.
So, we can apply (Composition ✓/ Product) limit law

$$\lim_{x\to 2} \cos\left(\pi\frac{3x^2-4x+1}{4}\right) = \cos\left(\lim_{x\to 2} \boxed{\pi\frac{3x^2-4x+1}{4}}\right)$$

Since g is (not continuous/ continuous ✓) at 2, we have

$$\lim_{x\to 2} \pi \frac{3x^2-4x+1}{4} = \pi \frac{3\boxed{2}^2 - 4\boxed{2}+1}{4} = \boxed{\frac{5\pi}{4}}$$

and, therefore

$$\lim_{x\to 2} \cos\left(\pi \frac{3x^2-4x+1}{4}\right) = \cos\left(\boxed{\frac{5\pi}{4}}\right) = \boxed{-\frac{\sqrt{2}}{2}}$$

(c)

$$\lim_{x\to e^3} (\ln x+5)^2 = \boxed{64}$$

Justification: Notice that $(\ln x+5)^2 = f(g(x))$, *where*

$$f(x) = \boxed{x^2} \quad \textit{and} \quad g(x) = \boxed{\ln x+5}$$

The function f is (not continuous/ continuous ✓) on $(-\infty,\infty)$.
The function g is a (product/ sum ✓) of two continuous functions, $\ln x$ *and* $\boxed{5}$. *Therefore, g is (not continuous/ continuous ✓) on its domain* $(\boxed{0},\boxed{\infty})$.
So, we can apply (Composition ✓/ Product) limit law

$$\lim_{x\to e^3} (\ln x+5)^2 = \left(\lim_{x\to e^3} (\ln x+5)\right)^{\boxed{2}}$$

Since g is (not continuous/ continuous ✓) at e^3, we have

$$\lim_{x\to e^3} (\ln x+5) = \ln\left(\boxed{e^3}\right)+5 = \boxed{3}+5 = \boxed{8}$$

and, therefore

$$\lim_{x\to e^3} (\ln x+5)^2 = \left(\boxed{8}\right)^2 = \boxed{64}$$
