

Three points correlators

Kaons oscillations like

Emanuele Rosi, October 2023

Abstract

Calculation of flavour non-singlet meson oscillations through the insertion of intermediate operators and stochastic sources in QCD with open boundary conditions.

1 Wick Contractions

We want to calculate three points correlators of this two types:

$$\begin{aligned} G_d(x_0, y_0, z_0) &= \sum_{\vec{x}, \vec{y}, \vec{z}} \left\langle \bar{\psi}_4(x) \Gamma_A \psi_1(x) \bar{\psi}_3(y) \Gamma_D \psi_2(y) \bar{\psi}_1(y) \Gamma_B \psi_4(y) \bar{\psi}_2(z) \Gamma_C \psi_3(z) \right\rangle^{\text{sea}} \\ G_c(x_0, y_0, z_0) &= \sum_{\vec{x}, \vec{y}, \vec{z}} \left\langle \bar{\psi}_4(x) \Gamma_A \psi_1(x) \bar{\psi}_3(y) \Gamma_B \psi_4(y) \bar{\psi}_1(y) \Gamma_D \psi_2(y) \bar{\psi}_2(z) \Gamma_C \psi_3(z) \right\rangle^{\text{sea}} \end{aligned} \quad (1)$$

the subscripts c and d refer to *connected* and *disconnected* correlators. Picture 1 shows the correlators in a simple representative way for $x_0 > y_0 > z_0$. The Wick contractions acting on the correlators are:

$$\begin{aligned} G_d(x_0, y_0, z_0) &= \sum_{\vec{x}, \vec{y}, \vec{z}} \left\langle \text{Tr} [\Gamma_A S_1(x, y) \Gamma_B S_4(y, x)] \cdot \text{Tr} [\Gamma_C S_3(z, y) \Gamma_D S_2(y, z)] \right\rangle^{\text{sea}} \\ G_c(x_0, y_0, z_0) &= - \sum_{\vec{x}, \vec{y}, \vec{z}} \left\langle \text{Tr} [\Gamma_A S_1(x, y) \Gamma_D S_2(y, z) \Gamma_C S_3(z, y) \Gamma_B S_4(y, x)] \right\rangle^{\text{sea}} \end{aligned} \quad (2)$$

I generate N_{noise} stochastic spinors in the timeslice x_0 and N_{noise} stochastic spinors in the timeslice z_0 . I refer to the formers with η^1 and the latters with η^2 . These stochastic spinors have again a Dirac index (α, β, \dots) and a colour index (a, b, \dots) . The properties of the previous work by Tomasz Korzec are generalized:

$$\begin{aligned} \langle \eta_{a\alpha}^1(u) \rangle^{\text{noise}} &= \langle \eta_{b\beta}^2(u) \rangle^{\text{noise}} = 0 \\ \langle \eta_{a\alpha}^{1*}(u) \eta_{b\beta}^1(v) \rangle^{\text{noise}} &= \delta_{a,b} \delta_{\alpha,\beta} \delta_{\vec{u}, \vec{v}} \delta_{u_0, x_0} \delta_{v_0, x_0} \\ \langle \eta_{a\alpha}^{2*}(u) \eta_{b\beta}^2(v) \rangle^{\text{noise}} &= \delta_{a,b} \delta_{\alpha,\beta} \delta_{\vec{u}, \vec{v}} \delta_{u_0, z_0} \delta_{v_0, z_0} \end{aligned} \quad (3)$$

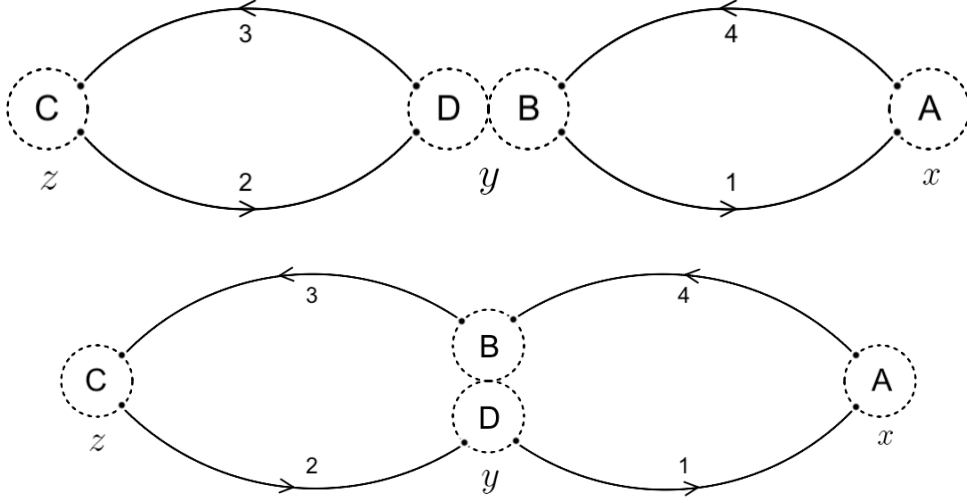


Figure 1: On the top: graph of disconnected Wick contraction. On the bottom: graph of connected Wick contraction.

where the symbol $\langle \cdot \rangle^{\text{noise}}$ refers to the average over N_{noise} vectors and $u, v \in \Lambda$ are lattice points. I define derived stochastic vectors:

$$\begin{aligned}\zeta_j^{(i,\pm)}(u) &= \sum_v S_{(i,\pm)}(u,v) \eta^j(v) \\ \xi_{j,X}^{(i,\pm)}(u) &= \sum_v S_{(i,\pm)}(u,v) \gamma_5 \Gamma_X^\dagger \eta^j(v)\end{aligned}\tag{4}$$

A brief discussion about their indices could clarify the notation:

- $j = 1, 2$ is the stochastic vector index. It tells you whenever to use $\eta^{(1)}$ or $\eta^{(2)}$.
- $X = A, B, C, D$ is the matrix index. It tells you to use $\Gamma_X^\dagger \in \{\Gamma_A^\dagger, \Gamma_B^\dagger, \Gamma_C^\dagger, \Gamma_D^\dagger\}$.
- $i = 1, 2, 3, 4$ refers to the propagator index. In principle, the ψ_i s could be four different quarks.
- the sign \pm is referred to the twisted mass parameter. If you use maximally twisted mass QCD - or maximally twisted Osterwalder-Seiler regularization - you should remember that the propagator is obtained by the inversion:

$$\sum_w \left(D_W^{(i)} \pm i \gamma_5 \mu^{(i)} \right) (u, w) S_{(i,\pm)}(w, v) = \delta_{u,v}$$

and I will use the following property: $\gamma_5 S_{(i,\pm)}^\dagger(u, v) \gamma_5 = S_{(i,\mp)}(v, u)$ that explain the need of \pm symbol.

- the symbols i and \pm are referred to the same propagator. For this reason they are coupled as (i, \pm) .

It could be easily checked that the contractions in (2) can be obtained by the following formulae:

$$G_d(x_0, y_0, z_0) = \sum_{\vec{y}} \left\langle \left\langle \left(\gamma_5 \xi_{1,A}^{(1,-)}(y) \right)^\dagger \Gamma_B \zeta_1^{(4,+)}(y) \cdot \left(\gamma_5 \xi_{C,2}^{(3,-)}(y) \right)^\dagger \Gamma_D \zeta_2^{(2,+)}(y) \right\rangle^{\text{noise}} \right\rangle^{\text{sea}}$$

$$G_c(x_0, y_0, z_0) = - \sum_{\vec{y}} \left\langle \left\langle \left(\gamma_5 \xi_{1,A}^{(1,-)}(y) \right)^\dagger \Gamma_D \zeta_2^{(2,+)}(y) \cdot \left(\gamma_5 \xi_{C,2}^{(3,-)}(y) \right)^\dagger \Gamma_B \zeta_1^{(4,+)}(y) \right\rangle^{\text{noise}} \right\rangle^{\text{sea}}$$

Then, to evaluate the previous couple of Wick contraction **I need only four quantites** for each couple of stochastic spinors:

$$\xi_{1,A}^{(1,-)}(y) \quad \zeta_1^{(4,+)}(y) \quad \xi_{C,2}^{(3,-)}(y) \quad \zeta_2^{(2,+)}(y)$$

The path to evaluate the correlators is:

- ▷ For each Gauge-sea configuration evaluate $2N_{\text{noise}}$ spinors - N_{noise} for η^1 and N_{noise} for η^2 .
- ▷ For each noise spinor evaluate the quantites ζ and ξ needed. Sometimes just a few number of them are needed.
- ▷ Evaluate the correlators and evaluate the noise average.
- ▷ Iterate the procedure and calculate the sea average.