CS 3823 - Theory of Computation: Homework Assignment 1

Parker Hix

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Set Theory

i. A is not a subset of B. B does not contain y or z.

ii. $\{y, z\}$

iii. $\{(x, a), (x, b), (x, x), (y, a), (y, b), (y, x), (z, a), (z, b), (z, x)\}$

iv. $\{\emptyset, \{x, y, z\}, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}\}$

Induction

We want to show that $f(n) = 11^{n+2} + 12^{2n+1}$ is divisible by 133 for all integers n. In other words, we want to show that $f(n) = 133x$ for some integer x.

Base Case:

Let $n = 0$. Then

$$f(0) = 11^{0+2} + 12^{2\cdot 0+1} = 11^2 + 12^1 = 121 + 12 = 133.$$

Clearly, $133 = 133 \cdot 1$, so $x = 1$ is an integer. Therefore, the base case is true.

Inductive Hypothesis:

Assume that for some integer k, the statement is true, i.e.,

$$f(k) = 11^{k+2} + 12^{2k+1} = 133z$$

for some integer z. This implies

$$11^{k+2} = 133z - 12^{2k+1}.$$

Inductive Step:

We need to show that

$$f(k+1) = 11^{(k+1)+2} + 12^{2(k+1)+1}$$

is also divisible by 133. Simplify $f(k+1)$:

$$f(k+1) = 11^{k+3} + 12^{2k+3}.$$

We can rewrite 11^{k+3} using the inductive hypothesis:

$$11^{k+3} = 11 \cdot 11^{k+2}.$$

Substitute 11^{k+2} from the inductive hypothesis:

$$11^{k+3} = 11 \cdot (133z - 12^{2k+1}) = 133 \cdot 11z - 11 \cdot 12^{2k+1}.$$

Now, add 12^{2k+3} to this:

$$f(k+1) = 11^{k+3} + 12^{2k+3} = 133 \cdot 11z - 11 \cdot 12^{2k+1} + 12^{2k+3}.$$

Notice that

$$12^{2k+3} = 12^2 \cdot 12^{2k+1} = 144 \cdot 12^{2k+1}.$$

Thus,

$$f(k+1) = 133 \cdot 11z - 11 \cdot 12^{2k+1} + 144 \cdot 12^{2k+1} = 133 \cdot 11z + (144 - 11) \cdot 12^{2k+1}.$$

$$f(k+1) = 133 \cdot 11z + 133 \cdot 12^{2k+1}.$$

$$f(k+1) = 133(11z + 12^{2k+1}).$$

Since $11z + 12^{2k+1}$ is an integer, $f(k+1)$ is divisible by 133. Therefore, the inductive step is true.

Thus, by induction, $f(n) = 11^{n+2} + 12^{2n+1}$ is divisible by 133 for all integers n.