Report 3

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The aim of this report is to compare the speed of convergence to the result of the Black Scholes formula, for pricing an American Call, of a simple Binomial Model and of the Leisen Reimer method. The parameters for our analysis are:

$$S = 100;\ \ K = 100;\ \ t = 1 \text{ Year};\ \ r = 0.01;\ \ \sigma = 0.2;$$

In this particular case there is no dividend yield.
The first thing to do is compute the Black Scholes formula for this parameters:

$$price^{call}(S,T) = \mathrm{N}(\mathrm{d}_1)\mathrm{S} - \mathrm{N}(\mathrm{d}_2)\mathrm{K}\mathrm{e}^{-rt} \tag{1}$$

$$\mathrm{d}_1 = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r + \frac{\sigma^2}{2}\right)\right] \tag{2}$$

$$\mathrm{d}_2 = \frac{1}{\sigma\sqrt{\mathrm{t}}}\left[\ln\left(\frac{S}{K}\right) + t\left(r - \frac{\sigma^2}{2}\right)\right] \tag{3}$$

Replacing the parameters inside the formulas we obtain the following result:

$$B\&S : 8.433319$$

Now I have to analyze the convergence of a simple Binomial model and the Leisen Reimer methods to this value.

Both methods depends on the number of steps, so to analyze the convergence I can fix the above parameters and just increase them. There is one important restriction with Leisen-Reimer method and is that the number of steps must be odd, in practice this means that during the next analysis all even numbers of steps are transformed into the previous (odd) ones. In the table below I reported the estimation of the call price using the Binomial model and using the Leisen-Reimer method respect to the number of steps, also I reported the error respect to the target value. From the table is clear that the Leisen-Reimer method converges much faster, in fact after just six steps it gets a lower error (0,0029) compare to the one of the Binomial model after 200 steps (0,0081). Moreover the Leisen-Reimer method, after 200 steps, reaches an error in the order of 10^{-6}.

Steps	Binomial model	Leisen-Reimer	Error-BM	Error-LR
2	7,7697	8,4032	0,6635	0,0300
4	8,1336	8,4209	0,2996	0,0123
6	8,2562	8,4265	0,1770	0,0067
10	8,3499	8,4303	0,0833	0,0029
20	8,4122	8,4324	0,0210	0,0008
50	8,4395	8,4331	-0,0061	0,00015
76	8,4425	8,4332	-0,0092	6,654E-05
100	8,4430	8,4332	-0,0097	3,886E-05
200	8,4414	8,4333	-0,0081	9,899E-06

In order to highlight better the convergence of these two methods I reported in the plots below the estimation respect to the *B&S* value (8.433319). In these plots the green line is the estimation of the Binomial model, the blu one is the estimation of the Leisen-Reimer method while the dashed red line is the *B&S* value. First I consider a "*transition*" phase of 25 steps (Fig. 1,Fig. 2). From these figures we can notice that the Leisen-Reimer method converges quite fast to the *B&S* value while the Binomial model has a strong oscillatory behaviour. After I analyzed the behaviour at the "*end*" of my analysis, in fact I consider a maximum number of iterations of 200. From the plots below (in Fig. 3,Fig. 4) we can notice that Leisen-Reimer method has a nice convergence, in fact the difference from the *B&S* value is in the order of 10^{-5} and is still decreasing. Instead, in the case of the Binomial model, the estimation are very oscillatory, but are still good since the error are in the order of 10^{-3}. Of course if we had to decide between one of these two, the Leisen-Reimer method is the one to choose, in fact it converge a lot faster and more accurately to the BS value.

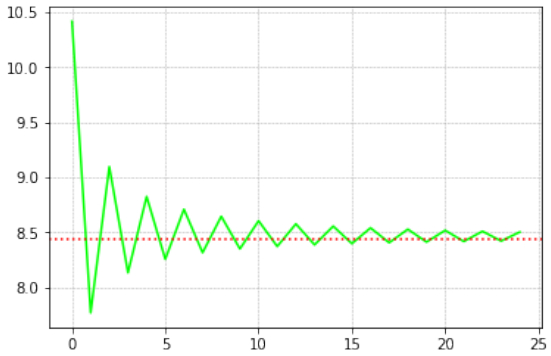

Figure 1: Binomial model in the first 25 steps

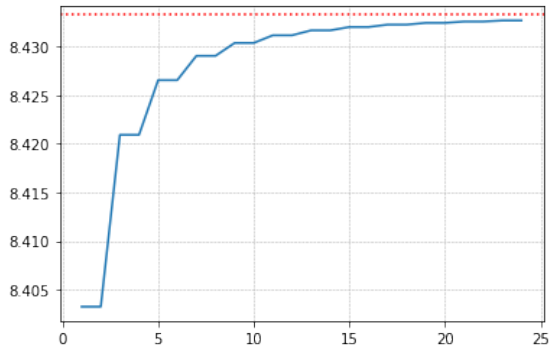

Figure 2: Leisen-Reimer method in the first 25 steps

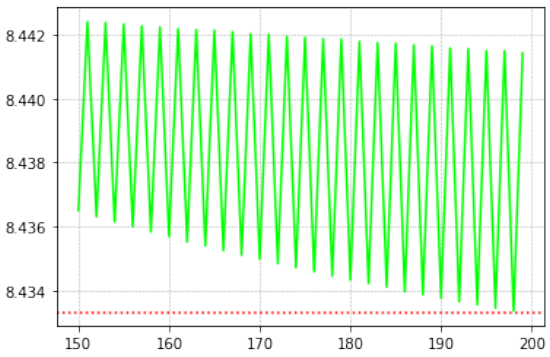

Figure 3: Binomial model from 150 to 200 steps

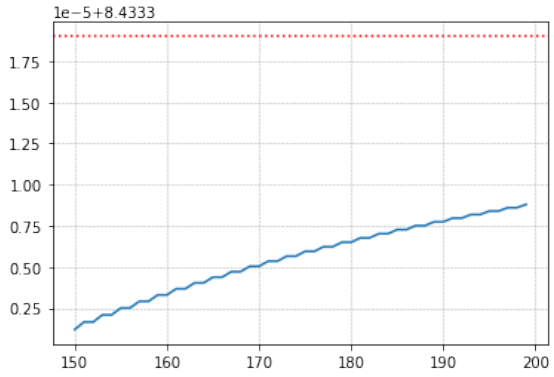

Figure 4: Leisen-Reimer method from 150 to 200 steps