Technische Universität Graz
Fraunhofer Austria Research GmbH

Geometrisches 3D-Modellieren in der Computergrafik

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http://www.cgv.tugraz.at, http://www.fraunhofer.at

Assignment #4
Deadline:
June 25th, 2023

Task 4: Bézier Techniques

The points

$$B_0(1|1), \quad B_1(2|4), \quad B_2(4|5), \quad B_3(5|3), \quad B_4(7|5)$$

are the control points of a Bézier curve defined on the parameter domain $[0, 1]$.

1. Evaluate the curve at parameter $t_0 = \frac{1}{2}$ and sketch the curve in rough strokes.
2. If one divides the Bézier curve in the curve point of the parameter t_0, the result is two partial curves. Specify the Bézier points of both partial curves.
3. Represent the Bézier curve with the control points B_0, ..., B_4 by a Bézier curve with degree 5. Determine the necessary Bézier points of the degree elevated curve (geometrically or algebraically).
4. How must the points of a curve connecting in the point B_4 be chosen, so that the junction at B_4 is
   (a) G^1 continuous,
   (b) C^1 continuous,
   (c) C^2 continuous?

Please prepare a PDF containing your submission. This task will be evaluated with 4 points.

Task 5: Bézier Surface Design

The Utah Teapot is designed using bicubic Bézier surfaces (see Frank Crow, „The Origins of the Teapot“, IEEE Computer Graphics and Applications, 7:8–19, 1987).

1. The geometric data is stored in the template `teapot.ecs`. Implement a function in this file that can evaluate bicubic Bézier surfaces at arbitrary parameter values `(u, v)`.
2. Use the function from (1.) to evaluate the Utah teapot and create a high-resolution, polygonal model.
3. Analogous to the function from (1.), create a function which returns the mean curvature of a Bézier surface at arbitrary parameter values `(u, v)`.
4. Extend the program from (2.) and colorize the Utah teapot with vertex colors according to its mean curvature.

The submission of this task can be done as ECS file (The programming language *Euclides* and its documentation can be downloaded at http://euclides.cgv.tugraz.at/.) or as JavaScript/X3DOM file. This task will be evaluated with 8 points. Up to 3 additional points will be awarded for functionality, readability, documentation, bug reports, etc.

Appendix: teapot.ecs

```
import 'blas'
import 'cadpolyface'
import 'cadpolyfacetools'
import 'color'
import 'io'
import 'math'

//
// The teapot vertices in a x-y-z-sequence.
//
const teapotVS = [
    1.4, 0.0, 2.4,              1.4, -0.784, 2.4,           0.784, -1.4, 2.4,
    0.0, -1.4, 2.4,             1.3375, 0.0, 2.53125,       1.3375, -0.749, 2.53125,
    0.749, -1.3375, 2.53125,    0.0, -1.3375, 2.53125,      1.4375, 0.0, 2.53125,
    1.4375, -0.805, 2.53125,    0.805, -1.4375, 2.53125,    0.0, -1.4375, 2.53125,
    1.5, 0.0, 2.4,              1.5, -0.84, 2.4,            0.84, -1.5, 2.4,
    0.0, -1.5, 2.4,             -0.784, -1.4, 2.4,          -1.4, -0.784, 2.4,
    -1.4, 0.0, 2.4,             -0.749, -1.3375, 2.53125,   -1.3375, -0.749, 2.53125,
    -1.3375, 0.0, 2.53125,      -0.805, -1.4375, 2.53125,   -1.4375, -0.805, 2.53125,
    -1.4375, 0.0, 2.53125,      -0.84, -1.5, 2.4,           -1.5, -0.84, 2.4,
    -1.5, 0.0, 2.4,             -1.4, 0.784, 2.4,           -0.784, 1.4, 2.4,
    0.0, 1.4, 2.4,              -1.3375, 0.749, 2.53125,    -0.749, 1.3375, 2.53125,
    0.0, 1.3375, 2.53125,       -1.4375, 0.805, 2.53125,    -0.805, 1.4375, 2.53125,
    0.0, 1.4375, 2.53125,       -1.5, 0.84, 2.4,            -0.84, 1.5, 2.4,
    0.0, 1.5, 2.4,              0.784, 1.4, 2.4,            1.4, 0.784, 2.4,
    0.749, 1.3375, 2.53125,     1.3375, 0.749, 2.53125,     0.805, 1.4375, 2.53125,
    1.4375, 0.805, 2.53125,     0.84, 1.5, 2.4,             1.5, 0.84, 2.4,
    1.75, 0.0, 1.875,           1.75, -0.98, 1.875,         0.98, -1.75, 1.875,
    0.0, -1.75, 1.875,          2.0, 0.0, 1.35,             2.0, -1.12, 1.35,
    1.12, -2.0, 1.35,           0.0, -2.0, 1.35,            2.0, 0.0, 0.9,
    2.0, -1.12, 0.9,            1.12, -2.0, 0.9,            0.0, -2.0, 0.9,
    -0.98, -1.75, 1.875,        -1.75, -0.98, 1.875,        -1.75, 0.0, 1.875,
    -1.12, -2.0, 1.35,          -2.0, -1.12, 1.35,          -2.0, 0.0, 1.35,
    -1.12, -2.0, 0.9,           -2.0, -1.12, 0.9,           -2.0, 0.0, 0.9,
    -1.75, 0.98, 1.875,         -0.98, 1.75, 1.875,         0.0, 1.75, 1.875,
    -2.0, 1.12, 1.35,           -1.12, 2.0, 1.35,           0.0, 2.0, 1.35,
    -2.0, 1.12, 0.9,            -1.12, 2.0, 0.9,            0.0, 2.0, 0.9,
    0.98, 1.75, 1.875,          1.75, 0.98, 1.875,          1.12, 2.0, 1.35,
    2.0, 1.12, 1.35,            1.12, 2.0, 0.9,             2.0, 1.12, 0.9,
    2.0, 0.0, 0.45,             2.0, -1.12, 0.45,           1.12, -2.0, 0.45,
    0.0, -2.0, 0.45,            1.5, 0.0, 0.225,            1.5, -0.84, 0.225,
    0.84, -1.5, 0.225,          0.0, -1.5, 0.225,           1.5, 0.0, 0.15,
    1.5, -0.84, 0.15,           0.84, -1.5, 0.15,           0.0, -1.5, 0.15,
    -1.12, -2.0, 0.45,          -2.0, -1.12, 0.45,          -2.0, 0.0, 0.45,
    -0.84, -1.5, 0.225,         -1.5, -0.84, 0.225,         -1.5, 0.0, 0.225,
    -0.84, -1.5, 0.15,          -1.5, -0.84, 0.15,          -1.5, 0.0, 0.15,
    -2.0, 1.12, 0.45,           -1.12, 2.0, 0.45,           0.0, 2.0, 0.45,
    -1.5, 0.84, 0.225,          -0.84, 1.5, 0.225,          0.0, 1.5, 0.225,
    -1.5, 0.84, 0.15,           -0.84, 1.5, 0.15,           0.0, 1.5, 0.15,
    1.12, 2.0, 0.45,            2.0, 1.12, 0.45,            0.84, 1.5, 0.225,
    1.5, 0.84, 0.225,           0.84, 1.5, 0.15,            1.5, 0.84, 0.15,
    -1.6, 0.0, 2.025,           -1.6, -0.3, 2.025,          -1.5, -0.3, 2.25,
    -1.5, 0.0, 2.25,            -2.3, 0.0, 2.025,           -2.3, -0.3, 2.025,
    -2.5, -0.3, 2.25,           -2.5, 0.0, 2.25,            -2.7, 0.0, 2.025,
    -2.7, -0.3, 2.025,          -3.0, -0.3, 2.25,           -3.0, 0.0, 2.25,
    -2.7, 0.0, 1.8,             -2.7, -0.3, 1.8,            -3.0, -0.3, 1.8,
    -3.0, 0.0, 1.8,             -1.5, 0.3, 2.25,            -1.6, 0.3, 2.025,
    -2.5, 0.3, 2.25,            -2.3, 0.3, 2.025,           -3.0, 0.3, 2.25,
    -2.7, 0.3, 2.025,           -3.0, 0.3, 1.8,             -2.7, 0.3, 1.8,
    -2.7, 0.0, 1.575,           -2.7, -0.3, 1.575,          -3.0, -0.3, 1.35,
    -3.0, 0.0, 1.35,            -2.5, 0.0, 1.125,           -2.5, -0.3, 1.125,
    -2.65, -0.3, 0.9375,        -2.65, 0.0, 0.9375,         -2.0, -0.3, 0.9,
    -1.9, -0.3, 0.6,            -1.9, 0.0, 0.6,             -3.0, 0.3, 1.35,
    -2.7, 0.3, 1.575,           -2.65, 0.3, 0.9375,         -2.5, 0.3, 1.125,
    -1.9, 0.3, 0.6,             -2.0, 0.3, 0.9,             1.7, 0.0, 1.425,
    1.7, -0.66, 1.425,          1.7, -0.66, 0.6,            1.7, 0.0, 0.6,
    2.6, 0.0, 1.425,            2.6, -0.66, 1.425,          3.1, -0.66, 0.825,
    3.1, 0.0, 0.825,            2.3, 0.0, 2.1,              2.3, -0.25, 2.1,
    2.4, -0.25, 2.025,          2.4, 0.0, 2.025,            2.7, 0.0, 2.4,
    2.7, -0.25, 2.4,            3.3, -0.25, 2.4,            3.3, 0.0, 2.4,
    1.7, 0.66, 0.6,             1.7, 0.66, 1.425,           3.1, 0.66, 0.825,
```

```
    2.6, 0.66, 1.425,          2.4, 0.25, 2.025,          2.3, 0.25, 2.1,
    3.3, 0.25, 2.4,            2.7, 0.25, 2.4,            2.8, 0.0, 2.475,
    2.8, -0.25, 2.475,         3.525, -0.25, 2.49375,     3.525, 0.0, 2.49375,
    2.9, 0.0, 2.475,           2.9, -0.15, 2.475,         3.45, -0.15, 2.5125,
    3.45, 0.0, 2.5125,         2.8, 0.0, 2.4,             2.8, -0.15, 2.4,
    3.2, -0.15, 2.4,           3.2, 0.0, 2.4,             3.525, 0.25, 2.49375,
    2.8, 0.25, 2.475,          3.45, 0.15, 2.5125,        2.9, 0.15, 2.475,
    3.2, 0.15, 2.4,            2.8, 0.15, 2.4,            0.0, 0.0, 3.15,
    0.0, -0.002, 3.15,         0.002, 0.0, 3.15,          0.8, 0.0, 3.15,
    0.8, -0.45, 3.15,          0.45, -0.8, 3.15,          0.0, -0.8, 3.15,
    0.0, 0.0, 2.85,            0.2, 0.0, 2.7,             0.2, -0.112, 2.7,
    0.112, -0.2, 2.7,          0.0, -0.2, 2.7,            -0.002, 0.0, 3.15,
    -0.45, -0.8, 3.15,         -0.8, -0.45, 3.15,         -0.8, 0.0, 3.15,
    -0.112, -0.2, 2.7,         -0.2, -0.112, 2.7,         -0.2, 0.0, 2.7,
    0.0, 0.002, 3.15,          -0.8, 0.45, 3.15,          -0.45, 0.8, 3.15,
    0.0, 0.8, 3.15,            -0.2, 0.112, 2.7,          -0.112, 0.2, 2.7,
    0.0, 0.2, 2.7,             0.45, 0.8, 3.15,           0.8, 0.45, 3.15,
    0.112, 0.2, 2.7,           0.2, 0.112, 2.7,           0.4, 0.0, 2.55,
    0.4, -0.224, 2.55,         0.224, -0.4, 2.55,         0.0, -0.4, 2.55,
    1.3, 0.0, 2.55,            1.3, -0.728, 2.55,         0.728, -1.3, 2.55,
    0.0, -1.3, 2.55,           1.3, 0.0, 2.4,             1.3, -0.728, 2.4,
    0.728, -1.3, 2.4,          0.0, -1.3, 2.4,            -0.224, -0.4, 2.55,
    -0.4, -0.224, 2.55,        -0.4, 0.0, 2.55,           -0.728, -1.3, 2.55,
    -1.3, -0.728, 2.55,        -1.3, 0.0, 2.55,           -0.728, -1.3, 2.4,
    -1.3, -0.728, 2.4,         -1.3, 0.0, 2.4,            -0.4, 0.224, 2.55,
    -0.224, 0.4, 2.55,         0.0, 0.4, 2.55,            -1.3, 0.728, 2.55,
    -0.728, 1.3, 2.55,         0.0, 1.3, 2.55,            -1.3, 0.728, 2.4,
    -0.728, 1.3, 2.4,          0.0, 1.3, 2.4,             0.224, 0.4, 2.55,
    0.4, 0.224, 2.55,          0.728, 1.3, 2.55,          1.3, 0.728, 2.55,
    0.728, 1.3, 2.4,           1.3, 0.728, 2.4,           0.0, 0.0, 0.0,
    1.5, 0.0, 0.15,            1.5, 0.84, 0.15,           0.84, 1.5, 0.15,
    0.0, 1.5, 0.15,            1.5, 0.0, 0.075,           1.5, 0.84, 0.075,
    0.84, 1.5, 0.075,          0.0, 1.5, 0.075,           1.425, 0.0, 0.0,
    1.425, 0.798, 0.0,         0.798, 1.425, 0.0,         0.0, 1.425, 0.0,
    -0.84, 1.5, 0.15,          -1.5, 0.84, 0.15,          -1.5, 0.0, 0.15,
    -0.84, 1.5, 0.075,         -1.5, 0.84, 0.075,         -1.5, 0.0, 0.075,
    -0.798, 1.425, 0.0,        -1.425, 0.798, 0.0,        -1.425, 0.0, 0.0,
    -1.5, -0.84, 0.15,         -0.84, -1.5, 0.15,         0.0, -1.5, 0.15,
    -1.5, -0.84, 0.075,        -0.84, -1.5, 0.075,        0.0, -1.5, 0.075,
    -1.425, -0.798, 0.0,       -0.798, -1.425, 0.0,       0.0, -1.425, 0.0,
    0.84, -1.5, 0.15,          1.5, -0.84, 0.15,          0.84, -1.5, 0.075,
    1.5, -0.84, 0.075,         0.798, -1.425, 0.0,        1.425, -0.798, 0.0 ];

//
// The teapot patches; each patch consists of 4x4 Bezier points.
//
const teapotBP = [
    [1,2,3,4,           5,6,7,8,           9,10,11,12,        13,14,15,16],
    [4,17,18,19,        8,20,21,22,        12,23,24,25,       16,26,27,28],
    [19,29,30,31,       22,32,33,34,       25,35,36,37,       28,38,39,40],
    [31,41,42,1,        34,43,44,5,        37,45,46,9,        40,47,48,13],
    [13,14,15,16,       49,50,51,52,       53,54,55,56,       57,58,59,60],
    [16,26,27,28,       52,61,62,63,       56,64,65,66,       60,67,68,69],
    [28,38,39,40,       63,70,71,72,       66,73,74,75,       69,76,77,78],
    [40,47,48,13,       72,79,80,49,       75,81,82,53,       78,83,84,57],
    [57,58,59,60,       85,86,87,88,       89,90,91,92,       93,94,95,96],
    [60,67,68,69,       88,97,98,99,       92,100,101,102,    96,103,104,105],
    [69,76,77,78,       99,106,107,108,    102,109,110,111,   105,112,113,114],
    [78,83,84,57,       108,115,116,85,    111,117,118,89,    114,119,120,93],
    [121,122,123,124,   125,126,127,128,   129,130,131,132,   133,134,135,136],
    [124,137,138,121,   128,139,140,125,   132,141,142,129,   136,143,144,133],
    [133,134,135,136,   145,146,147,148,   149,150,151,152,   69,153,154,155],
    [136,143,144,133,   148,156,157,145,   152,158,159,149,   155,160,161,69],
    [162,163,164,165,   166,167,168,169,   170,171,172,173,   174,175,176,177],
    [165,178,179,162,   169,180,181,166,   173,182,183,170,   177,184,185,174],
    [174,175,176,177,   186,187,188,189,   190,191,192,193,   194,195,196,197],
    [177,184,185,174,   189,198,199,186,   193,200,201,190,   197,202,203,194],
    [204,204,204,204,   207,208,209,210,   211,211,211,211,   212,213,214,215],
    [204,204,204,204,   210,217,218,219,   211,211,211,211,   215,220,221,222],
    [204,204,204,204,   219,224,225,226,   211,211,211,211,   222,227,228,229],
    [204,204,204,204,   226,230,231,207,   211,211,211,211,   229,232,233,212],
    [212,213,214,215,   234,235,236,237,   238,239,240,241,   242,243,244,245],
    [215,220,221,222,   237,246,247,248,   241,249,250,251,   245,252,253,254],
```

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    [222,227,228,229,    248,255,256,257,    251,258,259,260,    254,261,262,263],
    [229,232,233,212,    257,264,265,234,    260,266,267,238,    263,268,269,242],
    [270,270,270,270,    279,280,281,282,    275,276,277,278,    271,272,273,274],
    [270,270,270,270,    282,289,290,291,    278,286,287,288,    274,283,284,285],
    [270,270,270,270,    291,298,299,300,    288,295,296,297,    285,292,293,294],
    [270,270,270,270,    300,305,306,279,    297,303,304,275,    294,301,302,271]];

//
// The resulting teapot polyfaces:
//
var teapotPatches = [ ];
//
// Iterate of all patches:
//
for(var patchIndex : patch in teapotBP) {
    var vertices = <| 16 * 3 |>;
    var colors   = <| 16 * 3 |>;
    var faces    = [ ];

    for (var vertexIndex : vertexRef in patch) {
        // copy vertex coordinates into new vector
        var x = teapotVS@(3*(vertexRef-1)+0);
        var y = teapotVS@(3*(vertexRef-1)+1);
        var z = teapotVS@(3*(vertexRef-1)+2);
        vertices@(3*vertexIndex+0) = x;
        vertices@(3*vertexIndex+1) = y;
        vertices@(3*vertexIndex+2) = z;
        // generate a "nice" color
        var index = integer(round(norm2(<x, y, z> - < 0.0, 0.0, 1.75>) * 25.0));
        var color = ColorMaps@"BlackBodyRadiation"@index;
        colors@(3*vertexIndex+0) = color@0;
        colors@(3*vertexIndex+1) = color@1;
        colors@(3*vertexIndex+2) = color@2;
    }
    // all patches have the same topology
    faces = [ [ 0,  1,  5,  4], [ 1,  2,  6,  5], [ 2,  3,  7,  6],
              [ 4,  5,  9,  8], [ 5,  6, 10,  9], [ 6,  7, 11, 10],
              [ 8,  9, 13, 12], [ 9, 10, 14, 13], [10, 11, 15, 14]];
    //
    teapotPatches@"back"(cadpolyface(vertices, faces, undefined, colors));
}

var scene = cadPolyFaceTools@"merge"(teapotPatches, "Teapot");
fileOut(scene@"toFileZippedX3D"());
```