

Meson Correlators

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Abstract

Calculation of non-singlet meson correlators with stochastic sources in QCD with open boundaries.
Version to be used with `openQCD-1.2`

1 Correlation function

We wish to calculate the correlation function

$$G(x_0, y_0) = \sum_{\mathbf{x}, \mathbf{y}} \langle \bar{q}_1(x) \Gamma_A q_2(x) \bar{q}_2(y) \bar{\Gamma}_B q_1(y) \rangle, \quad (1)$$

Where q_1 and q_2 are two different quark flavors with bare masses m_1 and m_2 (which might be equal). Γ is usually diagonal in color space and some matrix in spin space (γ_5 for pions). $\bar{\Gamma} \equiv \gamma_0 \Gamma^\dagger \gamma_0$. After the fermionic integration we are left with

$$G(x_0, y_0) = - \sum_{\mathbf{x}, \mathbf{y}} \langle \text{tr} [\Gamma_A S_2(x, y) \bar{\Gamma}_B S_1(y, x)] \rangle^{\text{gauge}}. \quad (2)$$

The trace acts in color and spin. The propagators S_i are defined in the usual way

$$\sum_y [D_W(x, y) + m_i] S_i(y, z) = \mathbb{1} \delta_{x, z}. \quad (3)$$

An efficient way to carry out the calculation is to use stochastic sources. We introduce noise vectors η with the properties

$$\begin{aligned} \langle \eta_{\alpha a}(u) \rangle^{\text{noise}} &= 0 \\ \langle \eta_{\alpha a}^*(u) \eta_{\beta b}(v) \rangle^{\text{noise}} &= \delta_{u_0, x_0} \delta_{v_0, x_0} \delta_{\mathbf{u}, \mathbf{v}} \delta_{\alpha, \beta} \delta_{a, b}, \end{aligned} \quad (4)$$

typical choices being Z_2 or Gaussian noise. We define two derived stochastic quantities

$$\zeta(u) = \sum_v [D_W + m_1]^{-1}(u, v) \eta(v) \quad (5)$$

$$\xi(u) = \sum_v [D_W + m_2]^{-1}(u, v) \gamma_5 \Gamma_A^\dagger \eta(v). \quad (6)$$

Eq. (2) can now be written as

$$G(x_0, y_0) = - \sum_{\mathbf{y}} \left\langle \left\langle \left(\bar{\Gamma}_B^\dagger \gamma_5 \xi(y) \right)^\dagger \zeta(y) \right\rangle^{\text{noise}} \right\rangle^{\text{gauge}}, \quad (7)$$

which can be evaluated at a cost of two inversions per stochastic vector and gauge field configuration. Of course gauge and noise averages can be interchanged.

- For each gauge configuration create N_{noise} random vectors.
- With each vector calculate both ζ and ξ ¹
- Calculate $\sum_{\mathbf{y}} \left(\bar{\Gamma}_B^\dagger \gamma_5 \xi(y) \right)^\dagger \zeta(y)$ for each gauge field and noise vector. Average over the noise vectors and carry out the usual autocorrelation analysis with respect to the gauge field average.

2 Input file

The format of the input files is based on the openQCD package. It is best explained by an example.

```
[Run name]
name          openQCD_run_name
output        mesons_run_name      # optional

[Directories]                                # as in openQCD
log_dir       ./log
loc_dir       ./cnfg
cnfg_dir      ./cnfg
dat_dir       ./dat

[Configurations]                             # as in openQCD ms1
first         1
last         10
step         2

[Random number generator]                    # as in openQCD
level         0
seed         73099
```

¹Note that for the pion case ($\Gamma_A = \gamma_5$) with $m_1 = m_2$ only one inversion per noise vector is required, because $\xi = \zeta$.

```

[Measurements]
nprop      2          # number of different quark lines
ncorr      3          # number of different correlators
nnoise     100        # number of noise vectors / conf
noise_type  U1        # noise type: U1 or Z2 or GAUSS
csw        1.4951     # Dirac operator options
cF         1.0        # common to all quark lines

##### For every quark line a Propagator
# section, nprop in total

[Propagator 0]
kappa      0.121951219512195 # hopping parameter
isp        0                # solver id

[Propagator 1]
kappa      0.12
isp        1

##### For every correlator a Correlator
# section, ncorr in total

[Correlator 0]
iprop      0 0          # quark lines
type       G5 G5       # Dirac structures src snk
x0         16          # source time slice

[Correlator 1]
iprop      0 0
type       G0G5 G5
x0         16

[Correlator 2]
iprop      0 1
type       G1 G1
x0         12

##### For every solver id used, one Solver
# section, as in openQCD

[Solver 0]
solver     DFL_SAP_GCR
nkV        8
isolv      1
nmr        4

```

```

ncy          5
nmx          128
res          1.0e-12

```

[Solver 1]

```

solver       DFL_SAP_GCR
nkx          8
isolv        1
nmr          4
ncy          6
nmx          128
res          1.0e-12

```

[SAP]

```

bs           4 4 4 4

```

[Deflation subspace]

```

bs           4 4 4 4
Ns           20

```

[Deflation subspace generation]

```

kappa        0.12
mu           0.01
ninv         10
nmr          4
ncy          4

```

[Deflation projection]

```

nkx          16
nmx          256
res          1.0e-2

```

In this case three correlation functions would be calculated for all values of y_0 :

$$\text{corr 0 : } \sum_{\mathbf{x}, \mathbf{y}} \langle \bar{q}_0(16a, \mathbf{x}) \gamma_5 q_0(16a, \mathbf{x}) \bar{q}_0(y_0, \mathbf{y}) \gamma_5 q_0(y_0, \mathbf{y}) \rangle , \quad (8)$$

$$\text{corr 1 : } \sum_{\mathbf{x}, \mathbf{y}} \langle \bar{q}_0(16a, \mathbf{x}) \gamma_0 \gamma_5 q_0(16a, \mathbf{x}) \bar{q}_0(y_0, \mathbf{y}) \gamma_5 q_0(y_0, \mathbf{y}) \rangle , \quad (9)$$

$$\text{corr 2 : } \sum_{\mathbf{x}, \mathbf{y}} \langle \bar{q}_0(12a, \mathbf{x}) \gamma_1 q_1(12a, \mathbf{x}) \bar{q}_1(y_0, \mathbf{y}) \gamma_1 q_1(y_0, \mathbf{y}) \rangle , \quad (10)$$

Where quark q_0 has a mass corresponding to $\kappa = 0.12195\dots$ and quark q_1 to $\kappa = 0.12$. The configurations 1,3,5,7,9 would be processed and 100×4 inversions would be required on each configuration. For every noise vector

correlator 0 needs one inversion, correlator 1 needs two, of which one is already performed for correlator 0. Correlator 2 needs two additional inversions. It would be cheaper by one inversion per noise source to calculate for correlator 1 the combination `type G5 G0G5` instead. See appendix A for supported Dirac structures and their input-file string.

3 Output file format

The output `*.mesons.dat` file is always written in little endian and has the structure

- 16-Byte header with
 - 4-byte Integer: Number of Correlators `ncorr`
 - 4-byte Integer: Number of noise vectors `nnoise`
 - 4-byte Integer: Number of time-slices `tvals` (=T usually)
 - 4-byte Integer: Noise type (0,1,2=Z2,Gauss,U1)
- `ncorr` \times 32 byte blocks, each block containing
 - 8-byte Double: first quark kappa
 - 8-byte Double: second quark kappa
 - 4-byte Integer: Dirac structure at x_0
 - 4-byte Integer: Dirac structure at y_0
 - 4-byte Integer: x_0
 - 4-byte Integer: 1: correlator is real, 0: complex
- `nconf` blocks each containing
 - 4-byte Integer: configuration number
 - `ncorr` blocks, each block containing
 - * `tvals` blocks, each block containing
 - `nnoise` \times 8-byte Double (or 16-byte Complex) correlator

The matlab script `./tests/ana_mesons.m` reads in and analyzes dat files created by the meson program.

4 Tests

A Interpolating operators

With hermitian $\gamma_0 \dots \gamma_5$ the results in table 1 are easily verified.

| Γ | $\gamma_5 \Gamma^\dagger$ | $\bar{\Gamma}^\dagger \gamma_5$ | Input file | Output file |
|---------------------|---------------------------|---------------------------------|------------|-------------|
| γ_0 | $-\gamma_0 \gamma_5$ | $\gamma_0 \gamma_5$ | G0 | 0 |
| γ_1 | $-\gamma_1 \gamma_5$ | $-\gamma_1 \gamma_5$ | G1 | 1 |
| γ_2 | $-\gamma_2 \gamma_5$ | $-\gamma_2 \gamma_5$ | G2 | 2 |
| γ_3 | $-\gamma_3 \gamma_5$ | $-\gamma_3 \gamma_5$ | G3 | 3 |
| γ_5 | $\mathbb{1}$ | $-\mathbb{1}$ | G5 | 5 |
| $\gamma_0 \gamma_1$ | $\gamma_2 \gamma_3$ | $\gamma_2 \gamma_3$ | G0G1 | 7 |
| $\gamma_0 \gamma_2$ | $-\gamma_1 \gamma_3$ | $-\gamma_1 \gamma_3$ | G0G2 | 8 |
| $\gamma_0 \gamma_3$ | $\gamma_1 \gamma_2$ | $\gamma_1 \gamma_2$ | G0G3 | 9 |
| $\gamma_0 \gamma_5$ | $\gamma_- 0$ | $-\gamma_0$ | G0G5 | 10 |
| $\gamma_1 \gamma_2$ | $\gamma_0 \gamma_3$ | $-\gamma_0 \gamma_3$ | G1G2 | 11 |
| $\gamma_1 \gamma_3$ | $-\gamma_0 \gamma_2$ | $+\gamma_0 \gamma_2$ | G1G3 | 12 |
| $\gamma_1 \gamma_5$ | γ_1 | $+\gamma_1$ | G1G5 | 13 |
| $\gamma_2 \gamma_3$ | $\gamma_0 \gamma_1$ | $-\gamma_0 \gamma_1$ | G2G3 | 14 |
| $\gamma_2 \gamma_5$ | γ_2 | $+\gamma_2$ | G2G5 | 15 |
| $\gamma_3 \gamma_5$ | γ_3 | $+\gamma_3$ | G3G5 | 16 |
| $\mathbb{1}$ | γ_5 | γ_5 | 1 | 6 |

Table 1: Supported Dirac structures