

# Time-series Analytics

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# Decomposition & Detrending

Let's change  
perspective



Photo by [Chokniti Khongchum](#) from [Pexels](#)

# Thinking ...

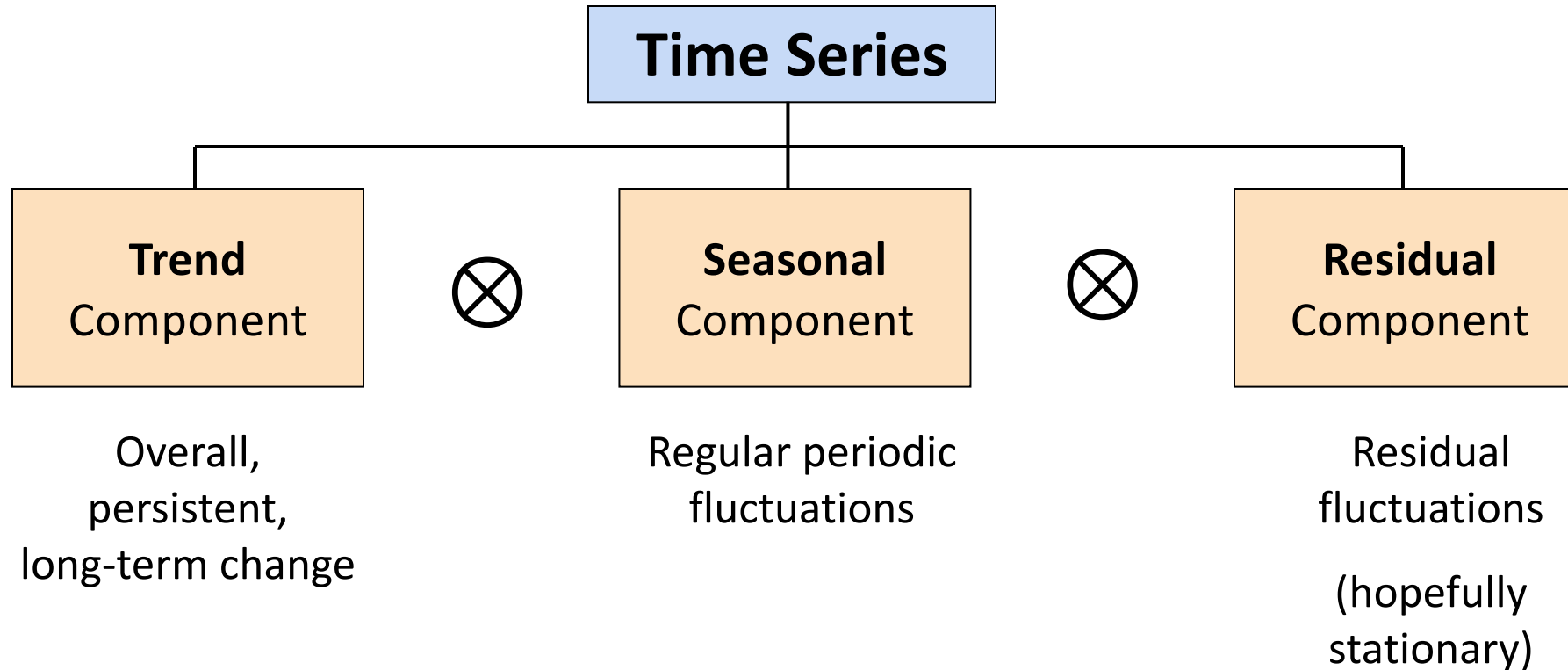
- If
  - stationary **implies** predictable
- and
  - changes in mean, variance
  - presence of seasonality
  - **implies** non-stationary
- **can we try to remove what causes of non-stationarity?**

# Thinking ...

- If
  - stationary **implies** predictable
- and
  - changes in mean, variance
  - presence of seasonality
  - **implies** non-stationary
- **can we try to remove what causes of non-stationarity?**
- **YES! Decomposing a time series**

# Time Series Decomposition

# Time-Series Components



 this character is a placeholder for various mathematical operation used to assemble the components

# Models to decompose time series:

- **Additive** model

$$X_t = m_t + s_t + Y_t$$

- **Multiplicative** model

$$X_t = m_t s_t Y_t$$

- Where

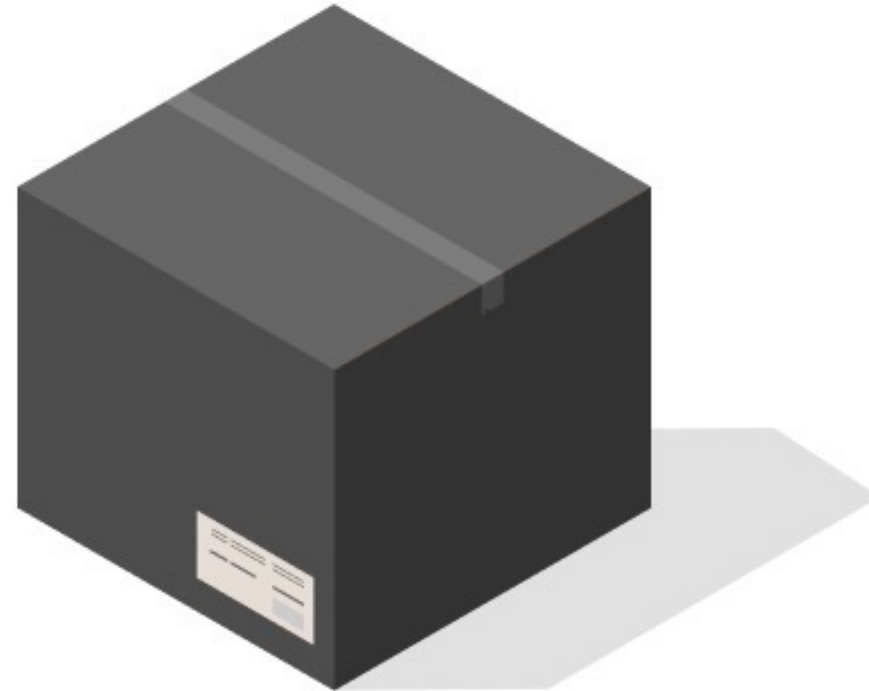
- $m_t$  is the trend component
- $s_t$  is the seasonal component
- $Y_t$  is the residual component

- **How to Choose** Between Additive and Multiplicative Decompositions

- The **additive** model is useful when the **seasonal** variation is relatively **constant over time**.
- The **multiplicative** model is useful when the **seasonal** variation **increases over time**.



Let's first go  
**black-box**



**Black box – we do not  
know anything**



- Use statsmodels as a *black-box* able to decompose a time-series
- Try both methods
  - Additive
  - Multiplicative

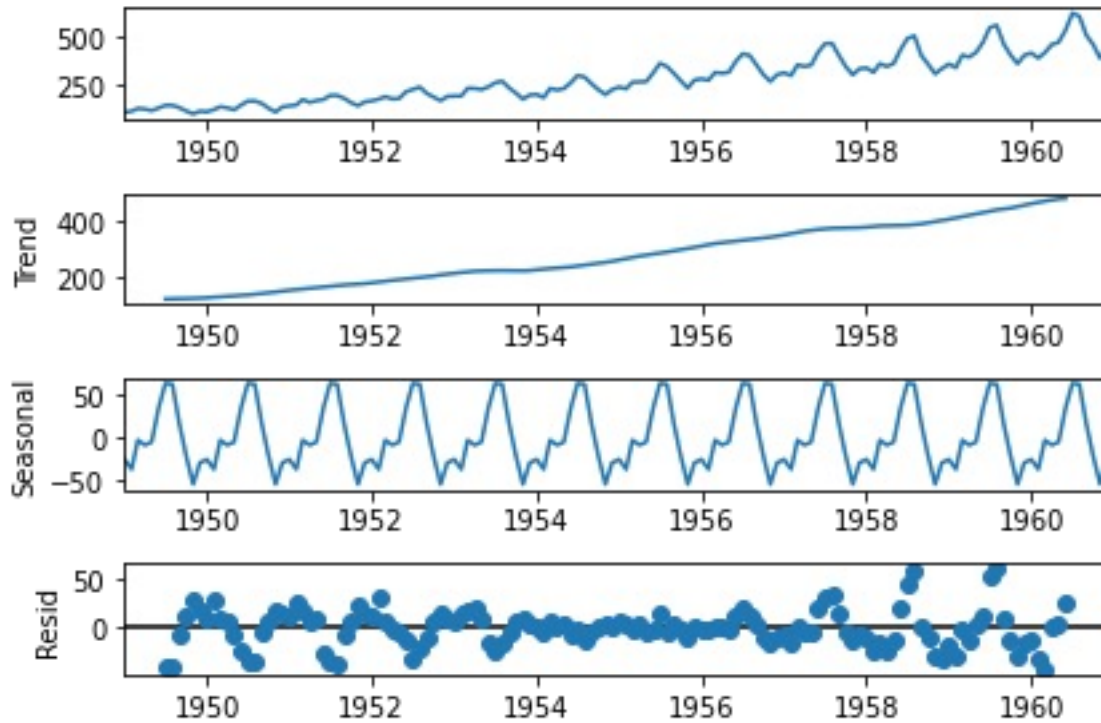


NOTE: for more information check out [https://www.statsmodels.org/stable/examples/notebooks/generated/statespace\\_seasonal.html](https://www.statsmodels.org/stable/examples/notebooks/generated/statespace_seasonal.html)

# Time series decomposition

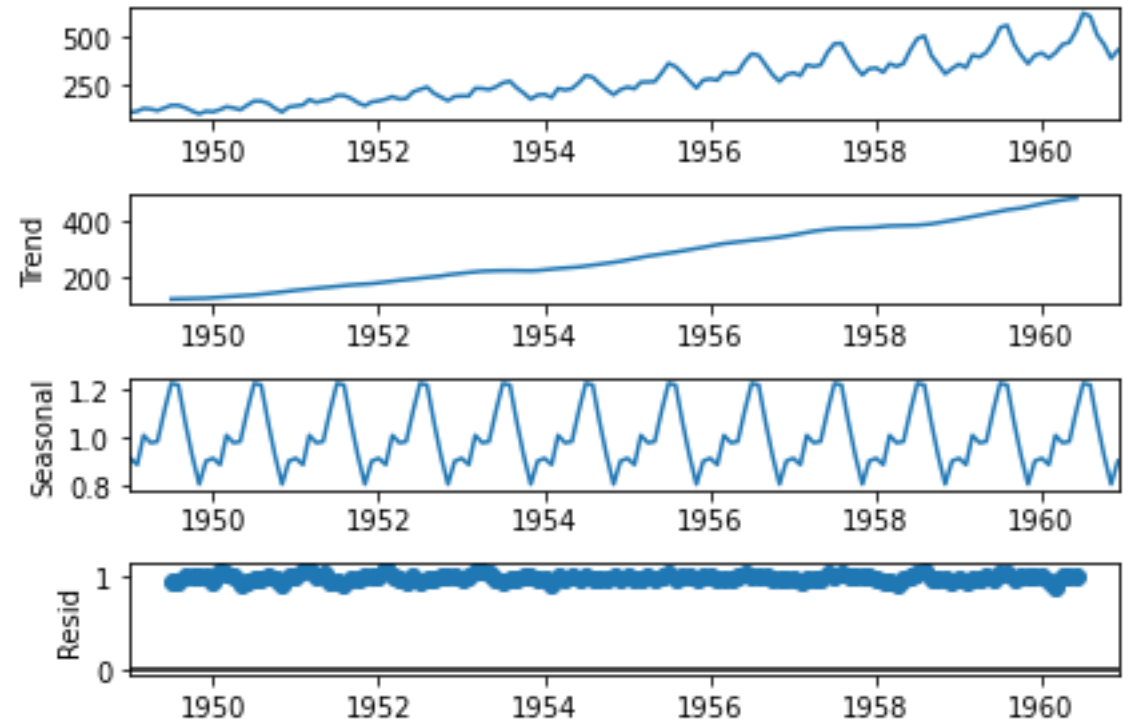
## A comparison

Additive



vs.

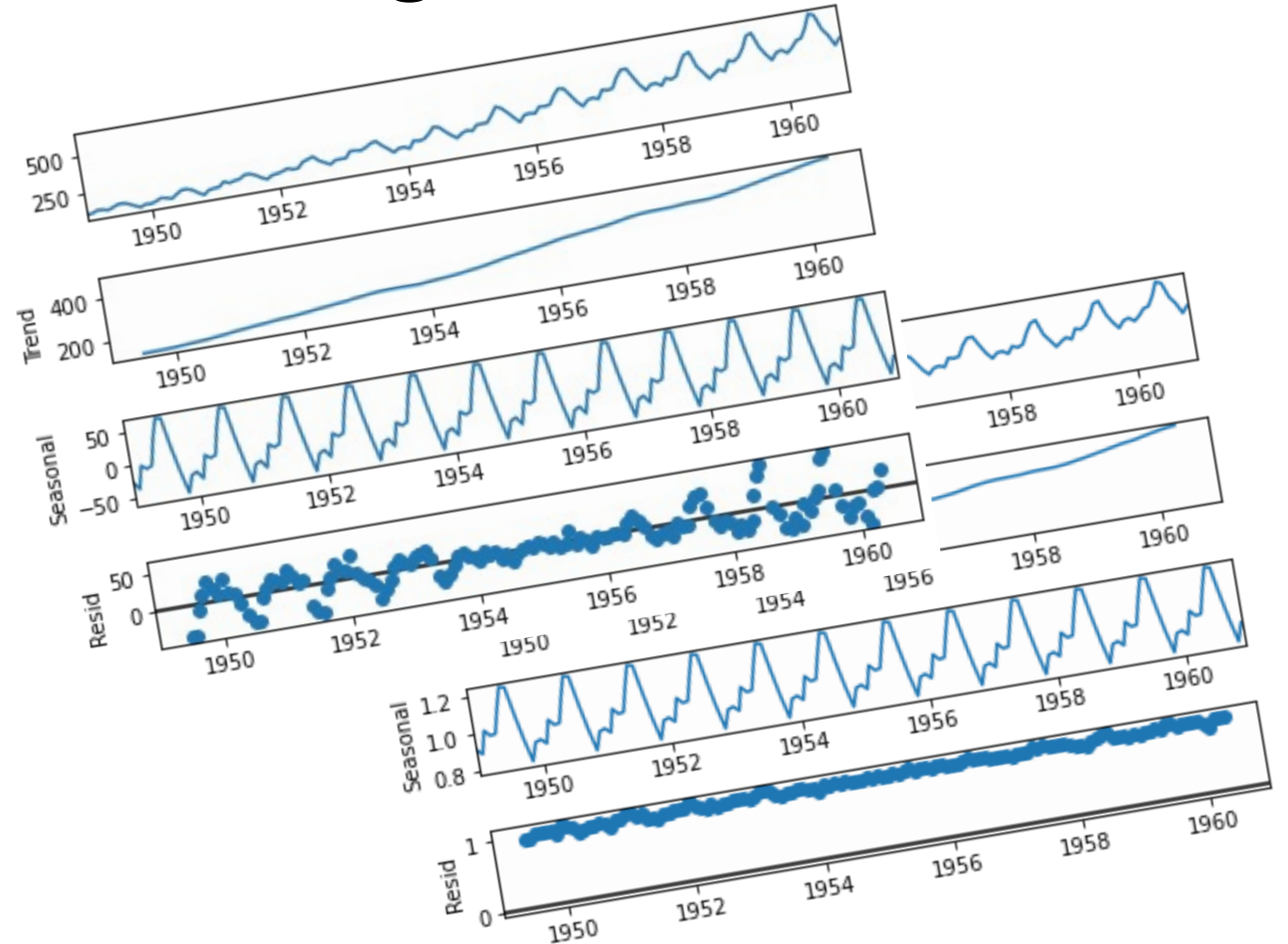
Multiplicative



# Time series decomposition

## Let's see if you are following ...

- Q: Is additive better than multiplicative?
- Q: Why?



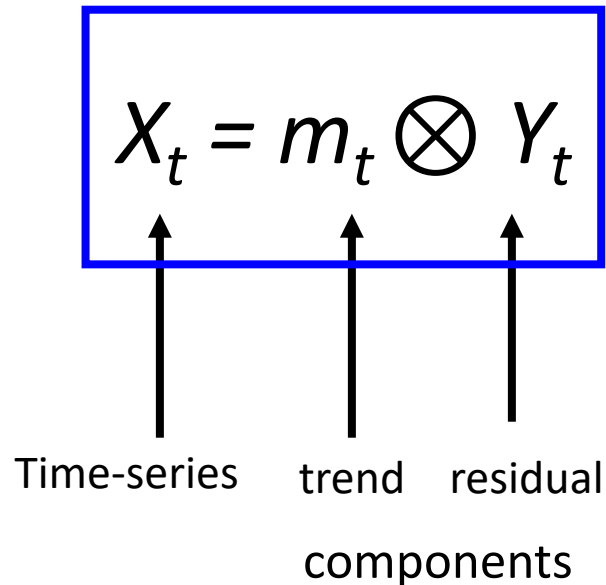
Let's now go  
**white-box**



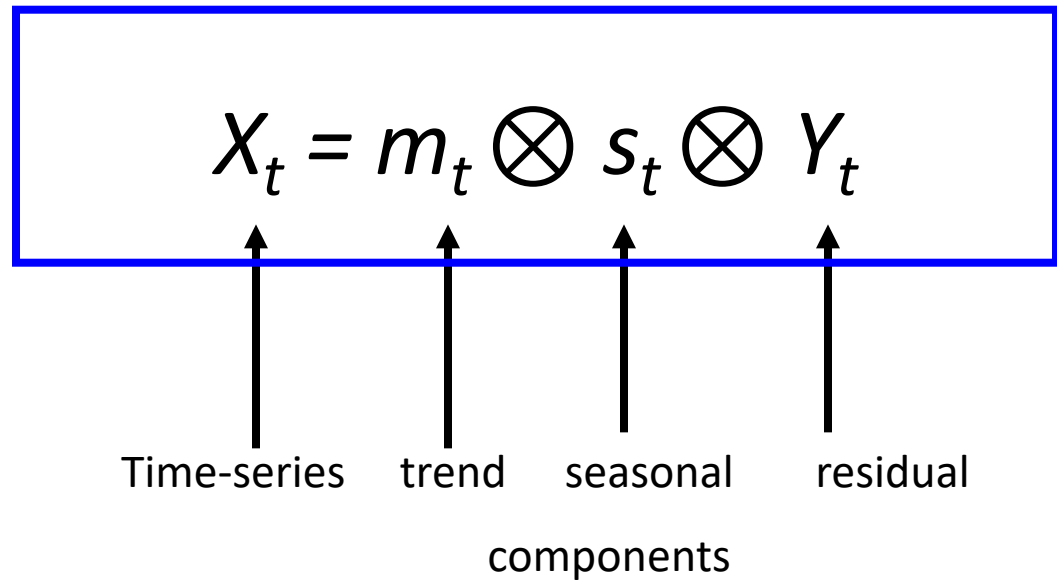
**White box - we know  
everything**

# Two simplified time series models

## Non-seasonal Decomposition Model with Trend



## Decomposition Model **with** Trend and **Seasonal** Components



# Non-seasonal Decomposition Models with Trend

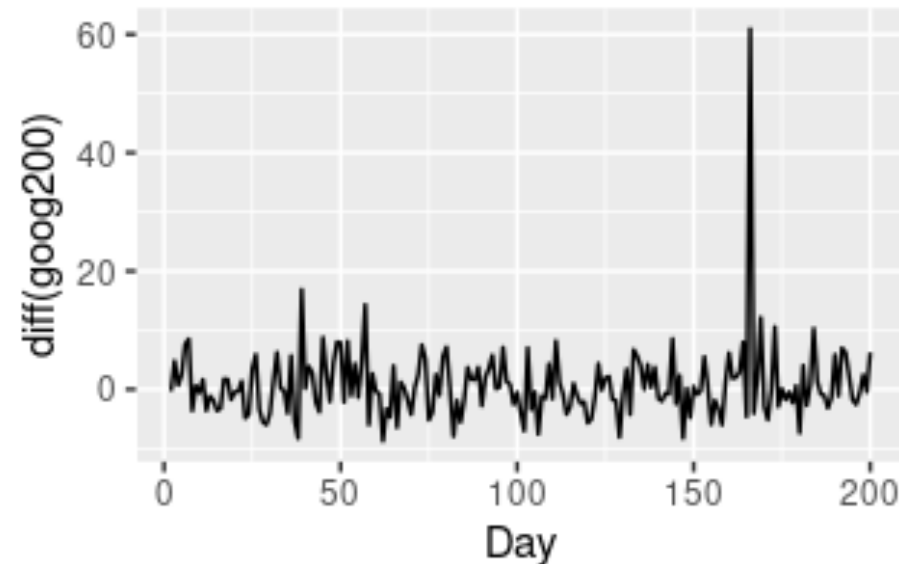
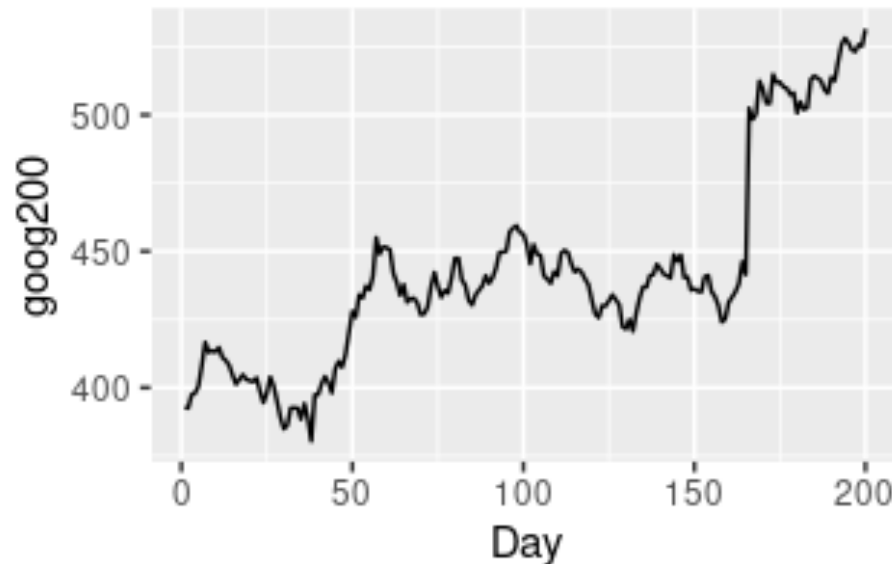
- There are two basic methods for estimating/removing trend:
- Method 1: *Trend elimination by differencing*
  - Differencing one or more times removes the trend
- Method 2: *Trend estimation by model fitting & removal*
  - first we estimate the trend fitting a model
  - then we remove it

# Non-seasonal Decomposition Models with Trend - Method 1

## Trend elimination by differencing

- Differencing of a time series  $\{X_t\}$  in discrete time  $t$  is the transformation of the series to a new time series  $\{D_t\}$  where the values are the differences between consecutive values of  $\{X_t\}$ .

$$d_t = x_t - x_{t-1}$$

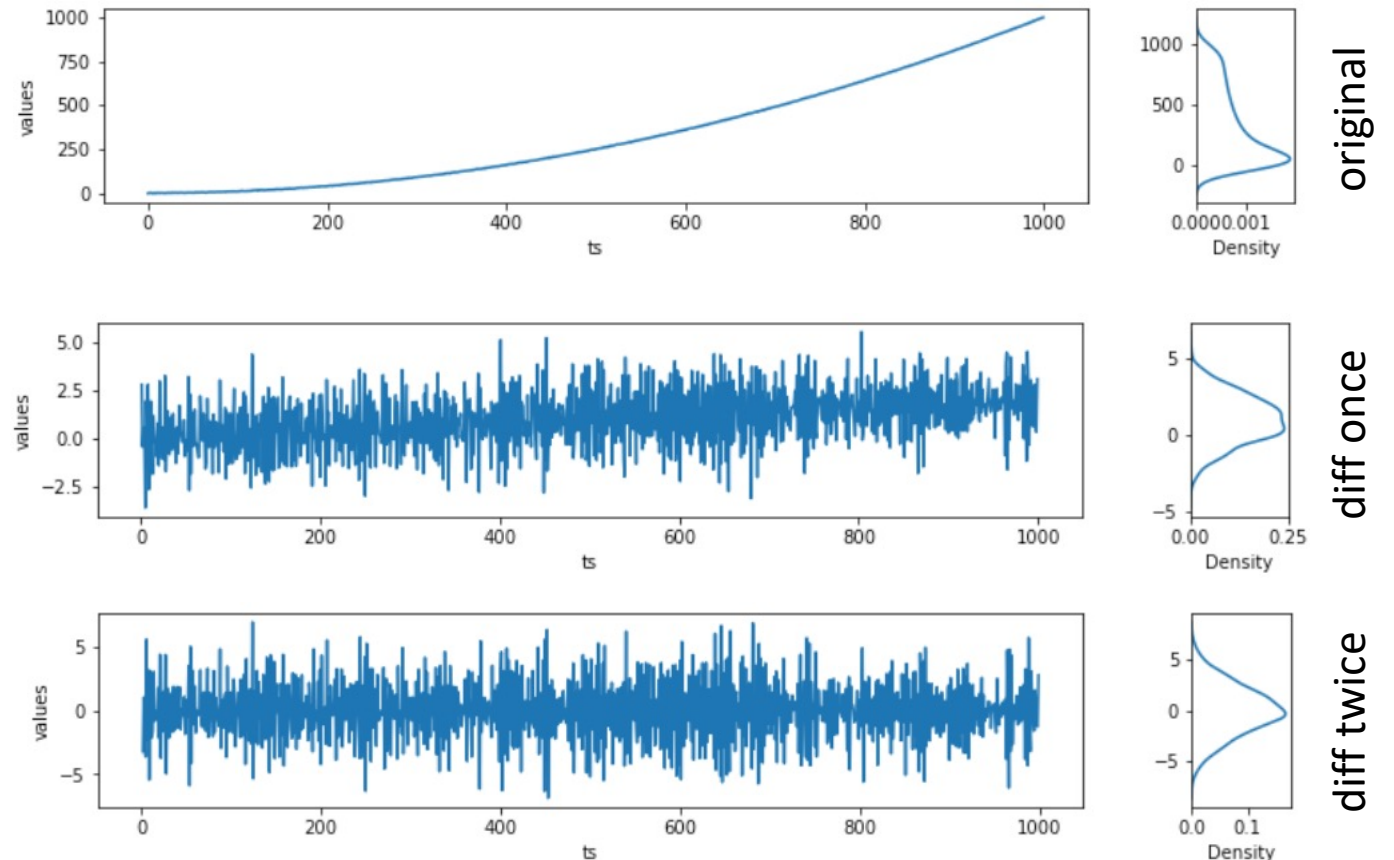




# Non-seasonal Decomposition Models with Trend - Method 1

## Trend elimination by differencing (cont.)

- If a trend is linear differencing once is sufficient to remove it
- If a trend is quadratic, you need two difference twice
- If a trend can be model with a polynomial of order  $n$ , then you need to difference  $n$  times



# Non-seasonal Decomposition Models with Trend - Method 1

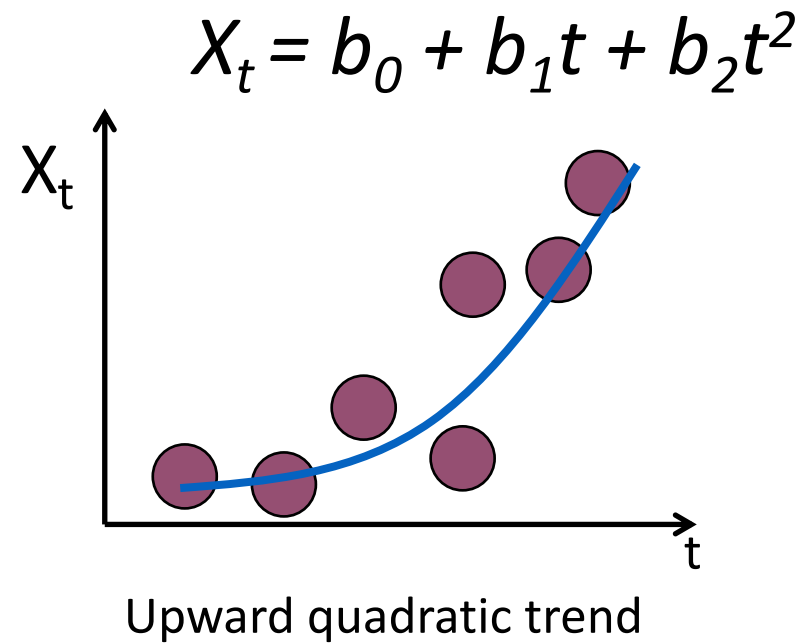
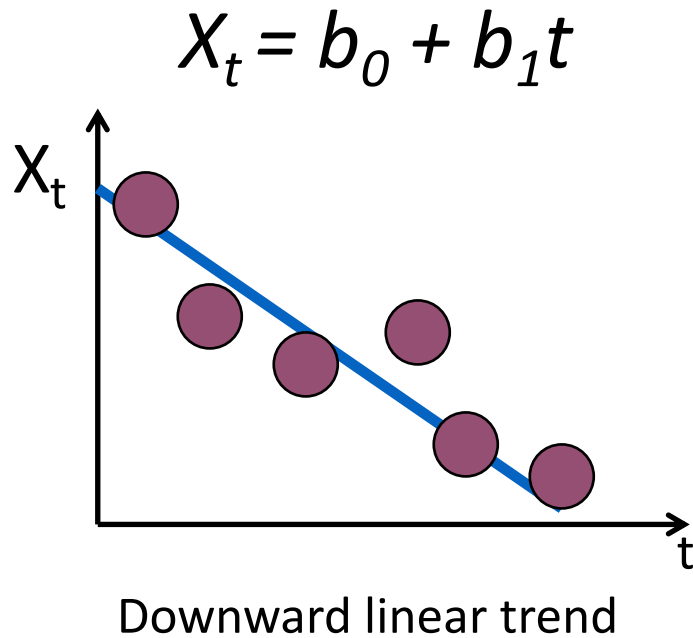
## Trend elimination by differencing

1. Generate synthetic data
2. Differencing one to remove a linear trend
3. Differencing twice to remove a quadratic trend



# Non-seasonal Decomposition Models with Trend - Method 2

## Trend estimation by model fitting & removal



# Non-seasonal Decomposition Models with Trend - Method 2

## Trend estimation by model fitting & removal

1. Generate synthetic data
2. Fit a linear regression
3. Fit a quadratic regression



# Non-seasonal Decomposition Models with Trend

## Combining Method 1 and 2

1. Generate synthetic data with quadratic trend
2. Differentiate
3. Observe the time-series still shows a trend
4. Detrend fitting a linear Regression



# Decomposition Models with Trend and Seasonality

There are four basic methods for estimating/removing the trend and seasonal components:

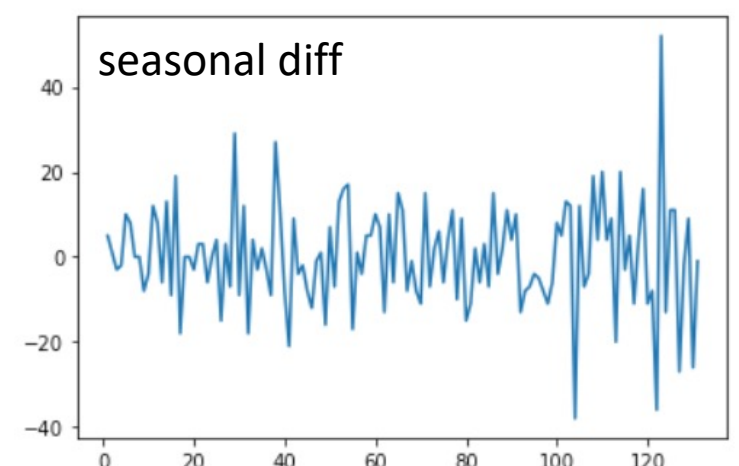
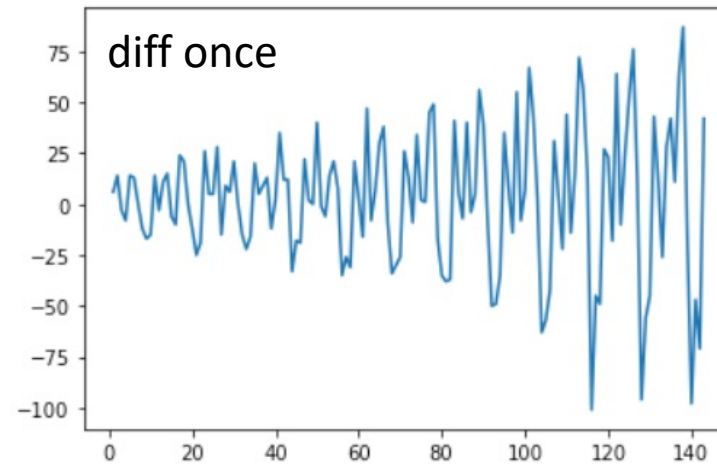
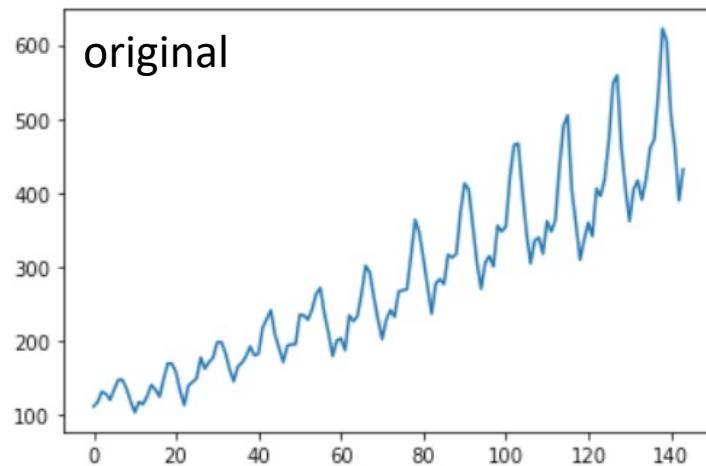
- Method 1 – Differencing
  - First we difference one or more time to remove the trend
  - Then we perform “**seasonal differencing**” to directly remove the season
- Method 2 - Filtering
  - First we estimate and remove the trend using a “**centered**” moving average
  - then we estimate and remove the seasonal component using “**periodic averages**”
- Method 3 - Joint-fit method
  - fitting a combined polynomial and **dynamic harmonic regression**

# Decomposition Models with Trend and Seasonality - Method 1

## Seasonal differencing

- Seasonal differencing of a time series  $\{X_t\}$  in discrete time  $t$  given the seasonality's period  $d$  is the transformation of the series to a new time series  $\{S_t\}$  where the values are the differences between the value of  $\{X_t\}$  at time  $t$  and the the value of  $\{X_t\}$  a period  $d$  before.

$$S_t = X_t - X_{t-d}$$



# Decomposition Models with Trend and Seasonality - Method 1

## Seasonal differencing

1. Generate synthetic data using a sine form
2. Apply "seasonal differencing"
3. Observe you removed the seasonality





# Decomposition Models with Trend and Seasonality - Method 2

## Estimate the trend using “centered” moving average

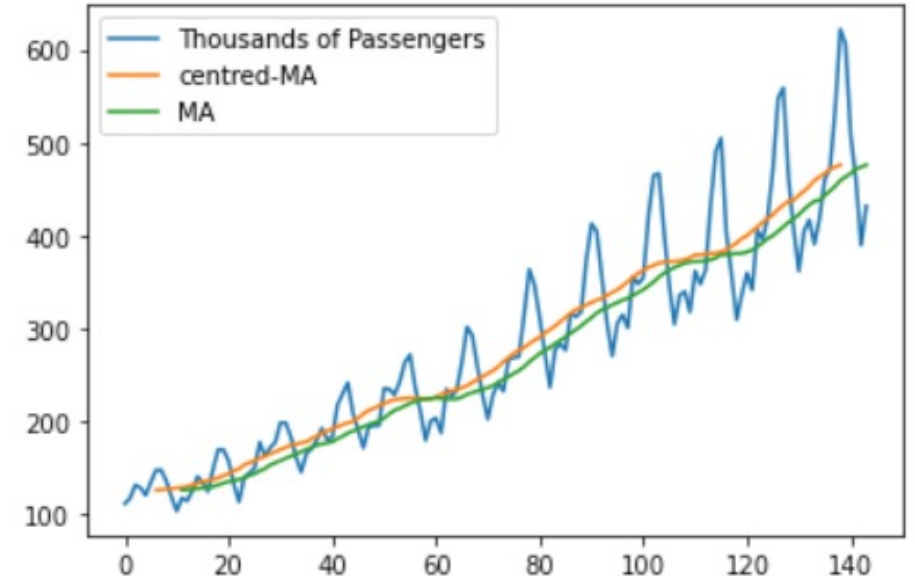
- Given the seasonality's period  $d$ 
  - If  $d$  is even, the «centered» moving average is defined as

$$\widehat{m}_t = (0.5x_{t-q} + x_{t-q+1} + \dots + x_{t+q-1} + 0.5x_{t+q})/d$$

- If  $d$  is odd, the «centered» moving average is defined as

$$\widehat{m}_t = (x_{t-q} + x_{t-q+1} + \dots + x_{t+q-1} + x_{t+q})/d$$

- Notes:
  - there are no values for either the first  $q$  or the last  $q$  data points, because we do not have enough observations on either side to define the moving average for those values of  $t$ .
  - This «centered» moving average is different from the «normal» moving average



# Decomposition Models with Trend and Seasonality - Method 2

## Estimate the seasonal component

- To estimate the seasonal component using “**periodic averages**”
  1. Divide the detrended value in seasons of length  $d$
  2. Compute the seasonal component values  $w_k$  by averaging each of the  $d$  points of the season (i.e.,  $k = 1, \dots, d$ )
  3. Compute adjusted the seasonal component values  $s_k$  to ensure that they add to zero

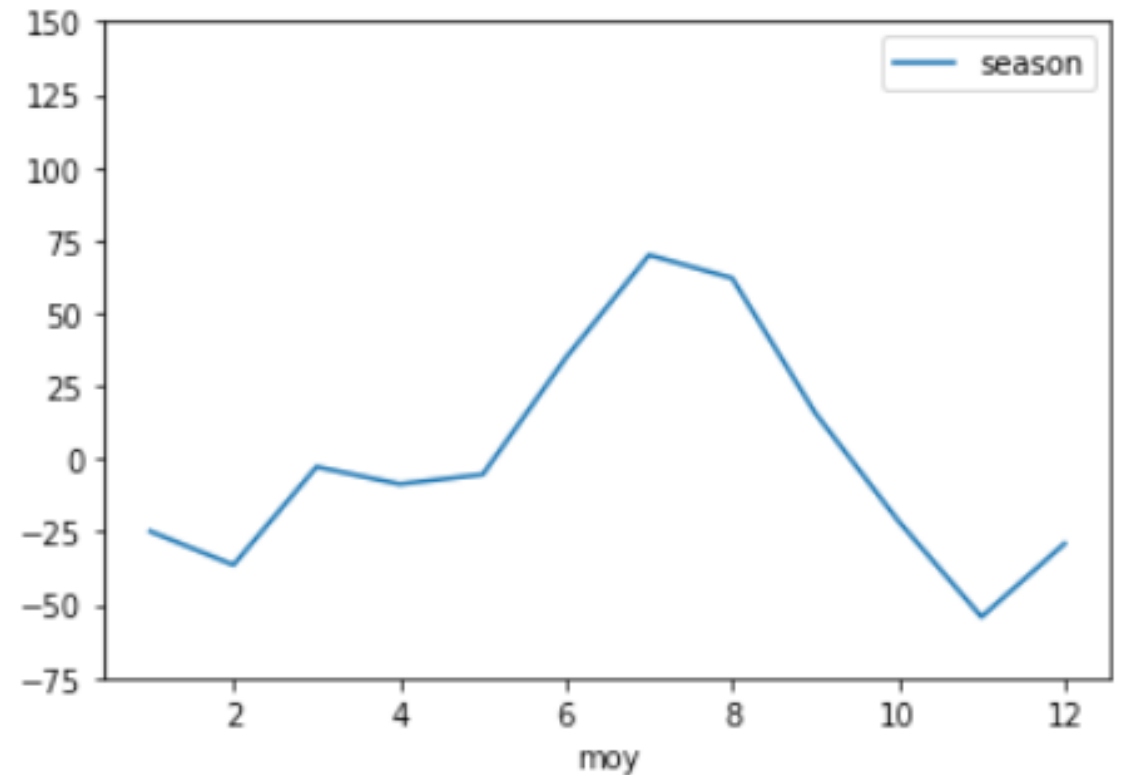
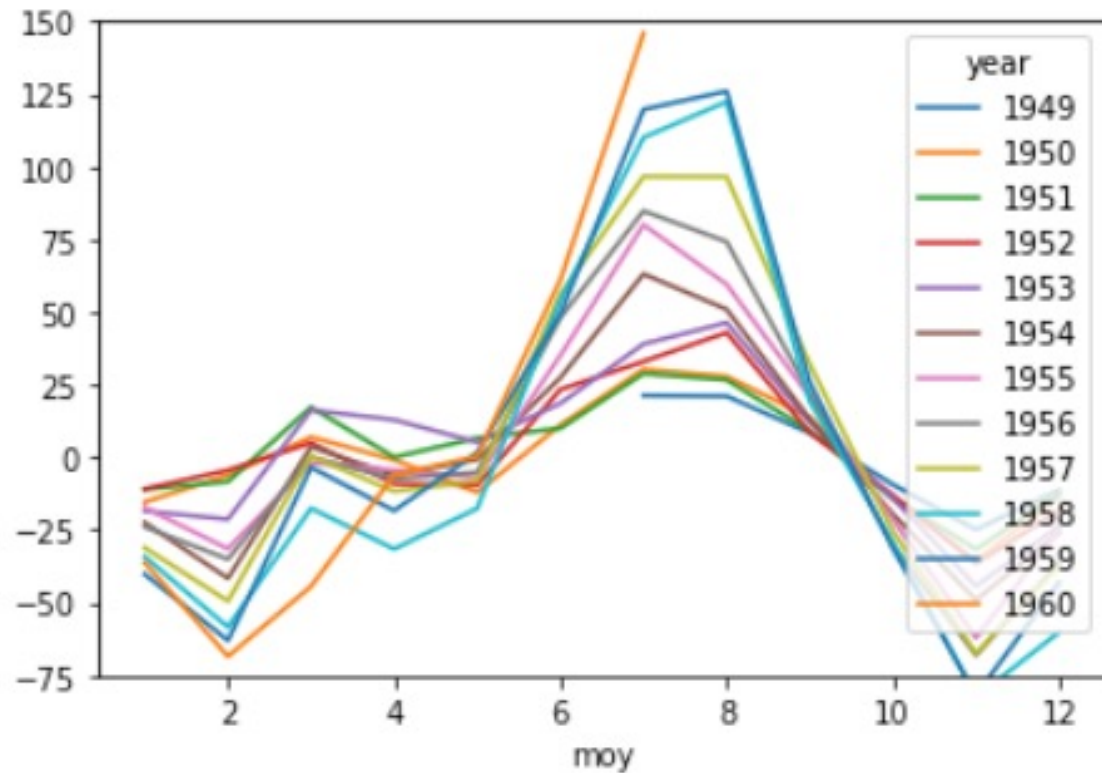
$$\hat{s}_k = w_k - d^{-1} \sum_{j=1}^d w_j, \quad k = 1, \dots, d$$

4. String together the adjusted seasonal component values in a sequence
5. Replace the sequence for each season

# Decomposition Models with Trend and Seasonality - Method 2

## Estimate the seasonal component

- The case of air travel time-series



# Decomposition Models with Trend and Seasonality - Method 2 Filtering

1. Load the time series
2. estimate the trend by the “centered” moving average
3. Remove the trend from the time series
4. Estimate the seasonal component using “**periodic averages**”
5. Remove it from the detrended series



# Decomposition Models with Trend and Seasonality - Method 3

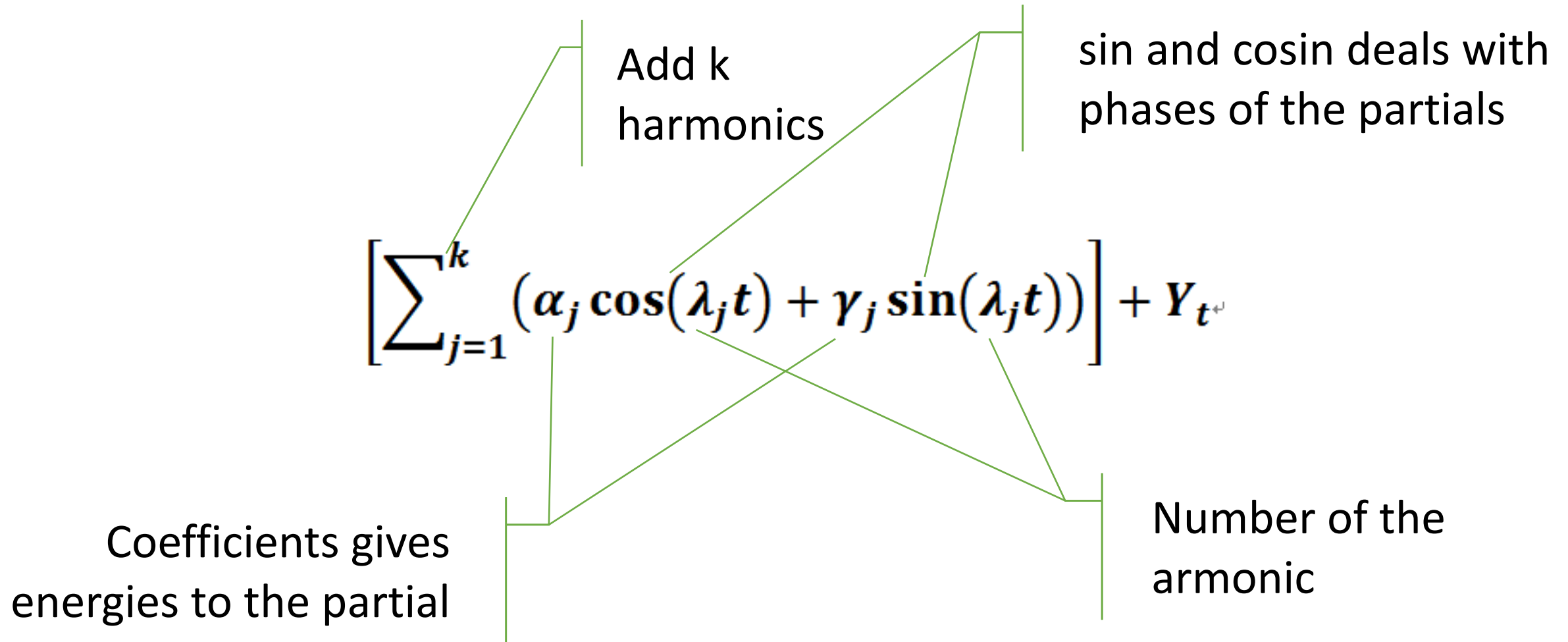
## Joint-fit method

- we can fit a combined polynomial linear regression and harmonic functions to estimate and then remove the trend and seasonal component simultaneously as the following

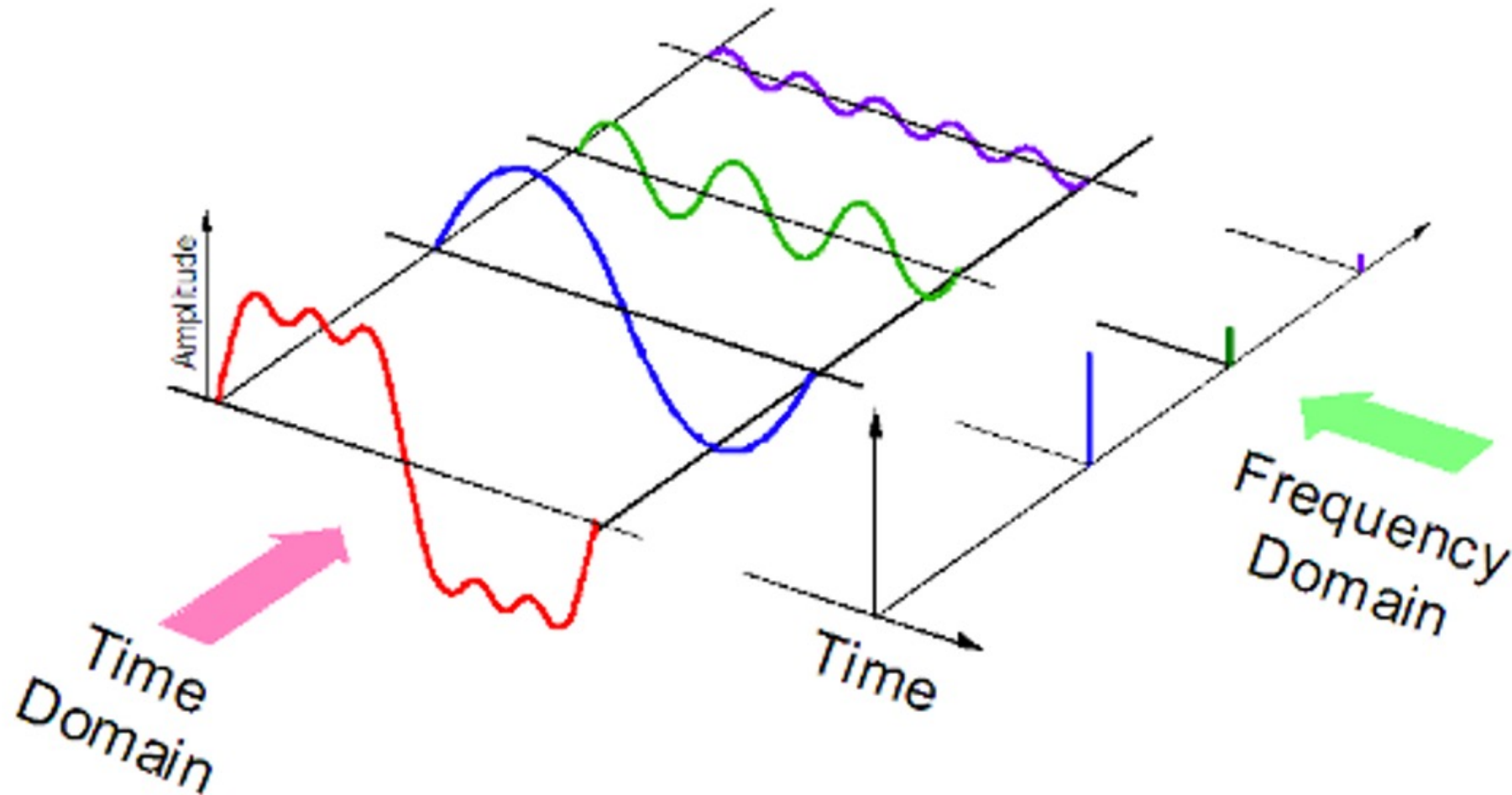
$$\begin{aligned} X_t &= m_t + s_t + Y_t \\ &= (\beta_0 + \beta_1 t + \beta_2 t^2) + \left[ \sum_{j=1}^k (\alpha_j \cos(\lambda_j t) + \gamma_j \sin(\lambda_j t)) \right] + Y_t \end{aligned}$$

# Decomposition Models with Trend and Seasonality - Method 3

## Anatomy of the harmonic function



# Decomposition Models with Trend and Seasonality - Method 3 Intuitively ....



# Decomposition & Detrending

## Quiz



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