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Nokia Corporation (natively Nokia Oyj) is a Finnish multinational telecommunications, information technology, and consumer electronics company, founded in 1865. Nokia's main headquarters are in Espoo, Finland, in the greater Helsinki metropolitan area. In 2020, Nokia employed approximately 92,000 people across over 100 countries, did business in more than 130 countries, and reported annual revenues of around €23 billion. Nokia is a public limited company listed on the Helsinki Stock Exchange and New York Stock Exchange. It is the world's 415th-largest company measured by 2016 revenues according to the Fortune Global 500, having peaked at 85th place in 2009. It is a component of the Euro Stoxx 50 stock market index.

The Market Cap is equal to 28.17B while the Enterprise Value is equal to 23.80B. Currently (18/03/2022-14:00) the price of an option is 5.33$, while at the open was 5.22$ and at the previous close 5.32$. In the graph below are reported the changing of the option price in the past 6 month and in the past year. We can see that if we consider 6 month we have a decrease in price of 0.37%, while in a year the price of the option increases of 31,28% (from 3.98$ to 5.33$). In the past year the price reached an high of 6.40$ and a minimum of 3.90$.

(a) Stock prices of the past 6 months

(b) Stock prices of the past year

Furthermore 0.01% of Shares are held by all insider while 10.10% are held by Institutions. Of these Institutions the top two are *Artisan Partners Limited Partnership* with 67,526,290 shares and *Optiver Holding B.v.* with 64,122,522. Another important number of share are held by the *Artisan International Value Fund* ,that is a mutual found, with 49,642,985 shares. For our analysis we must take into account the presence of dividends. If we look in the summary page there is no information about future dividend and the last dividend date is reported to be "Jul 29, 2019". Also if we check in the historical data there is no dividend in the last year, so I don't consider them for this pricing task.

For pricing I chose a maturity of 4 month, this because the closest date for calls is July 15, 2022 (More ore less 4 months from March 18). Since the current value (S_0) of the stock is 5.33$ and the first available strike *at the money* is equal to 6.00$, I chose a strike value of 5.65$($K$).

NOK220715C00004000	2022-03-17 2:48PM EDT	4.00	1.39	1.37	1.44	0.00	-	30	937	50.39%
NOK220715C00005000	2022-03-18 3:13PM EDT	5.00	0.62	0.59	0.64	0.00	-	252	5,618	39.06%
NOK220715C00006000	2022-03-18 3:01PM EDT	6.00	0.21	0.20	0.21	0.00	-	47	14,049	36.72%
NOK220715C00007000	2022-03-18 3:21PM EDT	7.00	0.07	0.07	0.09	-0.03	-30.00%	11	22,925	41.80%

Because I consider a maturity of 4 months I downloaded the past 4 months of daily data to have an approximation of the standard deviation of daily returns. To compute the daily returns we have just to apply the formula: $\frac{S_{t+1}-S_t}{S_t}$. If we take the standard deviation of the resulting vector we obtain an approximation of the daily volatility. In this case the daily volatility is equal to σ_d=0,022459. This imply that the annual volatility is equal to: $\sigma_y = \sqrt{252}\,\sigma_d$ = 0,356525.

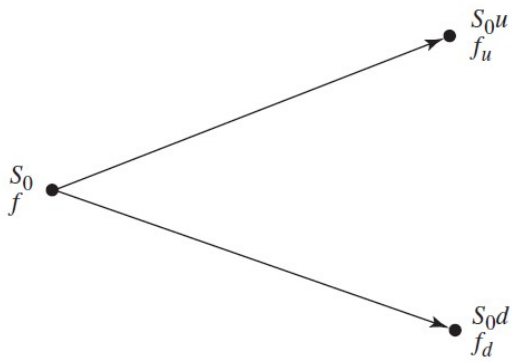

Figure 2: Stock and option prices in a general one-step tree

The next thing to do is to compute the u and d values of the Binomial model. These values represent respectively the percentage of increase and decrease of the stock price

and are calculated using the following formula: $u,d = \exp \pm(\sigma_y \sqrt{T})$, where T is the maturity, in our case 4 months so $\frac{4}{12}$ of a year. We obtain the following results:

$$u = 1.2285 \qquad\qquad d = 0.8139$$

Now I have to decide the value for the interest rate, *r*. Since there is no interest rate equal for 4 months in Libor I take the one of 3 months (even if the maturity is 4 months). In this case the USD Libor interest rate corresponding to a maturity of 3 months (18/03/2022) is equal to: 0.93400% = 0.0093400. Since Libor has a linear convention for the interesting rate, the capitalisation factor using the compounding rule is equal to 1+*r*T, while the discount factor is the inverse of the capitalisation factor. If we applied the formula we get a capitalisation factor equal to 1.0031 and a discount factor equal to 0.9969.

Now we can finally calculate the *risk-neutral probability*:

$$q = \frac{(1+rT)-d}{u-d} = 0.45637$$

Finally we can use the *risk-neutral probability* to apply the *risk neutral pricing formula* for pricing the Nokia call (recall that there are no dividends):

$$price = (1 + rT)(q(S_u - K)^+ + (1 - q)(S_d - k)^+) = 0.411$$

$$\text{recall: } K = 5.65 \text{ ; } S_u = S_0 \cdot u \text{ ; } S_d = S_0 \cdot d \text{ ; } S_0 = 5.33$$

I can compare the price I found with the one in the *yahoo.finance* site. The first strike *at the money* is 6.00 and have a Mid price of 0.205 ((Ask+Bid)/2), while the first strike *in the money* is 5.00 with a Mid price of 0.615. We obtained a price of 0.411 with a strike of 5.65, so is perfectly coherent with the market quote (is inside the interval 0.205-0.615). Moreover if we choose a strike of K=6.00 and we applied the same formula we obtain a *price* of 0.250, that is very close to the market quote corresponding to the same strike (there is a difference of 0.045). This difference is might be due to approximation errors, therefore it could be reduced by using more precise values in the analysis.

In the second part I have to consider a maturity of 6 month but, since the first available date for call options is October 21, 2022, I consider a maturity of 7 months. Now the pricing procedure is exactly the same as the previous one, considering this new maturity. As before the first strike price *at the money* is equal to 6.00, so I chose the same strike price for my analysis ($K = 5.65$).

NOK221021C00004000	2022-03-18 1:05PM EDT	4.00	1.48	1.45	1.51	-0.03	-1.99%	5	279	44.92%
NOK221021C00005000	2022-03-18 1:29PM EDT	5.00	0.79	0.73	0.82	-0.01	-1.25%	5	332	40.63%
NOK221021C00006000	2022-03-18 3:57PM EDT	6.00	0.36	0.35	0.37	-0.03	-7.69%	2,035	2,392	37.60%
NOK221021C00007000	2022-03-18 3:29PM EDT	7.00	0.17	0.16	0.18	-0.02	-10.53%	10	812	39.06%

Now, since I chose a maturity of 7 months, I have to download the daily data of the past 7 months in order to have an approximation of the daily volatility. In this case the daily volatility is equal to σ_d=0,019609, and the annual volatility is equal to: $\sigma_y = \sqrt{252}\,\sigma_d$ = 0,311283. To calculate the *u,d* values I have to apply the previous formula :*u,d* = $\exp \pm(\sigma_y\sqrt{T})$, with the new maturity *T*=$\frac{7}{12}$ year. We obtain the following results:

$$u = 1.2684 \qquad\qquad d = 0.7884$$

For the interest rate I chose the Libor interest rate corresponding to a maturity of 6 months (since there is not the corresponding one for 7 months). The interest rate, *r*, is equal to 1.28757% = 0.0128757 (18/03/2022). Now for T=7/12 there is a capitalisation factor equal to 1.0075 and a discount factor equal to 0.9925.

The *risk-neutral probability* for the new maturity is equal to:

$$q = \frac{(1+rT)-d}{u-d} = 0.45648$$

Finally we can use this new probability to apply the *risk neutral pricing formula* and we obtain:

$$price = (1 + rT)(q(S_u - K)^+ + (1 - q)(S_d - k)^+) = 0.5107$$

$$\text{recall: } K = 5.65 \text{ ; } S_u = S_0 \cdot u \text{ ; } S_d = S_0 \cdot d \text{ ; } S_0 = 5.33$$

Again we can compare the price with the one in the *yahoo.finance* site. The first strike *at the money* is 6.00 and have a Mid price of 0.360, while the first strike *in the money* is 5.00 and have a Mid price of 0.775. For a strike of 5.65 I found a price of 0.5107 so is coherent with the market quote (is inside the interval 0.360-0.775). Instead if I choose a strike *K*=6.00 I obtain a price of 0.350, that is very close to the market quote for the same strike (difference of 0.01).