Non-perturbative computation of Kaons oscillation amplitudes in Lattice QCD with $N_f = 2 + 1$ sea quarks and OBC

Master's Degree in Theoretical Physics

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Academic Year 2022/2023

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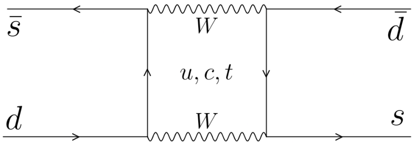
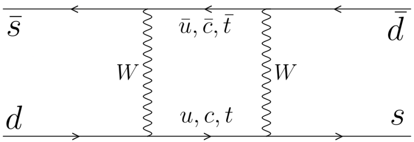
Kaons oscillations

SM oscillations

Kaons oscillations in Standard Model (SM) are mediated by Weak Interacitons.

- Two 1-loop order diagrams





Kaons oscillations

SM oscillations

Kaons oscillations in Standard Model (SM) are mediated by Weak Interacitons.

- Two 1-loop order diagrams
- An effective operator arises ($E \ll M_W$):

$$\langle \bar{K}^0 | \Theta_1 | K^0 \rangle \qquad \Theta_1 = \left[\bar{s}^a \gamma_\mu (1 - \gamma_5) d^a \right] \cdot \left[\bar{s}^b \gamma_\mu (1 - \gamma_5) d^b \right]$$

- Perturbative approach (PT) not allowed at energies $\lesssim \Lambda_{\text{QCD}}$

Vacuum insertion approximation (VIA):

$$\langle \bar{K}^0 | \left[\bar{s}^a \gamma_\mu (1-\gamma_5) d^a \right] \cdot \left[\bar{s}^b \gamma_\mu (1-\gamma_5) d^b \right] | K^0 \rangle \approx$$
$$\approx \langle \bar{K}^0 | \left[\bar{s}^a \gamma_\mu (1-\gamma_5) d^a \right] | 0 \rangle \langle 0 | \left[\bar{s}^b \gamma_\mu (1-\gamma_5) d^b \right] | K^0 \rangle +$$
$$\langle \bar{K}^0 | \left[\bar{s}^a \gamma_\mu (1-\gamma_5) d^b \right] | 0 \rangle \langle 0 | \left[\bar{s}^b \gamma_\mu (1-\gamma_5) d^a \right] | K^0 \rangle = \frac{8}{3} F_K^2 m_K^2$$

B_1 parameter (a.k.a. B_K) parametrizes deviation from VIA: $\langle \bar{K}^0 | \Theta_1 | K^0 \rangle = \frac{8}{3} B_1 F_K^2 m_K^2$

Kaons oscillations

Oscillations BSM

- B_1 parameter (a.k.a. B_K) parametrizes deviation from VIA
- Other operators $\{\Theta_i, \tilde{\Theta}_j\}$ appears in theories beyond the SM (BSM)
- They form a complete set of operators of dimension 6, composed by 4 quarks, closed under renormalization procedure
- New BSM *bag parameters*:

$$\langle \bar{K}^0 | \Theta_j^{[+],\text{ren}}(\mu) | K^0 \rangle = \xi_j \left(\frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right)^2 f_K^2 B_j(\mu) \qquad j \geq 2$$

- Non perturbative approach $\Longrightarrow$ *path integral*

Purpose of the work

Introduction

Non perturbative computation of bare bag parameters B_i on the lattice

$$\downarrow$$

Coordinated Lattice Simulations (CLS) gauge configurations with $N_f = 2 + 1$ sea quarks and open boundary conditions (OBC) in time direction

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Lattice regularizations of QFTs

Regularizations ingredients:

- Lattice space Λ of dimension $L \times L \times L \times T$
- Lattice spacing a: $L = a \cdot N_L$, $T = a \cdot N_T$
- Action discretization:

$$S^{\text{lat}}[a; \tilde{\phi}_1, \ldots, \tilde{\phi}_{N_{\text{fields}}}] \xrightarrow{a \to 0} S^{\text{cont}}[\phi_1, \ldots, \phi_{N_{\text{fields}}}]$$

- Troubles: finite volume effects & lattice spacing effects
- $O(a^n)$-improvement: $S^{\text{lat}}(a) = S^{\text{cont}} + o(a^n)$

Many regularizations have been developed

Gauge actions:

- Link variables

$$U_\mu(x) = \exp\left(i\int_x^{x+a\hat{\mu}} A_\nu(\omega)d\omega^\nu\right)$$

- Wilson loops are gauge invariant
- Plaquette, tree level improved Symanzik, Luscher Weisz

Fermion actions:

- Wilson-Dirac action Wilson term $r \in [-1, 1]$
- Sheikholeslami-Wohlert term:

$$D^W_{xy} \longmapsto D^W_{xy} + c_{SW}\frac{ia}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu}(x)\delta_{xy}$$

- Osterwalder-Seiler action (twist transformation)

- Gauge configuration exhibit long autocorrelation times as $a \to 0$
- Zero modes of D^W_{xy} make the sea quarks simulation inefficient

$$\Downarrow$$

Open boundary conditions (OBC) in time direction

$$F_{0i}(\vec{x}, x_4 = 0) = F_{0i}(\vec{x}, x_4 = T) = 0 \qquad \forall i = 1, 2, 3$$

$$\begin{aligned} P_+\psi(\vec{x}, 0) &= P_-\psi(\vec{x}, T) = 0 \\ \bar{\psi}(\vec{x}, 0)P_- &= \bar{\psi}(\vec{x}, T)P_+ = 0 \end{aligned} \qquad P_\pm = \frac{1}{2}\left(\mathbb{I} \pm \gamma_5\right)$$

- Restoration of connection of gauge fields space
- IR protection against zero modes

- Gauge action: Luscher-Weisz action

$$S_G[U] = \frac{1}{g_0^2}\left(\frac{5}{3}\sum_{\{P\}}\mathrm{Tr}\left(\mathbb{I} - U_P\right) - \frac{1}{12}\sum_{\{R\}}\mathrm{Tr}\left(\mathbb{I} - U_R\right)\right)$$

- Sea quarks: Dirac-Wilson + SW term

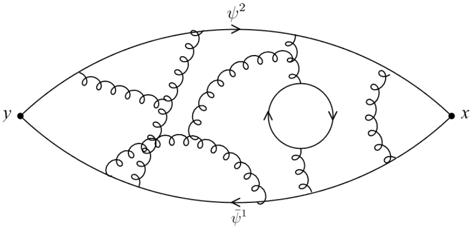
$$S^{\text{sea}}[U, f, \bar{f}] = \sum_{q=1}^{3}\sum_{x}\bar{f}_q(x)\left[D^{WD}(r_q) + \frac{ia}{4}c_{SW}\sigma_{\mu\nu}\hat{F}_{\mu\nu} + m_q^{\text{bare}}\right]f_q(x)$$

Valence actions

- Valence quarks: Osterwalder-Seiler at Maximal twist + SW term

$$S^{\text{val}}[U,\psi,\bar{\psi}] = a^4 \sum_q \sum_x \bar{\psi}_q(x) \left[D^{WD}(r_q) + \frac{ia}{4} c_{SW} \sigma_{\mu\nu} \hat{F}_{\mu\nu} + m^{\text{cr}} + i\gamma_5 \mu_q^{\text{bare}} \right] \psi_q(x)$$

- Flavours: d, d', s, s' with
  $\mu_s = \mu_{s'}$, $\mu_d = -\mu_{d'}$
  equiv. $r_d = r_s = r_{s'} = -r_{d'} = \pm 1$



- Bosonic ghosts $\{\phi_q, \bar{\phi}_q\}$ to cancel valence quarks determinant

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Frezzotti & Rossi Strategy

Why do we need flavor replicas d, d', s, s' ? Because of R. Frezzotti & G. Rossi strategy!

A rather long chain of transformations:

$$\text{Operators } \{\Theta_i^{[\pm]}\} \text{ in continuum QCD} \xrightarrow{\text{basis change}} \text{Operators } \{Q_i^{[\pm]}\} \text{ in continuum QCD}$$

$$\xrightarrow{\text{flavour replicas}} \text{P.E. operators } \{O_{i,[+]}^{\text{phys}}\} \xrightarrow{\text{maximal twist}} \text{P.E. operators } \{O_{i,[+]}^{\text{tw}}\}$$

Frezzotti & Rossi Strategy

Flavour replicas and twist: a simple example

$$\bar{K}^{0\prime} = \bar{s}'\gamma_5 d' = \bar{\chi}^3\gamma_5\chi^4 \qquad \bar{K}^{0\prime} = \bar{\psi}^3\gamma_5\psi^4$$

Maximal Twist $\chi^i = e^{i\gamma_5 r_i \frac{\pi}{4}}\psi^i$

$$\bar{K}^0 = \bar{s}\gamma_5 d = \bar{\chi}^1\gamma_5\chi^2 \qquad \bar{K}^0 = i\bar{\psi}^1\psi^2$$

Similar transformations act on operators $O_{i[+]}$ and map PE operators in PO ones.

Achievements:

Specific regularization of the valence quarks in order to obtain:

- $O(a)$-improved mixing amplitudes w/o the need for W.A.
- Absence of wrong chirality mixing between $\{O_i^{[+]}\}$ and $\{O_i^{[-]}\}$
- Blocks like renormalization matrix $[Z_{ij}]$ for parity odd operators

Two and three points correlators:

$$C_i(x_4, y_4, z_4) = \sum_{\vec{x},\vec{y},\vec{z}} \langle\Omega| T \left\{ \bar{K}^{0'}(x) O_{i[+]}(y) \bar{K}^0(z) \right\} |\Omega\rangle$$

$$G_{34}(x_4, y_4) = \sum_{\vec{x},\vec{y}} \langle\Omega| T \left\{ \bar{K}^{0'}(x) K^{0'}(y) \right\} |\Omega\rangle$$

$$G_{12}(x_4, y_4) = \sum_{\vec{x},\vec{y}} \langle\Omega| T \left\{ \bar{K}^{0}(x) K^{0}(y) \right\} |\Omega\rangle$$

$$X_{34}(x_4, y_4) = \sum_{\vec{x},\vec{y}} \langle\Omega| T \left\{ \bar{A}'_4(x) K^{0'}(y) \right\} |\Omega\rangle$$

$$X_{12}(x_4, y_4) = \sum_{\vec{x},\vec{y}} \langle\Omega| T \left\{ \bar{A}_4(x) K^{0}(y) \right\} |\Omega\rangle$$

Asymptotic behaviours

Asymptotic behaviours for $x_4 \gg y_4 \gg z_4$:

$$C_i(x_4, y_4, z_4) \approx \sum_{\vec{y}} \langle \Omega | \bar{K}^{0'} | \bar{K}^{0'} \rangle \langle \bar{K}^{0'} | O_{i[+]} | K^0 \rangle \langle K^0 | \bar{K}^0 | \Omega \rangle e^{-m_{K'}(x_4 - y_4) - m_K(y_4 - z_4)}$$

$$G_{34}(x_4, y_4) \approx \sum_{\vec{y}} \langle \Omega | \bar{K}^{0'} | \bar{K}^{0'} \rangle \langle \bar{K}^{0'} | K^{0'} | \Omega \rangle e^{-m_{K'}(x_4 - y_4)}$$

$$G_{12}(x_4, y_4) \approx \sum_{\vec{y}} \langle \Omega | \bar{K}^{0} | \bar{K}^{0} \rangle \langle \bar{K}^{0} | K^{0} | \Omega \rangle e^{-m_{K}(x_4 - y_4)}$$

$$X_{34}(x_4, y_4) \approx \sum_{\vec{y}} \langle \Omega | \bar{A}'_4 | \bar{K}^{0'} \rangle \langle \bar{K}^{0'} | K^{0'} | \Omega \rangle e^{-m_{K'}(x_4 - y_4)}$$

$$X_{12}(x_4, y_4) \approx \sum_{\vec{y}} \langle \Omega | \bar{A}_4 | \bar{K}^{0} \rangle \langle \bar{K}^{0} | K^{0} | \Omega \rangle e^{-m_{K}(x_4 - y_4)}$$

Bag parameters are recovered in continuum limit $a \to 0$ for $x_4 \gg y_4 \gg z_4$:

$$\mathcal{B}_1(a) \approx \frac{3}{8} \cdot \frac{C_1(x_4, y_4, z_4)}{X_{34}(x_4, y_4) X_{12}(y_4, z_4)} \xrightarrow{a \to 0} B_K$$

$$\mathcal{B}_i(a) \approx \frac{1}{\xi_i} \cdot \frac{\Lambda_{ij} C_j(x_4, y_4, z_4)}{A_{34}(x_4, y_4) A_{12}(y_4, z_4)} \xrightarrow{a \to 0} B_i$$

$\Rightarrow$ Need for 2 and 3 points correlation functions computation

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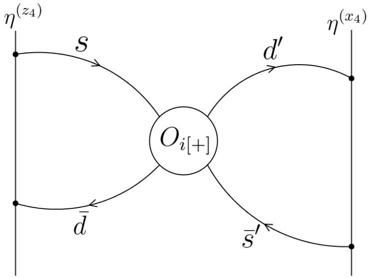
Noise spinors method

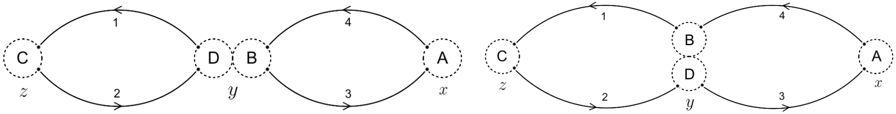
Mesonic sources in x_4 and z_4 are simulated through noise spinors $\eta^{(x_4)}$ and $\eta^{(z_4)}$:

- Two sets of N_n spinors randomly generated
- The sums $\sum_{\vec{x}}$ and $\sum_{\vec{z}}$ automatically implied when calculating the average over ηs
- Computational advantage
- Different Wick contractions are obtained by multiplying propagators from sources with Dirac matrices



Noise spinors method

Two types of diagrams:



Two types of noise spinors contractions:

$$G_d = \sum_{\vec{y}} \left\langle \left\langle \left(\gamma_5 \xi_{1,A}^{(3,-)}(y)\right)^\dagger \Gamma_B \zeta_1^{(4,+)}(y) \cdot \left(\gamma_5 \xi_{C,2}^{(1,-)}(y)\right)^\dagger \Gamma_D \zeta_2^{(2,+)}(y) \right\rangle^{\text{noise}} \right\rangle^{\text{sea}}$$

$$G_c = -\sum_{\vec{y}} \left\langle \left\langle \left(\gamma_5 \xi_{1,A}^{(3,-)}(y)\right)^\dagger \Gamma_D \zeta_2^{(2,+)}(y) \cdot \left(\gamma_5 \xi_{C,2}^{(1,-)}(y)\right)^\dagger \Gamma_B \zeta_1^{(4,+)}(y) \right\rangle^{\text{noise}} \right\rangle^{\text{sea}}$$

Simulation program tests

Tests:

- Gauge invariance test
- Gauge invariance + investigation about N_n
- Test on *Tree level improved Symanzik action*, quenched approximation
- Test on *Plaquette gauge action*, quenched approximation

Test #1: Gauge invariance

Gauge transformed propagators:

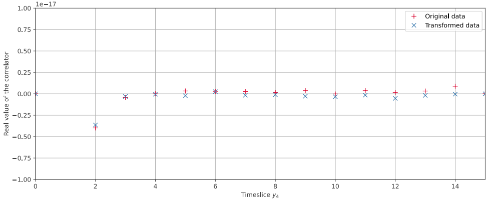
$$\left(S^{(f)}(y,x)[\Omega U \Omega^\dagger]\right)^{b,a}_{\beta,\alpha} = \Omega(y)^{b,c}\left(S^{(f)}(y,x)[U]\right)^{c,d}_{\beta,\alpha}\Omega^\dagger(x)^{d,a}$$

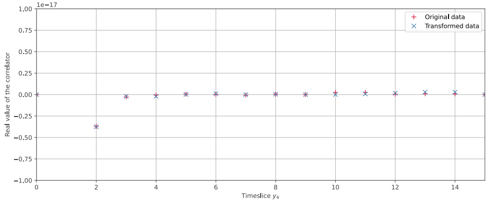
- Equation tested in three points integrated correlators $C_{i,[+]}$
- Link variables $U_\mu(x)$ are randomly generated and then transformed
- Pointlike sources instead of random spinors η

Test #2: Gauge invariance and investigation about N_n

- Equation tested in three points integrated correlators $C_{i,[+]}$
- Link variables $U_\mu(x)$ are randomly generated and then transformed
- Use of random spinors η
- Different number of N_n were tested. Estimator:

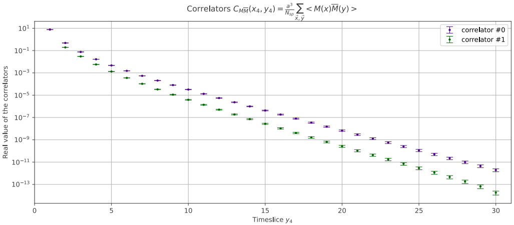
$$\varepsilon(N_n) = \frac{1}{N_T - 2} \sum_{y_4} \left(\frac{|C(y_4) - \tilde{C}(y_4)|}{C(y_4) + \tilde{C}(y_4)} \right)$$

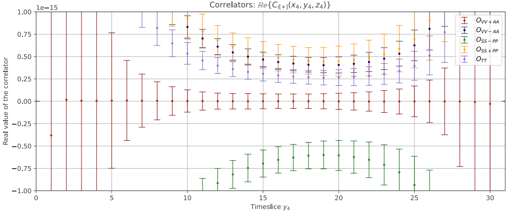




Test #3: tree level improved Symanzik

54 Gauge configurations - tree level improved Symanzik action
16x16x16x32 lattice, $\beta = 6.0$, OBCs, unphysical parameters.





$G_{12}(x_4, y_4), G_{34}(x_4, y_4)$

$C_{i[+]}(14a, y_4, a)$

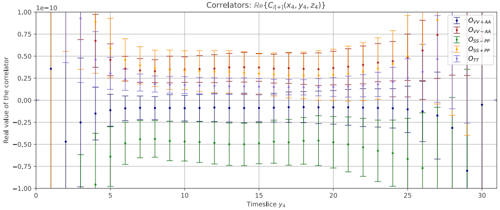
$M \neq M_{\text{crit}} \Rightarrow$ not maximal twist.

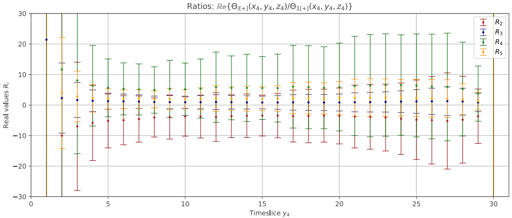
Test #4: Plaquette gauge action

4 Simulation Program

37 Gauge configurations - plaquette gauge action

16x16x16x32 lattice, $\beta = 6.0$, OBCs, maximal twist achieved





$$C_{i[+]}(14a, y_4, a)$$

$$\mathcal{R}_i(14a, y_4, a) = \Lambda_{ij} \frac{C_{i[+]}(14a, y_4, a)}{C_{1[+]}(14a, y_4, a)}$$

Future developments

- tests on faithful Gauge configurations with known parameters ($M_{\text{crit}}, c_{SW}, \mu_i s$)
- runs over the $N_f = 2 + 1$ CLS sea ensembles (many values of a)
- data analysis
- continuum limit extrapolation of B_i and R_i
- renormalization procedure

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Thank you for listening!

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