Discussion: Heterogeneous Beliefs Recovery by Darius Nik Nejad

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Idea

- **Problem**: Capturing the beliefs of investors without assuming rationality ex-post (e.g. a representative agent).
- **Idea**: Distribution of beliefs weighted by wealth (no common initial wealth) can be recovered from observing equilibrium prices.
- **Why**: Improves our understanding of asset prices and expectations.

State of literature

- **Classic approach**: recover beliefs of a representative agent with time-additive utility. E.g. Ross (2015), considering Jackwerth and Manner (2020) find that Ross's recovery doesn't work empirically.
- Relaxing the assumption of a representative agent:
- **Down-top approach**: e.g. surveys, analyst forecasts (representativeness decreases). E.g. De La O and Myers (2021).
- **Top-down approach**: Calibrating a parametric distribution to observable prices (might be too restrictive). E.g. Atmaz, Basak (2018).
- **This paper approach**: Recover the exact distribution of beliefs weighted by wealth from equilibrium prices – within a theoretical framework.

Framework

- Economy: continuous-time, competitive, frictionless, complete, in dynamic general equilibrium, with one risky asset.
- Agents are CRRAs holding different fixed beliefs regarding the future growth of aggregate consumption (which is modeled as a drift).
- No restriction on the shape of beliefs except being bounded and the recovery does not depend on any specific probability measure (only on their existence).

Intuition

- The equilibrium market price of risk is shaped by the discrepancy between an individual investor's belief and the average belief (weighted by consumption) across the entire market.
- The equilibrium interest rate is a function of the degree of dispersion in beliefs among investors.
- The distribution of consumption shares across different beliefs dynamically shifts over time due to investors whose beliefs align better with realized consumption, they get richer and have a larger influence on prices.

Recovery

1. The market price of risk and the interest rate are described by the mean and variance of the distribution of beliefs weighted by consumption. These first two moments are revealed by equilibrium prices.
2. All other moments of the distribution may then be obtained recursively from the first two.
3. Since a bounded distribution is entirely determined by its moments following from Hausdorff moment problem, we can recover the distribution of beliefs weighted by wealth.
4. A clever methodology is developed for simulating the approach on discrete data.

Extensions require additional regularities

- **Multidimensional beliefs** via multiple risky assets: requires invertible volatility matrix process at all times (challenging empirically).
- **Heterogeneous preferences**: requires all agents to agree on at least one element of the drift of the good available for consumption.
- **Stochastic beliefs**: requires pseudo-moments which the span of stochastic components can approximate any continuous function defined on the belief space arbitrarily well, which is a stronger assumption and only holds in certain cases. Think of the stochastic components being all cyclical (seasonality?) functions, their span wouldn't be dense in C.

Limits of application

- While the assumption of boundness is reasonable (limited optimism/pessimism), the continuous-time economy is essential for the recovery to be realistic.
- Parametrizing the set of sought distributions and minimizing the loss given discrete points is a clever twist to circumvent the non-uniform convergence of the time-derivative as Δt goes to zero.
- However, the loss is not necessarily convex and the ex-ante parametrization is an issue per se, something the literature is already struggling with empirically.

Suggestions

- Test methodology together with the extensions (simulation or data).
- Is there a specific optimal method for the parametrization of the loss in the methodology that works best?
- Heterogeneous information channels extension: in the baseline model private info is inconsequential given the recovery assumptions, what if information affects the recovery via heterogeneous signals?

Concluding remarks

- Insightful work.
- Recovery is supposed to be a benchmark that can be used to isolate interesting frictions. A full-fledged empirical application is outside of the scope of the paper.
- Providing deeper economic intuition also regarding the extensions would be helpful.
- Welfare costs and risks that could amplify when relaxing completeness (e.g. Martin, Papadimitriou 2022).