1 Power Spectrum

We divide the power spectrum this way:

$$P(k,a) = P_{\rm ad}(k,a) + P_{\rm iso}(k,a) \tag{1}$$

where:

- $$P_{\rm ad}(k,a) = 2\pi^2 A_s \frac{k^{n_s} c^{n_s+3}}{H_0^{n_s+3}} T^2(k) \left(\frac{D(a)}{D(a=1)}\right)^2 \tag{2}$$

  The growth factor in a universe with matter and dark energy is:

  $$\begin{aligned} D(a) &= \frac{5\Omega_m}{2}\frac{H(a)}{H_0}\int_0^a \frac{da'}{(a'H(a)/H_0)^3} \\ H(a) &= H_0\left(\frac{\Omega_m}{a^3} + \Omega_\Lambda\right)^{1/2} \end{aligned} \tag{3}$$

  By defining:

  $$\begin{aligned} \Omega_m(z) &= \frac{\Omega_{m,0}}{\Omega_{\Lambda,0}a^3 + \Omega_{m,0}} \\ \Omega_\Lambda(z) &= \frac{\Omega_{\Lambda,0}}{\Omega_{\Lambda,0} + \Omega_{m,0}/a^3} \end{aligned} \tag{4}$$

  we get the following approximation for the growth factor:

  $$D(z) = \frac{5\Omega_m(z)}{2}\left(\Omega_m(z)^{-4/7} - \Omega_\Lambda(z) + \left(1 + \frac{\Omega_m(z)}{2}\right)\left(1 + \frac{\Omega_\Lambda(z)}{70}\right)\right)^{-1} \tag{5}$$

  The transfer function is well fitted by:

  $$T(k) = \frac{\log(1+2.34q)}{2.34q}(1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4)^{-1/4} \tag{6}$$

  where:

  $$q \equiv \frac{1}{\Gamma}\left(\frac{k}{h\ Mpc^{-1}}\right) \quad \text{and} \quad \Gamma = \Omega_{m,0} h \exp(-\Omega_{b,0}(1 + \sqrt{2h}/\Omega_{m,0})) \tag{7}$$

- $$P_{\rm iso}(k,a) = \begin{cases} \frac{(\mathrm{f_{PBH}}\tilde{D}(a))^2}{\bar{n}_{\rm PBH}} & k < k_{\rm PBH} \\ 0 & k > k_{\rm PBH} \end{cases} \tag{8}$$

  where $k_{\rm PBH} = (2\pi^2\bar{n}_{\rm PBH}/\mathrm{f_{PBH}})^{1/3}$, $\bar{n}_{\rm PBH} = \mathrm{f_{PBH}}\frac{3H_0^2}{8\pi G}(\Omega_m - \Omega_b)/\mathrm{m_{PBH}}$ and:

  $$\begin{aligned} \tilde{D}(a) &= \left(1 + \frac{3\gamma}{2a_-}s\right)^{a_-} - 1 \qquad s = \frac{a}{a_{\rm eq}} \\ \gamma &= \frac{\Omega_m - \Omega_b}{\Omega_m} \qquad a_- = \frac{1}{4}(\sqrt{1+24\gamma} - 1) \end{aligned} \tag{9}$$

2 Halo Mass Function

The smoothed density variance is:

$$\sigma(R)^2 = \int_0^\infty \frac{dk}{2\pi^2} k^2 \; P(k) W(k,R)^2 \tag{10}$$

The window function is taken to be gaussian:

$$W(x,R) = \frac{1}{(2\pi)^{3/2} R^3} \exp(-x^2/2R^2) \tag{11}$$

with a Fourier transform:

$$W(k,R) = \exp(-k^2 R^2/2) \tag{12}$$

The relationship between the radius R and the mass of the halo is:

$$M(R) = (2\pi)^{3/2} R^3 \bar{\rho} \qquad \bar{\rho} = \rho_{\text{crit},0} \cdot \Omega_m \tag{13}$$

therefore:

$$W(k,M)^2 = \exp\left(-\frac{k^2}{2\pi} \left(\frac{M}{\bar{\rho}} \right)^{2/3} \right) \tag{14}$$

Finally the halo mass function is, considering ellipsoidal dynamics with Sheth-Tormen:

$$\frac{dn}{dM} = -A \sqrt{\frac{2}{\pi}} \sqrt{a} \; \nu \; \exp(-a\nu^2/2)(1 + (a\nu^2)^{-p}) \frac{\bar{\rho}}{M^2} \frac{\mathrm{d} \log(\sigma(M))}{\mathrm{d} \log(M)} \tag{15}$$

with $\nu = \frac{\delta_c}{\sigma(M)}$ and:

$$A = 0.32 \qquad a = 0.75 \qquad p = 0.3 \tag{16}$$

.We can rewrite it as:

$$\begin{aligned}
\frac{dn}{dM} &= -\sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma} \exp\left(-\frac{a\delta_c^2}{2\sigma^2} \right) (1 + (a\nu^2)^{-p}) \frac{\bar{\rho}}{M^2} \frac{\mathrm{d} \log(\sigma(M))}{\mathrm{d} \log(M)} \\
&= -\sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma} \exp\left(-\frac{a\delta_c^2}{2\sigma^2} \right) (1 + (a\nu^2)^{-p}) \frac{\bar{\rho}}{M^2} \frac{M}{\sigma} \frac{\mathrm{d}\sigma(M)}{\mathrm{d}M} \\
&= -\sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma^2} \exp\left(-\frac{a\delta_c^2}{2\sigma^2} \right) (1 + (a\nu^2)^{-p}) \frac{\bar{\rho}}{M} \frac{\mathrm{d}\sigma(M)}{\mathrm{d}M} \\
&= -\sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma^2} \exp\left(-\frac{a\delta_c^2}{2\sigma^2} \right) (1 + (a\nu^2)^{-p}) \frac{\bar{\rho}}{M} \frac{1}{2\sigma} \frac{\mathrm{d}(\sigma(M)^2)}{\mathrm{d}M} \\
&= -\sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma^3} \exp\left(-\frac{a\delta_c^2}{2\sigma^2} \right) (1 + (a\nu^2)^{-p}) \frac{\bar{\rho}}{M} \frac{1}{2} \frac{\mathrm{d}(\sigma(M)^2)}{\mathrm{d}M}
\end{aligned} \tag{17}$$

Now, the only dependence on M contained in $\sigma(M)^2$ is in $W(k,M)^2$, which is given by (12). Then:

$$\frac{\mathrm{d}(W(k,M)^2)}{\mathrm{d}M} = -\frac{k^2}{2\pi}\frac{2}{3}\frac{1}{\bar{\rho}^{2/3}M^{1/3}}W(k,M)^2 \tag{18}$$

Therefore the derivative of $\sigma(M)^2$ is:

$$\frac{\mathrm{d}(\sigma(M)^2)}{\mathrm{d}M} = -\frac{1}{3\cdot 2\pi}\frac{1}{\bar{\rho}^{2/3}}\frac{1}{M^{1/3}}\int_0^\infty \frac{dk}{\pi^2}k^4 \ P(k)W(k,M)^2 \tag{19}$$

Putting (16) in (14) we get:

$$\frac{dn}{dM} = \frac{1}{3\cdot(2\pi)^{3/2}}\frac{\delta_c}{\sigma^3}\frac{\bar{\rho}^{1/3}}{M^{4/3}}\exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)\ (1+(a\nu^2)^{-p})\int_0^\infty \frac{dk}{\pi^2}k^4 \ P(k)W(k,M)^2 \tag{20}$$

3 Cosmological Parameters

h	0.674
H_0	$100\ h\ \ km\ s^{-1}\ Mpc^{-1}$
$\rho_{\mathrm{crit},0}$	$1.260 \times 10^{11}\ M_\odot\ Mpc^{-3}$
Ω_m	0.315
Ω_Λ	0.685
Ω_b	0.049
A_s	$2.1\ \times 10^{-9}$
n_s	0.965
z_{eq}	3402
k_{eq}	$0.015\ h\ Mpc^{-1}$
G	$4.301\ \times 10^{-9}\ km^2\ Mpc\ M_\odot^{-1}\ s^{-2}$
δ_c	1.686

4 Results

$M_\star$	ϵ	$\mathrm{f_{PBH}}$	$\mathrm{m_{PBH}}$	$\mathrm{f_{PBH}m_{PBH}}$	$\rho_\star$
10^{10}	1	10^{-4}	$24 \cdot 10^{7}$	$2.4 \cdot 10^{4}$	
10^{10}	1	10^{-5}	$24 \cdot 10^{8}$	$2.4 \cdot 10^{4}$	
$10^{10.5}$	1	10^{-4}	$18 \cdot 10^{8}$	$1.8 \cdot 10^{5}$	
$10^{10.5}$	1	10^{-5}	$18 \cdot 10^{9}$	$1.8 \cdot 10^{5}$	
10^{10}	0.1	10^{-5}	$18 \cdot 10^{11}$	$1.8 \cdot 10^{7}$	
10^{10}	0.1	10^{-5}	$18 \cdot 10^{10}$	$1.8 \cdot 10^{6}$	
$10^{10.5}$	0.1	10^{-3}	$61 \cdot 10^{8}$	$6.1 \cdot 10^{6}$	
$10^{10.5}$	0.1	$3 \cdot 10^{-5}$	$61 \cdot 10^{10}$	$6.1 \cdot 10^{6}$	