Three points correlators

Kaons oscillations like

Emanuele Rosi, October 2023

Abstract

Calculation of flavour non-singlet meson oscillations through the insertion of intermediate operators and stochastic sources in QCD with open boundary conditions.

1 Wick Contractions

We want to calculate three points correlators of this two types:

$$\begin{aligned}
G_d(x_0,y_0,z_0) &= \sum_{\vec{x},\vec{y},\vec{z}} \left\langle \bar{\psi}_4(x)\Gamma_A\psi_1(x)\bar{\psi}_3(y)\Gamma_D\psi_2(y)\bar{\psi}_1(y)\Gamma_B\psi_4(y)\bar{\psi}_2(z)\Gamma_C\psi_3(z) \right\rangle^{\text{sea}} \\
G_c(x_0,y_0,z_0) &= \sum_{\vec{x},\vec{y},\vec{z}} \left\langle \bar{\psi}_4(x)\Gamma_A\psi_1(x)\bar{\psi}_3(y)\Gamma_D\psi_4(y)\bar{\psi}_1(y)\Gamma_B\psi_2(y)\bar{\psi}_2(z)\Gamma_C\psi_3(z) \right\rangle^{\text{sea}}
\end{aligned} \tag{1}$$

the subscripts c and d refer to *connected* and *disconnected* correlators. Picture 1 shows the correlators in a simple representative way for $x_0 > y_0 > z_0$. The Wick contractions acting on the correlators are:

$$\begin{aligned}
G_d(x_0,y_0,z_0) &= \sum_{\vec{x},\vec{y},\vec{z}} \left\langle \mathrm{Tr}\left[\Gamma_A S_1(x,y)\Gamma_B S_4(y,x)\right] \cdot \mathrm{Tr}\left[\Gamma_C S_3(z,y)\Gamma_D S_2(y,z)\right] \right\rangle^{\text{sea}} \\
G_c(x_0,y_0,z_0) &= -\sum_{\vec{x},\vec{y},\vec{z}} \left\langle \mathrm{Tr}\left[\Gamma_A S_1(x,y)\Gamma_B S_2(y,z)\Gamma_C S_3(z,y)\Gamma_D S_4(y,x)\right] \right\rangle^{\text{sea}}
\end{aligned} \tag{2}$$

There exists a simple way to evaluate these correlators. The method is based on the sheet by Tomasz Korzec for Meson correlators (2013) and it involves the use of stochastic spinors. I generate N_{noise} stochastic spinors in the timeslice x_0 and N_{noise} stochastic spinors in the timeslice z_0. I refer to the formers with η^1 and the latters with η^2. These stochastic spinors have a Dirac index $(\alpha,\beta,\cdots)$ and a colour index $(a,b,\cdots)$. The following properties must be satisfied:

$$\begin{aligned}
\langle \eta^1_{a\alpha}(u)\rangle^{\text{noise}} &= \langle \eta^2_{b\beta}(u)\rangle^{\text{noise}} = 0 \\
\langle \eta^{1*}_{a\alpha}(u)\eta^1_{b\beta}(v)\rangle^{\text{noise}} &= \delta_{a,b}\delta_{\alpha,\beta}\delta_{\vec{u},\vec{v}}\delta_{u_0,x_0}\delta_{v_0,x_0} \\
\langle \eta^{2*}_{a\alpha}(u)\eta^2_{b\beta}(v)\rangle^{\text{noise}} &= \delta_{a,b}\delta_{\alpha,\beta}\delta_{\vec{u},\vec{v}}\delta_{u_0,z_0}\delta_{v_0,z_0}
\end{aligned} \tag{3}$$

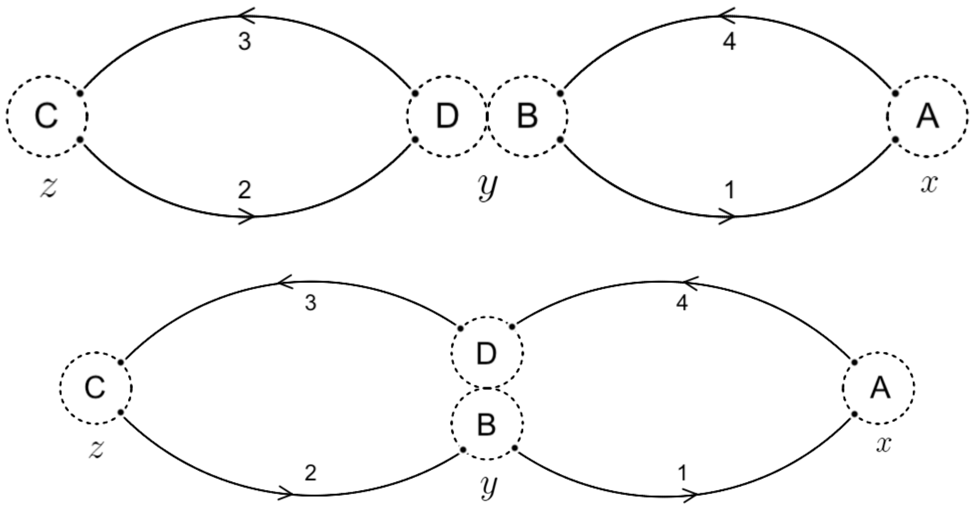

Figure 1: On the top: graph of disconnected Wick contraction. On the bottom: graph of connected Wick contraction.

where the symbol $\langle\,\cdot\,\rangle^{\text{noise}}$ refers to the average over N_{noise} vectors and $u, v \in \Lambda$ are lattice points. I define derived stochastic vectors:

$$\begin{aligned}\zeta_j^{(i,\pm)}(u) &= \sum_v S_{(i,\pm)}(u,v)\eta^j(v)\\ \xi_{j,X}^{(i,\pm)}(u) &= \sum_v S_{(i,\pm)}(u,v)\gamma_5\Gamma_X^\dagger\eta^j(v)\end{aligned} \tag{4}$$

A brief discussion about their indices could clarify the notation:

- $j = 1, 2$ is the stochastic vector index. It tells you whenever to use $\eta^{(1)}$ or $\eta^{(2)}$.
- $X = A, B, C, D$ is the matrix index. It tells you to use $\Gamma_X^\dagger \in \{\Gamma_A^\dagger, \Gamma_B^\dagger, \Gamma_C^\dagger, \Gamma_D^\dagger\}$.
- $i = 1, 2, 3, 4$ refers to the propagator index. In principle, the ψ_i s could be four different quarks.
- the sign $\pm$ is referred to the twisted mass parameter. If you use maximally twisted mass QCD - or maximally twisted Osterwalder-Seiler regularization - you should remember that the propagator is obtained by the inversion:

$$\sum_w \left(D_W^{(i)} \pm i\gamma_5\mu^{(i)}\right)(u,w)S_{(i,\pm)}(w,v) = \delta_{u,v}$$

  and I will use the following property: $\gamma_5 S_{(i,\pm)}^\dagger(u,v)\gamma_5 = S_{(i,\mp)}(v,u)$ that explain the need of $\pm$ symbol.

- the symbols i and $\pm$ are referred to the same propagator. For this reason they are coupled as $(i, \pm)$.

It could be easily checked that the contractions in (2) can be obtained by the following formulae:

$$G_d(x_0, y_0, z_0) = \sum_{\vec{y}} \left\langle \left\langle \left(\gamma_5 \xi_{1,A}^{(1,-)}(y)\right)^\dagger \Gamma_B \zeta_1^{(4,+)}(y) \cdot \left(\gamma_5 \xi_{C,2}^{(3,-)}(y)\right)^\dagger \Gamma_D \zeta_2^{(2,+)}(y) \right\rangle^{\text{noise}} \right\rangle^{\text{sea}}$$

$$G_c(x_0, y_0, z_0) = -\sum_{\vec{y}} \left\langle \left\langle \left(\gamma_5 \xi_{1,A}^{(1,-)}(y)\right)^\dagger \Gamma_B \zeta_2^{(2,+)}(y) \cdot \left(\gamma_5 \xi_{C,2}^{(3,-)}(y)\right)^\dagger \Gamma_D \zeta_1^{(4,+)}(y) \right\rangle^{\text{noise}} \right\rangle^{\text{sea}}$$

Then, to evaluate the previous couple of Wick contraction **I need only four quantites** for each couple of stochastic spinors:

$$\xi_{1,A}^{(1,-)}(y) \qquad \zeta_1^{(4,+)}(y) \qquad \xi_{C,2}^{(3,-)}(y) \qquad \zeta_2^{(2,+)}(y)$$

The path to evaluate the correlators is:

- ▷ For each Gauge-sea configuration evaluate $2N_{\text{noise}}$ spinors - N_{noise} for η^1 and N_{noise} for η^2.
- ▷ For each noise spinor evaluate the quantites ζ and ξ needed. Sometimes just a few number of them are needed.
- ▷ Evaluate the correlators and evaluate the noise average.
- ▷ Iterate the procedure and calculate the sea average.