De Morgan's Theorem

Why?

De Morgan's Theorem allows us to easily change a Boolean function from Sum of Products (SOP) form to Product of Sums (POS) form, or vice versa.

SOP to POS

Suppose we have $F = A \cdot B + \overline{A} \cdot C$. This function is in SOP form.

To convert it to POS form:

1. **Invert the Function:**

$$F' = (A \cdot B + \overline{A} \cdot C)'$$

2. **Apply De Morgan's Laws:**

$$F' = (A' + B') \cdot (A + C')$$

Now, the function is in POS form. To convert back to SOP form, apply De Morgan's laws again:

1. **Invert the Function:**

$$G = (A' + B') \cdot (A + C')$$

$$G' = [(A' + B') \cdot (A + C')]'$$

2. **Apply De Morgan's Laws Separately:**

$$G' = (A' + B')' + (A + C')'$$

$$= (A \cdot B) + (\overline{A} \cdot C)$$

Note: K-maps can also be used for this, but they may be slower and take up more space on paper. 😄

Dual of Functions

Finding the dual of a Boolean function is similar to converting between POS and SOP forms. I recommend using De Morgan's laws, but K-maps are an alternative if you prefer.

Example

Given $F = 1 + B$:

1. **Find the Dual:**

$$F' = (1 + B)' = 0 \cdot B' = 0$$

   (0 ANDed with anything is always 0. Note that this fact is useful for bit masking, which we might find useful later in the assembly project.)

Minterms

Minterms are product terms in which each variable appears exactly once, and they represent conditions where the Boolean function is true (output = 1).

How to Obtain SOM Canonical Form:

1. **Create a Truth Table:** List all possible combinations of input variables and their corresponding output values.
2. **Identify Minterms:** For each row in the truth table where the output is 1, write down the corresponding minterm.
3. **Combine Minterms:** Sum (OR) all the identified minterms to form the canonical expression.

Example

Given $F(A, B, C) = 1$ when $A = 0, B = 1, C = 0$.

- With the given information, one minterm is $\overline{A} \cdot B \cdot \overline{C}$.
- If given a full truth table, you can find all minterms and implement the function in SOP form.

Thus, a function H might be represented as:

$$H = (A \cdot B) + (\overline{B} \cdot A) + \dots$$

Maxterms

Maxterms are sum terms in which each variable appears exactly once, and they represent conditions where the Boolean function is false (output = 0).

How to Obtain POM Canonical Form:

1. **Create a Truth Table:** List all possible combinations of input variables and their corresponding output values.
2. **Identify Maxterms:** For each row in the truth table where the output is 0, write down the corresponding maxterm.
3. **Combine Maxterms:** Take the product (AND) of all the identified maxterms to form the canonical expression.

Example

Given $F(A, B, C) = 1$ when $A = 0, B = 1, C = 0$.

- With the given information, one maxterm is $A + \overline{B} + C$.
- If given a full truth table, you can find all maxterms and implement the function in POM form.

Thus, a function H might be represented as:

$$H = (A + B) \cdot (\overline{B} + A) \cdot \ldots$$

Note: Expressing functions in minterms or maxterms is considered a "canonical" form (standard, but not necessarily simplified).