Mincato Emanuele ID: 2019044 Date: 07/04/2022

Synopsys is an American electronic design automation company that focuses on silicon design and verification, silicon intellectual property and software security and quality. Their technology is present in self-driving cars, artificial intelligence, and internet of things consumer products.Synopsys was founded by Aart J de Geus and David Gregory in 1986 in Research Triangle Park, North Carolina. The company was initially established as Optimal Solutions with a charter to develop and market synthesis technology developed by the team at General Electric.

The Market Cap is equal to 49.60B while the Enterprise Value is equal to 51.39B. Currently (07/04/2022-14:00) the price of the asset underlying is 340.15$, while at the open was 323.30$ and at the previous close was 336.97$. This company does not pay dividends.

1. Part one

In the first part I have to check the behaviour of the "*Greeks*", each Greeks letter measures a different dimension to the risk in an option position and, in general, we have to manage them so that all risks are acceptable. For this analysis we consider the following parameters:

$$K = 100; \quad r = 0.01; \;\; \sigma = 0.2;$$

Moreover I considered a range for S from 60 to 140 and I chose as time to maturity, τ, the following values: 0.1, 0.2, 0.3, 0.5, 0.75, 1, 2, 3, 4, 5. After that I considered the effect of a volatility shock ($\pm$50%) on the *Greeks*, in fact I recomputed them considering a σ value of 0.1 and 0.3.

1.1. Delta

Delta (Δ) represents the rate of change between the option's price and the in the underlying asset's price. In other words, the price sensitivity of the option is relative to the underlying asset. The delta of a call option has a range between 0 and 1.

$$\Delta = \frac{\partial f}{\partial S}$$

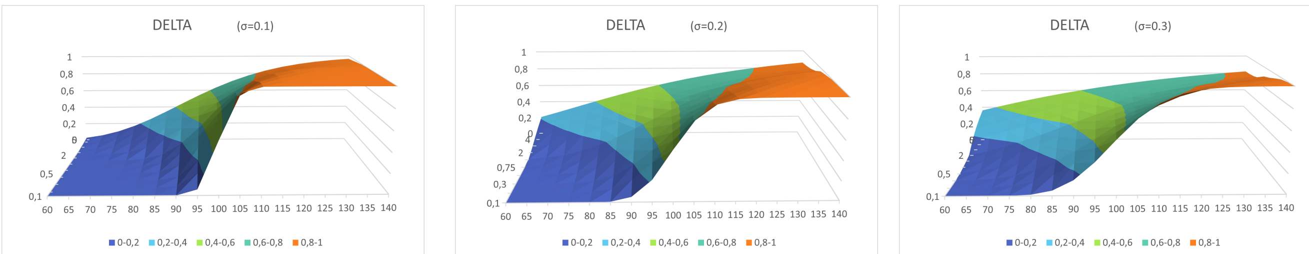


Figure 1: *Deltas* computed for 3 different volatility. Respectively σ=0.1, σ=0.2, σ=0.3

When we move away from maturity, the delta has a much smoother shape. The further we are from maturity, the flatter the curve appears. The delta converges faster and faster to its intrinsic value as time to maturity goes to zero. If the volatility increases the delta function becomes more smoother, while if the volatility decreases we get the opposite effect so we get a much more sharp function.

1.2. Gamma

Gamma (Γ) represents the rate of change between an option's delta and the underlying asset's price. This is called second-order (second-derivative) price sensitivity.

$$\Gamma = \frac{\partial^2 \Pi}{\partial S^2}$$

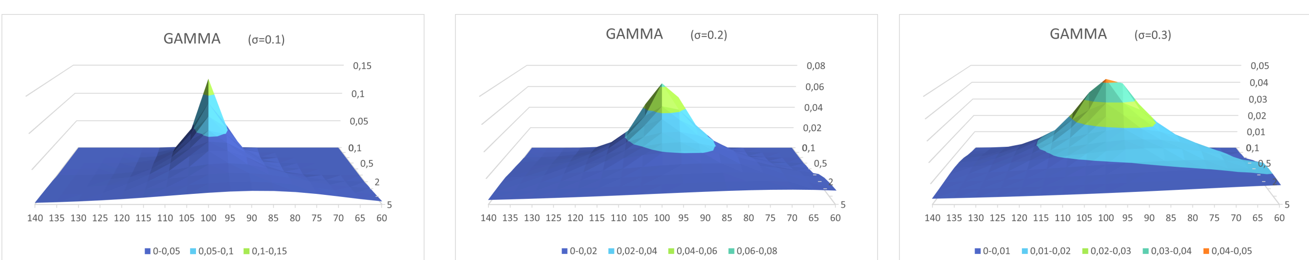


Figure 2: *Gammas* computed for 3 different volatility. Respectively σ=0.1,σ=0.2,σ=0.3

We have seen that delta get smoother for larger time to maturity. So we can expect a smoother gamma for a longer maturity, in fact we can see, from the above plot, that the gamma is almost flat for very far time to maturity, even if we change S. In this case if we decrease the volatility we get a sharper peak, this means that the delta function changes much faster around the strike and hardly changes when the value is far from it.

1.3. Vega

Vega (V) represents the rate of change between an option's value and the underlying asset's implied volatility. This is the option's sensitivity to volatility. When Vega is highly positive or highly negative, there is a high sensitivity to changes in volatility. If the Vega of an option position is close to zero, volatility changes have very little effect on the value of the position.

$$V = \frac{\partial f}{\partial \sigma}$$

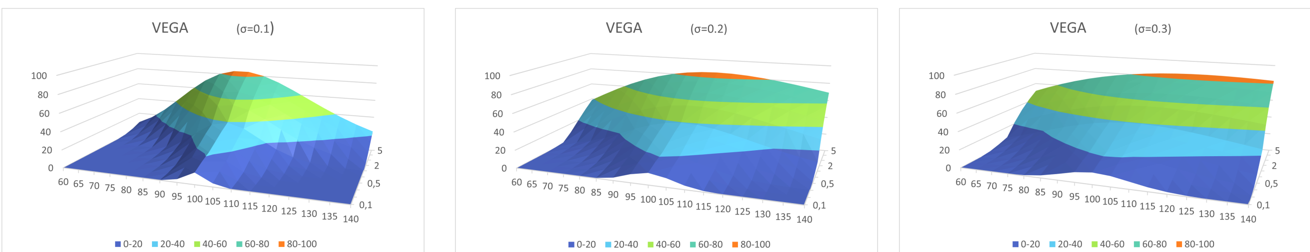


Figure 3: *Vegas* computed for 3 different volatility. Respectively σ=0.1, σ=0.2, σ=0.3

A higher volatility results in a wider vega curve, meaning that the vega function is more flat for a fixed time to maturity. As before this due to the fact that the ATM region becomes bigger as the volatility increases.

1.4. Theta

Theta (Θ) represents the rate of change between the option price and time, or time sensitivity, sometimes known as an option's time decay. Theta is usually negative for an option. This is because, as time passes with all else remaining the same, the option tends to become less valuable. The higher the volatility, the higher the option price and the time value, the more time value to lose, this imply that as the volatility increases also the absolute value of theta increases. In fact the values of theta with K=100 and τ = 1 Year are :

$$\sigma\text{=0.1: -2,487} \qquad \sigma\text{=0.2: -4,420} \qquad \sigma\text{=0.3: -6,333}$$

Furthermore the higher the volatility the wider is the ATM region, this means that the function assume "stronger" negative values on a wider area.

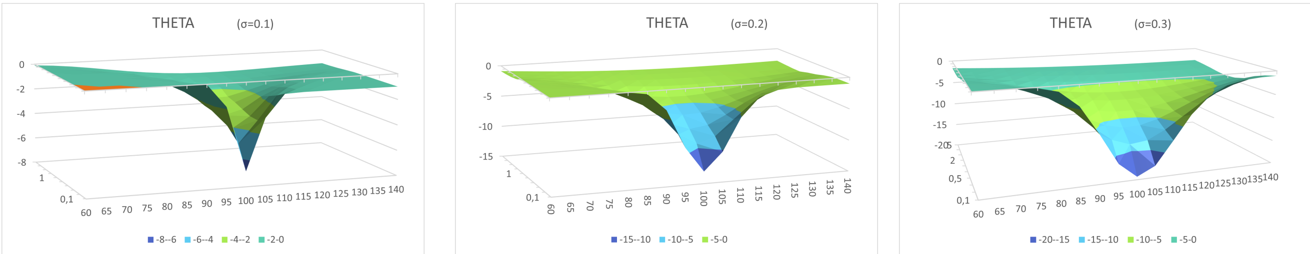


Figure 4: *Thetas* computed for 3 different volatility. Respectively σ=0.1, σ=0.2, σ=0.3

1.5. Rho

Rho (ρ) represents the rate of change between an option's value and a change in the interest rate.

$$\Delta = \frac{\partial f}{\partial r}$$

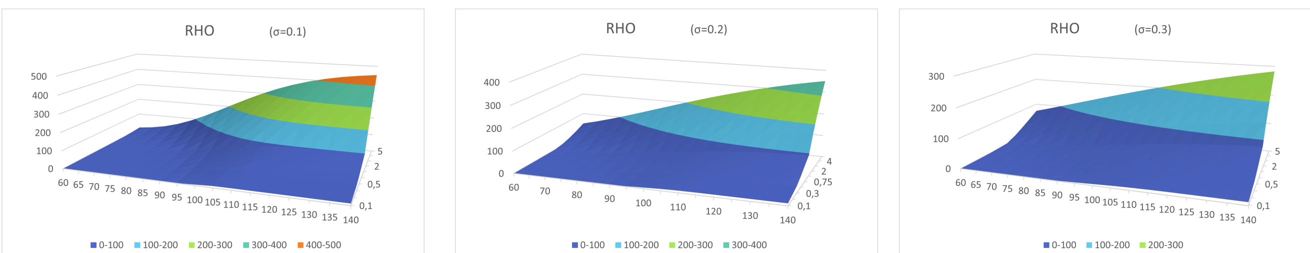


Figure 5: *Rhos* computed for 3 different volatility. Respectively σ=0.1, σ=0.2, σ=0.3

Rho increases as time to maturity increases. This imply that long-dated options are more sensitive to changes in interest rates than short-dated options. In this case there are not huge changes in the function related to the volatility shock, from the plots we can see that the function has slightly larger values when volatility decreases.

2. Part two

In this part I have to compute the implied volatility for a particular asset which does not pay dividends. I chose Synopsys, which I explained briefly above. The first thing to do is to chose different maturity time, *T*, for the option calls. I consider four different cases: May 20, 2022 (1 month), August 19, 2022 (4 months), January 20, 2023 (9 months) and January 19, 2024 (20 months). In order to find the implied volatility for a fixed maturity I used an iterative methods, the Newton-Raphson method:

$$\sigma_{n+1} = \sigma_n - \frac{V_{mkt} - V_{BS}(\sigma_n)}{\frac{\partial V_{BS}(\sigma_n)}{\partial \sigma}}$$

- V_{mkt} is the market price of the option.
- V_{BS} is the option price given by the Black-Scholes equation.
- σ is the volatility.

I ran the algorithm for 5000 iterations and I obtained the following results:

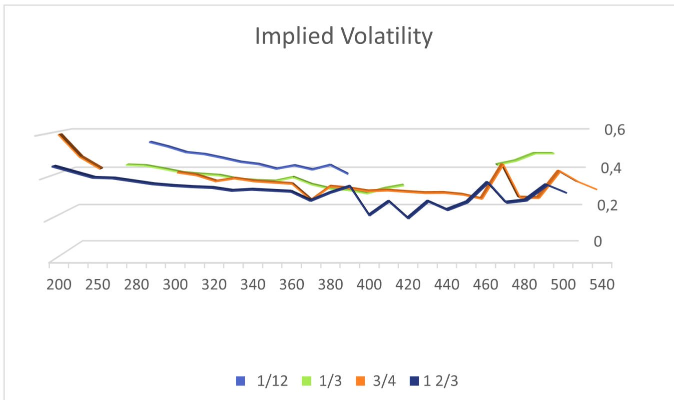
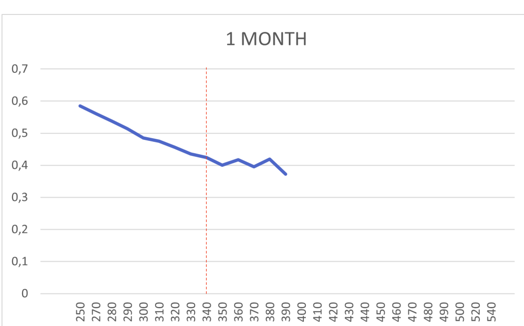
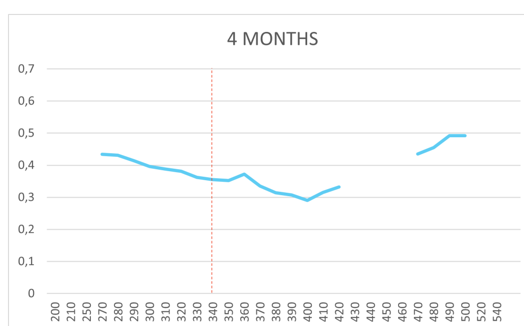
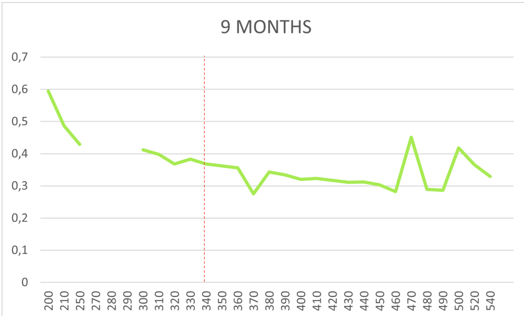


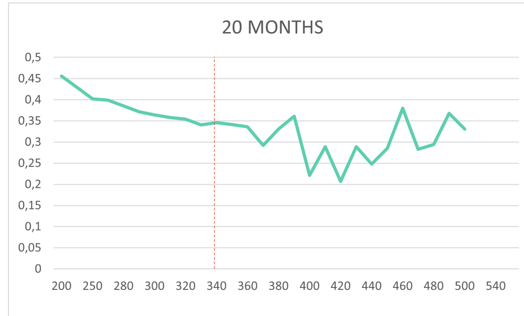
Figure 6: Implied volatility for different time maturity

The next thing to do is check for the presence of smile effects in the implied volatilities. The volatility smile is a pattern of implied volatility for a series of options that have the same underlying, the same expiration date, but different strike prices. The more an option is ITM or OTM, the greater its implied volatility becomes, this means that the implied volatility tend to be the lowest in the ATM region and should be almost flat around it. The volatility smile is not predicted by the Black-Scholes model, which consider a constant values for the volatility. To see the presence of the smile effect I plot separately the implied volatilities for each maturity, moreover I add a red vertical line that represent the current value of the underlying, S=340.15.

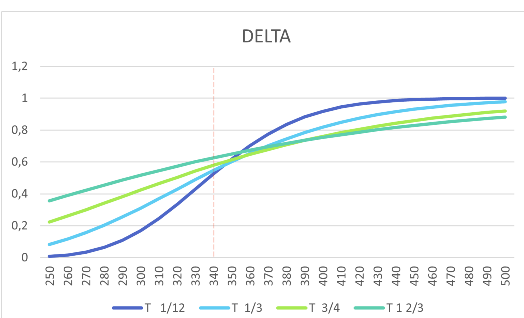


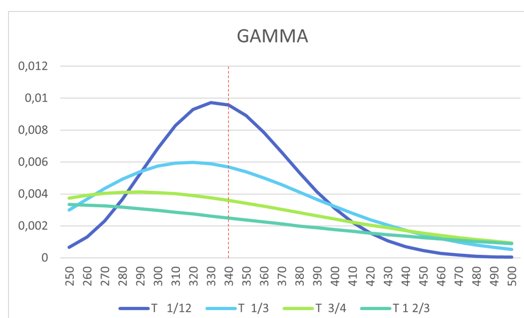


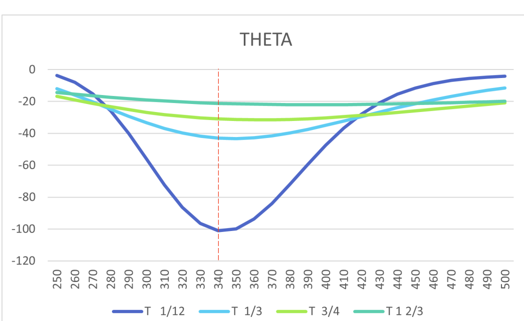


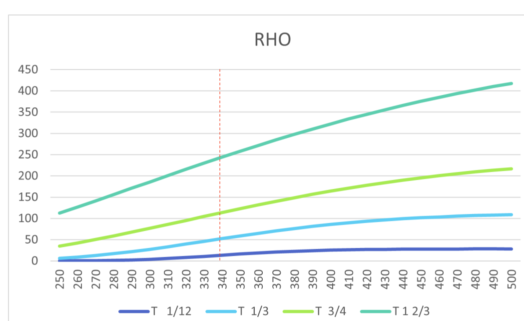


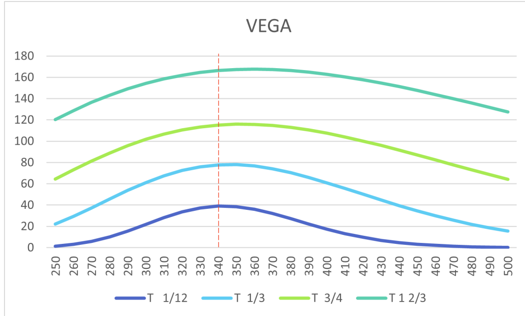
For all the four different maturities we can notice the presence of the smile effect. In particular for the case of 1 months there is a strong increment in the implied volatility in the left part (ITM options) while in the case for the 4 month we can see that the volatility increase both in the left and in the right part. Instead for the case of 9 months and 20 months the implied volatilities are almost flat near the current value of the underlying while they start to increase when they are very far from it. Moreover, when the strikes increases a lot, we have less reliable results for the implied volatility, in fact we can see from the case of 20 months that the behaviour of the implied volatility for high strikes is quite irregular.
Finally, the last thing to do for this part is to check how the smoothness of the *Greeks* changes as the time to maturity increases. Below I reported the different *Greeks* for each time to maturity and for different value of the underlying. As before the vertical red line represents the strikes ATM, so K=340.15 .









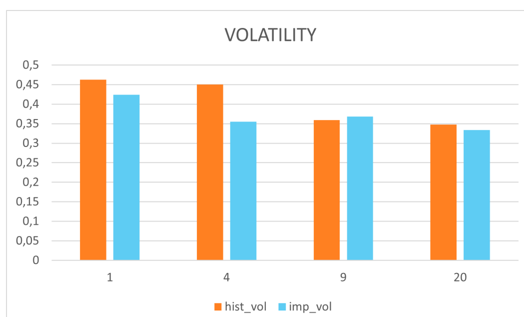


Looking at the plots is quite clear that as the time maturity increases the various *Greeks* functions becomes more smoother. Furthermore we can see that it is always around the ATM region that functions reach their peaks or that they have the maximum changes, unless for the rho case where is not so evident.

3. Part three

Finally for the same asset I have to compute the historical volatility and compare them with the implied one. To calculate the historical volatility I downloaded the same amount of past data for a particular maturity T so, for example, if I consider a maturity of 4 months I downloaded the past 4 months of daily data. After I downloaded the data I computed the daily returns, I applied the formula: $\frac{S_{t+1}-S_t}{S_t}$. If I take the standard deviation of the resulting vector I get an approximation of the daily volatility. To get the historical annual volatility I just multiplied the daily volatility by the square root of 252 (open days on the market for one year) : $\sigma_y = \sqrt{252}\,\sigma_d$. The table below shows historical volatility and implied volatility values with their differences for a particular maturity, T. For the the implied volatility I considered the case with a strike ATM, K=340.

T	vol_{hist}	vol_{imp}	diff%
1 month	46.25%	42.39%	3.86%
4 months	45.05%	35.50%	9.55%
9 months	35.88%	36.85%	-0.97%
20 months	34.80%	33.38%	1.42%



From the table we can see that historical volatility is almost everywhere slightly greater than implied volatility, unless unless for the case 4 months where there is a larger difference, 9.55%. Now that I have the historical volatility I can calculate the price for a call option using the Black-Scholes formula, with strike ATM ($S = K = 340.15$). For r I used the interest rate for the US market provided in the moodle page. In the table below I reported the price that I obtained using the *B&S* formula with the historical volatility corresponding to a particular maturity. Here $price_{mkt}$ is the the Mid price ((Ask+Bid)/2) ATM provided by *Yahoo*.

T	1 month	4 months	9 months	20 months
r	0.9616%	0.9723%	1.732%	2.653%
vol$_{hist}$	46.25%	45.05%	35.88%	34.80%
$B\&S$	18.23	35.69	43.96	66.79
$price_{mkt}$	16.8	28.35	42.2	64.5

As can be seen from the table, the pricing for different maturities with the historical volatility are quite similar to the ones using implied volatility. The biggest differences in price are the 1 month and 4 month cases, but this is not surprising, in fact they are the cases in which the historical volatility are more different from the implied ones.