Mincato Emanuele ID: 2019044 Date: 26/04/2022

1. Assets

1.1. Uber

Uber Technologies, Inc. (Uber) is an American mobility company as a service provider. It is based in San Francisco with operations in approximately 72 countries and 10,500 cities. Its services include ride-hailing, food delivery, package delivery, couriers, freight transportation, electric bicycle and motorized scooter rental via a partnership with Lime. In the fourth quarter of 2021, Uber had 118 million monthly active users worldwide and generated an average of 19 million trips per day. In the United States, as of January 2022, Uber had a 71% market share for ride-sharing and a 27% market share for food delivery.

The Market Cap is equal to 63.491B while the Enterprise Value is equal to 68.35B. Currently (26/04/2022-13:00) the price of the asset underlying is 32.44$, while at the open was 30.59$ and at the previous close was 30.83$. This company does not pay dividends.

1.2. Lyft

Lyft, Inc. is an American transport service support provider that develops, markets, and operates a mobile app, offering ride-hailing, vehicles for hire, motorized scooters, a bicycle-sharing system, rental cars, and food delivery. It is based in San Francisco, California and operates in 645 cities in the United States and 10 cities in Canada. Lyft does not own any vehicles; instead, it receives a commission from each booking. Fares are quoted to the customer in advance but vary using a dynamic pricing model based on the local supply and demand at the time of the booking. As of 2022, with a 29% market share, Lyft is the second-largest ridesharing company in the United States after Uber.

The Market Cap is equal to 11.82B while the Enterprise Value is equal to 10.08B. Currently (26/04/2022-13:00) the price of the asset underlying is 33.91$, while at the open was 32.82$ and at the previous close was 33.16$. This company does not pay dividends.

2. Equibalanced Portfolio

The first thing I did was build an equibalanced Portfolio of the two assets, for a 6-months time window. To do that I used an iterative approach starting from 6 months ago up to today. I assumed to invest (six month ago) 1,000,000$ in the Portfolio and to split them in the two assets , so 500,000$ for Uber shares and 500,000$ for Lyft shares. Then I calculated the number of shares (both for Uber and Lyft) that I could buy with that money. To obtain the value of the Portfolio for the next day I just multiply the number of shares I got by the price (Adjusted close) of the respective assets. When I obtained the new value of the Portfolio I assumed to re-invest it completely in the two assets, so I splitted it in two and I calculated again the number of shares I can buy. Then I multiplied the number of shares by the price (next day) of the two assets to get the new Portfolio value. I repeated this process for all days in the past six months. In this case the Portfolio is equibalanced because each day I equally split the value of my Portfolio in the two assets. Starting six months ago with an investment of 1,000,000$, and continuing to reinvest it every day in a balanced way, I ended up with an today's Portfolio value of 713,203.61$. Now that I have built my Portfolio I consider it as a single asset for further analysis. Finally, after I obtained all the daily values of my Portfolio, I computed the daily return of the past 6 months. To do that I applied the formula: $\frac{S_{t+1}-S_t}{S_t}$. If we take the standard deviation of the resulting vector we obtain an approximation of the daily volatility. In this case the daily volatility is equal to σ_d=0,03528.

3. Parametric Normal VaR

Value at Risk (VaR) is typically defined as the maximum loss which should not be exceeded during a specific time period with a given confidence level. The measurement is often applied to an investment Portfolio for which the calculation gives a confidence interval about the likelihood of exceeding a certain loss threshold. This first approach, better known as the variance-covariance method, assumes that asset returns are normally distributed around the mean. Also it assumes that asset returns have constant standard deviation over time.

To calculate the Var of my Portfolio I computed the mean and the standard deviation of his historical returns, respectively m=-0,0021 and σ=0,03528. Finally the daily VaR is simply a function of the standard deviation and the desired confidence level. It can be expressed as:

$$VaR = z * \sigma * \sqrt{T}$$

Where the parameter z links the quantile of the normal distribution and the standard deviation: z = 1.645 for p = 95%, z = 2.33 for p = 99% and z = 2.58 for p = 99.5%. In this case T is the time horizon we are taking into account. This formula is true if the mean is equal to zero. This is not my case so I adapted it to consider the mean of the daily returns, but the results doesn't change a lot since it is almost zero (m=-0,0021).

In the Table below I reported the percentile of the Value at Risk for a given confidence interval and time horizon. For graphic reason I just reported the first three days and the last three. If we want the absolute loss we can simply multiply the results by the value we want to invest in the Portfolio. For example, if we assume to invest 1,000,000$ in the Portfolio we are sure with a confidence level of 95% that the day after we can lost at most 55,937$ (1000000*0.0559) or that in 100 days we can lost at most 799847$ (1000000*0.7998) with a confidence level of 99% and so on.

The last point of the report asks to verify the additivity of VaR for all methods. I used the exact same procedure to compute the VaR for Uber and Lyft, again for difference confidence rate and for a time

Days	LYFT			UBER			Portfolio		
	95%	99%	99.5%	95%	99%	99.5%	95%	99%	99.5%
1	5.65%	8.07%	8.96%	5.97%	8.53%	9.47%	5.59%	8.00%	8.88%
2	7.99%	11.42%	12.67%	8.53%	12.07%	13.39%	7.91%	11.31%	12.56%
3	9.78%	13.98%	15.52%	10.50%	14.78%	16.40%	9.69%	13.85%	15.38%
...	....	....	....	....	....	....	....	....	....
...	....	....	....	....	....	....	....	....	....
98	55.90%	79.91%	88.71%	61.00%	84.47%	93.76%	55.37%	79.18%	87.90%
99	56.18%	80.32%	89.16%	61.31%	84.90%	94.23%	55.66%	79.58%	88.34%
100	56.46%	80.73%	89.61%	61.62%	85.33%	94.71%	55.94%	79.98%	88.79%

Table 1

horizon of 100 days. The results are reported in the table [1]. To verify the additivity I added together the Value at Risk of the two different assets and I compared it with that of the Portfolio. From the plots below we can see that if we consider the Portfolio as single assets we have a lower VaR then considering it as the sum of two different assets.

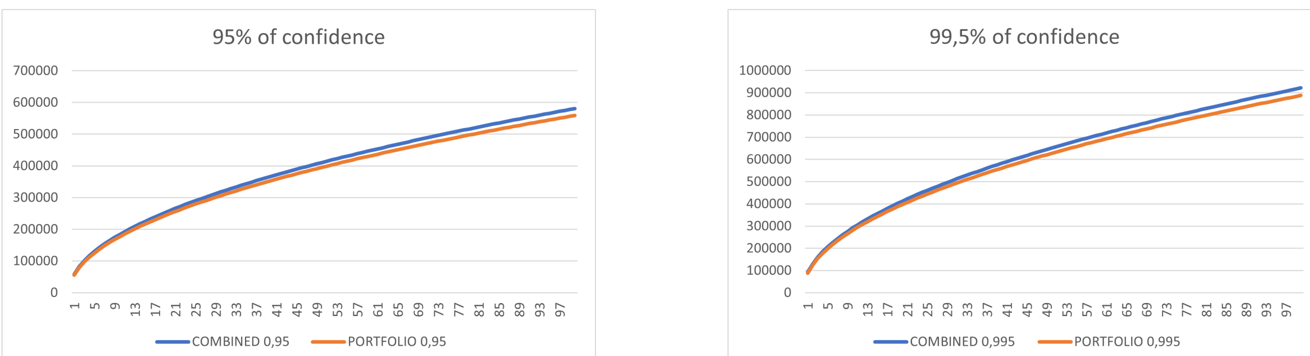

Figure 1: VaR for a time horizon of 100 days

4. Exponentially weighted moving average

The problem with the previous method is that are assigned the same weight to all returns, this imply that a daily return of six months ago is as important as yesterday's daily return. Instead, when we calculate the variance, we want to give more importance to more recent events. To solve this problem we can use an Exponential Weighted Moving Average (EWMA), as the name suggests we are applying exponential weighting to daily returns. The first thing to do is to define the smoothing parameter, in this case we considered a lamda value of 0.95. Then the weights are obtained with the following formula:

$$w_T = (1\text{-}\lambda) * (\lambda)^T$$

where T starts form 0 and represents how many days in the past the daily returns were calculated. Here to calculate the return I used the formula: $\ln \frac{S_t}{S_{t-1}}$. Next, to calculate the new variance, I multiplied the respective weights to the square of the daily returns. Now that I have the new 'weighted' variance of my daily return I calculated the VaR exactly as I did previously. The results are reported in the table below [2]. Here, respect to the previous approach, I have obtained smaller VaR. In fact, previously, we have a VaR for my Portfolio, with a time horizon of 2 days and a confidence interval of 99%, of 11.31% while now the VaR is 10.97% of my investment. This holds true also for other time horizon and other confidence level. This implies that, in this particular case, if we give more importance to recent daily return, we get a lower VaR for our Portfolio.

Days	LYFT			UBER			Portfolio		
	95%	99%	99.5%	95%	99%	99.5%	95%	99%	99.5%
1	5.31%	7.62%	8.46%	5.83%	8.36%	9.29%	5.41%	7.76%	8.62%
2	7.50%	10.77%	11.97%	8.25%	11.83%	13.14%	7.65%	10.97%	12.19%
3	9.19%	13.19%	14.66%	10.10%	14.49%	16.09%	9.36%	13.44%	14.93%
...	....	....	....	....	....	....	....	....	....
...	....	....	....	....	....	....	....	....	....
98	52.52%	75.40%	83.78%	57.72%	82.80%	91.97%	53.52%	76.81%	85.34%
99	52.79%	75.79%	84.21%	58.02%	83.22%	92.44%	53.79%	77.20%	85.77%
100	53.06%	76.17%	84.63%	58.31%	83.64%	92.91%	54.06%	77.59%	86.21%

Table 2

As before, if we add the VaR of the two different assets calculated using EWMA we get higher values for the VaR than considering the Portfolio as a single asset. We can see this behaviour in the plots below.

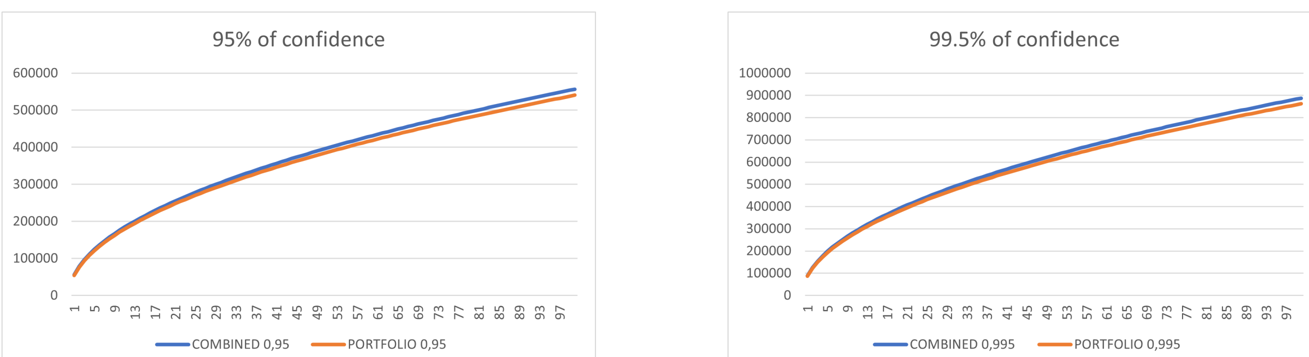

Figure 2: VaR for a time horizon of 100 days using EWMA

5. Monte Carlo VaR

In this section I used Monte Carlo simulation in order to estimate the Value at Risk of my Portfolio. Monte Carlo Simulation, also known as the Monte Carlo Method, is a mathematical technique, which is used to estimate the possible outcomes of an uncertain event. I used Monte Carlo methods to simulate possible final values for my Portfolio, then I calculated the respective loss, basically by subtracting the value of the investment from the simulated value obtained with MC, and finally I built a distribution of them to estimate the VaR. One important parameter of the MC methods is the number of simulation, *N*, obviously as the number of simulations increases I will get better results but it will take more time. To compute a new estimation of the value of my Portfolio for the next day I used the following formula:

$$V_{T+1} = V_T * e^{\frac{var}{2} + rnd\_val}$$

where V_T is today's value of my Portfolio, basically the amount of money I assume to invest, for this analysis I assumed an investment of 1,000,000$. While *rnd_val* is equal to the standard deviation of daily returns multiply by a random value pick from a normal distribution with mean 0 and var 1. Finally I multiply the results by the square root of the time horizon. In the table below [3, 4] I reported the Var calculated for different time horizon with different confidence level and for different number of iteration. The greater the number of iterations, the better the Var estimate should be. Also in Figure[3] I reported all the estimated percentage of increment/decrement of my Portfolio value, generated by 10000 MC simulation (data are group by a range of 0.01). From the plot we can see that the estimation have a bell-shape distribution around the mean that is equal to 1, it represent the actual investment in the Portfolio.

N = 100	Portfolio		COMBINED	
VaR	Percent loss	Absolute loss	Percent loss	Absolute loss
95%	23.03%	230261.42	24.26%	242629.9
99%	26.94%	269411.66	35.09%	350904
99,50%	29.59%	295904.74	36.54%	365407.4

N = 1000	Portfolio		COMBINED	
VaR	Percent loss	Absolute loss	Percent loss	Absolute loss
95%	25.86%	258640.02	25.16%	251566.7
99%	32.66%	326603.72	33.80%	337981.8
99,50%	35.13%	351333.44	37.63%	376286.7

N = 10000	Portfolio		COMBINED	
VaR	Percent loss	Absolute loss	Percent loss	Absolute loss
95%	25.84%	258414.22	26.55%	265528
99%	36.51%	365140.75	36.61%	366104.8
99,50%	40.96%	409581.71	40.02%	400212.2

Table 3: VaR of MC simulations considering T = 21

Finally, the last thing to do for the MC simulation is to check the additivity of the VaR. To do that I calculated in the same way the VaR for the two assets (Uber and Lyft) and then I sum up the values I got. But in this case of course there is no additivity since all the simulation are generated randomically, nevertheless the VaR I obtained for the two assets are similar to the ones of the single Portfolio, as we can see from the table [3,4]

N = 100	Portfolio		COMBINED	
VaR	Percent loss	Absolute loss	Percent loss	Absolute loss
95%	43.64%	436358.9	42.02%	420247.3
99%	71.19%	711943.7	60.78%	607783.5
99,50%	73.10%	731006.9	63.29%	632904.1

N = 1000	Portfolio		COMBINED	
VaR	Percent loss	Absolute loss	Percent loss	Absolute loss
95%	44.80%	447977.7	43.57%	435726.4
99%	56.57%	565694.2	58.54%	585401.6
99,50%	60.85%	608527.4	65.17%	651747.7

N = 10000	Portfolio		COMBINED	
VaR	Percent loss	Absolute loss	Percent loss	Absolute loss
95%	44.76%	447586.6	45.99%	459908
99%	63.24%	632442.3	63.41%	634112.1
99,50%	70.94%	709416.3	69.32%	693187.9

Table 4: VaR of MC simulations considering T = 63

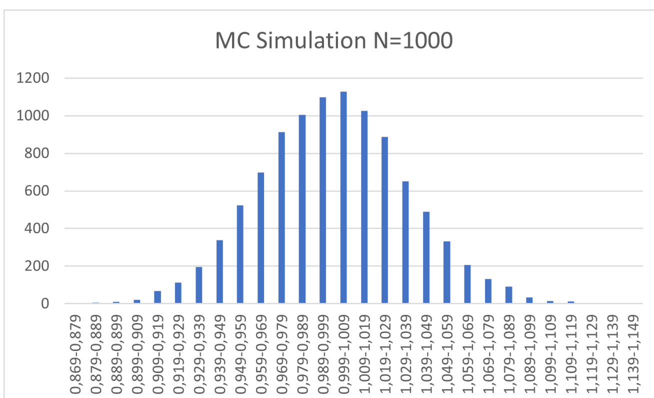

Figure 3: Distribution of 10000 MC simulation with T = 21

6. Historical VaR

In this section I computed the Value at Risk for my Portfolio using the historical value of the returns. This methodology is based on the assumption that the pattern of historical returns is indicative of the pattern of future returns. In this analysis I consider a time windows of 6 months for the daily returns, thus I have 126 scenarios or cases that will be representative of what will happen tomorrow. The first thing I did was sort the distribution of historical returns in ascending order (from the worst to the best). Finally I just select a confidence level to calculate the VaR, so for example if we consider a confidence level of 95%, then our VaR estimate corresponds to the 5st percentile of the probability distribution of daily returns. The historical VaR is estimated only from the data, for this reason is a non-parametric approach. In the table below [5] I reported the results, I considered one-day time horizon with difference confidence rate. Also in this case I checked the presence of additivity in the VaR. From the results reported in the table it seems that the VaR considering the Portfolio as a single assets are lower than considering it as the sum of the Var of the two assets. The advantage of this method is that is a simple and a fast way to calculate VaR, but the main drawback is that the assumption that the past represents the immediate future well is a strong assumption in the real world.

	Portfolio		COMBINED	
VaR	Percent loss	Absolute loss	Percent loss	Absolute loss
95%	6.78%	67794.085	6.81%	68124,719
99%	7.61%	76143.048	8.14%	81419,262
99,50%	7.80%	78044.953	8.60%	85961,660

Table 5

7. Historical simulations VaR

Finally in this last section I reported the VaR calculated using historical simulation. As the previous approach, it involves using past data as a guide to what will happen in the future. Since I considered the daily return of the past six month I can generate 126 possible scenario based on them, for simulate what can happen tomorrow. To generate the *i-th* scenario in the historical simulation I used the following formula:

$$\textit{Value under i-th scenario} = v_n * \frac{v_i}{v_{i-1}}$$

Where v_i is the closed price of a single asset (Uber/Lyft) and v_n is the closed price of the same asset at the last day (Today). Then I used the values I have obtain under the *i-th* scenario, for both assets, to calculate the Portfolio values under that scenario. Finally I computed the losses of my Portfolio, as the previous cases I considered an investment of 1,000,000$. When I found all the losses I constructed a distribution of them, so I can estimate the VaR by considering a percentile, based on the confidence level I want to have, of that distribution.

The last thing to do is to check to additivity of the VaR using this approach. In the table below I reported the estimation of the VaR for one day time horizon, both in the case in which I have considered the Portfolio as a single asset and in the case in which the two VaRs add up. From the table we can see that the estimation are very similar, the only things we can say is that the combined ones are a lit bigger than the single ones.

	Portfolio		COMBINED	
VaR	Percent loss	Absolute loss	Percent loss	Absolute loss
95%	5.17%	51682.852	5,46%	54561.236
99%	6.17%	61671.655	6,44%	64383.620
99,50%	6.60%	65975.849	6,74%	67443.899

Table 6

8. Conclusion

In this report I used different approach to estimate the Value at Risk of an equibalanced Portfolio. In more or less all of these methods, the VaR estimate are similar, whether the Portfolio is considered as a single asset or as the sum of the two assets. In some cases there are greater differences, of the order of 1%, between the two cases, for example in the historical VaR. In this case there is a difference of 0.80% between the two (Portfolio 7.80% - Combined 8.60%) with a confidence level of 99.5%. This could make a huge difference if we consider a large investment, such as in my analysis where I considered a 1 million dollars investment. Nevertheless is better to consider the higher value between the two in order to have an upper-bound of what I can at most lost in a finite time-horizon.