



Fair oversampling technique using heterogeneous clusters

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ABSTRACT

Class imbalance and group (e.g., race, gender, and age) imbalance are acknowledged as two reasons in data that hinder the trade-off between fairness and utility of machine learning classifiers. Existing techniques have jointly addressed issues regarding class imbalance and group imbalance by proposing fair oversampling techniques. Unlike the common oversampling techniques, which only address class imbalance, fair oversampling techniques significantly improve the abovementioned trade-off, as they can also address group imbalance. However, if the size of the original clusters is too small, these techniques may cause classifier overfitting. To address this problem, we herein develop a fair oversampling technique using data from heterogeneous clusters. The proposed technique generates synthetic data that have class-mix features or group-mix features to make classifiers robust to overfitting. Moreover, we develop an interpolation method that can enhance the validity of generated synthetic data by considering the original cluster distribution and data noise. Finally, we conduct experiments on five realistic datasets and three classifiers, and the experimental results demonstrate the effectiveness of the proposed technique in terms of fairness and utility.

1. Introduction

Machine learning (ML) classifiers have been widely used in many fields, as they improve the utility of various decision-making systems, such as loan screening, disease diagnosis, and recidivism prediction [1]. Due to the increased impact of such systems on people's lives, there are concerns regarding the fairness of the decisions based on classifiers [2]. Without proper intervention, classifiers can unintentionally discriminate against groups (e.g., race and gender); thus, worsening their fairness [3]. Therefore, improving the trade-off between fairness and utility is important for incorporating ML classifiers into decision-making systems.

Data imbalance is a major factor in causing the aforementioned trade-off, and it often occurs in ML pipelines. Group imbalance in data can cause the learning of classifiers to be biased toward the majority group (e.g., male) and against the minority group (e.g., female) [2,4]. Moreover, learned classifiers consequently have different prediction accuracies for different groups, and thus their fairness can deteriorate. Many studies have addressed this issue and have aimed to improve the trade-off between fairness and overall classification accuracy [5–12]. Class imbalance is also a common problem, and it occurs in situations where data collection is difficult, such as rejected applicants in loan screening or certain diseases in disease diagnosis. In addition, learning classifiers based on class imbalanced data is a challenging task, as the accuracy of the classifiers for minority classes is significantly deteriorated [13]. To overcome this challenge, existing studies have proposed various class balancing techniques [14–21]. Both group and class imbalance are frequent problems in ML; however, only a few studies addressed them simultaneously.

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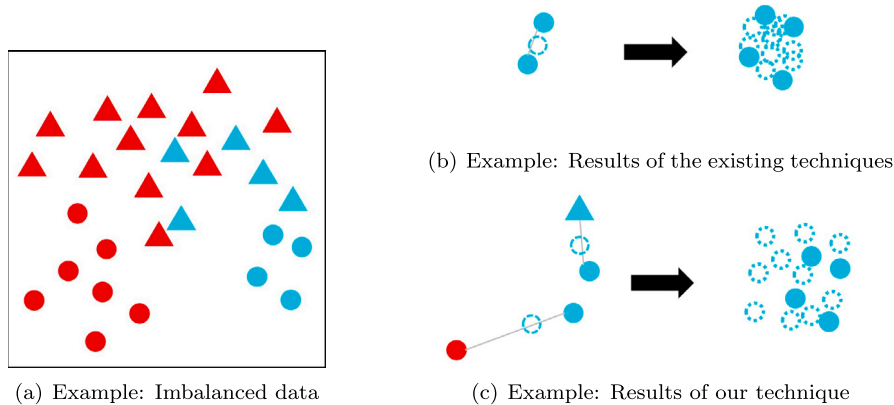


Fig. 1. A toy example of fair oversampling techniques for correcting class imbalance and group imbalance in data. The triangles and circles represent a majority class and a minority class, respectively. Red and blue represent the majority group and minority group, respectively. (a) Four clusters can be identified, with the red triangle cluster having the most data and the blue circle cluster having the least data. (b) Example of the synthetic data generated by existing fair oversampling techniques, which are based on homogeneous clusters, resulting in a lack of diversity in synthetic data. Such synthetic data can cause the overfitting of ML classifiers. (c) Example of the synthetic data generated by the proposed fair oversampling technique, which is based on heterogeneous clusters; thus, ensuring diversity in synthetic data. Such synthetic data can improve the generalization performance of ML classifiers.

In this work, we aim to address both group imbalance and class imbalance to improve the trade-off between fairness and utility in classifiers. To this end, we develop a technique for balancing cluster imbalances, where each cluster is composed of a class and a group (see Fig. 1(a) that shows a toy example of cluster imbalance).

Previous studies [5,10–12,22] have proposed various techniques to balance cluster imbalances to improve the abovementioned trade-off. Among them, fair oversampling techniques [10–12] are a particularly promising approach as they are classifier-agnostic and can provide new, useful information for classifier learning. However, an open problem remains in the field as existing fair oversampling techniques are based on homogeneous clusters, making it difficult to generate diverse synthetic data when the size of observed clusters is small. (Fig. 1(b) illustrates this phenomenon, showing that a cluster composed of a minority class and a minority group has the smallest number of observed data and that the resulting cluster loses diversity using existing techniques.) This problem can cause overfitting, resulting in poor generalization performance of the learned classifiers.

In this paper, we propose a fair oversampling technique using heterogeneous clusters to improve the trade-off between fairness and utility in classifiers. Our technique can effectively generate robust synthetic data even when the size of observed clusters is small by through the consideration of heterogeneous clusters. To consider the heterogeneous clusters in generating synthetic data, we develop a sampling method that forms pairs of data from heterogeneous clusters. This method enhances the diversity in the synthetic data even in the case of oversampling clusters with a small size of observations (as illustrated in Fig. 1(c)). Furthermore, we present a linear interpolation method to ensure that the synthetic data follow the distribution of the original clusters. Based on our interpolation method, we present a practical oversampling algorithm that can consider both distances to and classes of nearest neighbors of the original data to enhance the validity of the generated synthetic data. We have conducted experiments with a wide range of real-world datasets and multiple classifiers. The experimental results demonstrate that the proposed technique outperforms the existing ones in terms of the trade-off between fairness and utility of different measures.

The rest of this paper is organized as follows. We describe works related to our study in Section 2 and describe the problem formulation and definition of fairness metrics in Section 3. Section 4 shows a presentation of the proposed technique, which generates synthetic data to improve utility and fairness, and Section 5 shows the experimental settings and results. The conclusions are mentioned in Section 6.

2. Related works

2.1. Fairness in ML classifiers

Fairness in machine learning is crucial to ensure unbiased and equitable outcomes, regardless of sensitive attributes (e.g., gender or race). Fairness in unsupervised [23] and supervised learnings [24], particularly in classification [3,8], has received considerable attention due to its wide range of applications and potential for systemic discrimination [2]. **There are two definitions of fairness: individual fairness [8] and group fairness [3,8]. Individual fairness [8,23] aims to ensure that similar individuals should receive similar outcomes from classifiers, while group fairness [3,8] considers parity over different groups based on sensitive attributes (e.g., sex or race).** While individual fairness is not practical because of the difficulty in defining an appropriate similarity measure [25], several mathematical definitions for group fairness are often used in real-world regulations: statistical parity [8], equal opportunity [3], and equalized odds [3]. Thus, group fairness has more attention in fair ML.

To account for the fairness and utility of classifiers, many studies have proposed different approaches, including pre-processing [7–12], in-processing [5,6,26], and post-processing [3,27,28]. Pre-processing approaches aim to ensure fairness at the data level by

mitigating potential biases in training data. In-processing approaches attempt to incorporate constraints into learning algorithms to improve fairness. Post-processing approaches manipulate the predictions of classifiers to correct unfairness in predictions. Among various approaches, pre-processing approaches have become popular due to their classifier-agnostic nature.

One of the main challenges of the abovementioned existing pre-processing approaches is that they consider the overall classification accuracy and thus cannot address the class imbalance problem in training data. In this paper, we aim to develop a pre-processing approach that addresses this challenge.

2.2. Class imbalance in ML

Class imbalance often occurs in the cases in which it is difficult to collect data for a particular class (e.g., credit risk screening, case diagnosis, and spam determination). Thus, the distribution of classes in training data is often not very balanced. ML classifiers trained on such data may have worse accuracy for minority classes, as the algorithms of classifiers are constructed based on the theory of empirical risk minimization and tend to favor majority classes [29].

To address the class imbalance problem, many studies have proposed different approaches: data level [14–17], algorithmic level [18,19], and hybrid learning [20,21]. Most existing studies focus on data-level approaches, as they can be applied regardless of the used ML classifiers. Data level approaches can be further divided into two techniques: undersampling and oversampling. Oversampling techniques are widely used because of concerns that the undersampling techniques can worsen classifier performance due to the loss of a large amount of training data. The simplest oversampling technique is to duplicate the instances of minority classes for balancing class distribution. However, this technique can cause overfitting of ML classifiers due to the lack of diversity in generated data.

To tackle this problem, the Synthetic Minority Oversampling Technique (SMOTE) has been proposed [14]. SMOTE can generate synthetic data that can bring new information to learning classifiers. The algorithm of SMOTE is simple: (1) randomly select an instance from the minority class, (2) randomly select an instance from the K nearest neighborhoods of that instance, (3) and generate a synthetic instance based on the interpolation between the two instances. Due to the great success of SMOTE against class imbalances, many extensions of SMOTE have been proposed to improve its performance under different scenarios [15–17,30–32], such as techniques that generate synthetic data near the class boundary [15,16], techniques that do not generate synthetic data according to the class density of the neighborhoods [30,31], and techniques that generate data in sparse regions to avoid duplication [17,32]. Since clustering methods are effective for grouping data and detecting outliers [33–35], some studies have applied these methods to oversampling techniques [17,36].

While the above techniques aim to improve the utility, we aim to have a technique that can improve the trade-off between fairness and utility. Thus, we develop an oversampling technique that can correct the class imbalance and group imbalance based on SMOTE.

2.3. Fair class balancing techniques

Class imbalance and fairness problems in ML often occur simultaneously. For example, in the context of loan approval, the data may reflect a disproportionately small number of rejected applicants as well as a disproportionately small number of women. If left unresolved, this problem can result in erroneous predictions regarding socially significant classes or groups, thereby exacerbating the trade-off between the utility and fairness of classifiers.

To address this issue, fair class balancing techniques have been developed that aim to address both fairness and class imbalance simultaneously [5,10–12,22]. Two main approaches have been proposed to achieve this goal. The first is to modify classifier losses at the algorithmic level to achieve fair class balancing [5,22]. This approach has been shown to be effective in improving the trade-off between utility and fairness. The second approach involves oversampling techniques that achieve fair class balancing in a model-independent manner. One example of this type of technique is [12], which is based on the idea of oversampling to correct imbalances between groups in minority classes. Another example is [11], which is a modified version of [17] that aims to address cases where sensitive attributes are inaccessible due to individual privacy concerns [37]. FSMOTE [10] is a fair oversampling technique that aims to equalize the number of each class and each group by oversampling all but the largest clusters for clusters composed of classes and groups. FSMOTE has been empirically shown to improve the trade-off between utility and fairness over existing techniques because it can correct class and group imbalances simultaneously.

Our oversampling technique is related to FSMOTE and aims to correct class and group imbalances simultaneously. Since the FSMOTE algorithm is based on homogeneous clusters, synthetic data for small clusters may not be properly generated, which may degrade the generalization performance of the classifier. Our technique addresses this problem by generating robust synthetic data based on heterogeneous clusters, thereby improving the generalization performance of the classifier.

3. Preliminaries

We consider the setting in which one predicts the class label $y \in Y$ from the D -dimensional feature vector $x \in \mathbb{R}^D$ based on the training dataset $\mathcal{T} = \{(x^i, y^i)\}_{i=1}^N$, where N is the number of observed instances. In this study, the class label is assumed to be binary $Y \in \{0, 1\}$. We also assume that each instance is assigned to exactly one group $g \in G$ based on the protected attributes (e.g., race and gender) included in its feature vector. We consider M categorical groups $G \in [M]$, where groups are prespecified by ML practitioners.

For example, ML practitioners can consider groups based on race (e.g., white and black) or intersectional groups of race and gender (e.g., white women, white men, black women, and black men).

The goal of fair ML is to minimize disparity (i.e., unfairness) while maintaining utility with respect to the prediction \hat{Y} of Y . There are two metrics to assess disparity: individual fairness and group fairness. Since the latter is taken into account in many regulations and existing studies, we also consider only group fairness for the disparity.

The disparity based on the group fairness metric F is defined as follows:

$$|\max_{g \in G} F_g - \min_{g \in G} F_g|, \quad (1)$$

where F_g represents a group fairness metric for a group g . A value of 0 in Eq. (1) indicates that there is no difference in the outcome of the classifier between the groups, which is considered a fair outcome.

We introduce three group fairness metrics that are used in many studies: statistical parity [8], equal opportunity [3], and equalized odds [3].

Definition 1. Statistical parity measures the difference between the positive prediction rate for a group g and that of all instances.

$$F_g = P(\hat{Y} = 1) - P(\hat{Y} = 1 | G = g). \quad (2)$$

A negative value of Eq. (2) indicates that the group g has a higher positive prediction rate than the all instances, and conversely, a negative value of Eq. (2) indicates that the group has a lower positive prediction rate than the all instances.

Definition 2. Equal opportunity measures the difference between the true positive rate for a group g and that of all instances.

$$F_g = P(\hat{Y} = 1 | Y = 1) - P(\hat{Y} = 1 | Y = 1, G = g). \quad (3)$$

A negative value of Eq. (3) indicates that the group g has a higher true positive rate than the all instances, and conversely, a negative value of Eq. (3) indicates that the group has a lower true positive rate than the all instances.

Definition 3. Equalized odds measure the difference between the true positive and false positive rates for a group g and those of all instances.

$$F_g = P(\hat{Y} = 1 | Y = y) - P(\hat{Y} = 1 | Y = y, G = g). \quad (4)$$

A negative value of Eq. (4) indicates that the group g has a higher true positive rate and a higher true negative rate than the all instances, and conversely, a negative value of Eq. (4) indicates that the group has a lower true positive rate and a lower true negative rate than the all instances.

All the above fairness metrics are related to the outputs of trained classifiers. However, data imbalances are known to have a significant impact on fairness. Therefore, it is important to quantitatively assess data imbalances. Thus, we introduce a statistical metric to investigate data imbalances. Concerning fairness, class imbalances but also group imbalances are considered important. Therefore, we evaluate imbalances on a cluster basis. Each cluster $C_{y,g}$ is the intersection of class y and group g , i.e., $C_{y,g} = \{i | y^i = y, g^i = g\}$.

We quantify the degree of imbalance of a cluster $C_{y,g}$ as the size difference between the majority cluster and cluster $C_{y,g}$. The cluster with the maximum number of instances is the majority cluster. Then, the degree of imbalance of a cluster $C_{y,g}$ is defined as follows:

$$S - |C_{y,g}|, \quad (5)$$

where $S = \max_{y \in Y, g \in G} |C_{y,g}|$ represents the maximum cluster size among all clusters.

The larger the difference in the size of clusters, the more biased a classifier's predictions can be. We believe that this fact affects the trade-off between fairness and utility of the classifiers. In the next section, we describe how to correct this imbalance by applying the proposed oversampling technique to improve fairness and utility.

4. Method

In this section, we present the newly developed fair oversampling technique to address fairness in class imbalanced data. The main working procedure involves the following three steps: (1) Randomly select an instance i from a cluster $C_{y,g}$ that should be oversampled. (2) Randomly select an instance j from the heterogeneous clusters of $C_{y,g}$ using our sampling method. (3) Generate a synthetic instance by interpolating x^i and x^j with our interpolation method. The details of Step (2) and Step (3) are described in Section 4.1 and Section 4.2, respectively.

Before describing the details of each step, we first discuss the state-of-the-art algorithm of the fair oversampling technique, Fair SMOTE (FSMOTE) [10]. FSMOTE is an extension of the SMOTE algorithm [14], and its goal is to address the cluster imbalances Eq. (5). The main working steps of the FSMOTE algorithm are as follows: (1) Randomly select an instance i from a cluster $C_{y,g}$ that

should be oversampled. (2) Randomly select an instance j from the set of **K nearest neighbors (KNN)** of that instance i that belongs to the same cluster. (3) Generate a synthetic instance by interpolating x^i and x^j . The formula for generating a new synthetic instance (x^{new}, y^{new}) is as follows:

$$x^{new} = x^i + w \times (x^j - x^i), \quad y^{new} = y^j \quad (6)$$

where w is the interpolation weight drawn from the uniform distribution $U(0, 1)$.

As seen from the above steps, FSMOTE generates synthetic instances based on homogeneous clusters. Therefore, if the cluster is too small (e.g., a cluster of a minority class belongs to a minority group), FSMOTE cannot generate diverse synthetic data, and the synthetic data may worsen the performance of classifiers. The proposed oversampling technique can avoid this problem by considering data from heterogeneous clusters. Therefore, the synthetic data generated by the proposed technique is diverse and can provide new information useful for learning classifiers. The following sections describe the details of the proposed algorithm.

4.1. Sampling instance pairs from heterogeneous clusters

In this section, we explain our sampling method to select instance pairs for generation from heterogeneous clusters. We first define the heterogeneous clusters.

We consider two heterogeneous clusters H_y and H_g . Given a target cluster $C_{y,g}$ that should be oversampled, H_y is the cluster that has the different class and same group (i.e., $H_y = \{j \mid y^j \neq y, g^j = g\}$), and H_g is the cluster that has the same class and different group (i.e., $H_g = \{j \mid y^j = y, g^j \neq g\}$). **We use the cluster H_y for sampling intra-group pairs and the cluster H_g for sampling intra-class pairs.**

The intra-group instance pairs are used to generate synthetic instances that have class-mix features. The class-mix features can mitigate the spurious correlation between classes and features [38], allowing the classifier to learn the class boundary more correctly and improve the prediction accuracy of each class in a given group. The intra-class instance pairs are used to generate synthetic instances that have group-mix features. The group-mix features can mitigate the correlation between a certain class and protected attributes [26], thus promoting the independence of classifier predictions and protected attributes and improving group fairness. Thus, synthetic data with these features can improve the generalization performance of classifiers.

Our sampling method randomly selects intra-group or intra-class instance pairs for a target cluster $C_{y,g}$ with the probability $p_{y,g}$ and $1 - p_{y,g}$. We define the probability $p_{y,g}$ as follows:

$$p_{y,g} = \frac{|H_y|}{|H_y| + |H_g|}. \quad (7)$$

According to the Bernoulli distribution with $p_{y,g}$ (i.e., $\text{Bernoulli}(p_{y,g})$), we select which pairs to use each time we generate new synthetic data. This allows us to sample more data from heterogeneous clusters with a larger number of observed data.

4.2. Interpolation between the heterogeneous instances

In this section, we describe an interpolation method between two instances belonging to heterogeneous clusters. Using the proposed interpolation method, we can generate a valid synthetic instance using the instance i of the target cluster $C_{y,g}$ and the instance j of the heterogeneous cluster H_y or H_g . In general, generating a valid synthetic instance by the existing interpolation method Eq. (6) is difficult, as the distance between x^i and x^j is large. To address this issue, we normalize the features of the generated synthetic instance using the distance between x^i and its K nearest neighbors $\text{KNN}(i)$.

We propose a data interpolation method for a pair of instances from heterogeneous clusters.

$$x^{new} = x^i + w^i \times (x^j - x^i) \times \frac{\max_k d(i, \text{KNN}(i))}{d(i, j)}, \quad (8)$$

where $d(i, j)$ is the distance between x^i and x^j , and w^i is the interpolation weight for x^i . We use the euclidean distance as d in this paper. In the second term on the RHS of Eq. (8), we normalize the vector distance between x^i and x^j and multiply it by the maximum distance between x^i and the instance of $\text{KNN}(i)$. Thus, the generated synthetic data can follow the original distribution of the original cluster and maintain validity (further insights can be found in Appendix). Moreover, the distance to the most distant neighbors can ensure more diversity in the synthetic data [32]. Hence, we can generate synthetic instances that are robust to overfitting and that can preserve the original distribution of the cluster.

We also propose a different distribution of the interpolation weight from the existing one, which is generated from the uniform distribution $U(0, 1)$. However, the proposed technique is based on heterogeneous clusters and is prone to overlapping between classes. Since overlapping between classes has been shown to reduce classifier performance, avoiding this phenomenon is important for oversampling [15,30]. Thus, we use a KNN-based local density that evaluates the overlap ratio of each instance. The local density Δ^i of an instance i is the fraction of its KNN that is different from the class y^i .

$$\Delta^i = \frac{|\{k \mid x^k \in \text{KNN}(x^i), y^k \neq y^i\}|}{K}. \quad (9)$$

Intuitively, if Δ^i is small (large), an instance i is likely (not) to be overlapped. Using this Δ^i , we denote the distribution $U(0, \Delta^i)$ that w^i follows, allowing us to avoid the overlapping of the data generated based on the interpolation with heterogeneous clusters.

The summary of our algorithm is shown in Algorithm 1.

Algorithm 1 Proposed fair oversampling algorithm.

Input: Training dataset \mathcal{T} , the number of nearest neighbors K
 composes clusters from the training dataset $\{C_{y,g} \mid y \in Y, g \in G\}$.
 calculate maximum cluster size $S = \max_{y \in Y, g \in G} |C_{y,g}|$.
 create an empty set to add synthetic data \mathcal{T}^{new} .
for $C_{y,g} \in \mathcal{T}$ **do**
 compose heterogeneous clusters $H_y = \{j \mid y^j \neq y, g^j = g\}$ and $H_g = \{j \mid y^j = y, g^j \neq g\}$.
 for $s = 1, \dots, S - |C_{y,g}|$ **do**
 randomly sample x^i from $C_{y,g}$.
 calculate $p_{y,g}$ using Eq. (7).
 draw $b \sim \text{Bernoulli}(p_{y,g})$.
 if b is True **then**
 randomly sample x^j from H_y .
 else
 randomly sample x^j from H_g .
 end if
 generate synthetic data x^{new} using Eq. (8) with (x^i, x^j) .
 $\mathcal{T}^{new} = \mathcal{T}^{new} \cup (x^{new}, y)$
end for
end for
return $\mathcal{T} \cup \mathcal{T}^{new}$

Table 1

Description of the 5 benchmark datasets with their characteristics, including the number of instances N , dimensions of the feature vector D , number of groups M , class distribution, and group distribution.

Dataset	N	D	M	Class distribution	Group distribution
Adult	48842	12	4	37155/11687	3915/28735/3165/13027
German	1000	30	4	300/700	490/139/200/171
Heart	297	18	2	160/137	201/96
COMPAS	7214	14	4	3251/3963	3932/828/1887/567
C&C	1994	146	2	1386/608	1385/609

Finally, we discuss the computational complexity of the proposed technique. In the proposed technique, generating each new instance costs $O(SD + S \log S)$, where S is the maximum cluster size and D is the dimension of a feature vector. Then, the proposed technique generates $S - |C_{y,g}| (\approx S)$ instances for each of $|Y| \times |G|$ clusters. Thus, the computational complexity of the proposed technique is $O(|Y||G|(SD + S \log S))$. This computational complexity is the same as the state-of-the-art technique, FSMOTE [10].

5. Experiments

5.1. Datasets

In this work, we adopt five datasets that have been widely investigated in previous fairness studies, and their characteristics are summarized in Table 1.

The first dataset is the Adult dataset [39]. The task is to predict whether a person's income is more than 50k per year. We use four groups based on the intersection of gender (male/female) and race (black/white). The second dataset is the German Credit dataset [39]. The task is to predict whether an individual is a good or bad credit risk. We form four groups based on the intersection of gender (male/female) and age (over 30/under 30). The third is the Heart Disease dataset [40]. The task is to predict whether a patient has heart disease. We use two groups based on gender (male/female). The fourth dataset is the COMPAS dataset [41]. The task is to predict recidivism based on a defendant's risk score. We use four groups based on the intersection of gender (male/female) and race (black/white). The fifth dataset is the Community and Crime dataset [39]. The task is to predict whether a community has a high (above the 70 percentile) crime rate. We form two groups based on the percentage of Black (higher/lower than the median).

5.2. Experimental settings

To verify the effectiveness of the proposed technique, we select 5 techniques for comparison purposes. Two of these techniques (FBSMOTE and FADASYN) are newly developed in this study. FBSMOTE and FADASYN are simple extensions of existing techniques (Borderline-SMOTE [15] and ADASYN [16]). However, they may outperform the state-of-the-art technique, FSMOTE, because they consider heterogeneous clusters in their data generation. We consider these techniques to confirm the effectiveness of our technique in depth. The summary of the comparison techniques is as follows:

- Original: this technique does not modify the imbalances and uses the original training dataset as is.

Table 2
Confusion matrix.

		Predicted	
		Positive	Negative
Actual	Positive	TP	FN
	Negative	FP	TN

Table 3

Experiment results using the LR classifier. The technique achieving the lowest (best) fairness or highest (best) utility is shown in bold.

Dataset	Technique	Utility	Fairness		
		BAcc	SP	E. Opp.	E. Odds
Adult	Original	0.7631	0.222	0.1848	0.1365
	SMOTE	0.8208	0.4118	0.2727	0.2739
	FSMOTE	0.8124	0.2754	0.1439	0.1081
	FBSMOTE	0.7769	0.3315	0.2676	0.2321
	FADASYN	0.8018	0.3458	0.1957	0.2043
	Ours	0.8125	0.261	0.0642	0.0678
German Credit	Original	0.624	0.265	0.2449	0.2886
	SMOTE	0.6936	0.2707	0.2855	0.2407
	FSMOTE	0.6891	0.1943	0.236	0.2008
	FBSMOTE	0.6671	0.2072	0.2729	0.2139
	FADASYN	0.68	0.1882	0.2275	0.2272
	Ours	0.6883	0.1265	0.1581	0.1508
Heart	Original	0.8235	0.3229	0.2116	0.1304
	SMOTE	0.8221	0.3557	0.2206	0.1458
	FSMOTE	0.8278	0.2675	0.2268	0.1403
	FBSMOTE	0.8227	0.2507	0.2341	0.1249
	FADASYN	0.8137	0.2319	0.2519	0.1293
	Ours	0.822	0.2369	0.1572	0.09648
COMPAS	Original	0.6537	0.3983	0.2806	0.377
	SMOTE	0.6626	0.3763	0.264	0.3527
	FSMOTE	0.6574	0.1267	0.1519	0.117
	FBSMOTE	0.5884	0.1774	0.2039	0.1804
	FADASYN	0.6506	0.1232	0.088	0.1026
	Ours	0.6551	0.1064	0.1301	0.1012
C&C	Original	0.8221	0.5854	0.3956	0.3587
	SMOTE	0.8477	0.6509	0.3167	0.3974
	FSMOTE	0.797	0.3339	0.06308	0.05502
	FBSMOTE	0.7872	0.3254	0.03998	0.04942
	FADASYN	0.7845	0.299	0.08312	0.05936
	Ours	0.7698	0.2839	0.0003	0.0071

- SMOTE¹ [14]: this technique is the most widely used oversampling technique for correcting class imbalance.
- FSMOTE² [10]: this technique is the state-of-the-art fair oversampling technique for correcting the cluster imbalance Eq. (5).
- FBSMOTE: we adapt Borderline-SMOTE [15] to balance the cluster imbalance Eq. (5), called Fair Borderline-SMOTE (FBSMOTE). This technique was developed in the scope of this work. The implementation of FBSMOTE applies Borderline-SMOTE³ to each cluster.
- FADASYN: we adapt ADASYN [16] to balance the cluster imbalance Eq. (5), called Fair ADASYN (FADASYN). This technique was developed in the scope of this work.
The implementation of FADASYN applies ADASYN⁴ to each cluster.

The nearest neighbor number K of all the techniques in our experiment is fixed at 5. The optimal imbalance ratio is not obvious and its exploration is outside the scope of our study. Therefore, we fix the parameters related to the imbalance ratio set in each technique. Also, for a fair comparison, we will not implement the method of removing noisy instances after oversampling as proposed in [10].

¹ https://imbalanced-learn.org/stable/references/generated/imblearn.over_sampling.SMOTE.html.

² <https://github.com/joyfullyac/Fair-SMOTE>.

³ https://imbalanced-learn.org/stable/references/generated/imblearn.over_sampling.BorderlineSMOTE.html.

⁴ http://glemaitre.github.io/imbalanced-learn/auto_examples/over-sampling/plot_adasyn.html.

Table 4

Experiment results using the SVM classifier. The technique achieving the lowest (best) fairness or highest (best) utility is shown in bold.

Dataset	Technique	Utility BAcc	Fairness		
			SP	E. Opp.	E. Odds
Adult	Original	0.7402	0.2141	0.2174	0.1525
	SMOTE	0.8081	0.4464	0.3957	0.3526
	FSMOTE	0.8084	0.3478	0.2878	0.245
	FBSMOTE	0.7817	0.4398	0.4687	0.3762
	FADASYN	0.8045	0.441	0.4694	0.3809
	Ours	0.8108	0.2959	0.1127	0.1369
German Credit	Original	0.6172	0.2555	0.2157	0.2901
	SMOTE	0.6681	0.2098	0.2215	0.1807
	FSMOTE	0.6631	0.2288	0.1788	0.2549
	FBSMOTE	0.6617	0.1995	0.2089	0.2248
	FADASYN	0.6591	0.2159	0.1778	0.239
	Ours	0.6774	0.184	0.1981	0.1679
Heart	Original	0.801	0.3442	0.2611	0.1559
	SMOTE	0.812	0.3461	0.2303	0.1511
	FSMOTE	0.7835	0.2797	0.2347	0.1448
	FBSMOTE	0.7824	0.2631	0.2252	0.1732
	FADASYN	0.804	0.3017	0.243	0.1547
	Ours	0.7798	0.208	0.1857	0.09634
COMPAS	Original	0.66	0.3436	0.2554	0.3086
	SMOTE	0.659	0.2681	0.1873	0.2467
	FSMOTE	0.6686	0.2812	0.2278	0.2373
	FBSMOTE	0.5882	0.2281	0.3754	0.2224
	FADASYN	0.6419	0.2476	0.2791	0.2262
	Ours	0.6675	0.2104	0.1825	0.1682
C&C	Original	0.8237	0.6509	0.4787	0.4439
	SMOTE	0.8479	0.7049	0.3655	0.4634
	FSMOTE	0.8013	0.3445	0.0626	0.05982
	FBSMOTE	0.7823	0.3104	0.05512	0.03622
	FADASYN	0.787	0.3162	0.06698	0.06116
	Ours	0.7594	0.2429	0.1105	0.0593

In all our experiments, we use three different classifiers: Logistic Regression (LR), Support Vector Machine (SVM), and Naive Bayes (NB). All classifiers are used as implemented in the Python library scikit-learn [42] with default parameters.

We focus on the fairness and utility metrics in all experiments. For fairness, we use the disparity Eq. (1) based on the three group fairness metrics defined in Equations (2) to (4): statistical parity (SP), equal opportunity (E. Opp.), and equalized odds (E. Odds). For utility, we use the balanced accuracy score (BAcc), which is widely used in imbalanced classification problems, as the accuracy score (the fraction of right predictions) is no longer considered to be an excellent utility metric. BAcc is the mean of the true positive rate and true negative rate.

$$BAcc = \frac{1}{2} \left(\frac{TP}{TP + FN} + \frac{TN}{TN + FP} \right), \quad (10)$$

where TP, TN, FP, and FN are defined in Table 2.

To evaluate the proposed technique, we use 5-fold cross-validation for all experiments. Each metric is reported in the form of an average. The hyperparameters of the classifiers and oversampling techniques are fixed for all runs.

5.3. Results

Tables 3, 4 and 5 show the performance of 6 comparative techniques in terms of 5 datasets using the LR, SVM, and NB classifiers, respectively. Based on the results shown in these three tables, we can obtain the following conclusions.

- (1) The fair oversampling techniques (FSMOTE, FBSMOTE, FADASYN, and Ours) can improve all three fairness metrics well compared with the other techniques. This fact demonstrates that fair oversampling techniques which address group and class imbalances improve fairness.
- (2) The BAcc. of the oversampling techniques is better than the original technique (i.e., a technique without any modification of data imbalance). This fact demonstrates that correcting for class imbalance by generating synthetic data is effective with regard to improving the accuracy of classifier predictions for each class.
- (3) The fairness of SMOTE is severely compromised. This fact is because SMOTE exacerbated group imbalance by only addressing class imbalance. That is, SMOTE ignores minority groups, as it randomly selects instances within the minority class.

Table 5

Experiment results using the NB classifier. The technique achieving the lowest (best) fairness or highest (best) utility is shown in bold.

Dataset	Technique	Utility	Fairness		
		BAcc	SP	E. Opp.	E. Odds
Adult	Original	0.7182	0.4222	0.1655	0.2555
	SMOTE	0.7	0.3829	0.1818	0.245
	FSMOTE	0.6885	0.1783	0.05498	0.07438
	FBSMOTE	0.669	0.1618	0.07282	0.08856
	FADASYN	0.6306	0.1243	0.07212	0.09606
	Ours	0.6585	0.1164	0.0264	0.0502
German Credit	Original	0.6674	0.2687	0.2568	0.2433
	SMOTE	0.6602	0.2477	0.291	0.2246
	FSMOTE	0.6634	0.1552	0.1903	0.1643
	FBSMOTE	0.6469	0.153	0.22	0.1693
	FADASYN	0.6424	0.1449	0.1854	0.147
	Ours	0.6376	0.1232	0.1441	0.1314
Heart	Original	0.8086	0.2761	0.1622	0.0831
	SMOTE	0.8154	0.2915	0.1616	0.09466
	FSMOTE	0.7933	0.1931	0.1702	0.09852
	FBSMOTE	0.796	0.2167	0.2716	0.1639
	FADASYN	0.8044	0.2014	0.2395	0.1331
	Ours	0.7944	0.2026	0.1548	0.08062
COMPAS	Original	0.6241	0.62	0.5289	0.6121
	SMOTE	0.6137	0.6806	0.6147	0.669
	FSMOTE	0.61	0.1491	0.1501	0.13278
	FBSMOTE	0.5705	0.1456	0.1974	0.1429
	FADASYN	0.6004	0.1262	0.1159	0.1114
	Ours	0.6161	0.088	0.1728	0.1059
C&C	Original	0.7176	0.2941	0.1172	0.1092
	SMOTE	0.7238	0.3043	0.1159	0.1137
	FSMOTE	0.6999	0.2248	0.07886	0.06744
	FBSMOTE	0.7138	0.2249	0.1329	0.08966
	FADASYN	0.7244	0.2463	0.1187	0.07464
	Ours	0.6943	0.2138	0.07538	0.05788

- (4) The three fair oversampling techniques (FSMOTE, FBSMOTE, and FADASYN) do not perform as the proposed technique. These techniques generate synthetic data based on homogeneous clusters; thus, limiting their generation area to the inside of the observed cluster regions. Thus, they cannot generate effective samples when the observed clusters are small. This phenomenon results in the overfitting of classifiers and worsens their performances.
- (5) The proposed technique is fairer than other techniques and can preserve balanced accuracy (BAcc.). The results of the proposed technique are stable regardless of the datasets or classifiers, which demonstrates the effectiveness of the proposed technique for generating data by considering heterogeneous clusters. FBSMOTE and FADASYN also consider heterogeneous clusters and sometimes achieve the best fairness. However, our proposed technique that is based on a new interpolation method can often produce better results than FBSMOTE and FADASYN, because it can generate more diverse and valid synthetic data.

6. Conclusion

Ensuring ML fairness in class imbalanced environments is an open challenge in real-world applications, and the existing fair oversampling techniques that address this challenge are based on homogeneous clusters, where each cluster is the intersection of a single group and a single class. We could find that these techniques may result in the overfitting of ML classifiers if the observed clusters are small enough. In this work, we developed an oversampling technique that is robust to overfitting through the use of heterogeneous clusters. The proposed technique is based on the interpolation of instances from a cluster and its heterogeneous clusters while considering the valid coverage of the cluster. To achieve this, we presented a sampling method for selecting instance pairs to generate diverse synthetic instances. Moreover, we introduced an interpolation method to ensure the validity of the generated synthetic instances. Therefore, synthetic instances generated by the proposed technique are more diverse and valid than those generated by existing techniques. In addition, we discussed the performance of fairness oversampling techniques using five real-world datasets and basic three classifiers (Logistic Regression, Support Vector Machine, and Naive Bayes). The experimental results showed that the proposed technique outperforms existing techniques in the trade-off between fairness and utility, and that this performance is not limited to the chosen classifier algorithm.

In future works, we plan to study how our technique can affect performance in more complex classifiers (e.g., deep learning classifiers and classifiers with fairness constraints). We also plan to study the cases in which groups are not categorical (e.g., the proportion of each race in a given area).

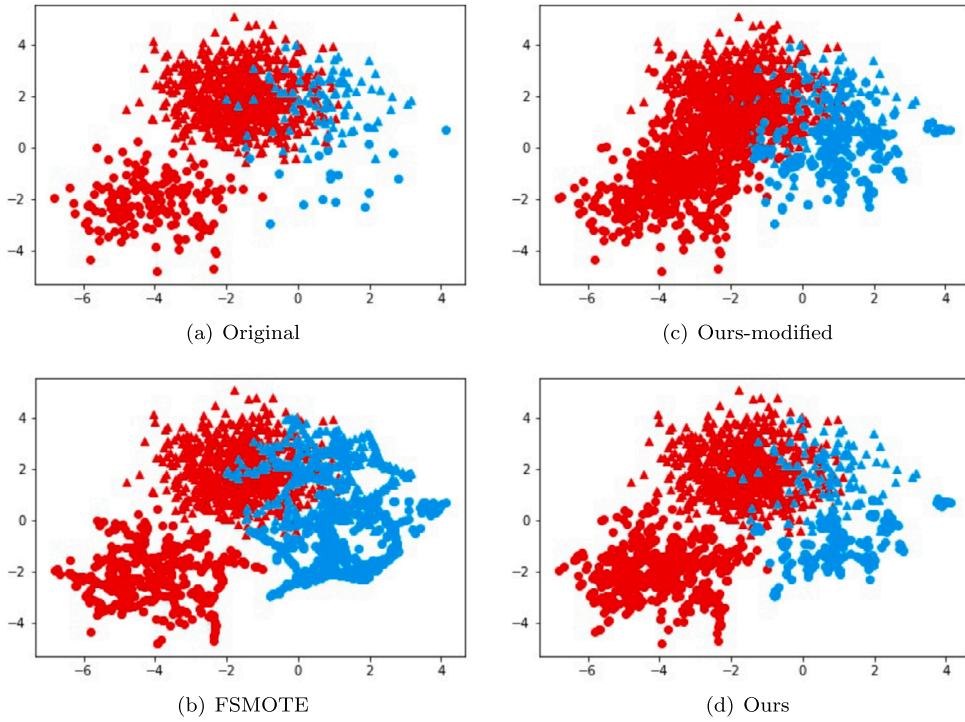


Fig. 2. Artificial data with two features. The triangles and circles represent a majority class and a minority class, respectively. The red and blue points represent the majority group and minority group, respectively. (a) Distribution of original features. (b) Distribution of synthetic features generated by FSMOTE. (c) Distribution of synthetic features generated by the modified proposed technique in which the interpolation method is replaced by the existing interpolation method (Eq. (6)) instead of the proposed interpolation method (Eq. (8)). (d) Distribution of synthetic features generated by the proposed technique.

CRediT authorship contribution statement

Ryosuke Sonoda: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Ryosuke Sonoda reports a relationship with Fujitsu Ltd that includes: employment.

Data availability

The authors do not have permission to share data.

Appendix A. Investigation of the effectiveness of the proposed oversampling technique: an empirical analysis

In this section, we investigate the performance of our technique to provide a better understanding of these algorithms. Our technique can generate robust synthetic data by considering heterogeneous clusters, which can further improve the utility and fairness of the classifier over existing techniques. To accomplish this, our technique forms pairs from instances belonging to different classes but the same groups or different groups but the same classes of clusters. Also, our proposed interpolation method (Eq. (8)) can generate synthetic data in a way to follow the distribution of the original clusters.

We conducted experiments to evaluate the performance of various fair oversampling techniques on artificial data consisting of four clusters. Fig. 2(a) shows the original data distribution. Fig. 2(b) shows the synthetic data distribution generated by FSMOTE. It can be seen that FSMOTE does not generate synthetic data in appropriate locations when the size of the observed clusters is small. In particular, the synthetic data for the minority group (blue points) has overlap between classes, which deteriorates the performance of the classifier [15,30]. We also tested a slightly modified version of our technique (Ours-modified), which generates synthetic data based on heterogeneous clusters using the existing interpolation method Eq. (6) instead of our interpolation method Eq. (8). Fig. 2(c) shows that while the Ours-modified technique generates diverse data, it fails to maintain the original distribution. This results in the overlap between classes, which may exacerbate the trade-off between utility and fairness. In contrast, Fig. 2(d) shows that our technique can generate robust synthetic data by considering heterogeneous clusters while preserving the original distribution.

As a result, our oversampling technique can effectively address both class and group imbalance problems and enhance the generalization performance of the classifier, thus improving the trade-off between utility and fairness.

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