Break-Ground:

Stars and functions

Two young mathematicians discuss stars and functions.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley, did you know I like looking at the stars at night?

Riley: Stars are freaking awesome balls of nuclear fire whose light took thousands of years to reach us.

Devyn: I know! But did you know that the best way to see a very dim star is to look **near** it but **not exactly at** it? It's because then you can use the "rods" in your eye, which work better in low light than the "cones" in your eyes.

Riley: That's amazing! Hey, that reminds me of when we were talking about the two functions

$$f(x) = \frac{x^2 - 3x + 2}{x - 2} \qquad \text{and} \qquad g(x) = x - 1,$$

which we now know are completely different functions.

Devyn: Whoa. How are you seeing a connection here?

Riley: If we want to understand what is happening with the function

$$f(x) = \frac{x^2 - 3x + 2}{x - 2},$$

at $x = 2$, we can't do it by setting $x = 2$. Instead we need to look **near** $x = 2$ but **not exactly at** $x = 2$.

Devyn: Ah ha! Because if we are **not exactly at** $x = 2$, then

$$\frac{x^2 - 3x + 2}{x - 2} = x - 1.$$

Problem 1 *Let* $f(x) = \dfrac{x^2 - 3x + 2}{x - 2}$ *and* $g(x) = x - 1$*. Which of the following is true?*

Learning outcomes: Consider values of a function at inputs approaching a given point.
Author(s):

Multiple Choice:

(a) $f(x) = g(x)$ *for every value of* x.

(b) *There is no* x*-value where* $f(x) = g(x)$.

(c) $f(x) = g(x)$ *when* $x \neq 2$. ✓

Problem 2 *When you evaluate*

$$f(x) = \frac{x^2 - 3x + 2}{x - 2},$$

at x*-values approaching (but not equal to)* 2*, what happens to the value of* $f(x)$*? The value of* $f(x)$ *approaches* $\boxed{1}$.

Problem 2.1 *Just from checking some values, can you be absolutely certain that your answer to the previous problem is correct?*

Multiple Choice:

(a) *yes*

(b) *no* ✓

Feedback(*`attempt`*): *Here you only have information about a few specific points on the graph. There are infinitely many* x*-values close to, but not equal to,* $x = 2$*. Hence we cannot be completely certain.*
