Non-perturbative computation of Kaon oscillation amplitudes in Lattice QCD with $N_f = 2 + 1$ sea quarks and OBC

Master's Degree in Theoretical Physics

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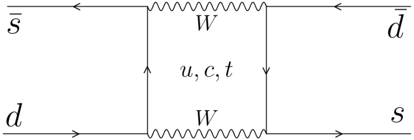
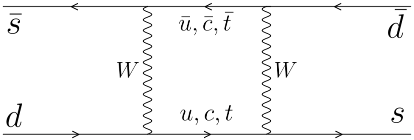
Kaon oscillations

SM oscillations

Kaon oscillations in Standard Model (SM) are mediated by Weak Interactions.

- Two 1-loop order diagrams





Kaons oscillations

SM oscillations

Kaons oscillations in Standard Model (SM) are mediated by Weak Interactions.

- Two 1-loop order diagrams
- Effective mixing operator ($E \ll M_W$):

$$\langle \bar{K}^0 | \Theta_1 | K^0 \rangle \qquad \Theta_1 = \left[\bar{s}^a \gamma_\mu (1 - \gamma_5) d^a \right] \cdot \left[\bar{s}^b \gamma_\mu (1 - \gamma_5) d^b \right]$$

- B_1 parameter parametrizes deviation from the Vacuum Insertion Approximation (VIA):

$$\langle \bar{K}^0 | \Theta_1 | K^0 \rangle = B_1 \langle \bar{K}^0 | \Theta_1 | K^0 \rangle_{\text{VIA}} = \frac{8}{3} B_1 F_K^2 m_K^2$$

Kaon oscillations

Oscillations BSM

- B_1 parameter (a.k.a. B_K) parametrizes deviation from VIA
- Other operators $\{\Theta_i, \tilde{\Theta}_j\}$ appear in theories beyond the SM (BSM) $[5]$
- They form a complete set of **operators of dimension 6 - composed by 4 quarks**, closed under renormalization procedure
- Definition of *bag parameters* B_j $[1]$:

$$\langle \bar{K}^0 | \Theta_j^{[+],\text{ren}}(\mu) | K^0 \rangle = \xi_j \left(\frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right)^2 f_K^2 B_j(\mu) \qquad j \geq 2$$

Purpose of the work

Non perturbative computation of bare bag parameters B_i

$$\downarrow$$

Path integral formalism - lattice QCD

$$\downarrow$$

Coordinated Lattice Simulations (CLS) gauge configurations with $N_f = 2 + 1$ sea quarks and open boundary conditions (OBC) in time direction

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2 Quantum Field Theories on the Lattice

Regularizations ingredients:

- Finite lattice space Λ: $L \times L \times L \times T$
- Lattice spacing a: $L = a \cdot N_L$, $T = a \cdot N_T$
- Action discretization:

$$S^{\text{lat}}[a; \tilde{\phi}_1, \ldots, \tilde{\phi}_{N_{\text{fields}}}] \xrightarrow{a \to 0} S^{\text{cont}}[\phi_1, \ldots, \phi_{N_{\text{fields}}}]$$

- $O(a^n)$-improvement: $S^{\text{lat}}(a) = S^{\text{cont}} + o(a^n)$
- Troubles: finite volume effects & lattice spacing effects

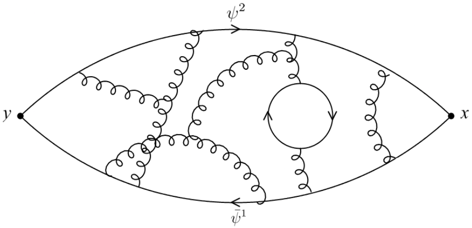
Sea vs Valence quarks

- Sea and valence quarks are regularized through different actions

  $\downarrow$

  Theoretical advantages in lattice QCD

- Sea sector: $N_f = 2 + 1$ quarks + gluons
  Valence sector: d, s quarks



$$G^{12}(x, y) = \langle M(x)\overline{M}(y) \rangle$$

Sea sector

- Gauge configuration exhibit **long autocorrelation times** as $a \to 0$
- Zero modes of Dirac Wilson operator D^W make the sea quarks simulation inefficient

$$\Downarrow$$

Open boundary conditions (OBC) in time direction $[4]$

- Gauge fields: Luscher-Weisz action
- Sea quarks: Dirac-Wilson action + Sheikholeslami-Wohlert term ($O(a)$-improved)

Valence sector

- Valence quarks: Osterwalder-Seiler at Maximal twist + SW term
- Flavours: d, d', s, s' with different signs Wilson parameters r_i
  $\Longrightarrow$ R. Frezzotti & G. Rossi strategy $[3]$

Achievements:

- $O(a)$-improved mixing amplitudes w/o the need for W.A.
- Absence of wrong chirality mixing between $\{O_i^{[+]}\}$ and $\{O_i^{[-]}\}$
- Blocks like renormalization matrix $[Z_{ij}]$ for parity odd operators $[2]$

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3 Bag parameters extrapolation

$$C_i(x_4, y_4, z_4) = \sum_{\vec{x},\vec{y},\vec{z}} \langle \bar{K}^{0'}(x) O_{i[+]}(y) \bar{K}^0(z) \rangle$$

$$G_{34}(x_4, y_4) = \sum_{\vec{x},\vec{y}} \langle \bar{K}^{0'}(x) K^{0'}(y) \rangle$$

$$G_{12}(x_4, y_4) = \sum_{\vec{x},\vec{y}} \langle \bar{K}^{0}(x) K^{0}(y) \rangle$$

$$X_{34}(x_4, y_4) = \sum_{\vec{x},\vec{y}} \langle A_4'(x) K^{0'}(y) \rangle$$

$$X_{12}(x_4, y_4) = \sum_{\vec{x},\vec{y}} \langle A_4(x) K^{0}(y) \rangle$$

Path integral allows to calculate only VEVs

$$\langle 0 | \cdots | 0 \rangle$$

$$\downarrow$$

Method based on asymptotic behaviours for large euclidean time separations

Bag parameters are recovered in continuum limit $a \to 0$ for $x_4 \gg y_4 \gg z_4$:

$$\mathcal{B}_1(a) \approx \frac{3}{8} \cdot \frac{C_1(x_4, y_4, z_4)}{X_{34}(x_4, y_4) X_{12}(y_4, z_4)} \xrightarrow{a \to 0} B_K$$

$$\mathcal{B}_i(a) \approx \frac{1}{\xi_i} \cdot \frac{\Lambda_{ij} C_j(x_4, y_4, z_4)}{G_{34}(x_4, y_4) G_{12}(y_4, z_4)} \xrightarrow{a \to 0} B_i$$

$$\mathcal{R}_i(a) \approx \Lambda_{ij} \cdot \frac{C_j(x_4, y_4, z_4)}{C_1(x_4, y_4, z_4)} \xrightarrow{a \to 0} R_i = \frac{B_i}{B_K}$$

$\Rightarrow$ Need for 2 and 3 points integrated correlation functions computation

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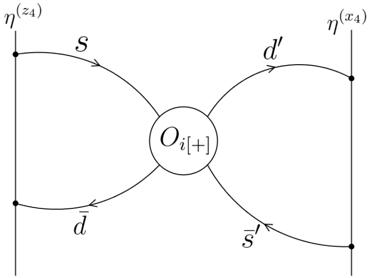
Noise spinors method

Mesonic sources in x_4 and z_4 are simulated through noise spinors $\eta^{(x_4)}$ and $\eta^{(z_4)}$:

- Two sets of N_n spinors randomly generated
- The sums $\sum_{\vec{x}}$ and $\sum_{\vec{z}}$ automatically implied when calculating the average over ηs
- Computational advantage
- Different Wick contractions are obtained by multiplying propagators from sources with Dirac matrices



Simulation program tests

Tests:

- Gauge invariance test
- Gauge invariance test + investigation about N_n
- Test on *Tree level improved Symanzik action*, quenched approximation
- Test on *Plaquette gauge action*, quenched approximation

Test #1: Gauge invariance

Gauge transformed propagators:

$$\left(S^{(f)}(y,x)[\Omega U \Omega^\dagger]\right)^{b,a}_{\beta,\alpha} = \Omega(y)^{b,c}\left(S^{(f)}(y,x)[U]\right)^{c,d}_{\beta,\alpha}\Omega^\dagger(x)^{d,a}$$

- Equation tested in three points integrated correlators $C_{i,[+]}$
- Link variables $U_\mu(x)$ are randomly generated and then transformed
- Pointlike sources instead of random spinors η

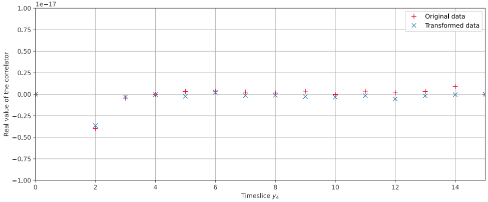
Relative errors: $\varepsilon_i \lesssim 10^{-13}$

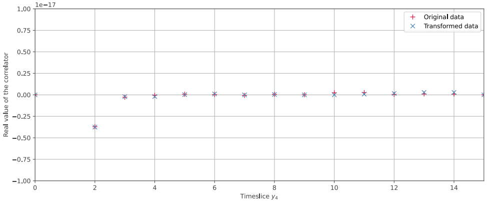
Mean relative error: $\bar{\varepsilon} = (1.313 \pm 1.634) \cdot 10^{-13}$

Test #2: Gauge invariance and investigation about N_n

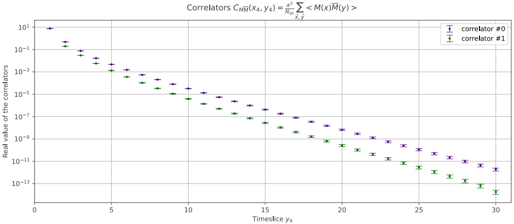
- Equation tested in three points integrated correlators $C_{i,[+]}$
- Link variables $U_\mu(x)$ are randomly generated and then transformed
- Use of random spinors η
- Different number of N_n were tested. Estimator:

$$\varepsilon(N_n) = \frac{1}{N_T - 2} \sum_{y_4} \left(\frac{|C(y_4) - \tilde{C}(y_4)|}{C(y_4) + \tilde{C}(y_4)} \right)$$

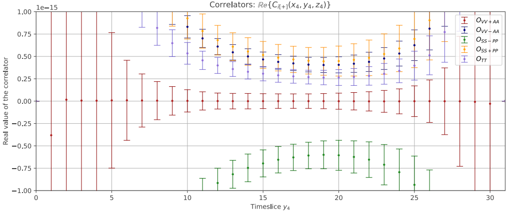




54 Gauge configurations - tree level improved Symanzik action
16x16x16x32 lattice, $\beta = 6.0$, OBCs, unphysical parameters.



$G_{12}(x_4, a), G_{34}(x_4, a)$

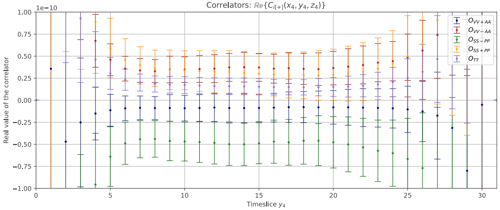


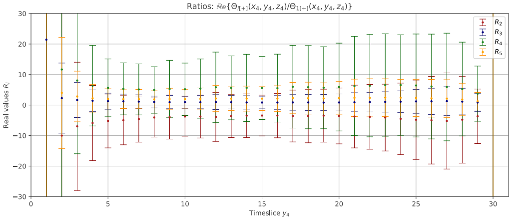
$C_{i[+]}(14a, y_4, a)$

Test #4: Plaquette gauge action

37 Gauge configurations - plaquette gauge action

16x16x16x32 lattice, $\beta = 6.0$, OBCs, maximal twist achieved





$$C_{i[+]}(14a, y_4, a)$$

$$\mathcal{R}_i(14a, y_4, a) = \Lambda_{ij} \frac{C_{i[+]}(14a, y_4, a)}{C_{1[+]}(14a, y_4, a)}$$

Future developments

- Numerical simulations with the $N_f = 2 + 1$ CLS sea ensembles
- Data analysis
- Renormalization procedure [2]
- Continuum limit extrapolation of B_i and R_i
- Comparison with other lattice results [6]

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Thank you for listening!

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⋆ Action regularizations

Many regularizations have been developed

Gauge actions:

- Link variables

$$U_\mu(x) = \exp\left(i \int_x^{x+a\hat{\mu}} A_\nu(\omega) d\omega^\nu\right)$$

- Wilson loops are gauge invariant
- Plaquette, tree level improved
  Symanzik, Luscher Weisz

Fermion actions:

- Wilson-Dirac action
  Wilson term $r \in [-1, 1]$
- Sheikholeslami-Wohlert term:

$$D^W_{xy} \longmapsto D^W_{xy} + c_{SW} \frac{ia}{4} \sigma_{\mu\nu} \hat{F}_{\mu\nu}(x) \delta_{xy}$$

- Osterwalder-Seiler action
  (twist transformation)

⋆ Sea sector

- Gauge action: Luscher-Weisz action

$$S_G[U] = \frac{1}{g_0^2}\left(\frac{5}{3}\sum_{\{P\}}\mathrm{Tr}\left(\mathbb{I} - U_P\right) - \frac{1}{12}\sum_{\{R\}}\mathrm{Tr}\left(\mathbb{I} - U_R\right)\right)$$

- Sea quarks: Dirac-Wilson + SW term

$$S^{\mathrm{sea}}[U, f, \bar{f}] = \sum_{q=1}^{3}\sum_{x}\bar{f}_q(x)\left[D^{WD}(r_q) + \frac{ia}{4}c_{SW}\sigma_{\mu\nu}\hat{F}_{\mu\nu} + m_q^{\mathrm{bare}}\right]f_q(x)$$

$\star$ Valence sector

Operators $\{\Theta_i^{[\pm]}\}$ in continuum QCD $\xrightarrow{\text{basis change}}$ Operators $\{Q_i^{[\pm]}\}$ in continuum QCD

$\xrightarrow{\text{flavour replicas}}$ P.E. operators $\{O_{i,[+]}^{\text{phys}}\}$ $\xrightarrow{\text{maximal twist}}$ P.E. operators $\{O_{i,[+]}^{\text{tw}}\}$

Flavour replicas and twist - an example $\left[\chi^j = \text{physical} \,\middle|\, \psi^j = \text{twisted}\right]$

$$\bar{K}^{0'} = \bar{s}'\gamma_5 d' = \bar{\chi}^3\gamma_5\chi^4 \qquad \text{Maximal Twist } \chi^i = e^{i\gamma_5 r_i \frac{\pi}{4}}\psi^i \qquad \bar{K}^{0'} = \bar{\psi}^3\gamma_5\psi^4$$

$$\bar{K}^0 = \bar{s}\gamma_5 d = \bar{\chi}^1\gamma_5\chi^2 \qquad\qquad \bar{K}^0 = i\bar{\psi}^1\psi^2$$

Similar transformations act on operators $O_{i[+]}$ and map PE operators in PO ones.

⋆ Asymptotic behaviours

Asymptotic behaviours for $x_4 \gg y_4 \gg z_4$:

$$C_i(x_4, y_4, z_4) \approx \sum_{\vec{y}} \langle \Omega | \bar{K}^{0'} | \bar{K}^{0'} \rangle \langle \bar{K}^{0'} | O_{i[+]} | K^0 \rangle \langle K^0 | \bar{K}^0 | \Omega \rangle e^{-m_{K'}(x_4 - y_4) - m_K(y_4 - z_4)}$$

$$G_{34}(x_4, y_4) \approx \sum_{\vec{y}} \langle \Omega | \bar{K}^{0'} | \bar{K}^{0'} \rangle \langle \bar{K}^{0'} | K^{0'} | \Omega \rangle e^{-m_{K'}(x_4 - y_4)}$$

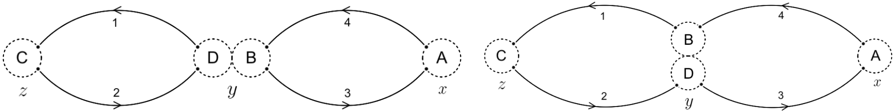
$$G_{12}(x_4, y_4) \approx \sum_{\vec{y}} \langle \Omega | \bar{K}^{0} | \bar{K}^{0} \rangle \langle \bar{K}^{0} | K^{0} | \Omega \rangle e^{-m_{K}(x_4 - y_4)}$$

$$X_{34}(x_4, y_4) \approx \sum_{\vec{y}} \langle \Omega | A'_4 | \bar{K}^{0'} \rangle \langle \bar{K}^{0'} | K^{0'} | \Omega \rangle e^{-m_{K'}(x_4 - y_4)}$$

$$X_{12}(x_4, y_4) \approx \sum_{\vec{y}} \langle \Omega | A_4 | \bar{K}^{0} \rangle \langle \bar{K}^{0} | K^{0} | \Omega \rangle e^{-m_{K}(x_4 - y_4)}$$

⋆ Noise spinors method

Two types of diagrams:

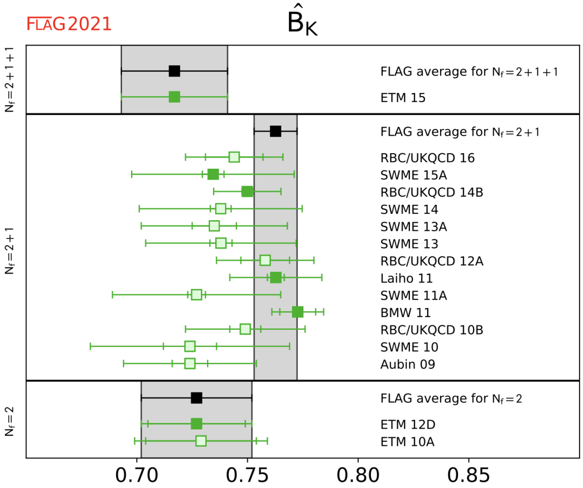


Two types of noise spinors contractions:

$$G_d = \sum_{\vec{y}} \left\langle \left\langle \left(\gamma_5 \xi_{1,A}^{(3,-)}(y)\right)^\dagger \Gamma_B \zeta_1^{(4,+)}(y) \cdot \left(\gamma_5 \xi_{C,2}^{(1,-)}(y)\right)^\dagger \Gamma_D \zeta_2^{(2,+)}(y) \right\rangle^{\text{noise}} \right\rangle^{\text{sea}}$$

$$G_c = -\sum_{\vec{y}} \left\langle \left\langle \left(\gamma_5 \xi_{1,A}^{(3,-)}(y)\right)^\dagger \Gamma_D \zeta_2^{(2,+)}(y) \cdot \left(\gamma_5 \xi_{C,2}^{(1,-)}(y)\right)^\dagger \Gamma_B \zeta_1^{(4,+)}(y) \right\rangle^{\text{noise}} \right\rangle^{\text{sea}}$$

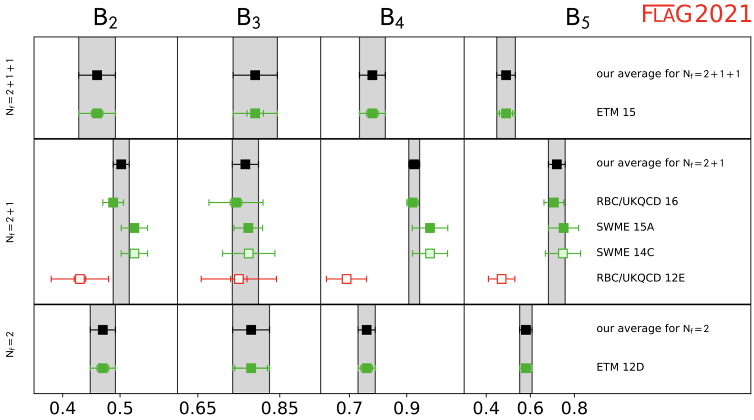
⋆ Lattice results: $\hat{B}_K$



Results for renormalization group invariant $\hat{B}_K$ [6]

- Different simulations use different regularizations
- Results depend on both B_K^{bare} and renormalisation
- Strong deviation between simulations with different number of sea quarks

⋆ Lattice results: $B_i, i \geq 2$



Results for renormalized bag parameters in $\overline{MS}$ scheme at $\mu = 3$ GeV