

# Aprendizado Automático

João Paulo Pordeus Gomes



# Support Vector Machines

# Support Vector Machines

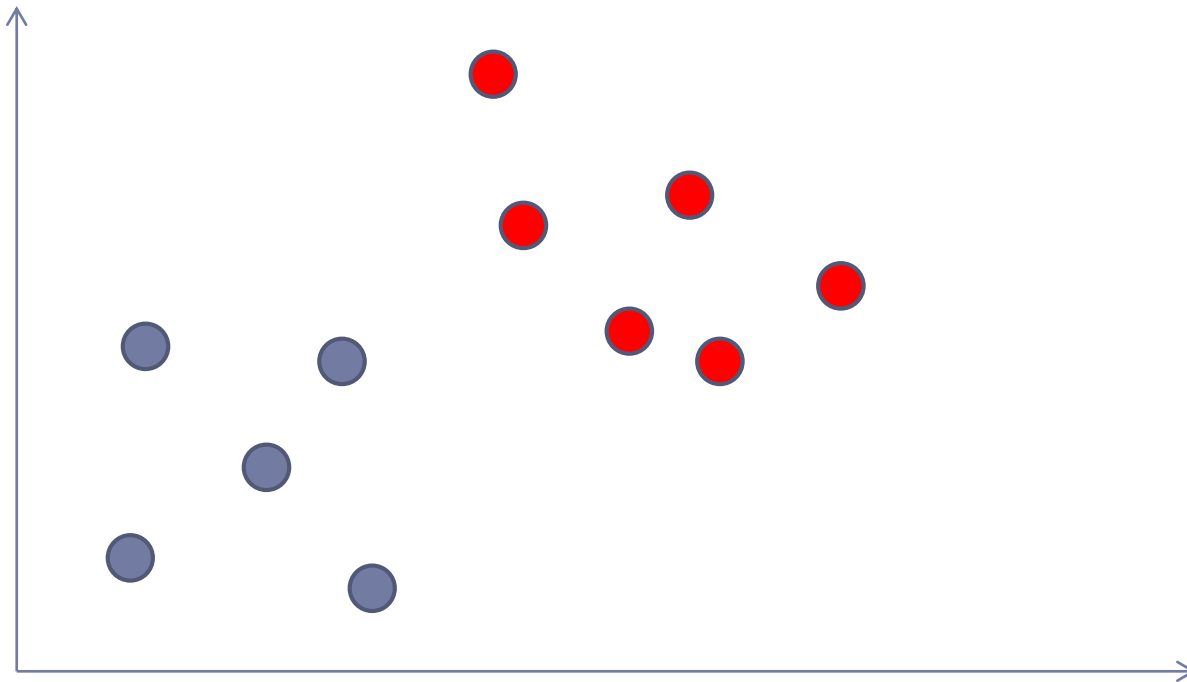
---

- ▶ Máquinas de Vetores Suporte
- ▶ Classificação e Regressão
- ▶ Busca encontrar a melhor superfície de separação entre os dados de diferentes classes



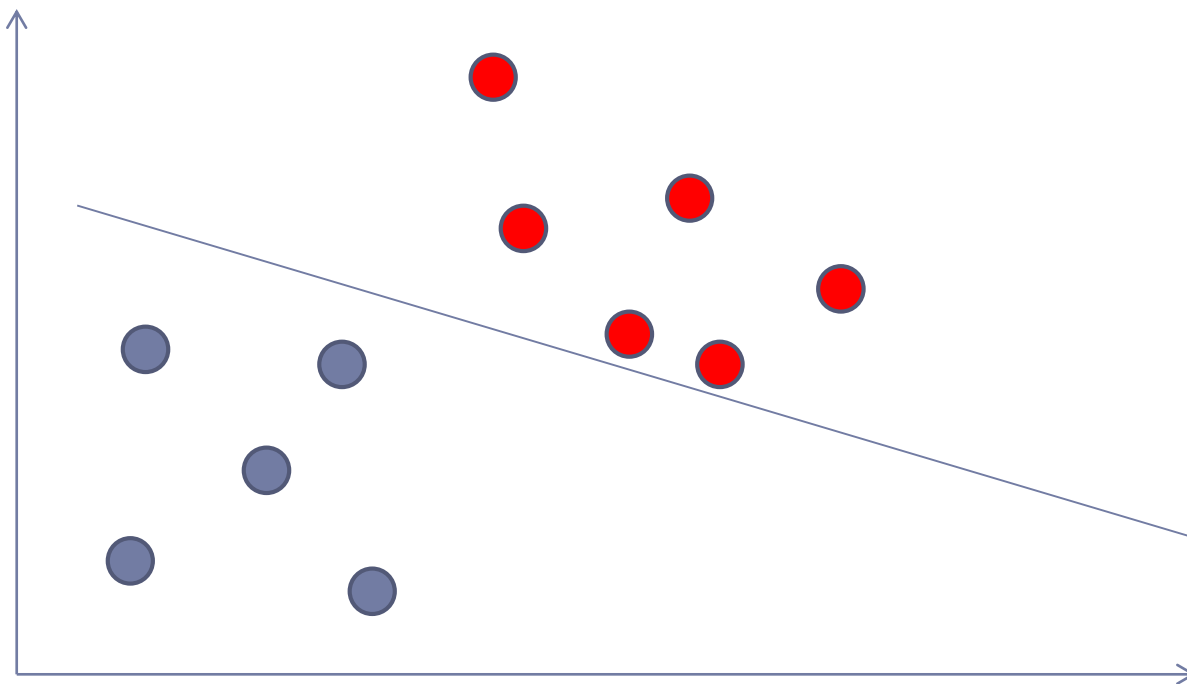
# Superfície de Separação

---



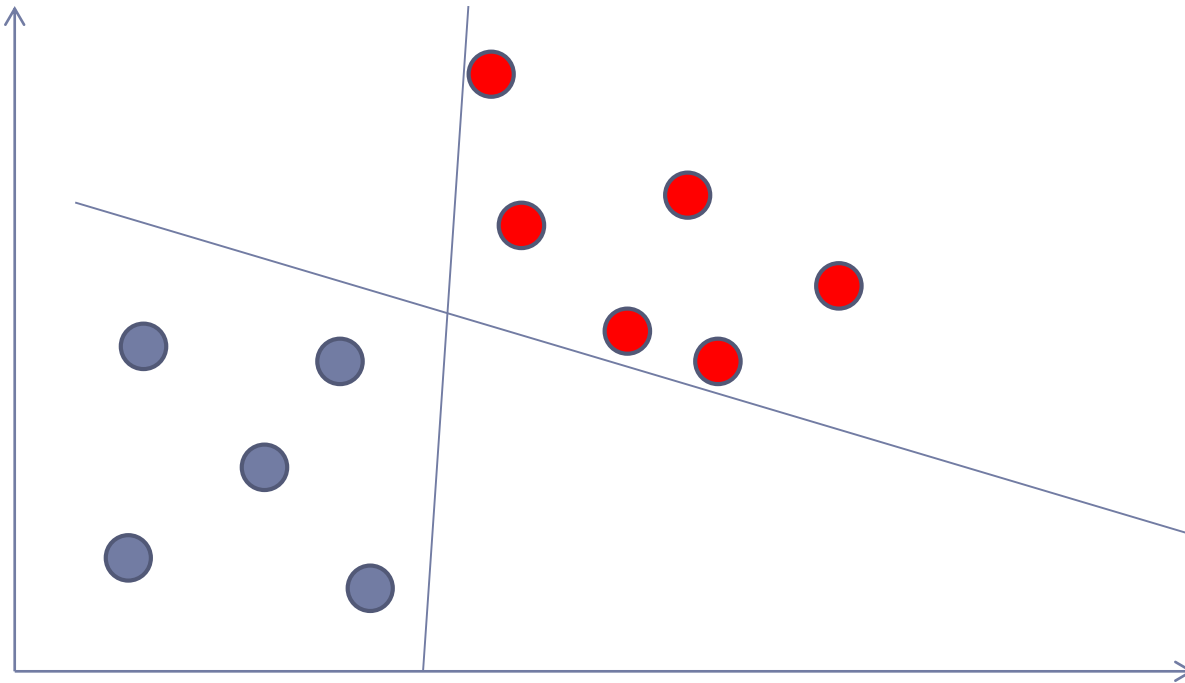
# Superfície de Separação

---



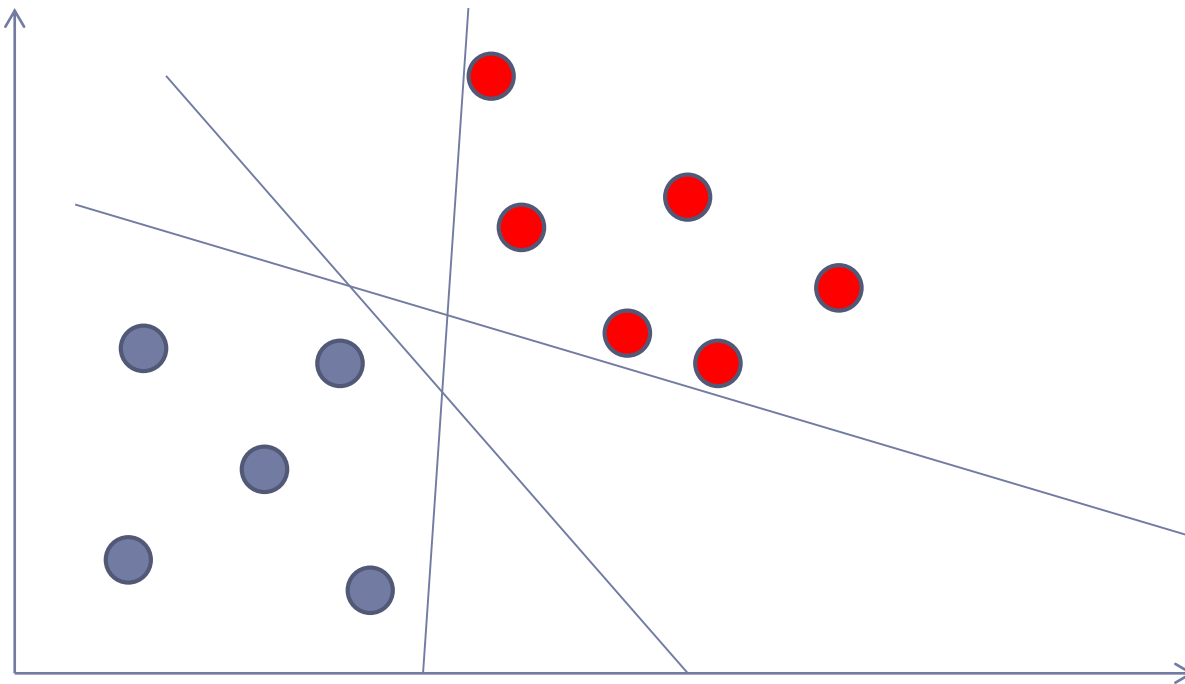
# Superfície de Separação

---



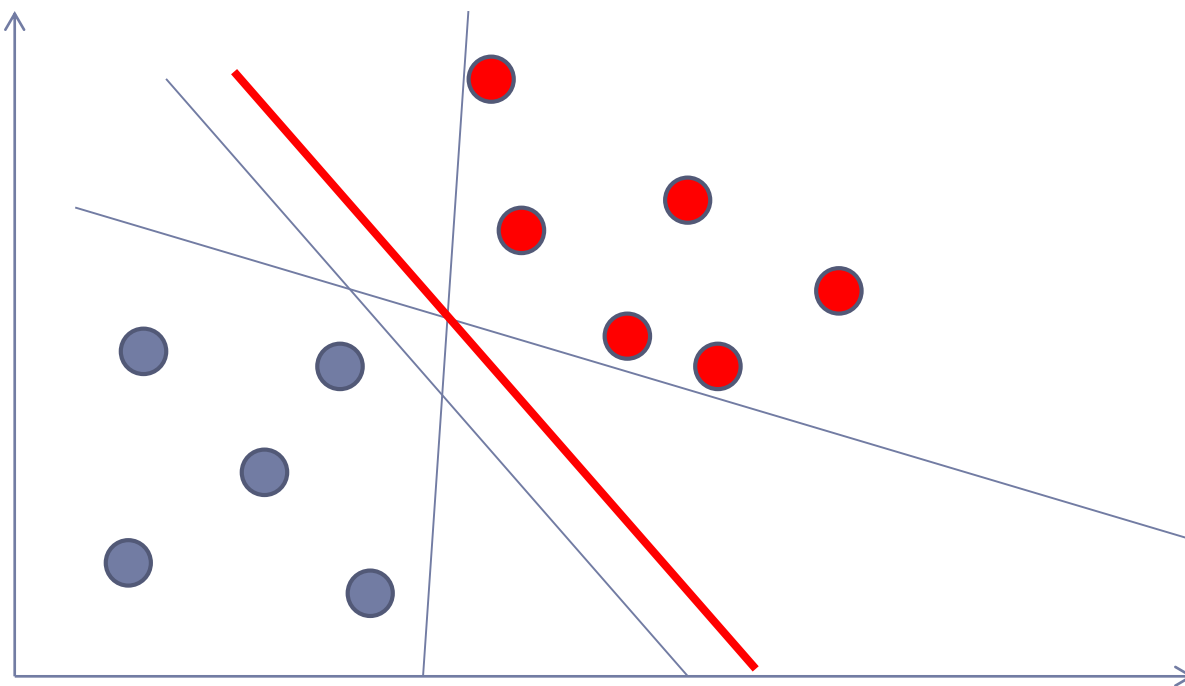
# Superfície de Separação

---



# Superfície de Separação

---





# Fundamentos Matemáticos

# Tópicos

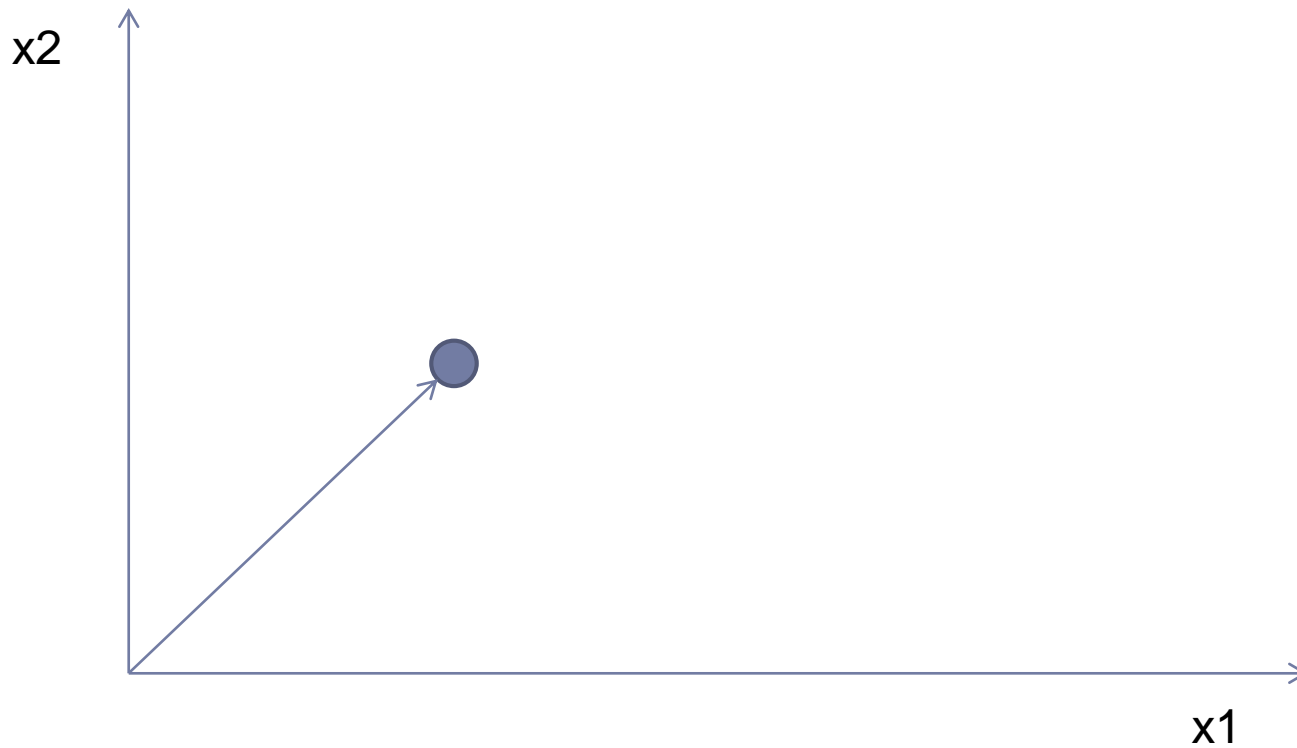
---

- ▶ Álgebra Linear
  - ▶ Representação Vetorial
  - ▶ Norma de um vetor
  - ▶ Produto Escalar
  - ▶ Vetor Perpendicular a um plano
  - ▶ Distância entre dois planos paralelos



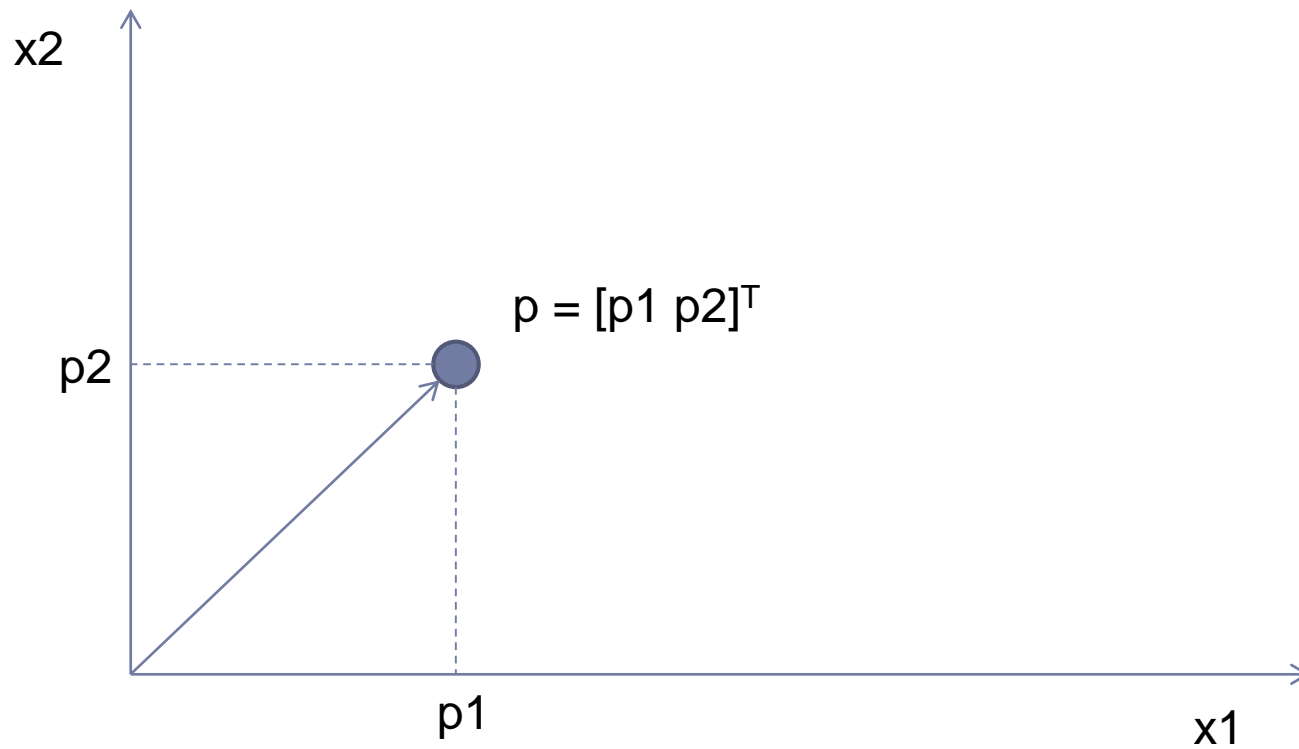
# Representação Vetorial

---



# Representação Vetorial

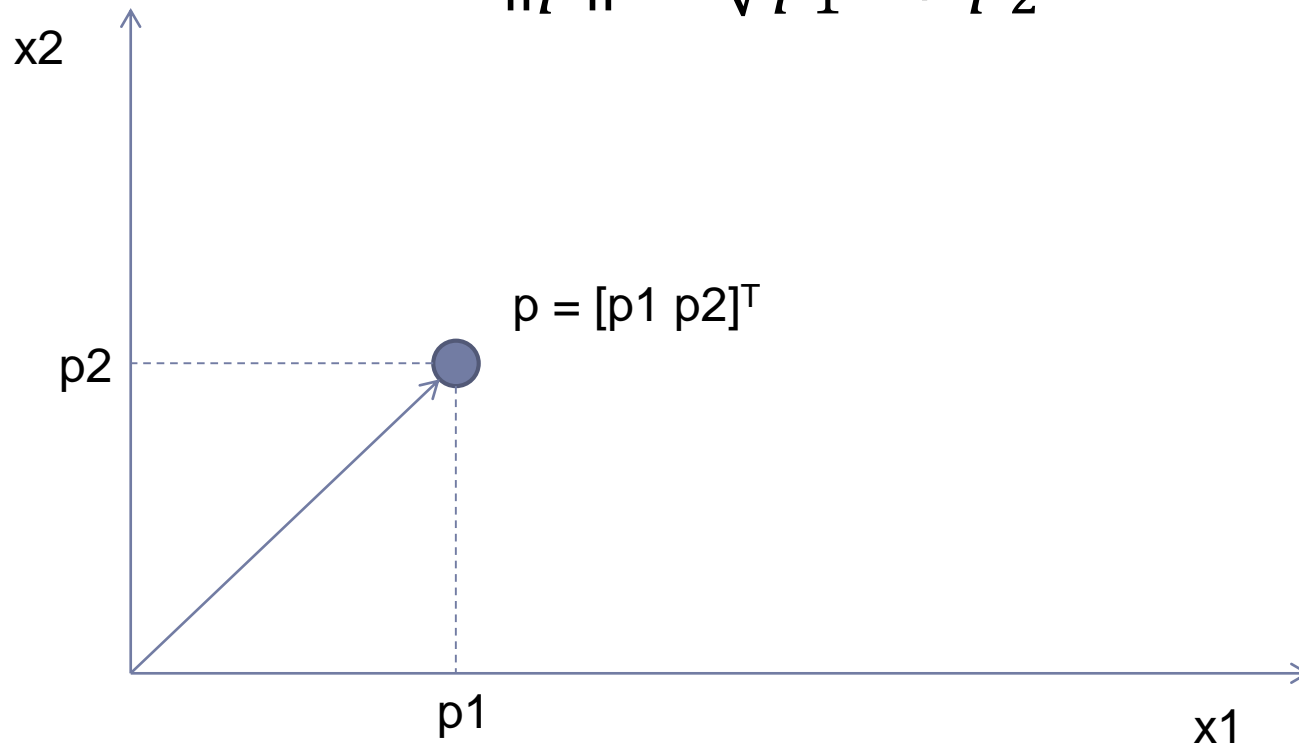
---



# Norma de um Vetor

---

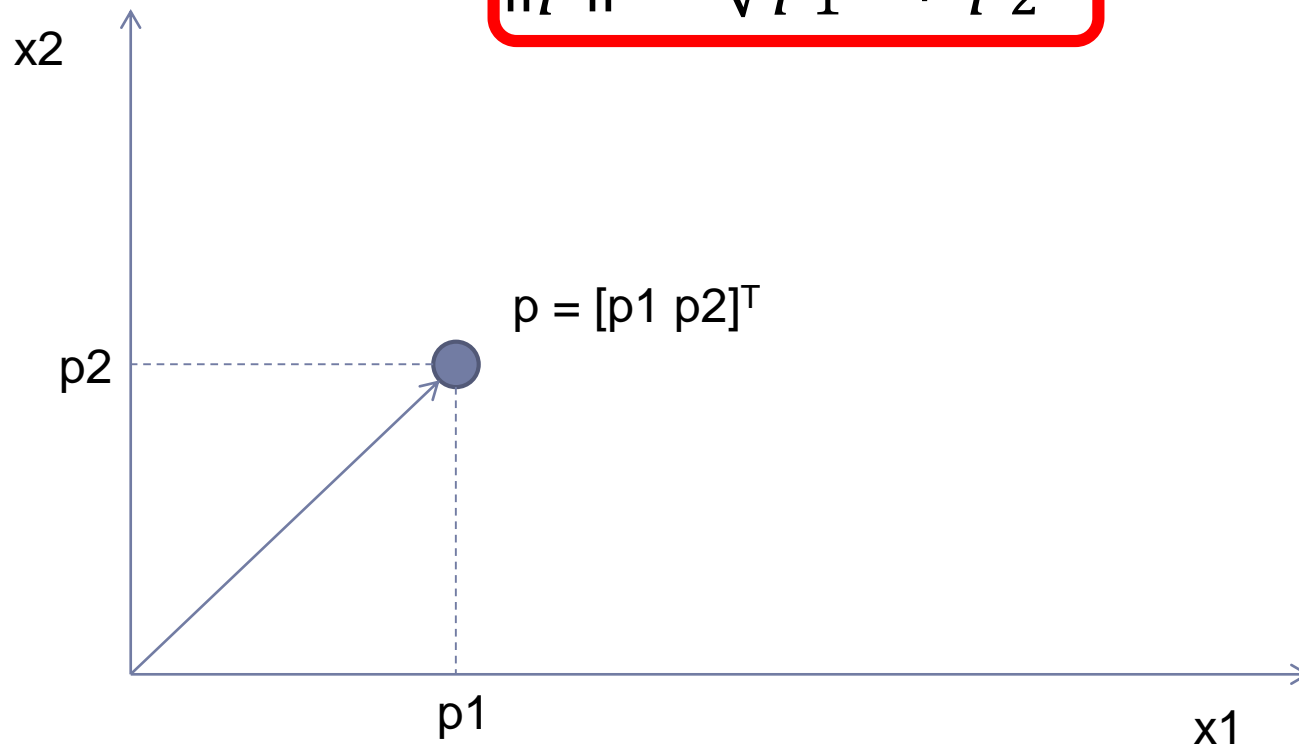
$$\|p\| = \sqrt{p_1^2 + p_2^2}$$



# Norma de um Vetor

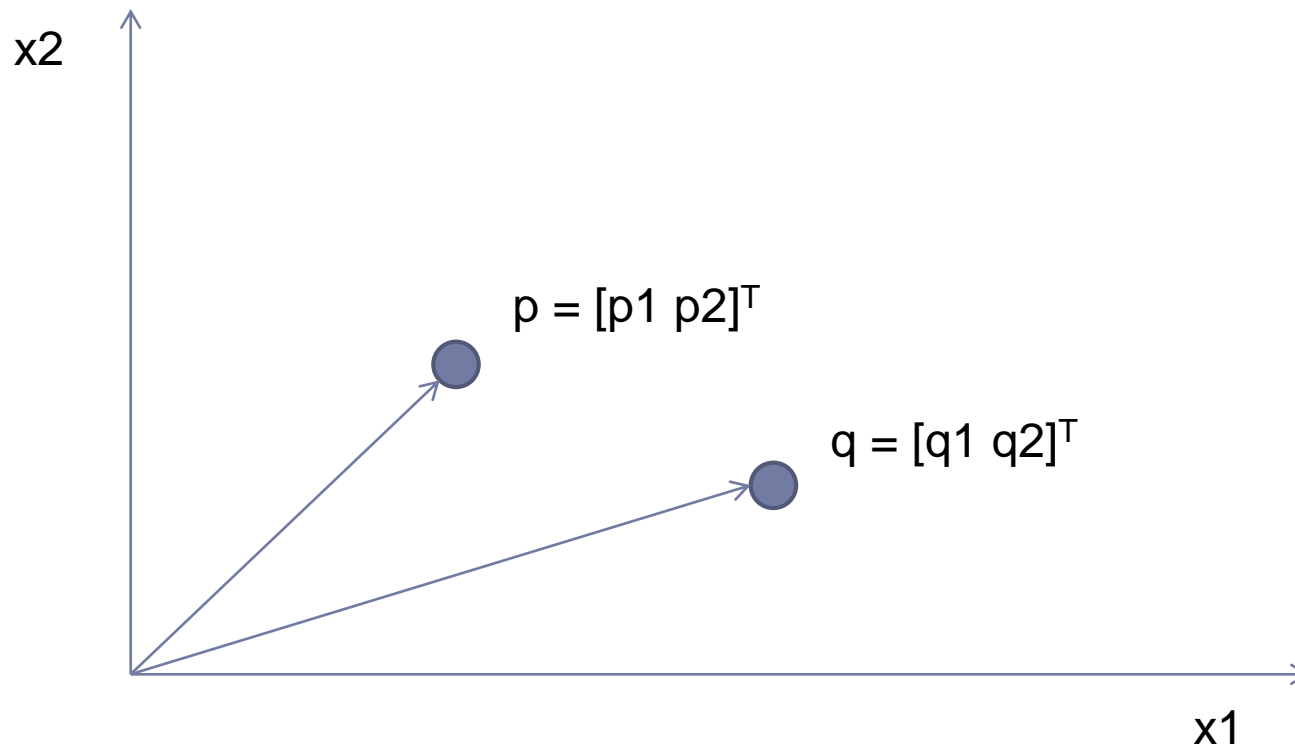
---

$$\|p\| = \sqrt{p_1^2 + p_2^2}$$



# Representação Vetorial

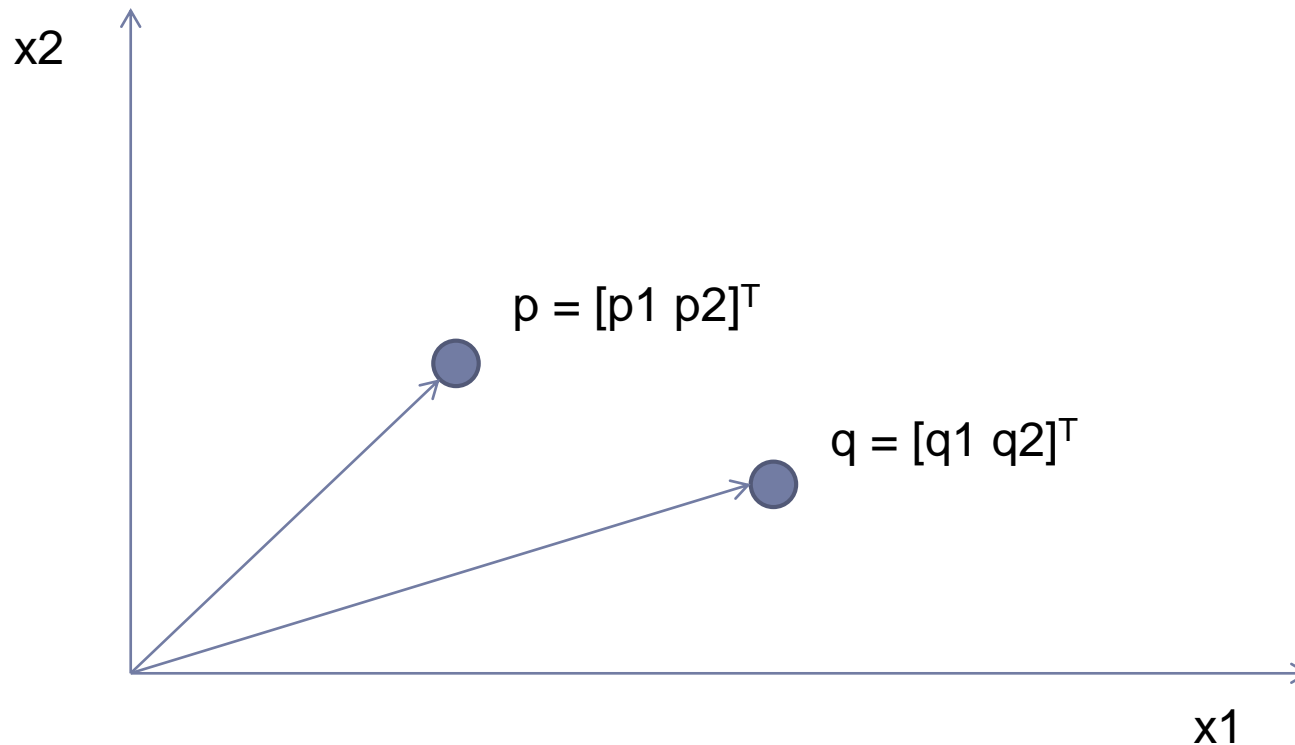
---



# Produto Escalar (Interno)

---

$$p \cdot q = \|p\| \|q\| \cos \theta$$

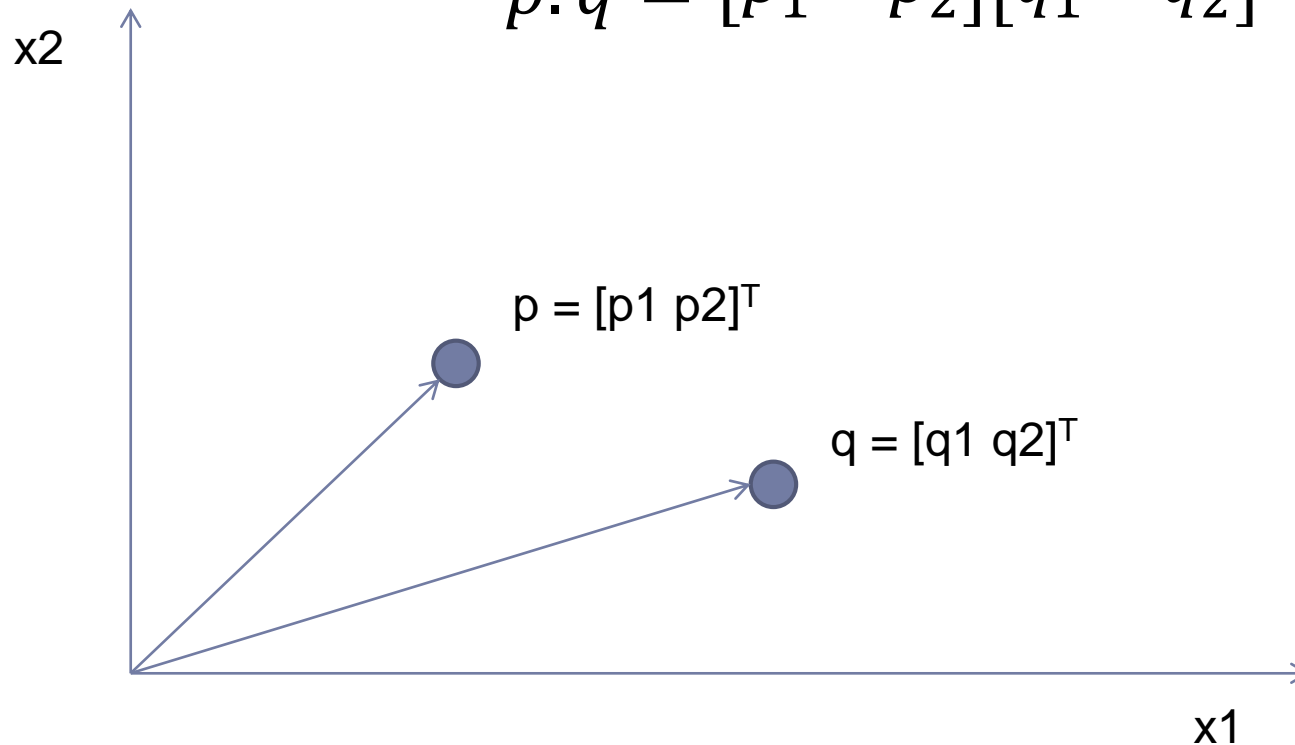




# Produto Escalar (Interno)

---

$$p \cdot q = \|p\| \|q\| \cos \theta$$
$$p \cdot q = [p_1 \quad p_2] [q_1 \quad q_2]^T$$

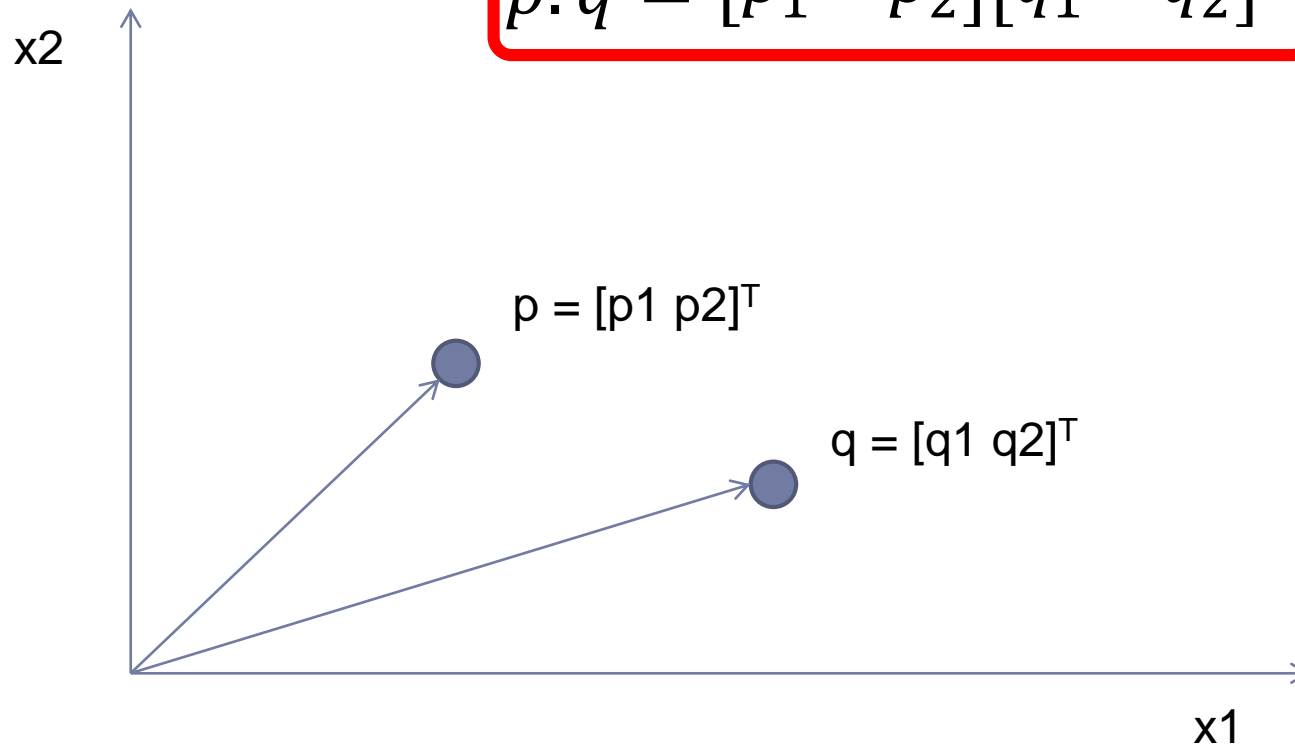


# Produto Escalar (Interno)

---

$$p \cdot q = \|p\| \|q\| \cos \theta$$

$$p \cdot q = [p_1 \quad p_2] [q_1 \quad q_2]^T$$



# Equação de um hiperplano

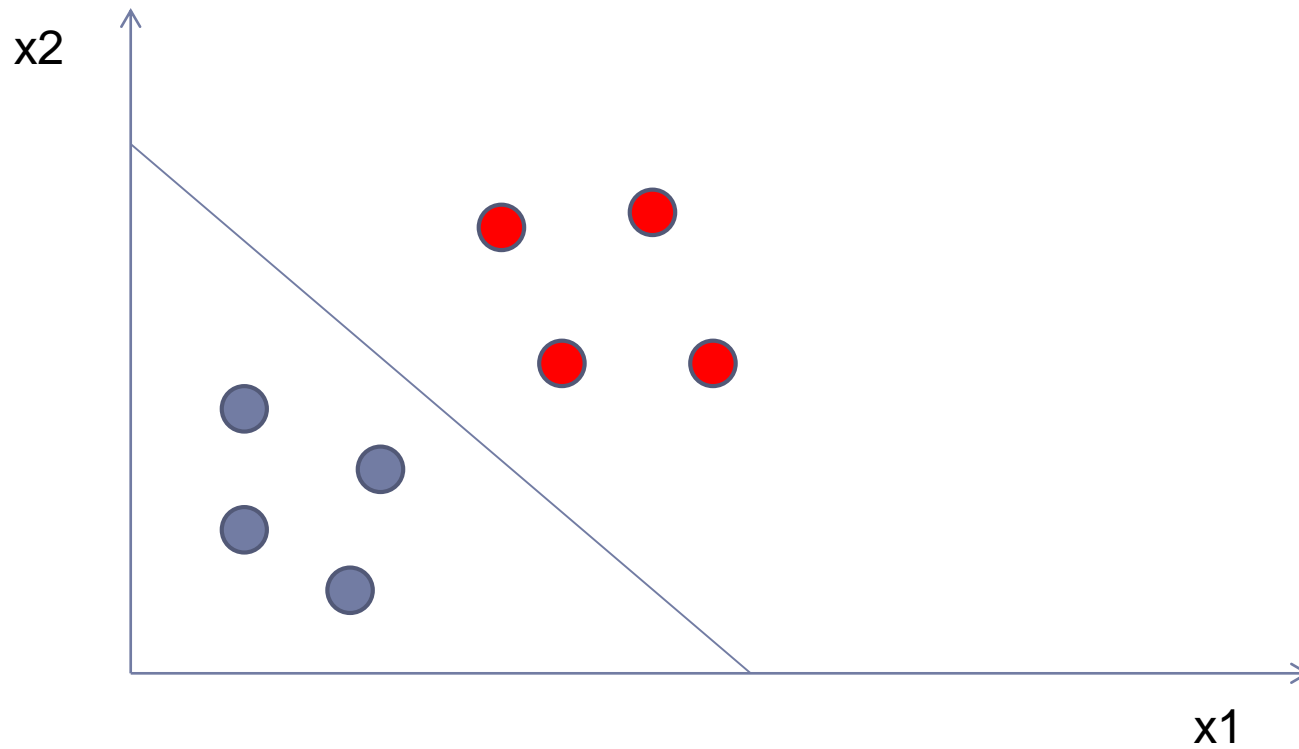
---

- ▶ Problemas de classificação
  - ▶ Hiperplano de separação



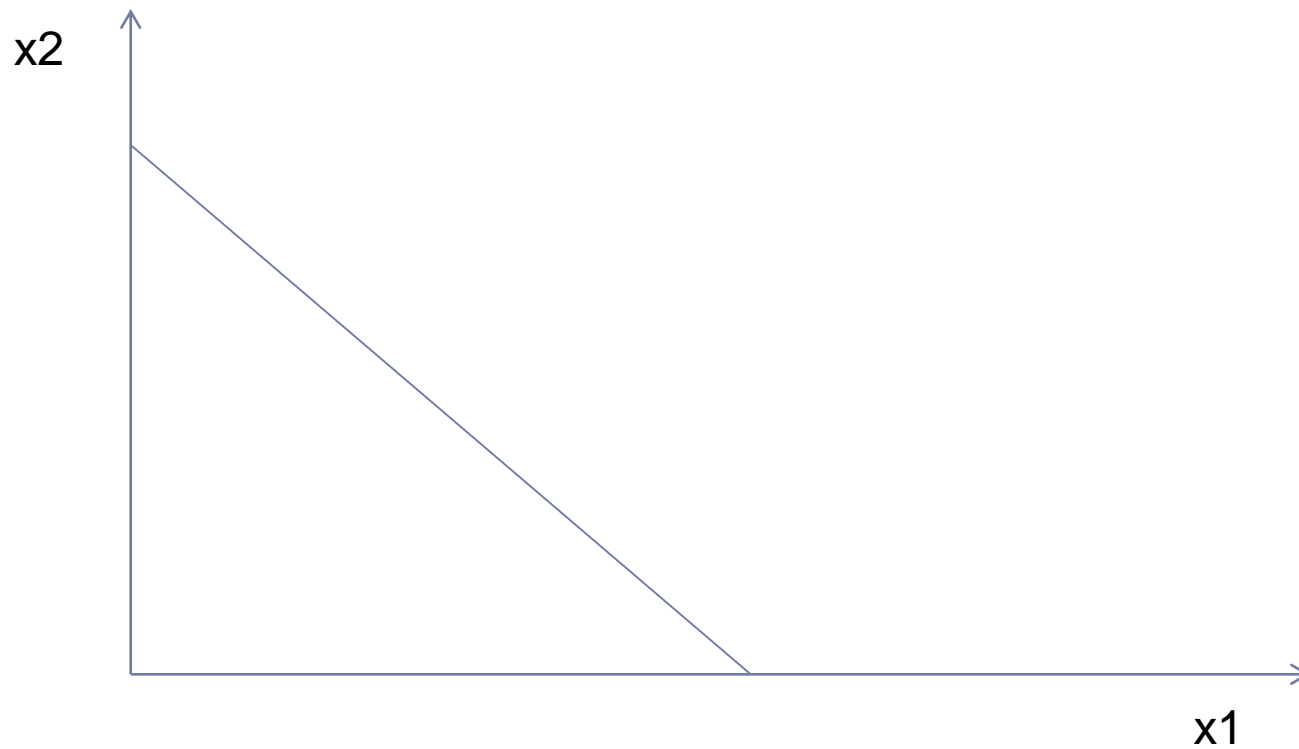
# Equação do Hiperplano

---



# Equação do Hiperplano

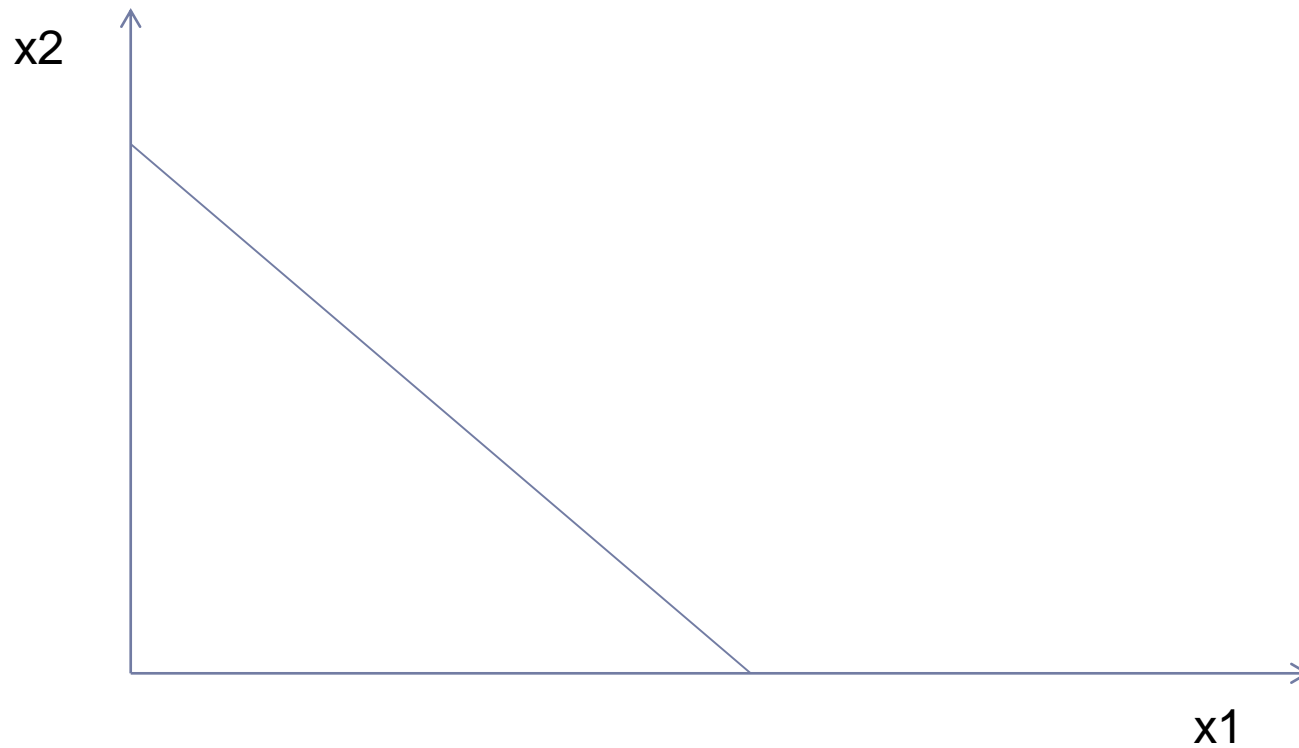
---



# Equação do Hiperplano

---

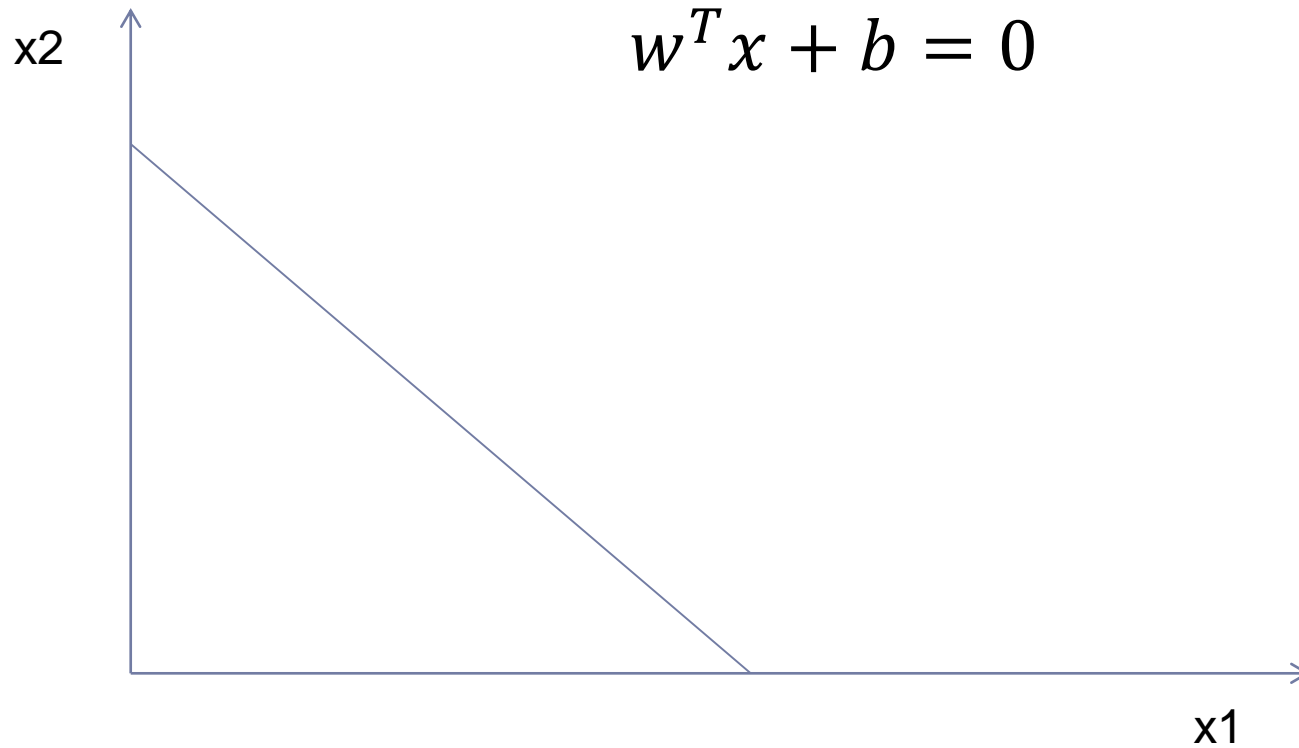
$$w_1x_1 + w_2x_2 + b = 0$$



# Equação do Hiperplano

---

$$w_1x_1 + w_2x_2 + b = 0$$
$$w^T x + b = 0$$

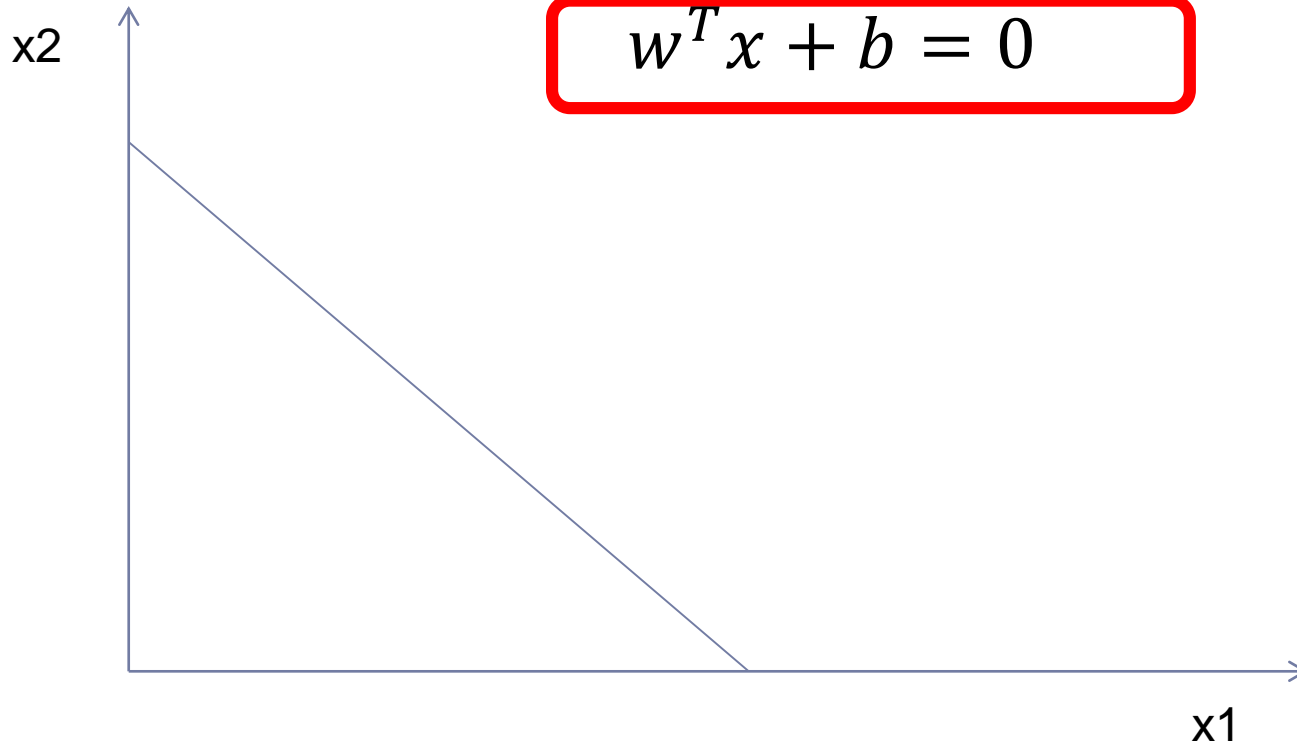


# Equação do Hiperplano

---

$$w_1x_1 + w_2x_2 + b = 0$$

$$w^T x + b = 0$$

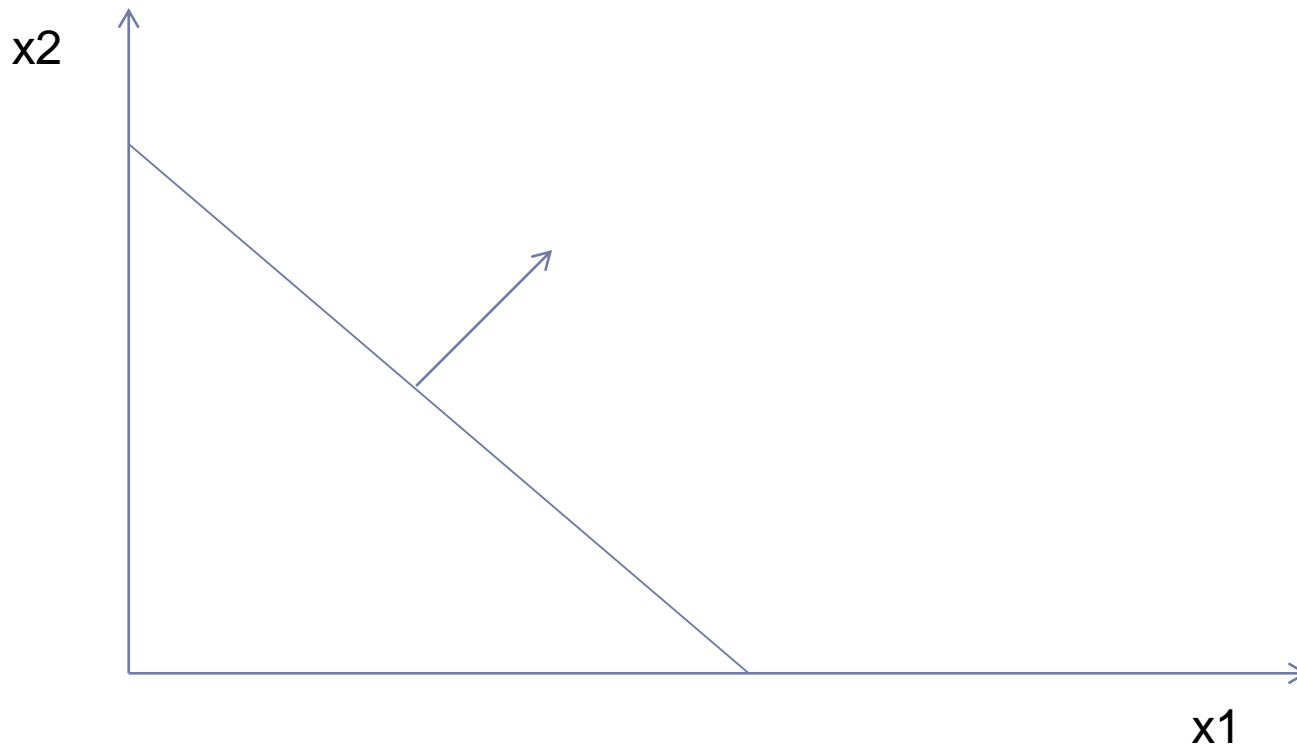




# Vetor perpendicular ao hiperplano

---

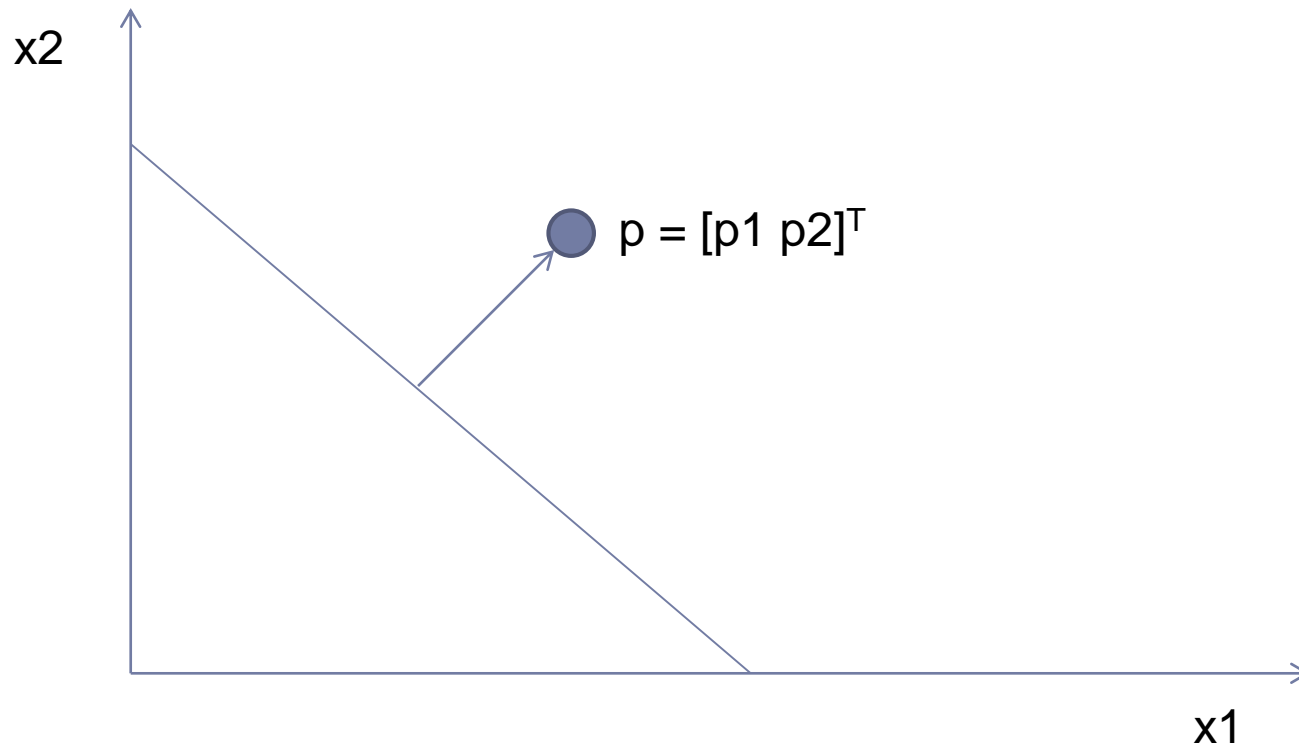
$$w^T x + b = 0$$



# Vetor perpendicular ao hiperplano

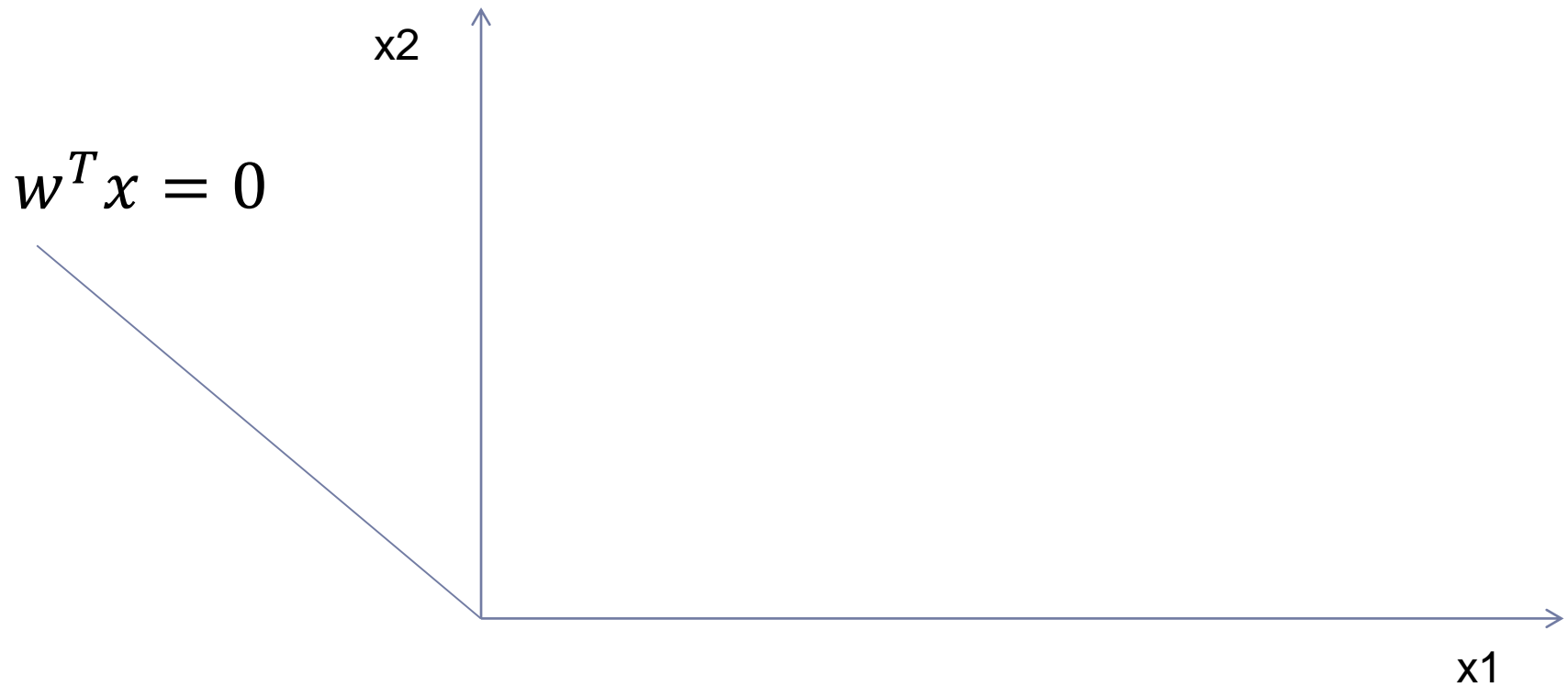
---

$$w^T x + b = 0$$



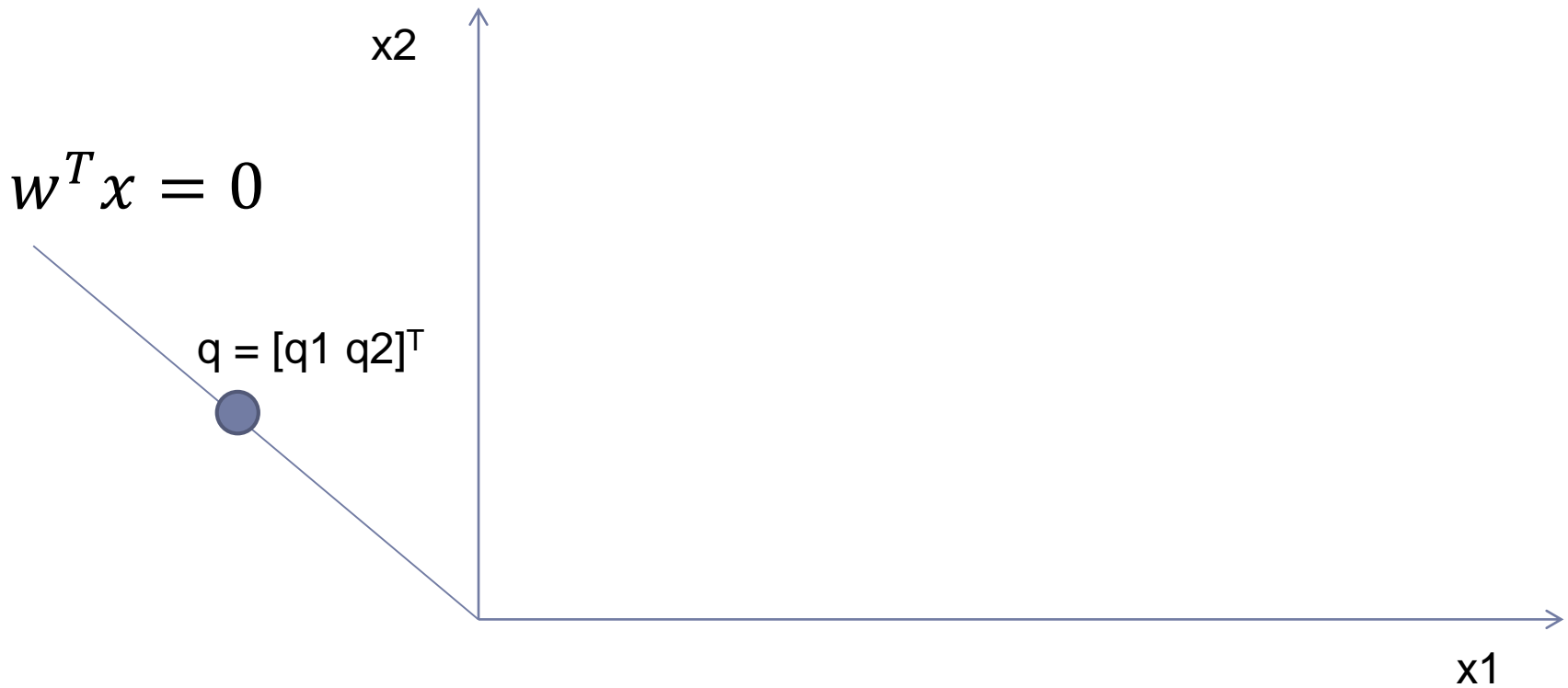
# Vetor perpendicular ao hiperplano

---



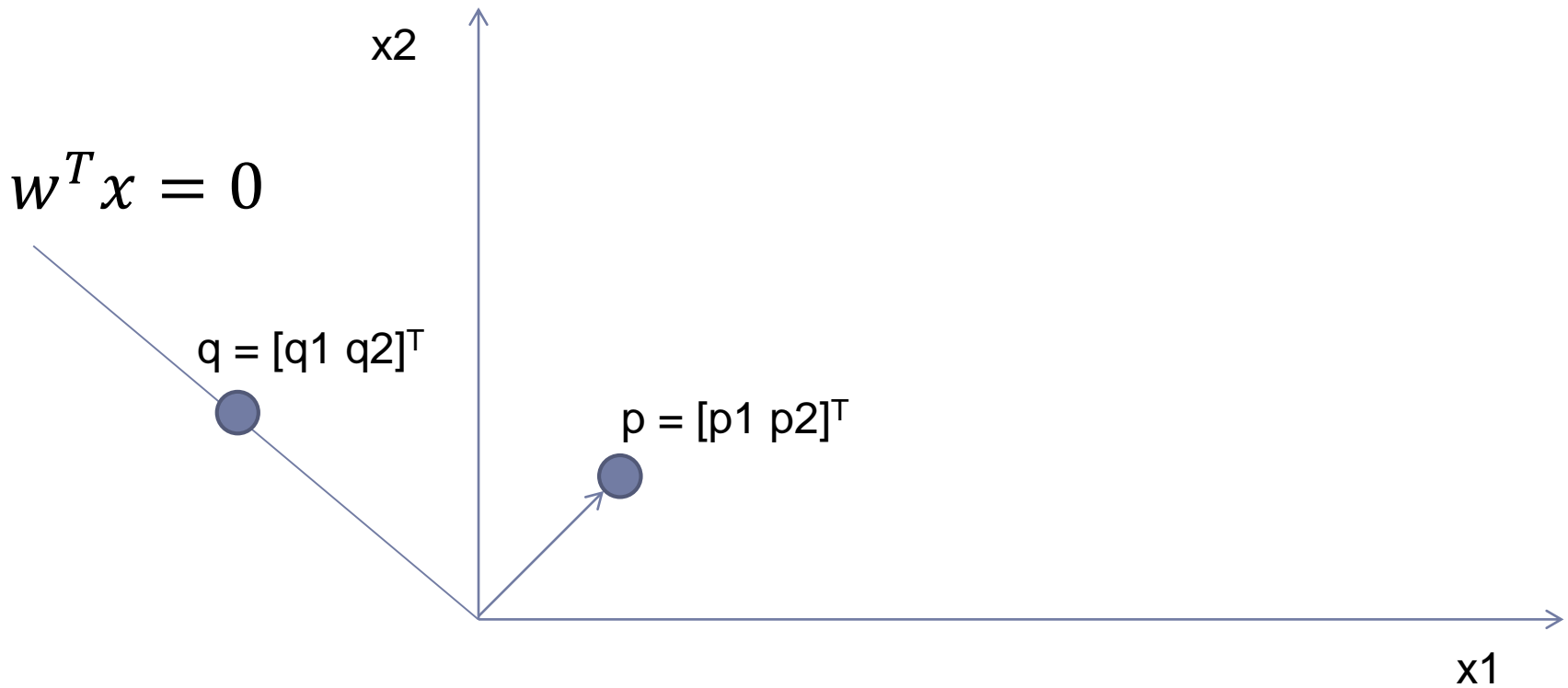
# Vetor perpendicular ao hiperplano

---



# Vetor perpendicular ao hiperplano

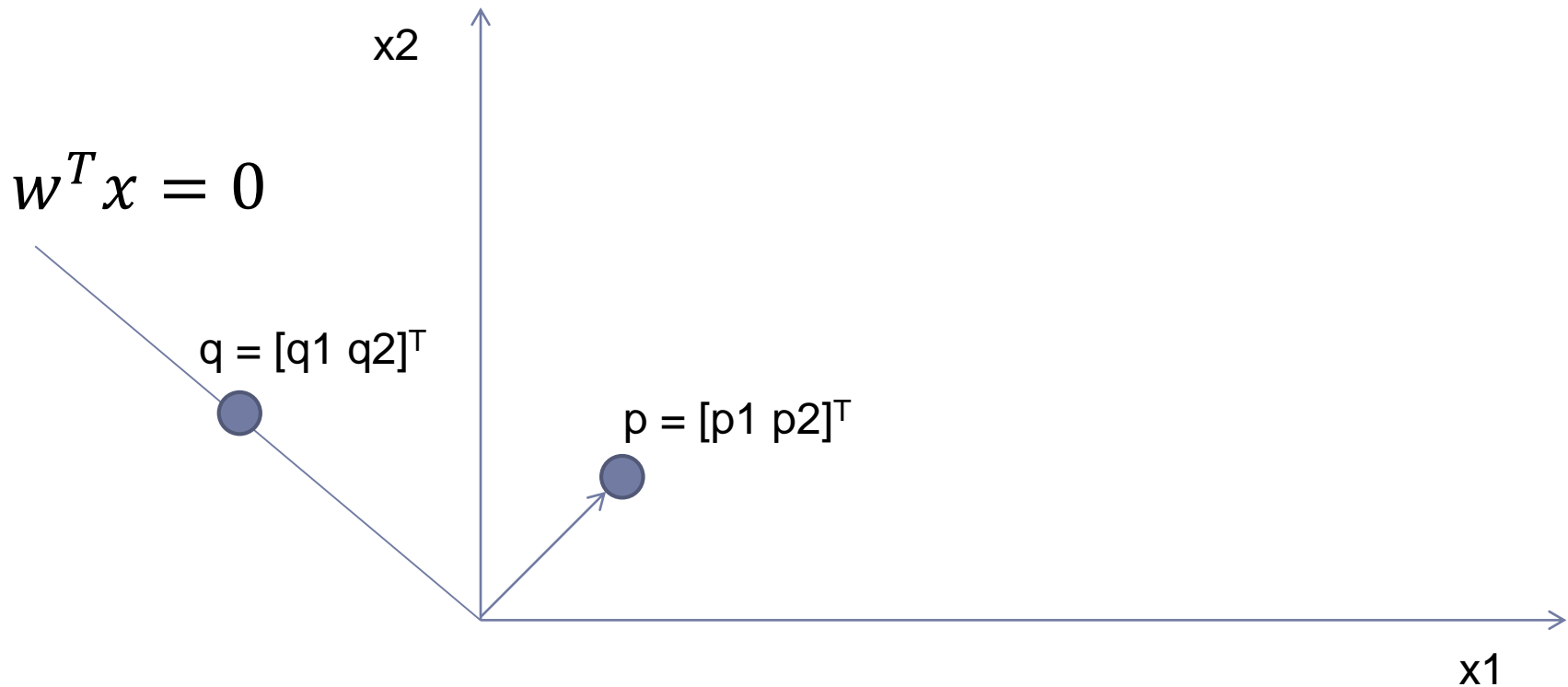
---



# Vetor perpendicular ao hiperplano

---

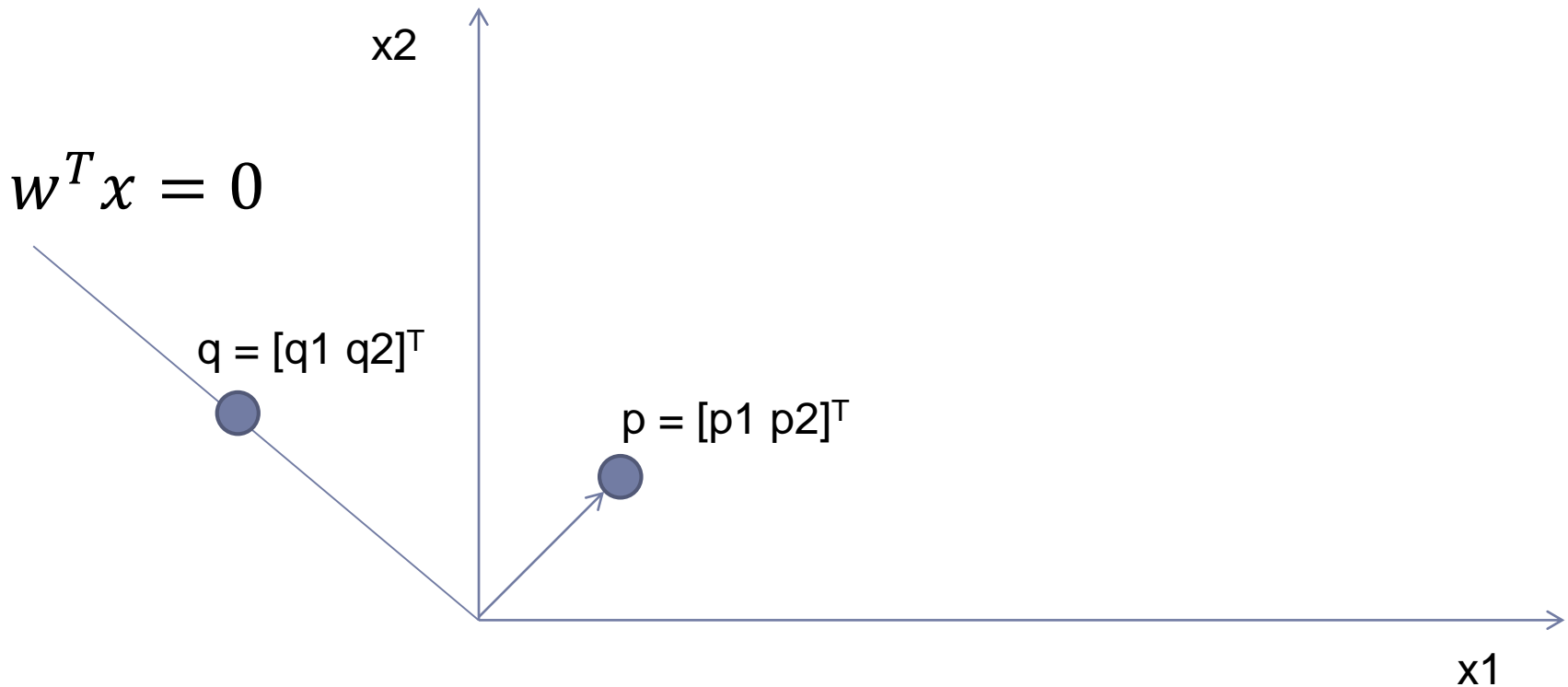
$$w_1 x_1 + w_2 x_2 = 0$$



# Vetor perpendicular ao hiperplano

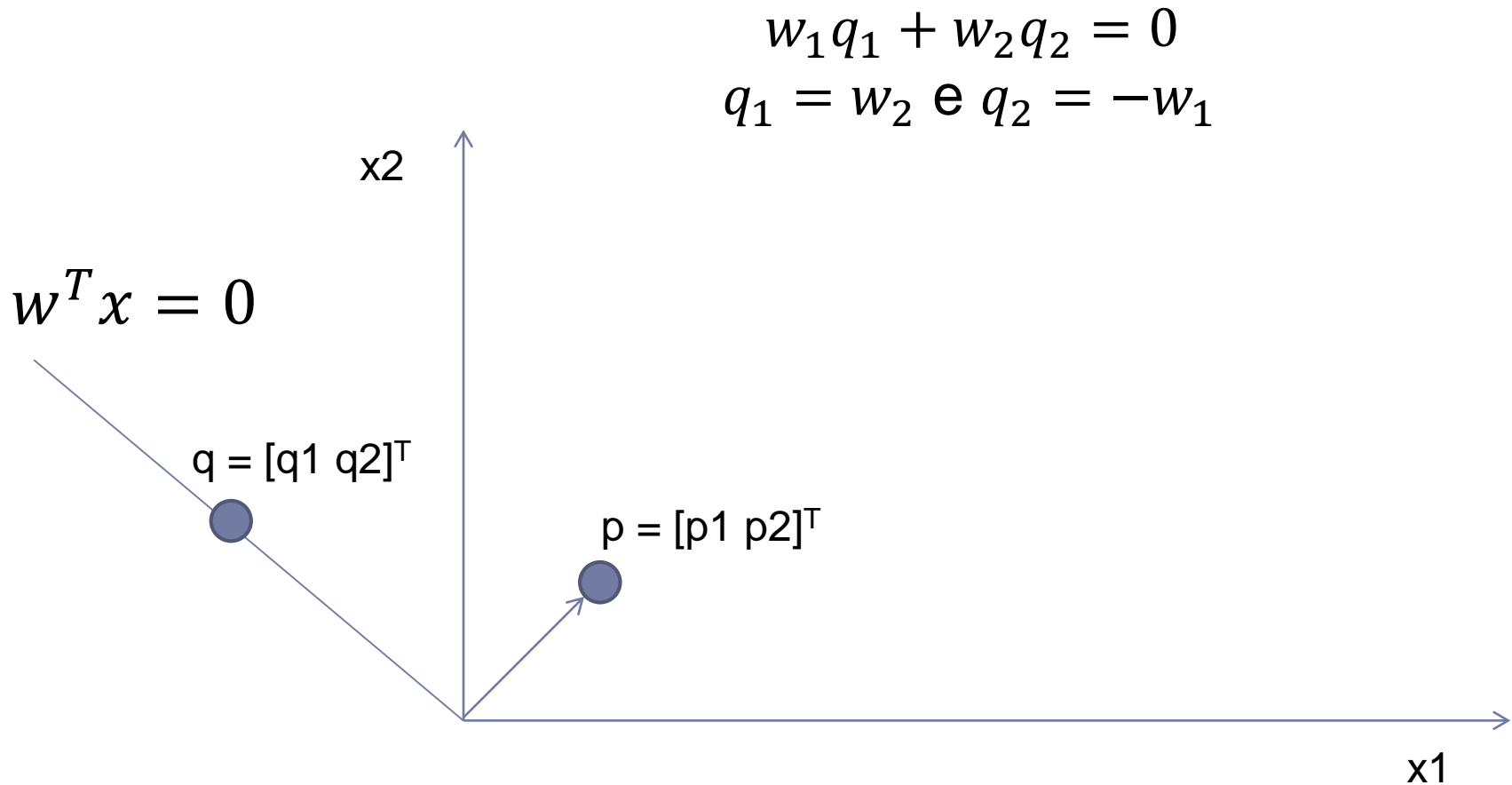
---

$$w_1 q_1 + w_2 q_2 = 0$$



# Vetor perpendicular ao hiperplano

---

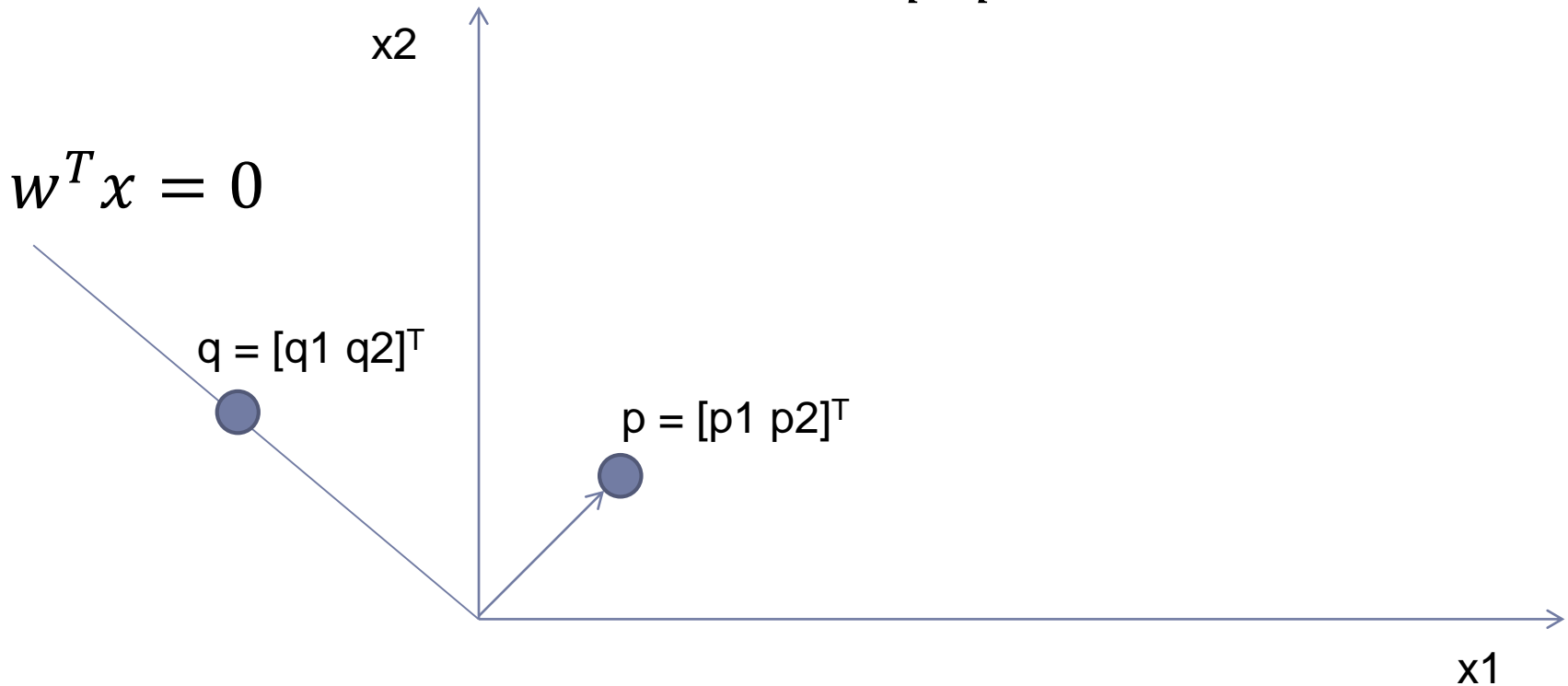




# Vetor perpendicular ao hiperplano

---

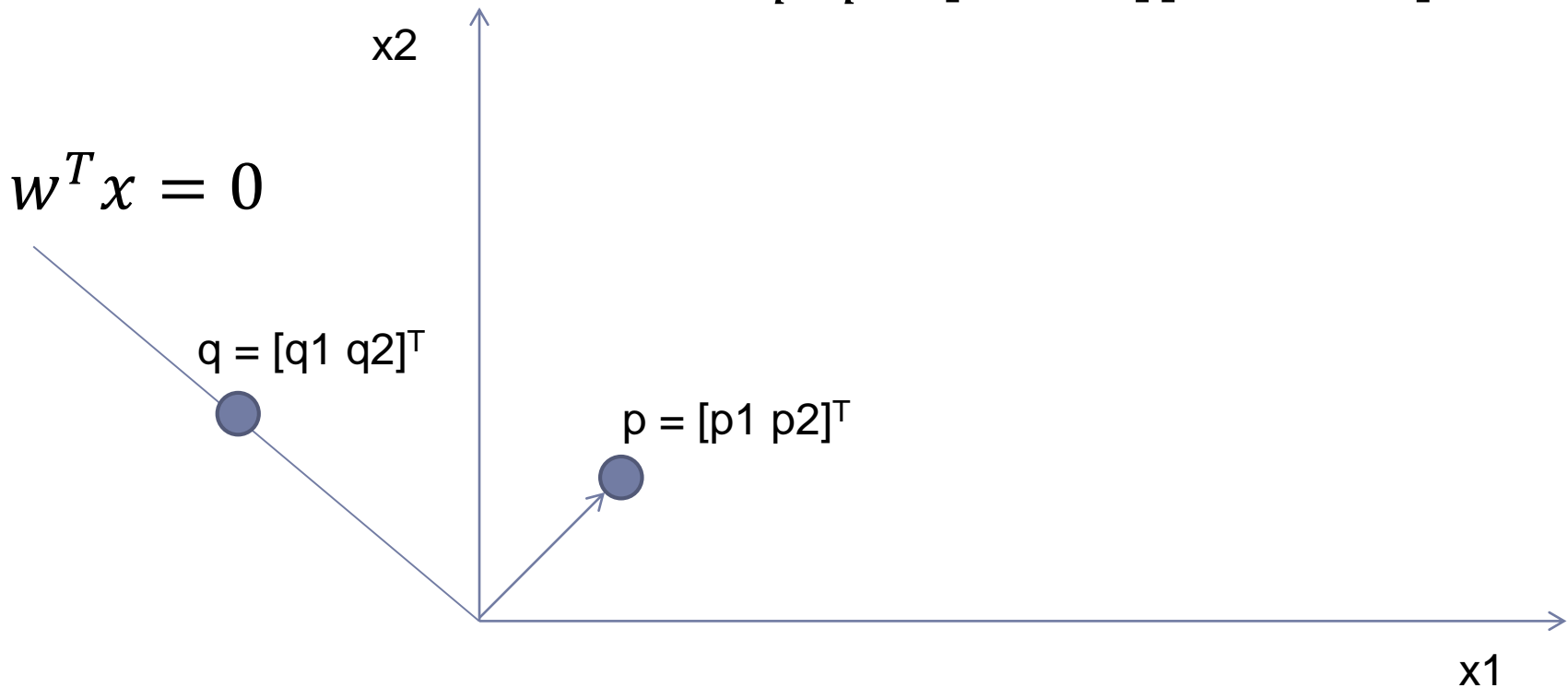
$$q_1 = w_2 \text{ e } q_2 = -w_1$$
$$p \cdot q = 0$$



# Vetor perpendicular ao hiperplano

---

$$q_1 = w_2 \text{ e } q_2 = -w_1$$
$$p \cdot q = [p_1 \quad p_2][w_2 \quad -w_1]^T = 0$$



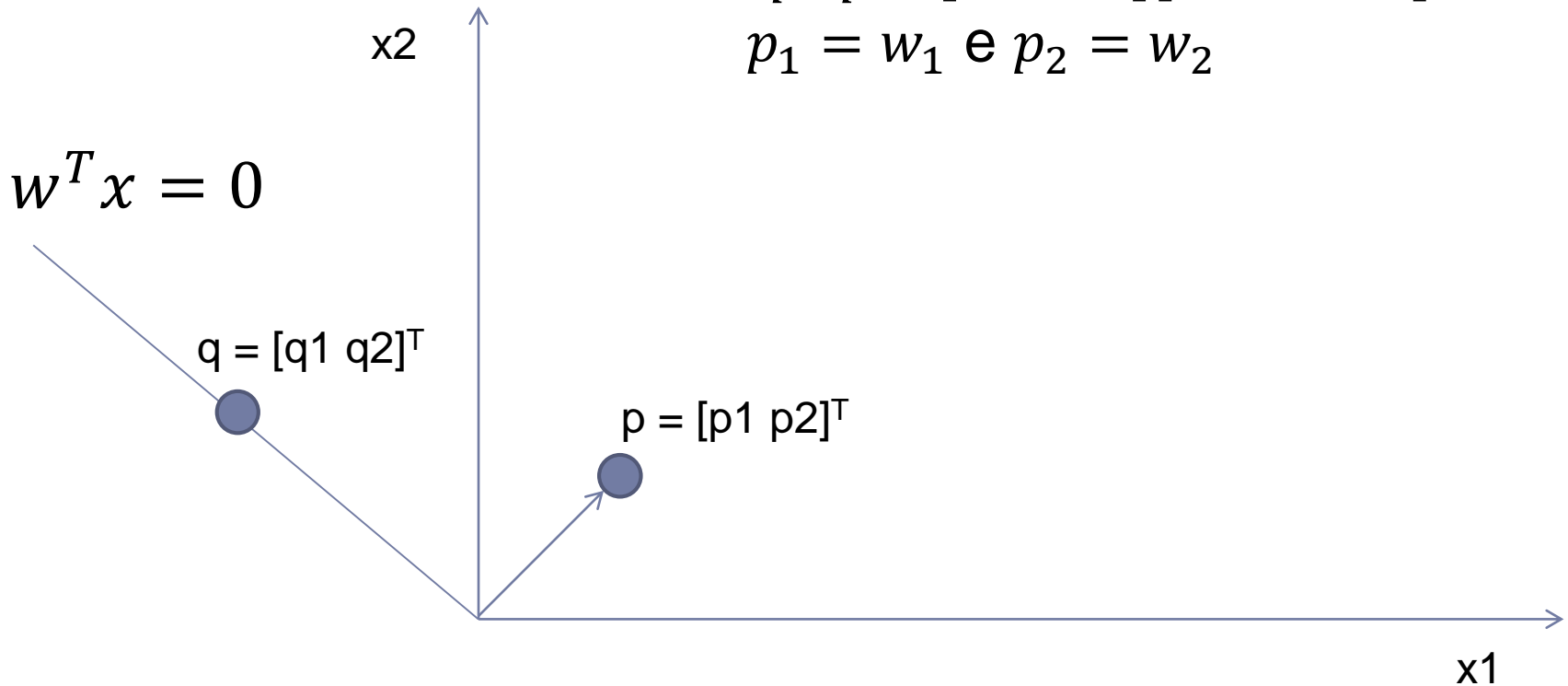
# Vetor perpendicular ao hiperplano

---

$$q_1 = w_2 \text{ e } q_2 = -w_1$$

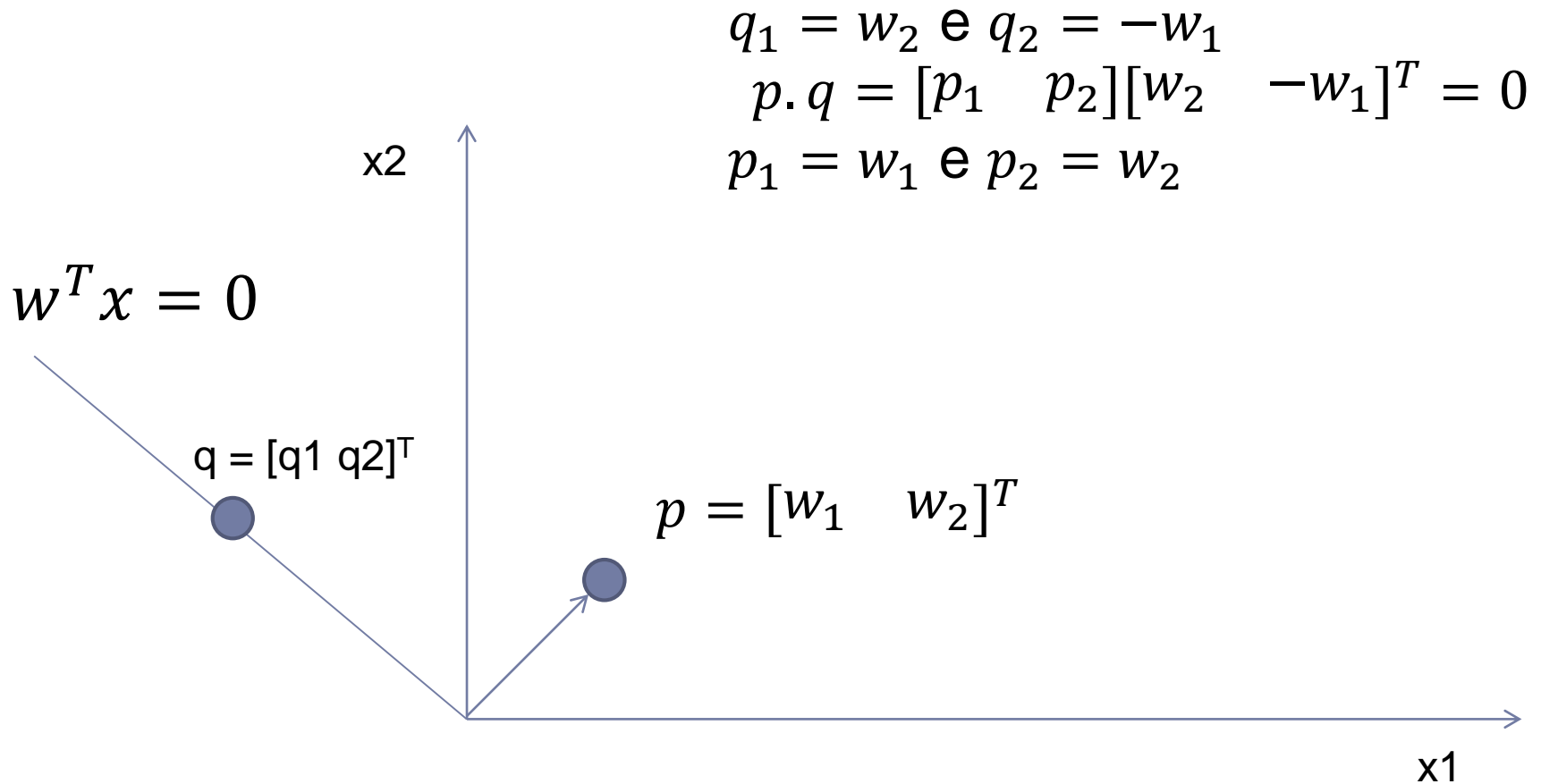
$$p \cdot q = [p_1 \ p_2][w_2 \ -w_1]^T = 0$$

$$p_1 = w_1 \text{ e } p_2 = w_2$$



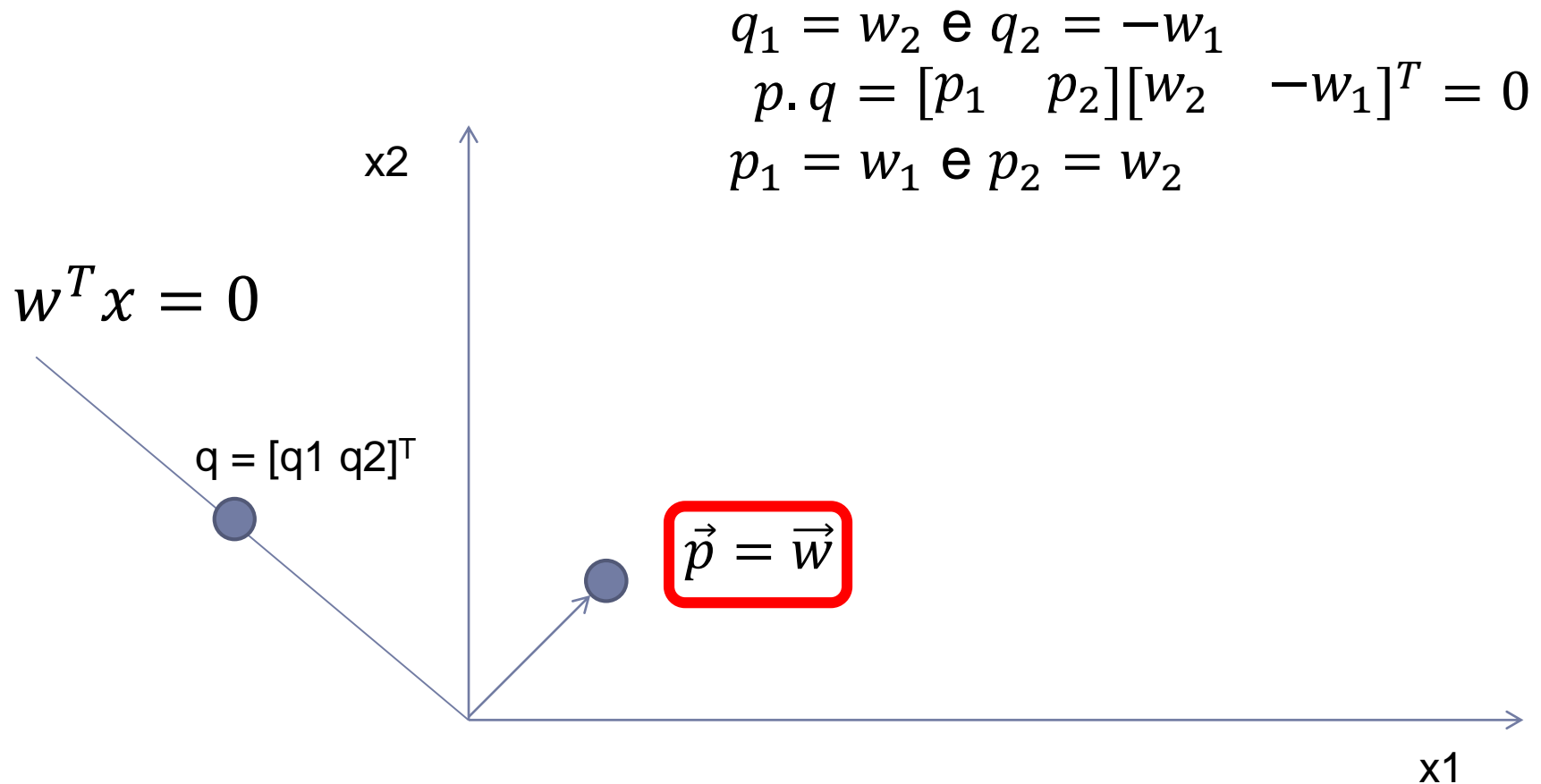
# Vetor perpendicular ao hiperplano

---



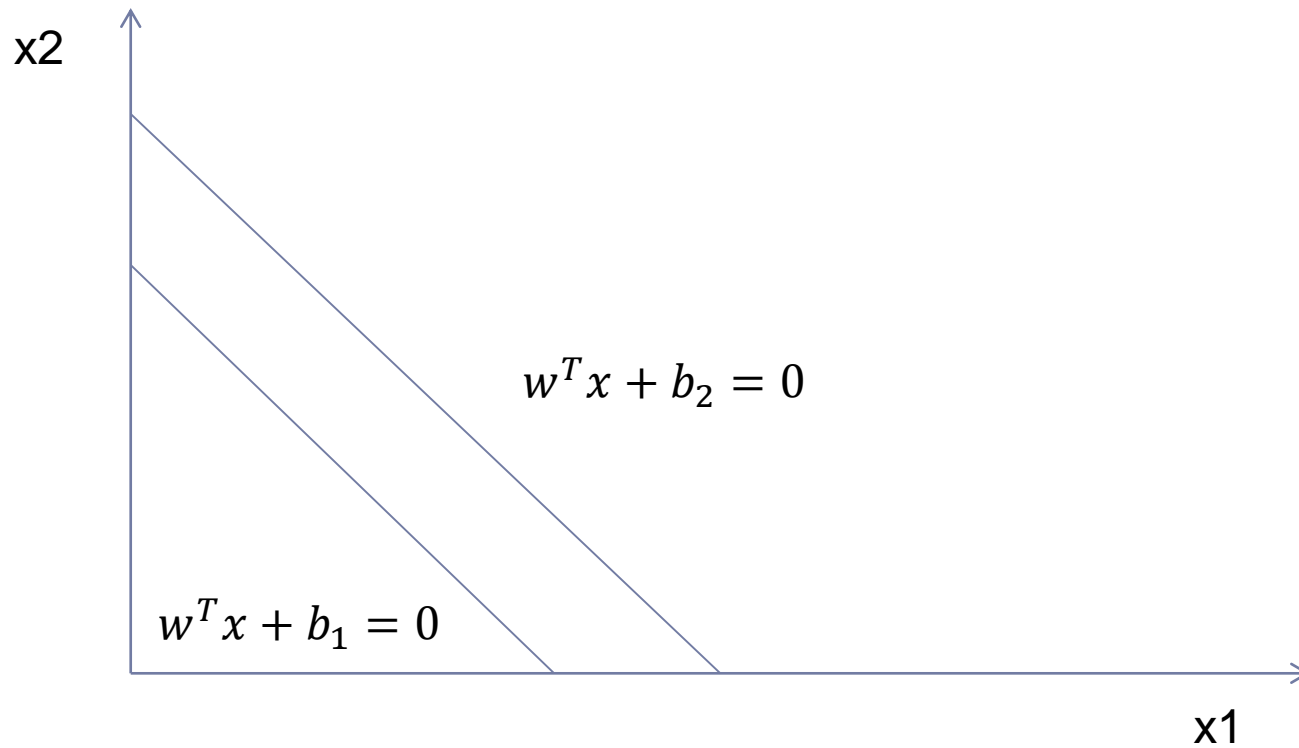
# Vetor perpendicular ao hiperplano

---



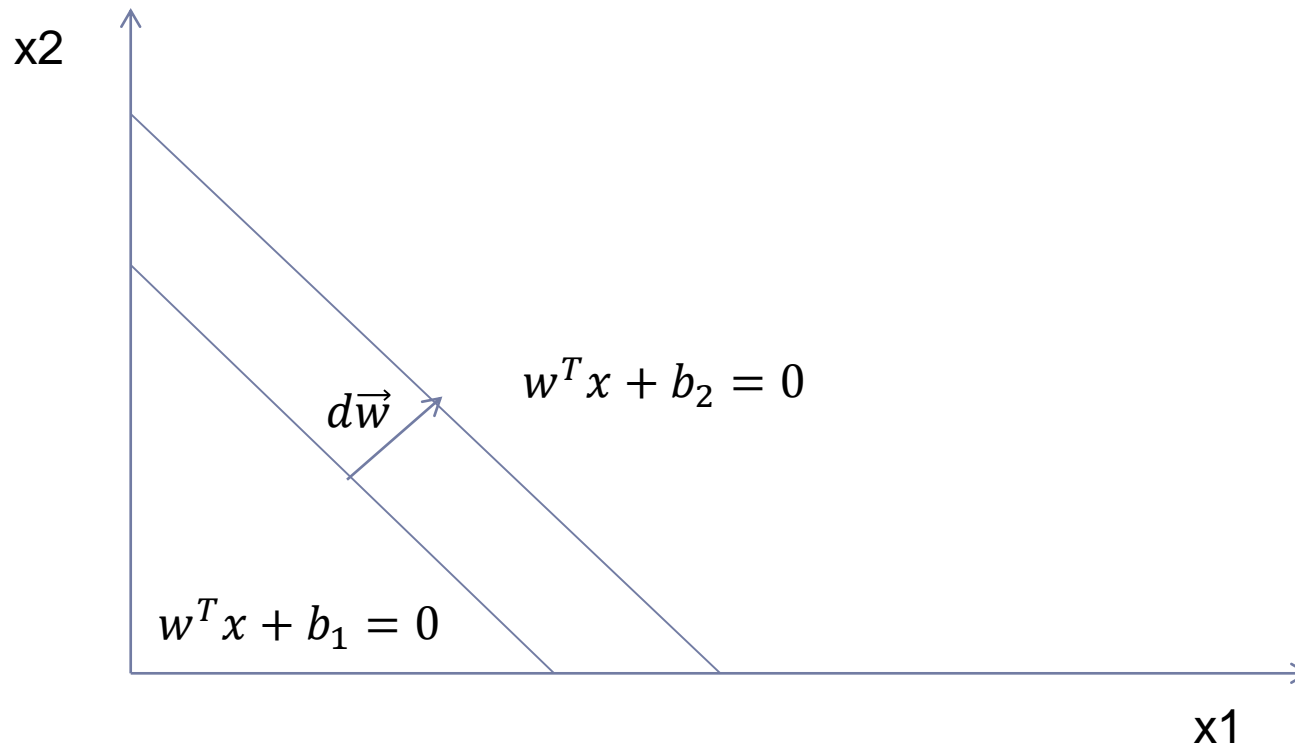
# Distância entre hiperplanos paralelos

---



# Distância entre hiperplanos paralelos

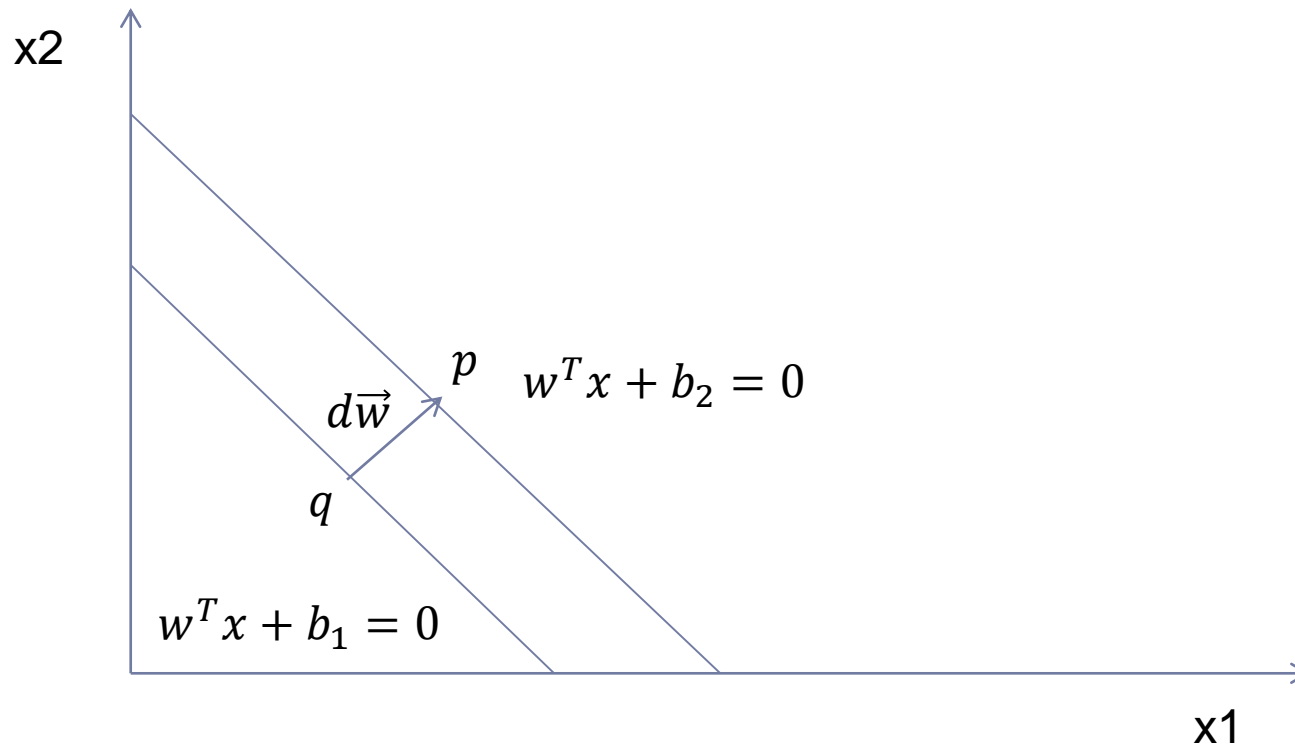
---



# Distância entre hiperplanos paralelos

---

$$p - q = d\vec{w}$$

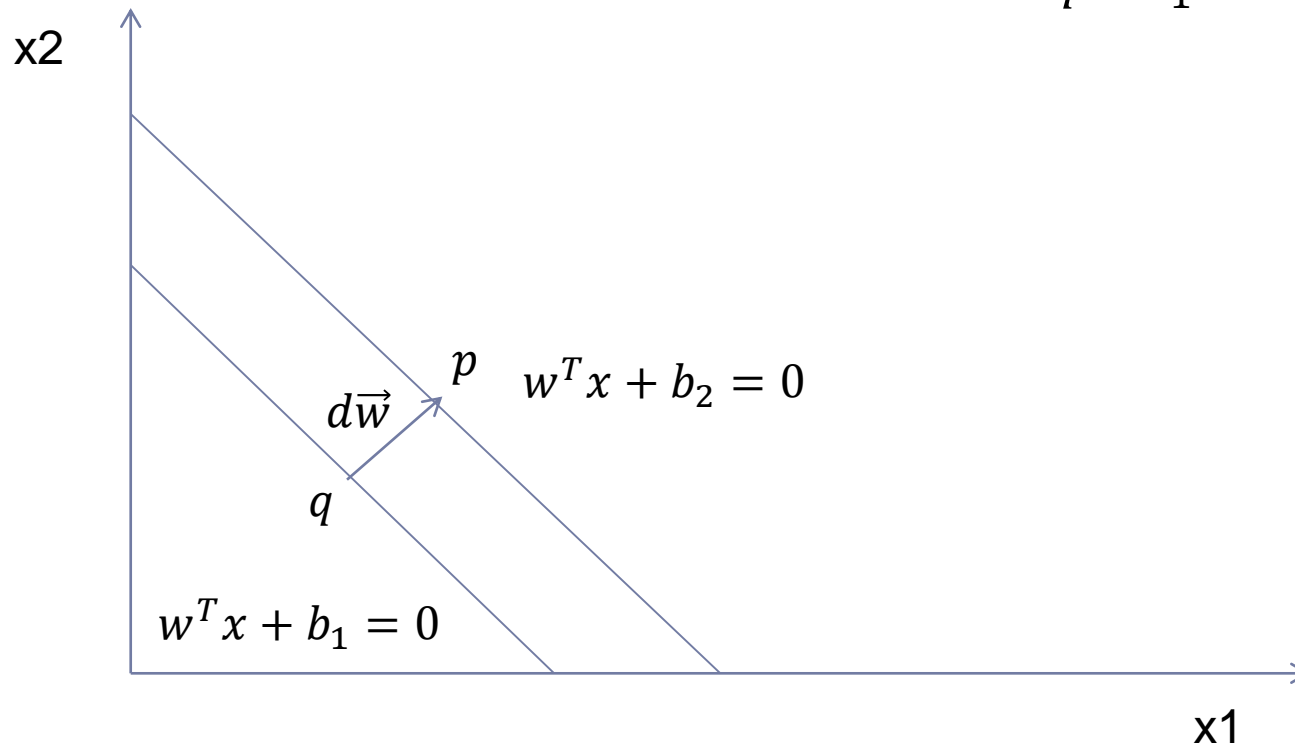




# Distância entre hiperplanos paralelos

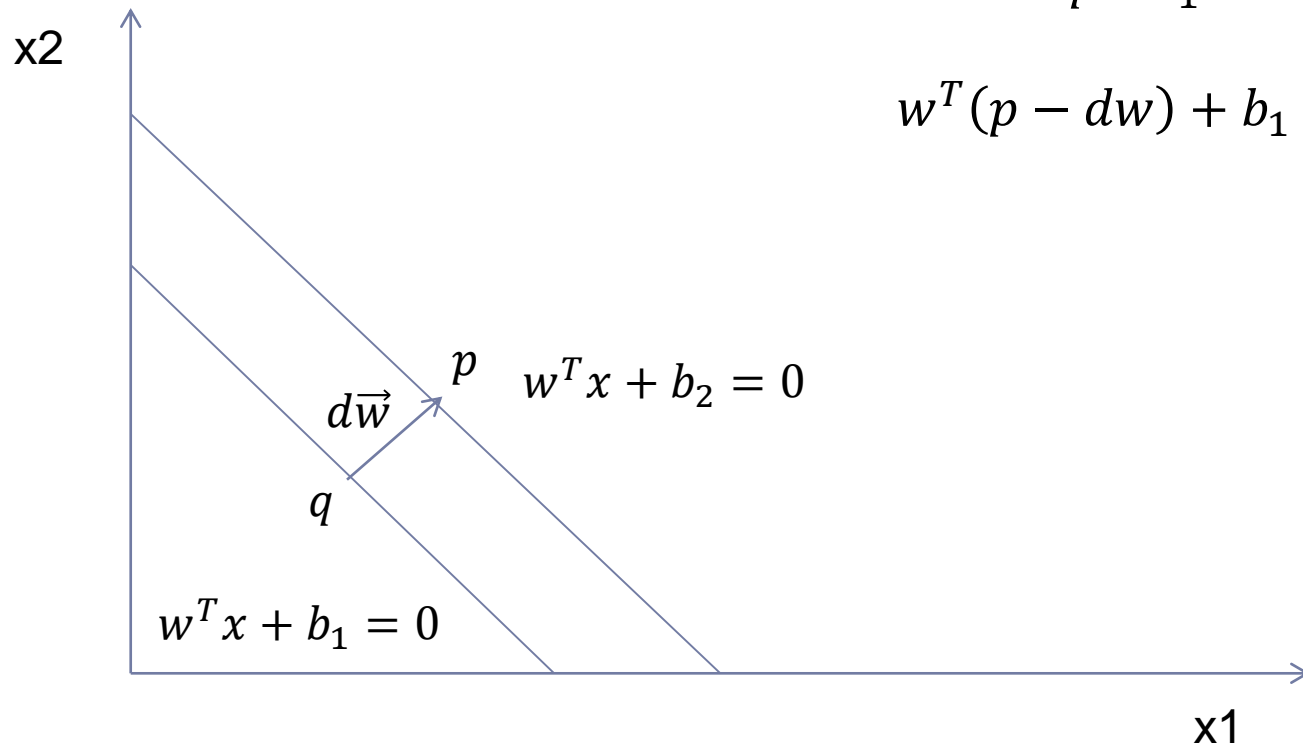
---

$$\begin{aligned} p - q &= dw \\ w^T q + b_1 &= 0 \end{aligned}$$



# Distância entre hiperplanos paralelos

---



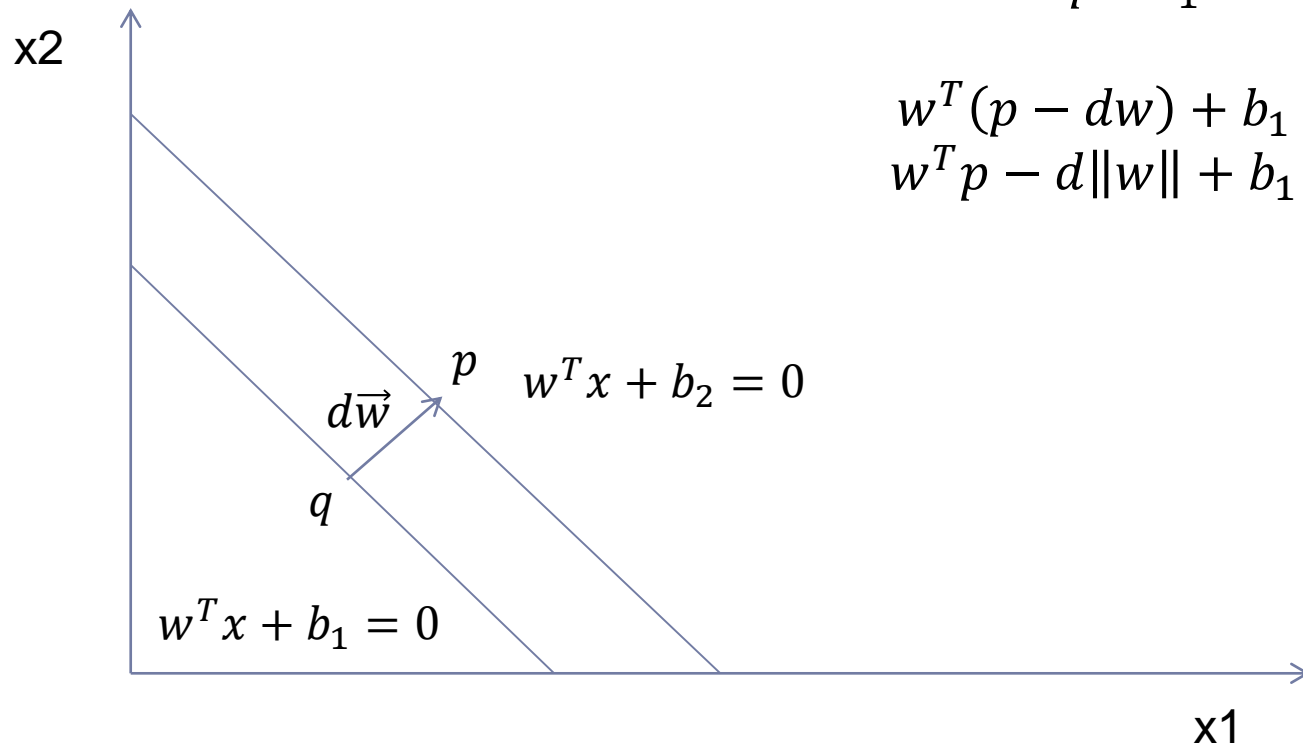
$$p - q = dw$$
$$w^T q + b_1 = 0$$

$$w^T (p - dw) + b_1 = 0$$



# Distância entre hiperplanos paralelos

---



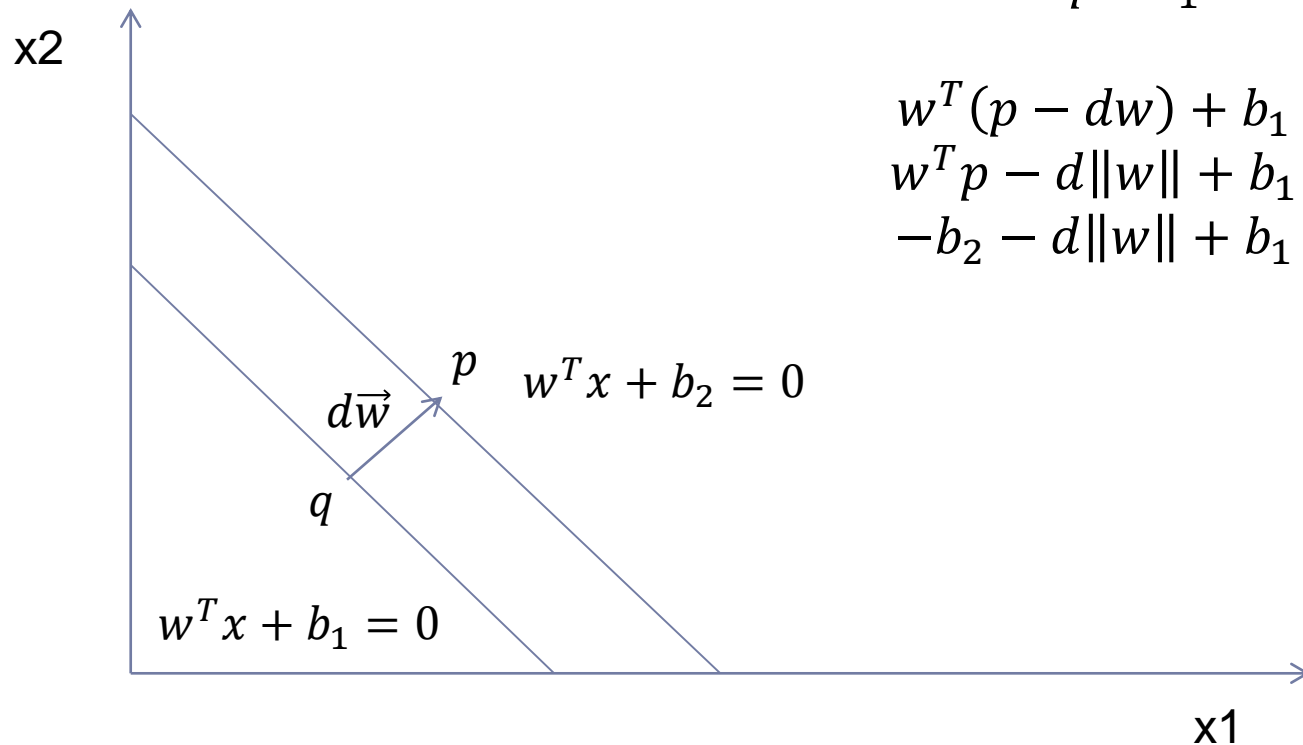
$$p - q = dw$$
$$w^T q + b_1 = 0$$

$$w^T(p - dw) + b_1 = 0$$
$$w^T p - d\|w\| + b_1 = 0$$



# Distância entre hiperplanos paralelos

---



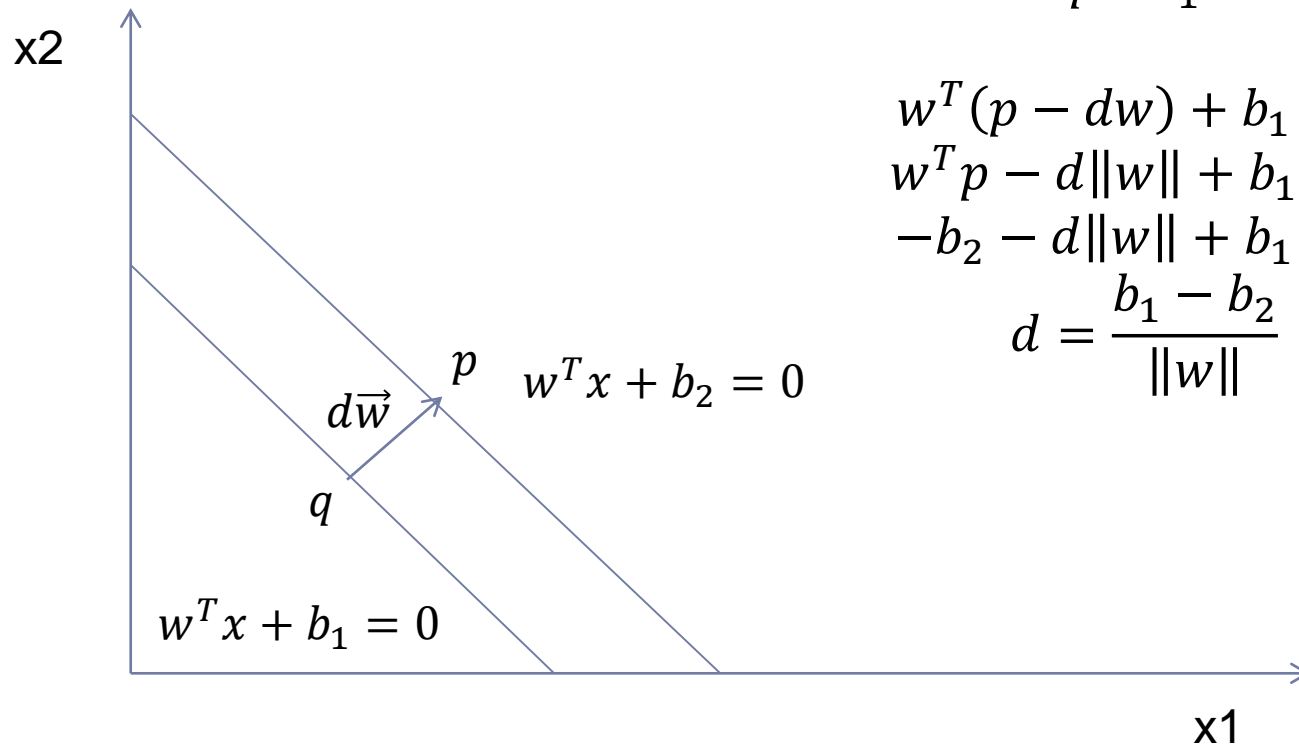
$$p - q = dw$$
$$w^T q + b_1 = 0$$

$$w^T(p - dw) + b_1 = 0$$
$$w^T p - d\|w\| + b_1 = 0$$
$$-b_2 - d\|w\| + b_1 = 0$$



# Distância entre hiperplanos paralelos

---



$$p - q = dw$$
$$w^T q + b_1 = 0$$

$$w^T(p - dw) + b_1 = 0$$

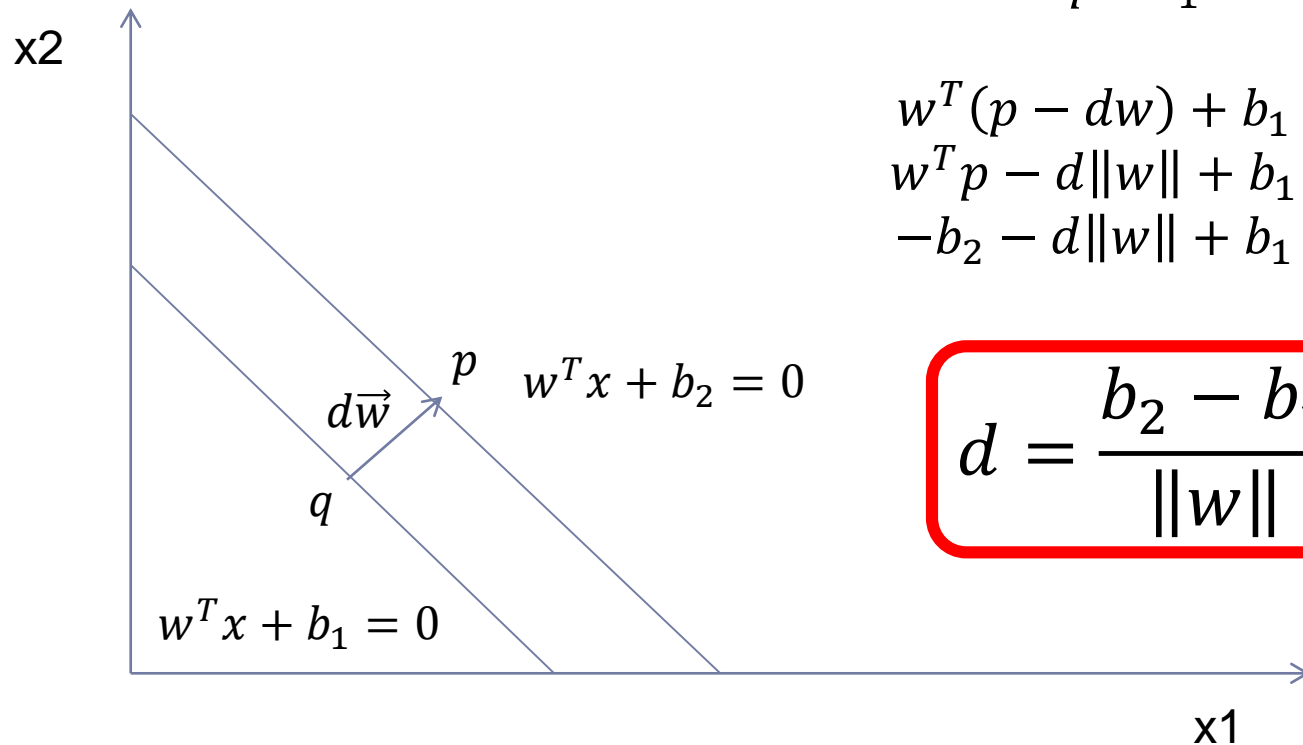
$$w^T p - d\|w\| + b_1 = 0$$

$$-b_2 - d\|w\| + b_1 = 0$$

$$d = \frac{b_1 - b_2}{\|w\|}$$

# Distância entre hiperplanos paralelos

---



$$p - q = dw$$
$$w^T q + b_1 = 0$$

$$w^T(p - dw) + b_1 = 0$$
$$w^T p - d\|w\| + b_1 = 0$$
$$-b_2 - d\|w\| + b_1 = 0$$

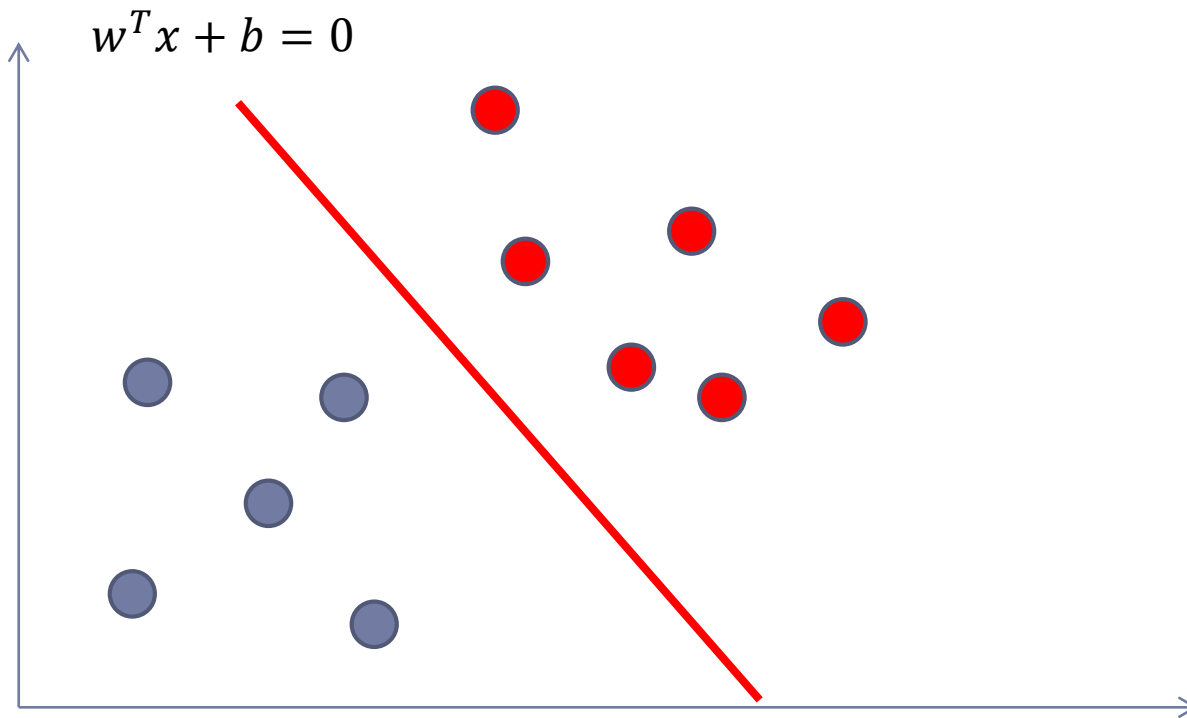
$$d = \frac{b_2 - b_1}{\|w\|}$$



# Definição do Problema

# Superfície de Separação

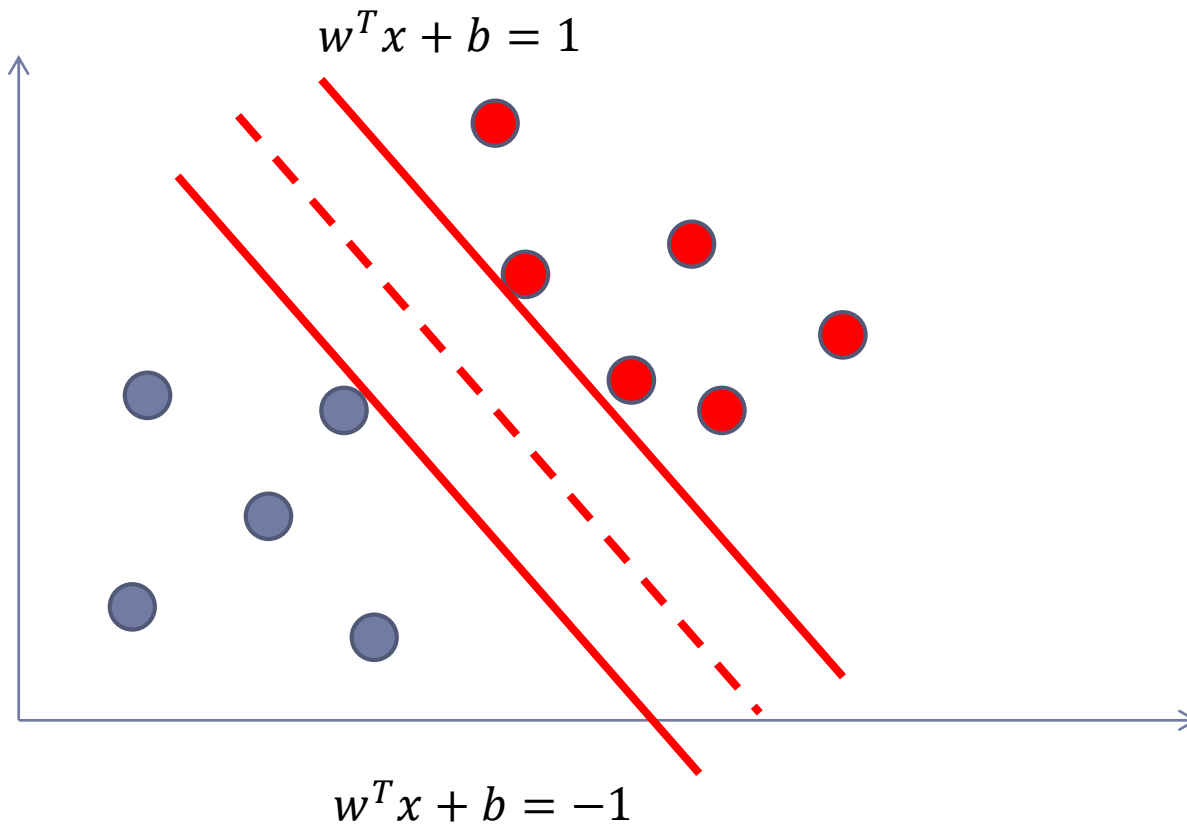
---





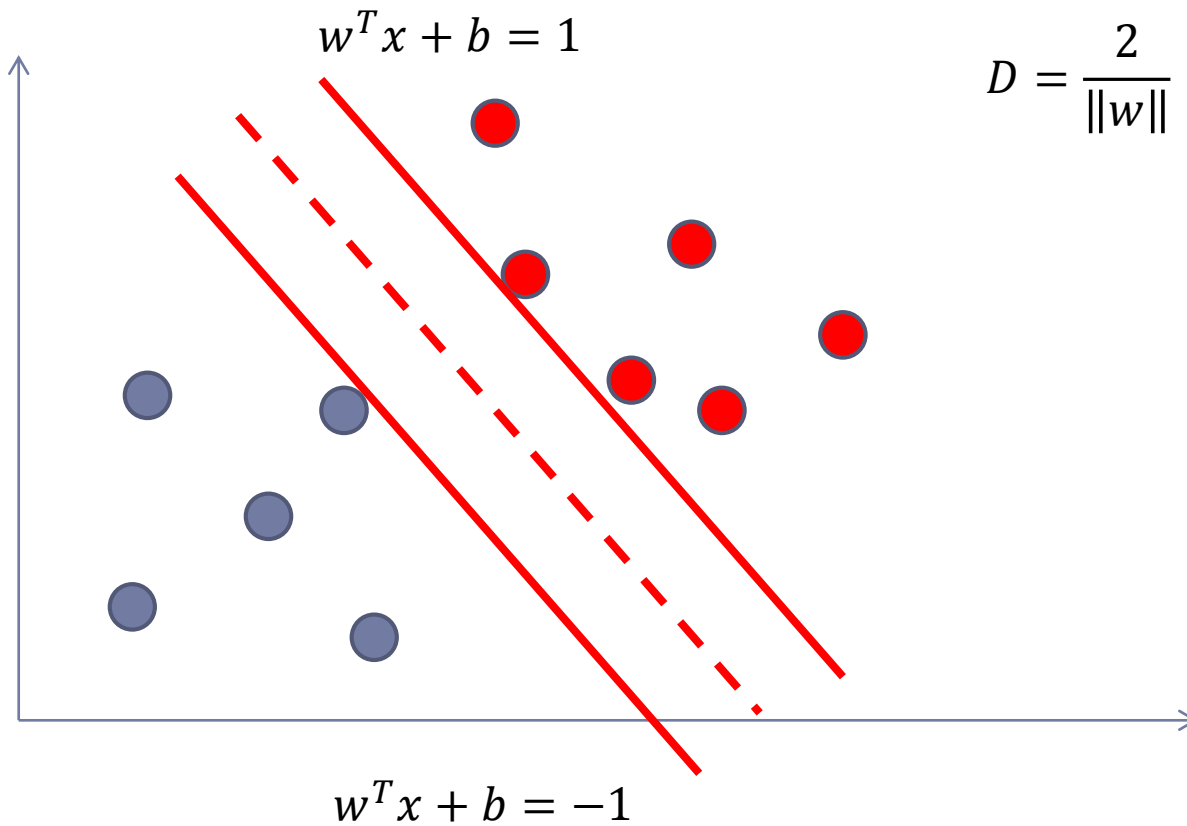
# Superfície de Separação

---



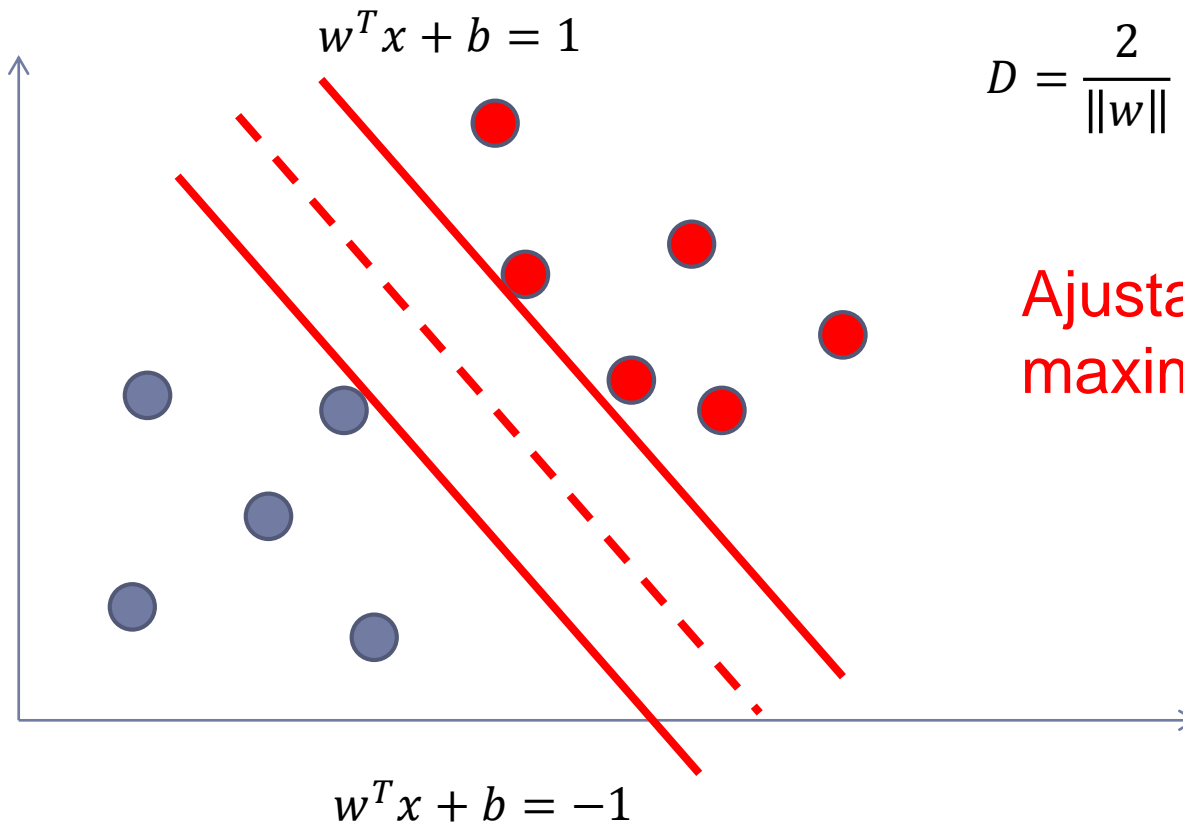
# Superfície de Separação

---



# Superfície de Separação

---

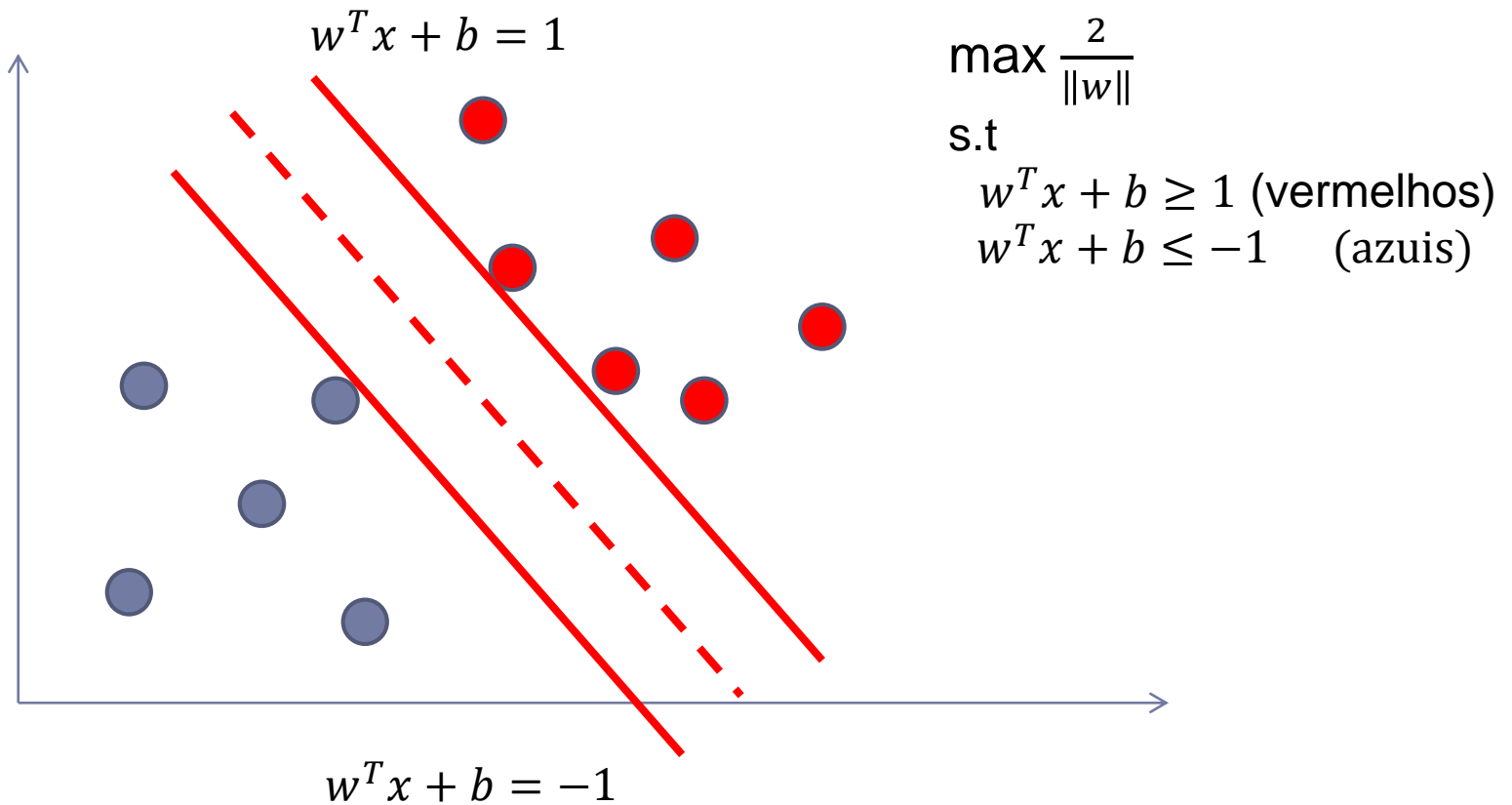


Ajustar  $w$  para  
maximizar  $D$



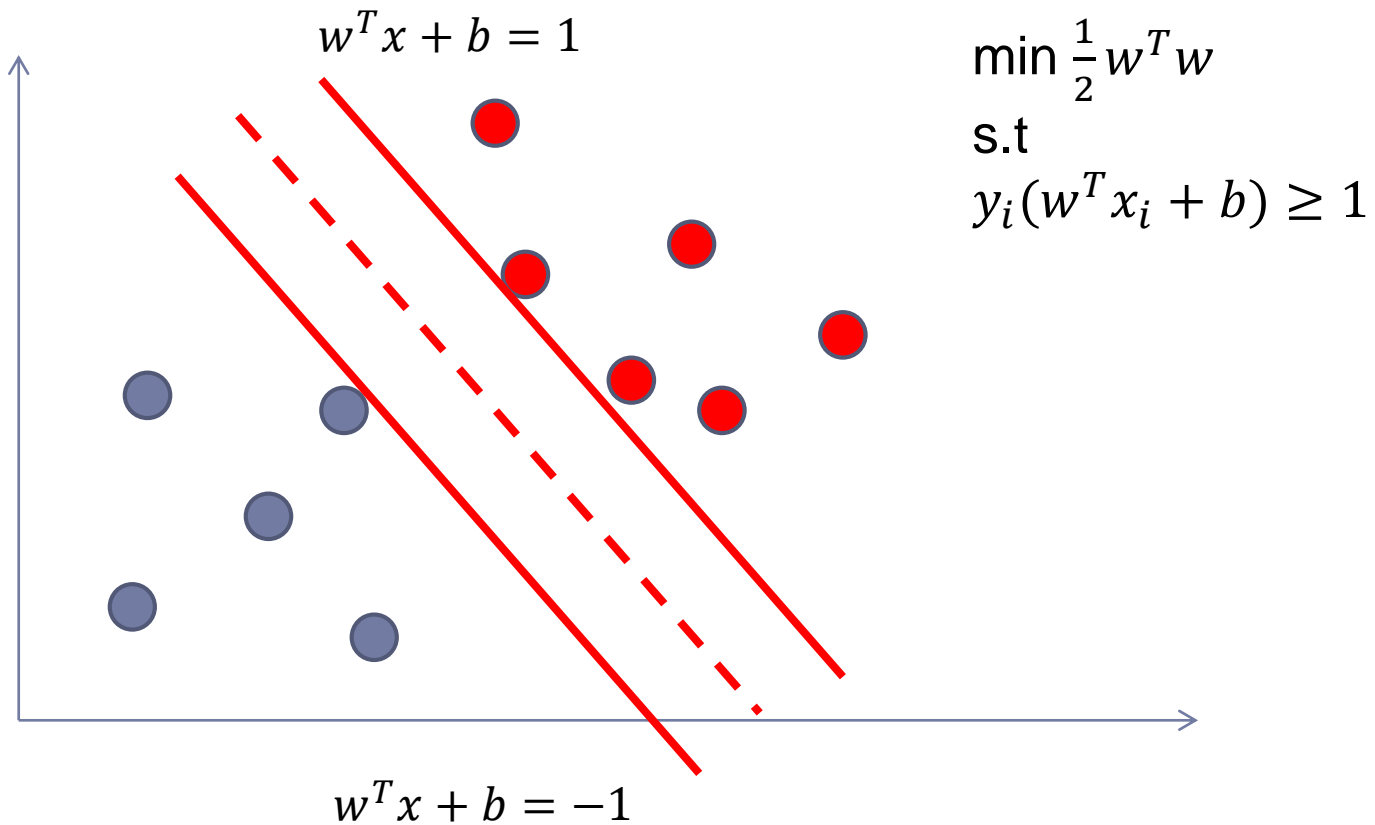
# Superfície de Separação

---



# Superfície de Separação

---





Dúvidas ?