Variance of a Mixture of distributions

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Formula

Obtained from the Stack Exchange question: What is the variance of the weighted mixture of two gaussians? (click on the question)

Formula

Let X_i with $i = 1, ..., n$ be independent random variables each with mean μ_i and variance σ_i^2.

Therefore the variance of $Y = \sum_{i=1}^{n} p_i X_i$, with $\sum_i p_i = 1$ and$\forall p_i > 0$ will be

$$Var(Y) = \sum_{i=1}^{n} p_i \sigma_i^2 + \left[\sum_{i=1}^{n} p_i \mu_i^2 - \left(\sum_{i=1}^{n} p_i \mu_i \right)^2 \right]$$

Y is basically a weighted linear combination of independent random variables.

Toy example

```
# Domain order:
# a - b - c - ab - bc - ac - abc
n_doms <- c(rep(1e3, 3), 1e2, 2e3, 4e2, 1e2) # Population of each domain
mu_s <- c(rep(5, 3), 4, 3, 1, 1) # Means
sd_s <- c(rep(9, 3), 4, 5, 6, 6) # Standard deviations
frame_sd (pop_d = n_doms, mu_domains = mu_s, sd_domains = sd_s)
```

```
## [[1]]
## [1] 8.13725
##
## [[2]]
## [1] 6.604257
##
## [[3]]
## [1] 6.648615
```