2 Towards a Mathematical Description of the Universe

In the previous lecture we learned that our Universe is expanding. Intuitively, then, at earlier times objects must have been close to one another and the far past conditions very hot and dense. During primordial times, particles collided frequently and they were all in thermal equilibrium at a temperature T. A useful relation, which we will not derive, relates the temperature of the Universe T to its age:

$$\frac{T}{1 \text{ MeV}} \approx \left(\frac{t}{1\ s}\right)^{-1/2} \tag{2.1}$$

This equation tells us that, for instance, the Universe had a temperature of about 1 MeV when it was just one second old. The rates of reactions were extremely high, and a lot of interesting phenomena took place in a short timeframe.

Above 100 GeV, all the particles of the Standard Model, colliding very frequently, were in equilibrium and therefore their abundances were roughly equal to one another. This can be taken as the initial condition for our Universe. Then, in just 10^{-9} seconds, the Universe expanded by a factor of 10^4 and the temperature dropped rapidly. During this short time the Universe went through successive evolutionary stages, that we will now try to keep track of.

At around 100 GeV the electroweak (EW) symmetry of the Standard Model was broken in what is called the *EW phase transition*, where the weak and electromagnetic forces decoupled (above 100 GeV there is no distinction between the two forces) and particles acquired their mass. The detailed dynamics of this transition are still an open subject of research, even though the basics of it were confirmed by the discovery of the Higgs boson.

As the temperature drops the energy drops, thus after a while the particle-antiparticle annihilation reaction will be favored over the reverse process of particle creation. Clearly, since it requires less energy to form light particles rather than heavier ones, the first particles to disappear in this way were the most massive quarks, followed shortly after by the massive bosons W and Z, the tau lepton and the Higgs. Finally, at around 150 MeV, the remaining quarks, at this point having a very low kinetic energy, could condense into hadrons (mesons and protons/neutrons), resulting in the *QCD phase transition* (the name "QCD" stands for Quantum ChromoDynamics, the theory at the heart of strong interactions).

Particles can fall out of thermal equilibrium when their interaction rate drops below the expansion rate of the Universe, in other words the Universe expands way too rapidly for the particles to interact. At that "moment" those particles will stop interacting with the environment, they *decouple*, creating a relic abundance. One of the most important decoupling events is the *neutrino decoupling* around one second after the Big Bang, which produced the *cosmic neutrino background* (CνB).

About 1 minute after the Big Bang, the temperature dropped enough to allow the creation of the first light nuclei Helium-4 and Lithium-7. This Big Bang nucleosynthesis produced very few elements heavier than lithium, because there are no stable nuclei with 5 or 8 nucleons that would be required in order to sustain the reaction. Heavier elements instead formed only later on in the insides of stars, via *stellar nucleosynthesis*, where densities allow for the creation of, for instance, carbon out of alpha particles via the 3α channel. Later, with the explosion of stars, these heavier elements, up until iron, spread throughout the Universe; instead, the origin of elements that lie beyond iron is still

debated, but they most likely formed via *explosive nucleosynthesis*, either with s-processes during the last phases of stellar evolution, or with r-processes in supernovae explosions or neutron star mergers.
About 370000 years after the Big Bang, the temperature dropped enough so that the first atoms could form. This process, known as *recombination*, when the photons stopped interacting with electrons (that in turn could begin to be captured by the nuclei) produced what we now see as a relic photon background, the cosmic microwave background (CMB), which we introduced in the last lecture. We have also mentioned how the CMB contains temperature fluctuations that, although very small ($\delta T/T \sim 10^{-5}$), contain very important information about the primordial Universe.

Now, if the cosmological principle held perfectly, if the distribution of matter in the Universe were perfectly homogeneous and isotropic, there would be no structures (galaxies, clusters etc...) now. Obviously, the inside of the Sun looks very different from the inside of my house, so at sufficiently small scales, inhomogeneities must exist. We then seem to need a mechanism which gives rise to these deviations from perfect uniformity. Where should we look for it?
A clue comes from noticing the following thing. It is expected that a general relativistic description of the Universe breaks down at very early times when the Universe is so dense and hot that quantum effects become important (that's why our discussion began at 100 GeV. At higher energies we don't have the slightest of ideas of what happened or what theory we should even use); in fact, a model based only on GR has a number of conceptual problems when applied to the early Universe (as we will see). A possible solution to these problems requires extending the GR model by introducing density perturbations created via quantum fluctuations at early times, which are now believed to be responsible for the formation of all the large scale structure that we see. These fluctuations are also responsible for the temperature anisotropies of the CMB. A popular theory that accomplishes the generation of those fluctuation seeds is the *inflationary theory* in which the Universe is supposed to have gone through a rapid expansion over very little amount of time driven by one (or more) quantum field, the *inflaton*.
Our understanding of the Universe is still very far from complete and, despite the inflationary paradigm, we are still unable to predict the initial conditions for structure formation from first principles, forcing us to rely on a set of parameters taken from observational data.
However, once the initial conditions have been specified, it is very straightforward to compute the way in which density perturbations evolve in time in a homogeneous background, as we will do. When the universe is matter "dominated" (we will see what this means), the perturbations will grow, because a region where the initial density is slightly higher than the mean will attract its sorroundings more strongly than average. Therefore overdense regions become more overdense over time, and underdense regions become more rarefied as matter will flow away from them. The exact rate at which perturbations grow will depend on the cosmological model, and in particular this growth will stop when dark energy comes to dominate the Universe. In any case, once a certain region is overdense enough, it will stop growing and start to collapse, which is the birth of large objects. In general, as we will see, dark matter starts growing earlier than baryonic matter, and nonlinear, quasi equilibrium dark matter objects are called *dark matter halos*; they are the potential wells into which the baryons will eventually fall and form galaxies (this is what we meant when we said that baryons need to be "gravitationally assisted" by dark matter in

order to form galaxies).

2.1 General Relativity

The study of Cosmology in general only requires a very basic level knowledge of general relativity. Practically, it is only sufficient to understand what the *metric* is and what it allows us to do, so that's what we will briefly recall here. Let's consider our usual Euclidian 3D space with coordinates $\vec{x} = (x, y, z)$, and two objects at locations $\vec{x}_1 = (x_1, y_1, z_1)$ and $\vec{x}_2 = (x_2, y_2, z_2) = \vec{x}_1 + d\vec{x} = (x_1 + dx, y_1 + dy, z_1 + dz)$ for a very small $d\vec{x}$. Clearly, their distance squared is:

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{2.2}$$

which can be written introducing a *metric tensor*, in this case a bit uninterestingly:

$$ds^2 = \sum_{i,j=1}^{3} \delta_{ij} dx^i dx^j \tag{2.3}$$

where, confusingly, $(x_1, x_2, x_3) \equiv (x, y, z)$, and the fact that the indices are upstairs is just for convenience for now. More importantly, we have introduced:

$$\delta_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \tag{2.4}$$

which is just a Kronecker delta, also written more handily as $\delta_{ij} = \mathrm{diag}(1, 1, 1)$. Basically, we can see that the metric tensor δ_{ij} contains the same information as ds^2, and indeed specifies it. This means that a given space will have its own metric (this word is interchangeable with metric tensor), which can be used to determine lengths and angles in the said space. In fact, given two vectors $\vec{A}$ and $\vec{B}$, we can find their norm (the "lengths" part) and the angle between them (the "angles" part):

$$\begin{aligned} \|\vec{A}\|^2 &= \sum_{i,j=1}^{3} \delta_{ij} A^i A^j \\ \vec{A} \cdot \vec{B} &= \sum_{i,j}^{3} \delta_{ij} A^i B^j = \|\vec{A}\| \; \|\vec{B}\| \; \cos\theta \end{aligned} \tag{2.5}$$

Under a coordinate change, the metric can change. We can see this very clearly when we go from cartesian to spherical coordinates, where the distance between the same two objects $\vec{x}_1 = (r, \theta, \phi)$ and $\vec{x}_2 = (r + dr, \theta + d\theta, \phi + d\phi)$ becomes:

$$ds^2_{\text{spherical}} = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 = \sum_{i,j=1}^{3} g_{ij}(\vec{x}) dx^i dx^j \tag{2.6}$$

where we have assumed, as before, $(x_1, x_2, x_3) \equiv (r, \theta, \phi)$ for convenience. The point is that now the metric tensor changes its form a bit:

$$g_{ij} = \begin{pmatrix} 1 & & \\ & r^2 & \\ & & r^2 \sin^2\theta \end{pmatrix} \tag{2.7}$$

However, it should be intuitive that simply switching around some coordinates does not alter the physical world, so that the two objects remain at the same physical distance. Renaming $ds^2 \to ds^2_{\text{cartesian}}$ in (2.3), mathematically this observation translates to:

$$ds^2_{\text{cartesian}} = ds^2_{\text{spherical}} \tag{2.8}$$

The metric tensor is fundamental even when it comes to describing the motion of test particles in an arbitrary space. In fact, let's start with the motion of a free particle in flat space, described by the simple Lagrangian:

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{m}{2}\delta_{ij}\dot{x}^i\dot{x}^j \tag{2.9}$$

Here we introduced a handy notation, called the *Einstein notation*, where repeated indices are summed over, i.e. $\delta_{ij}\dot{x}^i\dot{x}^j \equiv \sum_{ij}\delta_{ij}\dot{x}^i\dot{x}^j$. Critically, this line of reasoning can be extended to any given space that has a metric tensor of the form $g_{ij}(\vec{x})$:

$$\mathcal{L} = \frac{m}{2}g_{ij}(\vec{x})\dot{x}^i\dot{x}^j \tag{2.10}$$

Let's now try to obtain a general form of the equations of motion, which follow from solving the Euler-Lagrange equations:

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{x}^i} - \frac{\partial\mathcal{L}}{\partial x^i} = 0 \tag{2.11}$$

We then have:

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial\dot{x}^i} = mg_{ik}\dot{x}^k \implies \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{x}^i} &= m\frac{\partial g_{ik}}{\partial x^j}\dot{x}^j\dot{x}^k + mg_{ik}\ddot{x}^k \\ \frac{\partial\mathcal{L}}{\partial x^i} &= \frac{m}{2}\frac{\partial g_{jk}}{\partial x^i}\dot{x}^i\dot{x}^k \end{aligned} \tag{2.12}$$

Putting these together:

$$g_{ik}\ddot{x}^k + \left(\frac{\partial g_{ik}}{\partial x^j} - \frac{1}{2}\frac{\partial g_{jk}}{\partial x^i}\right)\dot{x}^j\dot{x}^k = 0 \tag{2.13}$$

but:

$$\frac{\partial g_{ik}}{\partial x^j}\dot{x}^j\dot{x}^k = \frac{1}{2}\left(\frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{ij}}{\partial x^k}\right)\dot{x}^j\dot{x}^k \tag{2.14}$$

Therefore (2.13) becomes:

$$g_{ik}\ddot{x}^k + \frac{1}{2}\left(\frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{ij}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^i}\right)\dot{x}^j\dot{x}^k = 0 \tag{2.15}$$

Now we multiply the whole equation by the inverse of the metric tensor g^{-1}, which is defined by raising its indices. Also, by definition of the inverse, g^{-1} satisfies:

$$g_{ij}g^{jk} = \delta^i_k \tag{2.16}$$

Again, for now, don't worry about the positioning of the indices, since it has no real meaning, i.e. we can just assume $\delta^i_j \equiv \delta_{ij}$ (soon we will understand why). However, when we include the dimension of time, upper or lower indices will have their differences. In any case, the equations of motion become:

$$\ddot{x}^i + \Gamma^i_{jk}\dot{x}^j\dot{x}^k = 0 \tag{2.17}$$

where we have introduced the *Christoffel symbols*:

$$\Gamma^i_{jk}(\vec{x}) = \frac{1}{2}g^{il}\left(\frac{\partial g_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l}\right) \tag{2.18}$$

The $\vec{x}$ dependence is present because $g_{ij} = g_{ij}(\vec{x})$. Equation (2.18) is called the *geodesics equation* and it is fundamental, because if someone hands us an arbitrary metric tensor, we can find the free motion of a test particle simply by solving it (in many cases it is far from simple. Actually, it's very boring).

Now, in general relativity, the direction of time is added, so that the coordinates span 4 dimensions instead of 3. By convention, dimension number 0 is reserved for time, whereas 1,2,3 are the usual spatial dimensions; for this reason, a general 4-vector will be of the form $x^\mu = (ct, \vec{x}) \equiv (ct, x^i)$, where $\mu = 0, 1, 2, 3$. Having said this, everything that we have said before can easily be applied to this new situation. We start with the invariant ds^2:

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu \tag{2.19}$$

of which the easiest example is flat space, or *Minkowski space*:

$$ds^2 = -c^2 dt^2 + \delta_{ij} dx^i dx^j \tag{2.20}$$

also referred to as $g_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1) \equiv (-, +, +, +)$. Given a certain $g_{\mu\nu}$ and two vectors A^μ and B^μ, we can use it to calculate lengths and angles as in (2.5):

$$\begin{aligned} A^2 &\equiv g_{\mu\nu} A^\mu A^\nu = A_\mu A^\mu \\ A \cdot B &\equiv g_{\mu\nu} A^\mu B^\nu = A^\mu B_\mu = A_\mu B^\mu = A^2 \ B^2 \ \cos\theta \end{aligned} \tag{2.21}$$

where now there is a difference between upper and lower indices, which are mapped from one to the other by the metric tensor:

$$\begin{aligned} A_\mu &= g_{\mu\nu} A^\nu \\ A^\mu &= g^{\mu\nu} A_\nu \end{aligned} \tag{2.22}$$

the inverse of $g_{\mu\nu}$ being defined as before:

$$g^{\mu\nu} g_{\nu\rho} = \delta^\mu_\rho \tag{2.23}$$

Now we are ready to understand why in Euclidian 3D space there is no distinction between upper and lower indices. It all has to do with the shape of the metric tensor, since, following the definition in (2.22), we have $A_i = \delta_{ij} A^j = A^i$. Following the same thread as before, the Lagrangian of a free particle can be written as:

$$\mathcal{L} = \frac{m}{2} g_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu \tag{2.24}$$

and the geodesic equation as:

$$\ddot{x}^\mu + \Gamma^\mu_{\nu\rho} \dot{x}^\nu \dot{x}^\rho = 0 \tag{2.25}$$

with:

$$\Gamma^\mu_{\nu\rho} = \frac{1}{2} g^{\mu\lambda} \left(\frac{\partial g_{\rho\lambda}}{\partial x^\nu} + \frac{\partial g_{\nu\lambda}}{\partial x^\rho} - \frac{\partial g_{\nu\rho}}{\partial x^\lambda} \right) \tag{2.26}$$

Finally, any given spacetime $g_{\mu\nu}$ has to satisfy the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{2.27}$$

where:

$$\begin{aligned} R &= R^{\mu\nu} R_{\mu\nu} \\ R_{\mu\nu} &= R^\rho_{\mu\rho\nu} \\ R^\rho_{\sigma\mu\nu} &= \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma} \end{aligned} \tag{2.28}$$

and $T_{\mu\nu}$, the *energy-momentum tensor*, encapsulates all the forms of energy present in that spacetime.

As an aside, we note that in the geodesic equation (2.25) the dots mean a derivative with respect to *proper time* τ:

$$\frac{d^2x^\mu}{d\tau^2} = -\Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} \tag{2.29}$$

The physical interpretation, and definition, of such an object, is the time measured by an observer at rest between any two events A and B. Clearly, in his reference frame, the events happen at the same spatial point, so that the distance ds^2 only regards a distance in time:

$$-c^2 d\tau^2 = ds^2 \tag{2.30}$$

This quantity is an invariant, and any given observer, who parametrizes the path of the observer in motion as $x(\sigma)$, calculates it in the following way:

$$\Delta\tau = \int_{\text{path}} \frac{1}{c}\sqrt{-ds^2} = \int_{\sigma_A}^{\sigma_B} \frac{d\sigma}{c}\sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma}} \tag{2.31}$$

Remember that in general relativity, we should be careful with assigning meaning to coordinates. The coordinate t should not be thought of as "time flowing", just as r is not a "distance from the center". The only physical objects in the theory are invariants, like ds^2, that everybody can agree on. As an example, we can think of the decay of a neutron. When physicists say, "a neutron decays on average in around 11 minutes", that lapse of time is the proper time $\Delta\tau$ of the neutron, that is the time measured in the position of the said particle. Any other observer whizzing by will measure their own *coordinate time* Δt, which itself depends on the speed and angle of the observer's velocity. Clearly, it doesn't make sense to compare the time Δt_1 and Δt_2 of two different observers, since one is not "more real" that the other. Time is in fact only a local concept. What the observers can agree on is that, calculating the proper time with equation (2.31), they will get the same result.

2.2 The Geometry of our Universe

We are now in a position to try to guess what the Universe's metric might look like. Even though this might seem like an impossible task, it turns out that the observations we listed in the first lecture decrease the number of possible alternatives rather drastically. Let's start with the most general metric we can write:

$$ds^2 = -c^2 g_{00}(t,\vec{x})dt^2 + 2g_{0i}(t,\vec{x})dt dx^i + g_{ij}(t,\vec{x})dx^i dx^j \tag{2.32}$$

Now, let's think for a moment about the $g_{00}(t,\vec{x})$ term. Since the Universe is homogeneous, it cannot possibly depend on $\vec{x}$, because in that case observers sitting in different points of space would measure time differently, violating homogeneity itself, so $g_{00} = g_{00}(t)$. Moreover, at this point we could define a new variable $dt' = \sqrt{-g_{00}(t)}dt$, so t is really just up to redefinition. Then, we know that all g_{0i} have to be zero, as a consequence of isotropy, because there cannot be a preferred spatial direction. So for now we can write (2.32) as:

$$ds^2 = -c^2 dt^2 + a^2(t)d\sigma^2 \tag{2.33}$$

where $a(t)$ is called the *scale factor* and it does not depend on r because, if that were the case, each observer would measure a different scale factor, violating homogeneity once again. We have also introduced the spatial metric:

$$d\sigma^2 = \gamma_{ij}(\vec{x})dx^i dx^j \tag{2.34}$$

Now, how do we find γ_{ij}? It is very convenient that homogeneous and isotropic spaces must have constant intrinsic curvature (that means constant R as introduced in (2.28)), and in general relativity there exist only 3 options: the curvature can be zero, positive or negative. These correspond to flat, spherical and hyperbolic spaces respectively, depicted here below:

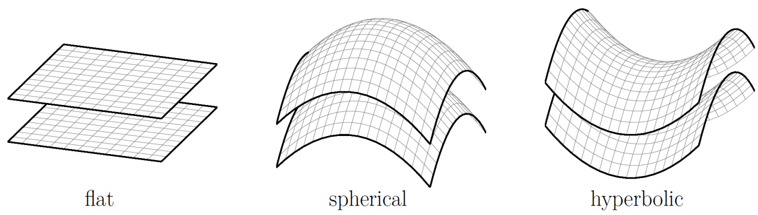

Figure 1: The only three spaces with constant curvature

Let's determine the metric for each of these cases:

- Flat space: this is simply Euclidian 3D space, the metric of which we defined before (2.2):

$$d\sigma^2 = d\vec{x}^2 = \delta_{ij}dx^i dx^j \tag{2.35}$$

  This type of space has constant curvature equal to 0.

- Spherical space: a 3-sphere with radius A is defined as an embedding in 4D Euclidian space:

$$d\sigma^2 = d\vec{x}^2 + du^2 \quad (x^1)^2 + (x^2)^2 + (x^3)^2 + u^2 = A^2 \tag{2.36}$$

  The 3-sphere has a positive constant curvature. It may be not that clear why (2.36) represents the metric on the sphere. To verify this, it is easier to operate in 2 spatial dimensions instead of 3, so that we start with the embedding condition:

$$(x^1)^2 + (x^2)^2 + (x^3)^2 = \tilde{A}^2 \tag{2.37}$$

  and the metric:

$$d\tilde{\sigma}^2 = d(x^1)^2 + d(x^2)^2 + d(x^3)^2 \tag{2.38}$$

  Since, for instance, the coordinate x^3 actually depends on the other two, we can rewrite $d\tilde{\sigma}^2$ as:

$$d\tilde{\sigma}^2 = d(x^1)^2 + d(x^2)^2 + \frac{(x^1dx^1 + x^2dx^2)^2}{\tilde{A}^2 - (x^1)^2 - (x^2)^2} \tag{2.39}$$

  Now introduce polar coordinates:

$$x^1 = \tilde{A}\sin\theta\cos\phi \quad x^2 = \tilde{A}\sin\theta\sin\phi \implies x^3 = \tilde{A}\cos\theta \tag{2.40}$$

  in terms of which we have:

$$d\tilde{\sigma}^2 = \tilde{A}^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2.41}$$

  which is precisely the metric on the 2-sphere.

- Hyperbolic space: a 3-hyperboloid with curvature A is defined as an embedding in Minkowski space:

$$d\sigma^2 = d\vec{x}^2 - du^2 \quad (x^1)^2 + (x^2)^2 + (x^3)^2 - u^2 = -A^2 \tag{2.42}$$

  The 3-hyperboloid has constant negative curvature.

We can now combine the metric of the spherical and hyperbolic spaces like this:

$$d\sigma^2 = d\vec{x}^2 \pm du^2 \quad \vec{x}^2 \pm u^2 = \pm A^2 \tag{2.43}$$

Now, the embedding condition $\vec{x}^2 \pm u^2 = \pm A^2$ implies $udu = \mp \vec{x} \cdot d\vec{x}$, which we can substitute in the metric $d\sigma^2$:

$$d\sigma^2 = d\vec{x}^2 \pm \frac{(\vec{x} \cdot d\vec{x})^2}{A^2 \mp \vec{x}^2} \tag{2.44}$$

To combine this metric with flat space (2.35) we write all three of them in one go as:

$$d\sigma^2 = d\vec{x}^2 + k\frac{(\vec{x} \cdot d\vec{x})^2}{A^2 - k\vec{x}^2} \qquad k = \begin{cases} 0 & \text{flat} \\ +1 & \text{spherical} \\ -1 & \text{hyperbolic} \end{cases} \tag{2.45}$$

In general, it is convenient to write the metric in spherical polar coordinates (r, ϕ, θ) using the coordinate change:

$$\begin{aligned} d\vec{x}^2 &= dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \\ \vec{x} \cdot d\vec{x} &= rdr \\ \vec{x}^2 &= r^2 \end{aligned} \tag{2.46}$$

So finally (2.45) becomes:

$$d\sigma^2 = \frac{dr^2}{1 - kr^2/A^2} + r^2 d\Omega^2 \tag{2.47}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the 2-sphere metric. Putting this in (2.33) we find:

$$ds^2 = -c^2dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2/A^2} + r^2 d\Omega^2\right) \tag{2.48}$$

However, by redefining $r/A = r'$, we have:

$$ds^2 = -c^2dt^2 + a^2(t)A^2\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right) \tag{2.49}$$

so that we can just decide to include A in the definition of $a(t)$, getting rid of it altogether. The following is then the metric of our Universe, or the so-called *Friedmann-Robertson-Walker metric* (FRW):

$$ds^2 = -c^2dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right) \tag{2.50}$$

This is the starting point in the study of cosmology, and in the next lecture we will analyze it in detail. We will understand what those coordinates mean and what can be deduced from them, but also how, by solving the Einstein equations, this metric governs the dynamical evolution of the Universe.