Break-Ground:

Equal or not?

Here we see a dialogue where students discuss combining limits with arithmetic.

Check out this dialogue between two calculus students (based on a true story):

Devyn: Riley, I've been thinking about limits.

Riley: So awesome!

Devyn: Think about

$$\lim_{x\to a} \left(f(x)+g(x)\right).$$

This is the number that $f(x)+g(x)$ gets nearer and nearer to, as x gets nearer and nearer to a.

Riley: You know it!

Devyn: So I think it is the same as

$$\lim_{x\to a} f(x) + \lim_{x\to a} g(x).$$

Riley: Yeah, that does make sense, since when you add two numbers, say

$$\text{(a number near 6)} + \text{(a number near 7)}$$

you get

$$\text{(a number near 13)}$$

Riley: Right! And I think the same reasoning will work for multiplication! So we should be able to say

$$\lim_{x\to a} \left(f(x)\cdot g(x)\right) = \left(\lim_{x\to a} f(x)\right)\cdot\left(\lim_{x\to a} g(x)\right).$$

Devyn: Yes, I think that's right! But what about *division*? Can we use similar reasoning to conclude

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}.$$

Learning outcomes:
Author(s):

Problem 1 *Give an argument (similar to the one above) supporting the idea that*

$$\lim_{x\to a} (f(x) \cdot g(x)) = \left(\lim_{x\to a} f(x)\right) \cdot \left(\lim_{x\to a} g(x)\right).$$

Free Response:

For the next problems, suppose L is a number near 1 and that M is a number near 0.

Problem 2 *Using the context above,*

$$\frac{\textit{large}}{\textit{small}} = ?$$

Multiple Choice:

(a) *"large"* ✓

(b) *"small"*

(c) *impossible to say*

Problem 3 *Using the context above,*

$$\frac{\textit{small}}{\textit{small}} = ?$$

Multiple Choice:

(a) *"large"*

(b) *"small"*

(c) *impossible to say* ✓