

# MF - first results

## 1 Population structure

We fix the following population parameters:

$$N = 70000$$

$$N_a = 10000$$

$$N_b = 10000$$

$$N_c = 10000$$

$$N_{ab} = 10000$$

$$N_{ac} = 10000$$

$$N_{bc} = 10000$$

$$N_{abc} = 10000$$

Then simulate the study variable  $Y$  following Lohr and Rao 2005.

$$Y_{ik} = \mu_k + \epsilon_{ik}$$

where  $k$  represents the domain,  $\mu_k$  is the average of domain  $k$  and  $\epsilon_{ik} \sim N(0, \sigma_k)$

### Scenario 1

$$[\mu_a, \mu_b, \mu_c, \mu_{ab}, \mu_{bc}, \mu_{ac}, \mu_{abc}] = [15, 15, 15, 15, 15, 15, 15]$$

$$[\sigma_a, \sigma_b, \sigma_c, \sigma_{ab}, \sigma_{bc}, \sigma_{ac}, \sigma_{abc}] = [1, 1, 1, 1, 1, 1, 1]$$

### Scenario 2

Tabella 1: Frames parameters

| frame | Mean | St. Dev. | CV   |
|-------|------|----------|------|
| A     | 15   | 1        | 0.07 |
| B     | 15   | 1        | 0.07 |
| C     | 15   | 1        | 0.07 |

Tabella 2: Frames parameters

| frame | Mean | St. Dev. | CV   |
|-------|------|----------|------|
| A     | 15   | 2.9      | 0.19 |
| B     | 15   | 2.9      | 0.19 |
| C     | 15   | 4        | 0.27 |

Tabella 3: Frames parameters

| frame | Mean | St. Dev. | CV   |
|-------|------|----------|------|
| A     | 15   | 2.9      | 0.19 |
| B     | 15   | 2.9      | 0.19 |
| C     | 15   | 1        | 0.07 |

$$[\mu_a, \mu_b, \mu_c, \mu_{ab}, \mu_{bc}, \mu_{ac}, \mu_{abc}] = [15, 15, 15, 15, 15, 15, 15]$$

$$[\sigma_a, \sigma_b, \sigma_c, \sigma_{ab}, \sigma_{bc}, \sigma_{ac}, \sigma_{abc}] = [1, 1, 4, 1, 4, 4, 4]$$

### Scenario 3

$$[\mu_a, \mu_b, \mu_c, \mu_{ab}, \mu_{bc}, \mu_{ac}, \mu_{abc}] = [15, 15, 15, 15, 15, 15, 15]$$

$$[\sigma_a, \sigma_b, \sigma_c, \sigma_{ab}, \sigma_{bc}, \sigma_{ac}, \sigma_{abc}] = [4, 4, 1, 4, 1, 1, 1]$$

## 2 Sampling design

Simple Random Sampling without replacement

$$n = 600; f_A = f_B = f_C = 0.005 = \frac{200}{40000}$$

Sampling sequence: A -i B -i C

**Screener design:** 1) without reallocation of  $n$ ; 2)  $n$  reallocated in the cheapest frame;  
3)  $n$  reallocated in frame B and then C

**Overlap design**

## 3 Results

Considero  $n_A = 100$ ,  $n_B = 200$ ,  $n_C = 300$  and Scenario 2:

Tabella 4: Number of discarded units

| frame | Min.   | Median | Mean | Max. |
|-------|--------|--------|------|------|
| A     | 125.00 | 150.00 | 150  | 172  |
| B     | 77.0   | 100.0  | 100  | 126  |

Tabella 5: Empirical Mean Squard error and Average cost

| Scenario | Empirical Mean Squared Error |          |          |          | Average cost |          |          |        |
|----------|------------------------------|----------|----------|----------|--------------|----------|----------|--------|
|          | Y_M                          | Y_str_1  | Y_str_2  | Y_str3   | Y_M          | Y_str_1  | Y_str_2  | Y_str3 |
| 1        | 19044.56                     | 3733.124 | 3073.348 | 3046.242 | 2000         | 919.5383 | 1256.292 | 1170.4 |
| 2        | 20833.18                     | 11513    | 8438.607 | 7805.059 | 2000         | 919.5383 | 1256.292 | 1170.4 |
| 3        | 20959.44                     | 10316.19 | 9307.697 | 9830.041 | 2000         | 919.5383 | 1256.292 | 1170.4 |

Tabella 6: Empirical Mean Squard error and Average cost

| Scenario | Empirical Mean Squared Error |          |          |          |          |
|----------|------------------------------|----------|----------|----------|----------|
|          | Y_M                          | $Y_{KA}$ | Y_str_1  | Y_str_2  | Y_str3   |
| 2        | 21472.71                     | 24152.71 | 9575.046 | 8119.957 | 7560.884 |