Meson Correlators

Tomasz Korzec

September 16, 2013

Abstract

Calculation of non-singlet meson correlators with stochastic sources in QCD with open boundaries.
Version to be used with `openQCD-1.2`

1 Correlation function

We wish to calculate the correlation function

$$G(x_0, y_0) = \sum_{\boldsymbol{x},\boldsymbol{y}} \left\langle \bar{q}_1(x)\Gamma_A q_2(x)\ \bar{q}_2(y)\bar{\Gamma}_B q_1(y) \right\rangle , \tag{1}$$

Where q_1 and q_2 are two different quark flavors with bare masses m_1 and m_2 (which might be equal). Γ is usually diagonal in color space and some matrix in spin space (γ_5 for pions). $\bar{\Gamma} \equiv \gamma_0 \Gamma^\dagger \gamma_0$. After the fermionic integration we are left with

$$G(x_0, y_0) = -\sum_{\boldsymbol{x},\boldsymbol{y}} \left\langle \mathrm{tr}\left[\Gamma_A S_2(x,y)\bar{\Gamma}_B S_1(y,x)\right] \right\rangle^{\mathrm{gauge}} . \tag{2}$$

The trace acts in color and spin. The propagators S_i are defined in the usual way

$$\sum_y [D_W(x,y) + m_i] S_i(y,z) = \mathbb{1}\ \delta_{x,z} . \tag{3}$$

An efficient way to carry out the calculation is to use stochastic sources. We introduce noise vectors η with the properties

$$\begin{aligned} \langle \eta_{\alpha a}(u) \rangle^{\mathrm{noise}} &= 0 \\ \langle \eta^*_{\alpha a}(u) \eta_{\beta b}(v) \rangle^{\mathrm{noise}} &= \delta_{u_0,x_0}\delta_{v_0,x_0}\delta_{\boldsymbol{u},\boldsymbol{v}}\delta_{\alpha,\beta}\delta_{a,b} , \end{aligned} \tag{4}$$

typical choices being Z_2 or Gaussian noise. We define two derived stochastic quantities

$$\zeta(u) = \sum_v [D_W + m_1]^{-1}(u,v)\eta(v) \tag{5}$$

$$\xi(u) = \sum_v [D_W + m_2]^{-1}(u,v)\gamma_5\Gamma_A^\dagger \eta(v) . \tag{6}$$

Eq. (2) can now be written as

$$G(x_0, y_0) = -\sum_{\boldsymbol{y}} \left\langle \left\langle \left(\bar{\Gamma}_B^\dagger \gamma_5 \xi(y) \right)^\dagger \zeta(y) \right\rangle^{\text{noise}} \right\rangle^{\text{gauge}} , \tag{7}$$

which can be evaluated at a cost of two inversions per stochastic vector and gauge field configuration. Of course gauge and noise averages can be interchanged.

- For each gauge configuration create N_{noise} random vectors.
- With each vector calculate both ζ and ξ[1]
- Calculate $\sum_{\boldsymbol{y}} \left(\bar{\Gamma}_B^\dagger \gamma_5 \xi(y) \right)^\dagger \zeta(y)$ for each gauge field and noise vector. Average over the noise vectors and carry out the usual autocorrelation analysis with respect to the gauge field average.

2 Input file

The format of the input files is based on the openQCD package. It is best explained by an example.

```
[Run name]
name          openQCD_run_name
output        mesons_run_name       # optional


[Directories]                       # as in openQCD
log_dir       ./log
loc_dir       ./cnfg
cnfg_dir      ./cnfg
dat_dir       ./dat

[Configurations]                    # as in openQCD ms1
first         1
last          10
step          2

[Random number generator]           # as in openQCD
level         0
seed          73099
```

[1]Note that for the pion case ($\Gamma_A = \gamma_5$) with $m_1 = m_2$ only one inversion per noise vector is required, because $\xi = \zeta$.

```
[Measurements]
nprop        2                    # number of different quark lines
ncorr        3                    # number of different correlators
nnoise       100                  # number of noise vectors / conf
noise_type   U1                   # noise type: U1 or Z2 or GAUSS
csw          1.4951               # Dirac operator options
cF           1.0                  # common to all quark lines


################################## For every quark line a Propagator
                                  # section, nprop in total
[Propagator 0]
kappa        0.121951219512195    # hopping parameter
isp          0                    # solver id

[Propagator 1]
kappa        0.12
isp          1

################################# For every correlator a Correlator
                                 # section, ncorr in total
[Correlator 0]
iprop        0 0                 # quark lines
type         G5 G5               # Dirac structures src snk
x0           16                  # source time slice

[Correlator 1]
iprop        0 0
type         G0G5 G5
x0           16

[Correlator 2]
iprop        0 1
type         G1 G1
x0           12

################################ For every solver id used, one Solver
                                # section, as in openQCD

[Solver 0]
solver       DFL_SAP_GCR
nkv          8
isolv        1
nmr          4
```

```
ncy          5
nmx          128
res          1.0e-12

[Solver 1]
solver       DFL_SAP_GCR
nkv          8
isolv        1
nmr          4
ncy          6
nmx          128
res          1.0e-12

[SAP]
bs           4 4 4 4

[Deflation subspace]
bs           4 4 4 4
Ns           20

[Deflation subspace generation]
kappa        0.12
mu           0.01
ninv         10
nmr          4
ncy          4

[Deflation projection]
nkv          16
nmx          256
res          1.0e-2
```

In this case three correlation functions would be calculated for all values of y_0:

$$\text{corr } 0: \quad \sum_{\boldsymbol{x},\boldsymbol{y}} \langle \bar{q}_0(16a,\boldsymbol{x})\gamma_5 q_0(16a,\boldsymbol{x})\ \bar{q}_0(y_0,\boldsymbol{y})\bar{\gamma}_5 q_0(y_0,\boldsymbol{y})\rangle \ , \tag{8}$$

$$\text{corr } 1: \quad \sum_{\boldsymbol{x},\boldsymbol{y}} \langle \bar{q}_0(16a,\boldsymbol{x})\gamma_0\gamma_5 q_0(16a,\boldsymbol{x})\ \bar{q}_0(y_0,\boldsymbol{y})\bar{\gamma}_5 q_0(y_0,\boldsymbol{y})\rangle \ , \tag{9}$$

$$\text{corr } 2: \quad \sum_{\boldsymbol{x},\boldsymbol{y}} \langle \bar{q}_0(12a,\boldsymbol{x})\gamma_1 q_1(12a,\boldsymbol{x})\ \bar{q}_1(y_0,\boldsymbol{y})\bar{\gamma}_1 q_0(y_0,\boldsymbol{y})\rangle \ , \tag{10}$$

Where quark q_0 has a mass corresponding to $\kappa = 0.12195\ldots$ and quark q_1 to $\kappa = 0.12$. The configurations 1,3,5,7,9 would be processed and 100×4 inversions would be required on each configuration. For every noise vector correlator 0 needs one inversion, correlator 1 needs two, of which one is already performed for correlator 0. Correlator 2 needs two additional inversions. It would be cheaper by one inversion per noise source to calculate for correlator 1 the combination `type G5 G0G5` instead. See appendix A for supported Dirac structures and their input-file string.

3 Output file format

The output *.mesons.dat file is always written in little endian and has the structure

- 16-Byte header with
  - 4-byte Integer: Number of Correlators `ncorr`
  - 4-byte Integer: Number of noise vectors `nnoise`
  - 4-byte Integer: Number of time-slices `tvals` (=T usually)
  - 4-byte Integer: Noise type (0,1,2=Z2,Gauss,U1)
- `ncorr` $\times$ 32 byte blocks, each block containing
  - 8-byte Double: first quark kappa
  - 8-byte Double: second quark kappa
  - 4-byte Integer: Dirac structure at x_0
  - 4-byte Integer: Dirac structure at y_0
  - 4-byte Integer: x_0
  - 4-byte Integer: 1: correlator is real, 0: complex
- `nconf` blocks each containing
  - 4-byte Integer: configuration number
  - `ncorr` blocks, each block containing
    - `tvals` blocks, each block containing
      - `nnoise` $\times$ 8-byte Double (or 16-byte Complex) correlator

The matlab script `./tests/ana_mesons.m` reads in and analyzes dat files created by the meson program.

4 Tests

A Interpolating operators

With hermitian $\gamma_0 \ldots \gamma_5$ the results in table 1 are easily verified.

Γ	$\gamma_5\Gamma^\dagger$	$\bar{\Gamma}^\dagger\gamma_5$	Input file	Output file
γ_0	$-\gamma_0\gamma_5$	$\gamma_0\gamma_5$	`G0`	0
γ_1	$-\gamma_1\gamma_5$	$-\gamma_1\gamma_5$	`G1`	1
γ_2	$-\gamma_2\gamma_5$	$-\gamma_2\gamma_5$	`G2`	2
γ_3	$-\gamma_3\gamma_5$	$-\gamma_3\gamma_5$	`G3`	3
γ_5	$\mathbb{1}$	$-\mathbb{1}$	`G5`	5
$\gamma_0\gamma_1$	$\gamma_2\gamma_3$	$\gamma_2\gamma_3$	`G0G1`	7
$\gamma_0\gamma_2$	$-\gamma_1\gamma_3$	$-\gamma_1\gamma_3$	`G0G2`	8
$\gamma_0\gamma_3$	$\gamma_1\gamma_2$	$\gamma_1\gamma_2$	`G0G3`	9
$\gamma_0\gamma_5$	γ_-0	$-\gamma_0$	`G0G5`	10
$\gamma_1\gamma_2$	$\gamma_0\gamma_3$	$-\gamma_0\gamma_3$	`G1G2`	11
$\gamma_1\gamma_3$	$-\gamma_0\gamma_2$	$+\gamma_0\gamma_2$	`G1G3`	12
$\gamma_1\gamma_5$	γ_1	$+\gamma_1$	`G1G5`	13
$\gamma_2\gamma_3$	$\gamma_0\gamma_1$	$-\gamma_0\gamma_1$	`G2G3`	14
$\gamma_2\gamma_5$	γ_2	$+\gamma_2$	`G2G5`	15
$\gamma_3\gamma_5$	γ_3	$+\gamma_3$	`G3G5`	16
$\mathbb{1}$	γ_5	γ_5	`1`	6

Table 1: Supported Dirac structures