Low-Complexity User Matching and Stream-Based Power Allocation for Multicarrier RSMA

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1 System Model

Let the pair of users $(i, j) \in \mathcal{P} = \{(i, j) \mid 1 \leq i < j \leq J\}$ is allocated to subcarrier n, where $n \in \mathcal{N}$ with $\mathcal{N}$ being the set of subcarrier. $\mathcal{J} = \{1, \ldots, j, \ldots, J\}$ is a set of users served by a base station. Assuming the precoding directions using low-complexity Zero Forcing (ZF) and a normalized unit noise power The private rates of users i and j using this subcarrier, denoted by $R^{\mathrm{p}}_{n,i}$ and $R^{\mathrm{p}}_{n,j}$, respectively, are given by:

$$
\begin{aligned}
R^{\mathrm{p}}_{n,i} &= \log_2 \left(1 + \|\mathbf{h}_{n,i}\|^2 \cdot \rho_n \cdot P_{n,i}\right), \\
R^{\mathrm{p}}_{n,j} &= \log_2 \left(1 + \|\mathbf{h}_{n,j}\|^2 \cdot \rho_n \cdot P_{n,j}\right).
\end{aligned} \tag{1}
$$

where $\rho_n = 1 - |\overline{\mathbf{h}}^H_{n,i}\overline{\mathbf{h}}_{n,j}|^2$. The commum rate must be equal to the minimum of the individual commum rates $R^{\mathrm{c}}_n = \min(R^{\mathrm{c}}_{n,i}, R^{\mathrm{c}}_{n,j})$, where:

$$
\begin{aligned}
R^{\mathrm{c}}_{n,i} &= \log_2 \left(1 + \frac{\left|\mathbf{h}^H_{n,i} \cdot \mathbf{f}^{\mathrm{c}}_n\right|^2 \cdot P_{n,\mathrm{c}}}{1 + \|\mathbf{h}_{n,i}\|^2 \cdot \rho_n \cdot P_{n,i}}\right), \\
R^{\mathrm{c}}_{n,j} &= \log_2 \left(1 + \frac{\left|\mathbf{h}^H_{n,j} \cdot \mathbf{f}^{\mathrm{c}}_n\right|^2 \cdot P_{n,\mathrm{c}}}{1 + \|\mathbf{h}_{n,j}\|^2 \cdot \rho_n \cdot P_{n,j}}\right),
\end{aligned} \tag{2}
$$

where the commom precoder direction is given by

$$
\mathbf{f}^{\mathrm{c}}_n = \frac{1}{\sqrt{2(1 + |\overline{\mathbf{h}}^H_{n,i}\overline{\mathbf{h}}_{n,j}|)}} (\overline{\mathbf{h}}_{n,i} + \overline{\mathbf{h}}_{n,j} \cdot e^{-j\angle \overline{\mathbf{h}}^H_{n,i}\overline{\mathbf{h}}_{n,j}}). \tag{3}
$$

Let the part of R^{c}_n allocated to the pair of users in a given subcarrier, satisfies $0 \leq C_{n,i} + C_{n,2} \leq R^{\mathrm{c}}_n$. The weighted sum rate can expressed as:

$$
W^{\mathrm{T}} = \sum_{n=1}^{N} W_n = \sum_{n=1}^{N} \sum_{i=1}^{J-1} \sum_{j=i+1}^{J} \left(u_i(R^{\mathrm{p}}_{n,i} + C_{n,i}) + u_j(R^{\mathrm{p}}_{n,j} + C_{n,j})\right) \cdot x_{i,j} \tag{4}
$$

Based on these assumptions, the studied problem can be formulated as in the following

$$\max_{\mathcal{P}_n, P_{n,i}, P_{n,j}, P_n^{\mathrm{c}}, C_{n,i}, C_{n,j} \forall n \in \mathcal{N}} W^{\mathrm{T}} \tag{5a}$$

subject to

$$\sum_{n=1}^{N} P_n = \sum_{n \in \mathcal{N}} (P_{n,1} + P_{n,2} + P_{n,\mathrm{c}}) \leq P^{\mathrm{T}}, \tag{5b}$$

$$P_n \geq 0, \tag{5c}$$

$$0 \leq C_{n,i} + C_{n,2} \leq R_n^{\mathrm{c}}, \tag{5d}$$

$$\sum_{i=1}^{J-1} \sum_{j=i+1}^{J} x_{i,j} = 1 \tag{5e}$$

$$x_{i,j} \in \{0, 1\}, \tag{5f}$$

$$\mathcal{P}_n \in \{(i,j)\} \; \forall i,j \in \mathcal{J} \text{ with } i \neq j. \tag{5g}$$

2 Low Complexity User Matching

In this work, we propose a low-complexity user matching algorithm designed to maximize the weighted sum rate in a downlink Rate-Splitting Multiple Access (RSMA) system. This approach allocates user pairs to subcarriers by considering both channel gain and orthogonality, thereby achieving efficient resource allocation while maintaining manageable computational complexity.

The algorithm follows a two-step process:

1. **Primary User Selection**: For each subcarrier, the user with the highest effective weighted channel gain is selected as the primary user. Specifically, given a weight vector $\mathbf{u}$ representing the priority or importance of each user, we compute a weighted gain $u_j \cdot \|\mathbf{h_{n,j}}\|$, $\forall j \in \mathcal{J}$. The user with the maximum value of this metric is selected as the primary user. This step ensures that high-priority users are more likely to be chosen, aligning the resource allocation with the weighted sum rate maximization goal.

2. **Secondary User Selection**: Once the primary user for each subcarrier is chosen, a secondary user is selected based on the degree of orthogonality with the primary user. We calculate the orthogonality between the primary user's channel vector $\mathbf{h_{n,j^\star}}$ and each other user's channel vector $\mathbf{h_{n,j}}$ $\forall j \neq j^\star$. The user with the highest orthogonality measure, defined by $\rho_n = 1 - |\overline{\mathbf{h}}_{n,j^\star}^H \overline{\mathbf{h}}_{n,j}|^2$ is selected as the secondary user. This pairing minimizes interference and enhances the overall rate by ensuring that selected users have channels that are as orthogonal as possible.

This user matching strategy combines weighted channel gain maximization with orthogonal pairing, effectively balancing rate and interference considerations. Notably, this algorithm avoids the high computational costs of exhaustive search methods and totally unimodular matrix based solution by simplifying the matching process into sequential selection steps. As a result, it achieves a low-complexity solution while still yielding high performance in terms of weighted sum rate, making it suitable for practical RSMA systems. The user matching process is detailed in algorithm 1.

3 Stream-Based Power Allocation

Firstly, fixing the pair of users obtained by the low-complexity user matching algorithm and assuming that for a given subcarrier n, we assume hereafter that $\mathbf{h}_{n,1}$ and $\mathbf{h}_{n,2}$ are the channel responses at subcarrier n for the users with the best and worst channel conditions in $\mathcal{P}_n$, respectively. So, if $\mathcal{P}_n = \{(i,j)\}$ and $\|\mathbf{h}'_{n,i}\| \geq \|\mathbf{h}'_{n,j}\|$, we have that $\mathbf{h}_{n,1} = \mathbf{h}'_{n,i}$ and $\mathbf{h}_{n,2} = \mathbf{h}'_{n,j}$. Similarly, we assume that $u_{n,1} = u_i$ and $u_{n,2} = u_j$ are the weights or priorities of the users in best and worst channel conditions assigned to subcarrier n, respectively. The papers [1], [2] and [3] are based in a commum precoder direction that is function of the objective variable. In this way, our objective is formulated an expression that the commum precoder direction depends only know variables. There is a work

Algorithm 1 Low Complexity User Matching for RSMA System

Require: channel frequency response $\mathbf{h}_{n,j}$, user weights u_j, number of users J, number of subcarriers N

Ensure: User pairs for each subcarrier

```
for n = 1 to N do
    Primary User Selection:
    for j = 1 to J do
        Compute weighted gain for user j on subcarrier n: G_{j,n} = u_j ||h_{n,j}||
    end for
    Select primary user j* as j* = arg max_j G_{n,j}
    Secondary User Selection:
    for j = 1 to J, j ≠ j* do
        Compute orthogonality measure between primary user j* and user j:
            O_{j*,j,n} = 1 - |h^H_{j*,n} h_{j,n}| / (||h_{j*,n}|| ||h_{j,n}||)
    end for
    Select secondary user as j** = arg max_{j≠j*} O_{j*,j,n}
    Assign pair (j*, j**) to subcarrier n
end for
```

that is based in this kind of precoder direction, but based on the mathematically definition of RSMA system model, the commum rate must be equal to the minimum of the individual commum rates, now we need to overcome the problem of min function to build an closed expression for analisys of the power allocation. By assuming the precoding directions using low-complexity Zero Forcing (ZF) and a normalized unit noise power, the expressions for the private and common rates can be rewritten as follows:

$$
\begin{aligned}
R_{n,1}^{\mathrm{c}} &= \log_2\left(1 + \frac{\left|\mathbf{h}_{n,1}^{H} \cdot \mathbf{f}_{n,\mathrm{c}}\right|^2 \cdot P_{n,c}}{1 + \|\mathbf{h}_{n,1}\|^2 \cdot \rho_n \cdot P_{n,1}}\right), \\
R_{n,2}^{\mathrm{c}} &= \log_2\left(1 + \frac{\left|\mathbf{h}_{n,2}^{H} \cdot \mathbf{f}_{n,\mathrm{c}}\right|^2 \cdot P_{n,c}}{1 + \|\mathbf{h}_{n,2}\|^2 \cdot \rho_n \cdot P_{n,2}}\right),
\end{aligned}
\tag{6}
$$

where,

$$
\mathbf{f}_n^{\mathrm{c}} = \frac{1}{\sqrt{2(1 + |\overline{\mathbf{h}}_{n,1}^{H} \overline{\mathbf{h}}_{n,2}|)}} (\overline{\mathbf{h}}_{n,1} + \overline{\mathbf{h}}_{n,2} \cdot e^{-j\angle \overline{\mathbf{h}}_{n,1}^{H} \overline{\mathbf{h}}_{n,2}}). \tag{7}
$$

Now, to ensure that the pair of users assigned to subcarrier n decode the common message, the achievable common rate is given by

$$
R_n^{\mathrm{c}} = \min\{R_{n,1}^{\mathrm{c}}, R_{n,2}^{\mathrm{c}}\} \tag{8}
$$

Let $C_{n,k}$ be the part of R_n^{c} allocated to user-k, and it satisfies $0 \leq C_{n.1} + C_{n,2} \leq R_n^{\mathrm{c}}$. The weighted sum rate can expressed as

$$
W_n = \sum_{k=1,2} u_{n,k}(R_{n,k}^{\mathrm{p}} + C_{n,k}) \tag{9}
$$

We assume that both users equally contribute to the composition of the commum rate, in this way we have $C_{n,1} = C_{n,2} = \frac{1}{2} \cdot R_n^{\mathrm{c}}$. Now we can rewrite the equation as follows

$$
W^{\mathrm{T}} = \sum_{n=1}^{N} W_n = \sum_{n=1}^{N} \left(u_{n,1} \cdot R_{n,1}^{\mathrm{p}} + u_{n,2} \cdot R_{n,2}^{\mathrm{p}} + \frac{u_{n,1} + u_{n,2}}{2} \cdot R_n^{\mathrm{c}}\right). \tag{10}
$$

Based on these assumptions, the studied problem can be reformulated as in the following:

$$\max_{P_{n,1},P_{n,2},P_{n,\mathrm{c}}\forall n\in\mathcal{N}} W^{\mathrm{T}} \tag{11a}$$

$$\text{subject to}$$

$$\sum_{n\in\mathcal{N}} P_n = \sum_{n\in\mathcal{N}} \left(P_{n,1}+P_{n,2}+P_{n,c}\right) \leq P^{\mathrm{T}}, \tag{11b}$$

$$P_n \geq 0, \tag{11c}$$

where (11a) is the total weighted sum rate in the system, (11b) and (11c) represents the power allocation constraint. For mathematical simplification, we assume that $P_{n,1} = P_{n,2} = \frac{1}{2}\cdot P_{n,\mathrm{p}}$. With these considerations, the commom rate in a given subcarrier will be given by:

$$R_n^{\mathrm{c}} = \min\left(\log_2\left(1+\frac{\left|\mathbf{h}_{n,1}^H\cdot\mathbf{f}_{n,\mathrm{c}}\right|^2\cdot P_{n,c}}{1+\left\|\mathbf{h}_{n,1}\right\|^2\cdot\rho_n\cdot P_{n,1}}\right), \log_2\left(1+\frac{\left|\mathbf{h}_{n,2}^H\cdot\mathbf{f}_{n,\mathrm{c}}\right|^2\cdot P_{n,c}}{1+\left\|\mathbf{h}_{n,2}\right\|^2\cdot\rho_n\cdot P_{n,2}}\right)\right) \tag{12}$$

considering that $\alpha = \left|\overline{\mathbf{h}}_{n,1}^H\mathbf{f}_{n,\mathrm{c}}\right| = \left|\overline{\mathbf{h}}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}\right|$, and then analyzing the lowest value, we have that:

$$\begin{gathered}
\frac{\left|\mathbf{h}_{n,1}^H\cdot\mathbf{f}_{n,\mathrm{c}}\right|^2\cdot P_{n,c}}{1+\left\|\mathbf{h}_{n,1}\right\|^2\cdot\rho_n\cdot P_{n,1}} > \frac{\left|\mathbf{h}_{n,2}^H\cdot\mathbf{f}_{n,\mathrm{c}}\right|^2\cdot P_{n,c}}{1+\left\|\mathbf{h}_{n,2}\right\|^2\cdot\rho_n\cdot P_{n,2}} \implies \\
\frac{\cancel{\alpha}\left\|\mathbf{h}_{n,1}\right\|^2\cdot \cancel{P_{n,c}}}{1+\left\|\mathbf{h}_{n,1}\right\|^2\cdot\rho_n\cdot\frac{P_{n,\mathrm{p}}}{2}} > \frac{\cancel{\alpha}\left\|\mathbf{h}_{n,2}\right\|^2\cdot \cancel{P_{n,c}}}{1+\left\|\mathbf{h}_{n,2}\right\|^2\cdot\rho_n\cdot\frac{P_{n,\mathrm{p}}}{2}} \implies \\
\left\|\mathbf{h}_{n,1}\right\|^2 + \cancel{\left\|\mathbf{h}_{n,1}\right\|^2\left\|\mathbf{h}_{n,2}\right\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}} > \left\|\mathbf{h}_{n,2}\right\|^2 + \cancel{\left\|\mathbf{h}_{n,1}\right\|^2\left\|\mathbf{h}_{n,2}\right\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}} \implies \\
\left\|\mathbf{h}_{n,1}\right\|^2 > \left\|\mathbf{h}_{n,2}\right\|^2
\end{gathered} \tag{13}$$

As previously assumed, $\|\mathbf{h}_{n,1}\| > \|\mathbf{h}_{n,2}\|$, then the inequality is true. We can now rewrite the objective function W_n as follows:

$$\begin{aligned}
W_n = u_{n,1}\cdot\log_2\left(1+\left\|\mathbf{h}_{n,1}\right\|^2\cdot\rho_n\cdot\frac{P_{n,\mathrm{p}}}{2}\right) + \frac{u_{n,2}-u_{n,1}}{2}\cdot\log_2\left(1+\left\|\mathbf{h}_{n,2}\right\|^2\cdot\rho_n\cdot\frac{P_{n,\mathrm{p}}}{2}\right) \\
+\frac{u_{n,1}+u_{n,2}}{2}\cdot\log_2\left(1+\left\|\mathbf{h}_{n,2}\right\|^2\cdot\rho_n\cdot\frac{P_{n,\mathrm{p}}}{2}+\left|\mathbf{h}_{n,2}^H\cdot\mathbf{f}_{n,\mathrm{c}}\right|^2\cdot P_{n,c}\right)
\end{aligned} \tag{14}$$

The idea is to model the problem with the two private and common power allocation variables, to analyze the convexity of the function, the Hessian matrix. To do this, the idea is to evaluate the convexity of the function W_n, because according to the [4], the sum of convex/concave functions is also a convex/concave function, in this sense, proving the convexity of W_n also proves the convexity of W^{T} below the mathematical proofs are presented.

$$H_{W_n} = \begin{bmatrix} \frac{\partial^2 W_n}{\partial P_{n,p}^2} & \frac{\partial^2 W_n}{\partial P_{n,p}\partial P_{n,c}} \\ \frac{\partial^2 W_n}{\partial P_{n,c}\partial P_{n,p}} & \frac{\partial^2 W_n}{\partial P_{n,c}^2} \end{bmatrix}$$

$$\left[\begin{array}{cccc}
-\frac{\|\mathbf{h}_{n,2}\|^4\rho_n^2 u_1}{4\ln(2)(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2})^2} & -\frac{\|\mathbf{h}_{n,2}\|^4\rho_n^2(u_1+u_2)}{8\ln(2)(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,c})^2} & -\frac{\|\mathbf{h}_{n,2}\|^4\rho_n^2(u_2-u_1)}{8\ln(2)(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2})^2} & -\frac{\|\mathbf{h}_{n,2}\|^2|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2(u_1+u_2)}{4\ln(2)(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,c})^2} \\
 & -\frac{\|\mathbf{h}_{n,2}\|^2|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2(u_1+u_2)}{4\ln(2)(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,c})^2} & & -\frac{|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4(u_1+u_2)}{2\ln(2)(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,c})^2}
\end{array}\right] \tag{15}$$

To analyze the concavity or convexity, check the definiteness of the Hessian matrix:

Convex Function: The function f is convex in a neighborhood of x if the Hessian matrix is positive semi-definite. This means all eigenvalues of are non-negative.

Strictly Convex Function: The function is strictly convex if the Hessian matrix is positive definite (all eigenvalues are positive).

Concave Function: The function f is concave in a neighborhood of x if the Hessian matrix is negative semi-definite (all eigenvalues are non-positive).

Strictly Concave Function: The function is strictly concave if the Hessian matrix is negative definite (all eigenvalues are negative).

To check definiteness, you can either calculate the eigenvalues directly or use the following criteria based on leading principal minors: For a 2×2 matrix, if $\frac{\partial^2 W_n}{\partial P_{n,p}^2} < 0$ and $det(H_{W_n}) > 0$, so, H_{W_n} is negative definite. It is easy to notice that $\frac{\partial^2 W_n}{\partial P_{n,p}^2} < 0$ is true, now we need to check if $det(H_{W_n}) > 0$.

$$
\begin{aligned}
&\frac{\partial^2 W_n}{\partial P_{n,p}^2}\cdot\frac{\partial^2 W_n}{\partial P_{n,c}^2}-\frac{\partial^2 W_n}{\partial P_{n,p}\partial P_{n,c}}\cdot\frac{\partial^2 W_n}{\partial P_{n,c}\partial P_{n,p}}>0\\
&\Bigg(\frac{\|\mathbf{h}_{n,2}\|^4\rho_n^2 u_1}{4\ln(2)\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}\right)^2}+\frac{\|\mathbf{h}_{n,2}\|^4\rho_n^2(u_1+u_2)}{8\ln(2)\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}\\
&+\frac{\|\mathbf{h}_{n,2}\|^4\rho_n^2(u_2-u_1)}{8\ln(2)\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}\right)^2}\Bigg)\cdot\frac{|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4(u_1+u_2)}{2\ln(2)\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}\\
&-\left(\frac{\|\mathbf{h}_{n,2}\|^2|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2(u_1+u_2)}{4\ln(2)\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}\right)^2>0\implies\\
&\left(\frac{\|\mathbf{h}_{n,2}\|^4\rho_n^2(u_1+u_2)}{8\ln(2)\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}\right)^2}+\frac{\|\mathbf{h}_{n,2}\|^4\rho_n^2(u_1+u_2)}{8\ln(2)\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}\right)\\
&\cdot\frac{|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4(u_1+u_2)}{2\ln(2)\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}\\
&-\frac{\|\mathbf{h}_{n,2}\|^4|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4(u_1+u_2)^2}{16\ln(2)^2\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^4}>0\implies\\
&\frac{\|\mathbf{h}_{n,2}\|^4|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4\rho_n^2(u_1+u_2)^2}{16\ln(2)^2\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}\right)^2\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}\\
&+\frac{\|\mathbf{h}_{n,2}\|^4|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4\rho_n^2(u_1+u_2)^2}{16\ln(2)^2\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^4}\\
&-\frac{\|\mathbf{h}_{n,2}\|^4|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4(u_1+u_2)^2}{16\ln(2)^2\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^4}>0\implies\\
&\frac{\|\mathbf{h}_{n,2}\|^4|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4\rho_n^2(u_1+u_2)^2}{16\ln(2)^2\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}\right)^2\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}\\
&+\frac{\|\mathbf{h}_{n,2}\|^4|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4(u_1+u_2)^2\left(\rho_n^2-1\right)}{16\ln(2)^2\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^4}>0.
\end{aligned}
\tag{16}
$$

As ρ_n is between 0 and 1, therefore the second term of the inequality is always negative, therefore, to evaluate whether the sum will be positive, we must only evaluate whether the first term is greater than the second.

$$
\frac{\cancel{\|\mathbf{h}_{n,2}\|^4|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4}\rho_n^2\cancel{(u_1+u_2)^2}}{\cancel{16\ln(2)^2}\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}\right)^2\cancel{\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}} >
$$

$$
\frac{\cancel{\|\mathbf{h}_{n,2}\|^4|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^4(u_1+u_2)^2}\left(\rho_n^2-1\right)}{\cancel{16\ln(2)^2}\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^{\cancel{4}}}. \implies
$$

$$
\frac{\rho_n^2}{\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}\right)^2} > \frac{\left(\rho_n^2-1\right)}{\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}. \implies
$$

$$
\rho_n^2\left(\frac{1}{\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}\right)^2}-\frac{1}{\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}\right) > \frac{-1}{\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}. \tag{17}
$$

Given that the term on the right of the equality is always negative, we must then demonstrate that the term on the left is positive.

$$
\begin{aligned}
\frac{1}{\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}\right)^2} &> \frac{1}{\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2} \implies \\
1 &> \frac{\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}\right)^2}{\left(1+\|\mathbf{h}_{n,2}\|^2\rho_n\frac{P_{n,\mathrm{p}}}{2}+|\mathbf{h}_{n,2}^H\mathbf{f}_{n,\mathrm{c}}|^2P_{n,\mathrm{c}}\right)^2}
\end{aligned} \tag{18}
$$

As on the right side of the equality the value of the numerator is being divided by a greater value, that is, let A/B given that $B > A$, then this result is less than 1. Thus, the inequality is true, which follows that the determinant is positive. In this way, it was shown that $\frac{\partial^2 W_n}{\partial P_{n,p}^2} < 0$ and $det(H_{W_n}) > 0$, therefore H_{W_n} is negative definite based on leading principal minors. Then, the objective function W_n is stricly concave function.

We can rewrite the optimization problem considering now that $P_{n,1} = P_{n,2} = \frac{P_{n,\mathrm{p}}}{2}$ and fixing the user pair variable.

$$
\max_{P_{n,\mathrm{p}},P_{n,\mathrm{c}}\forall n\in\mathcal{N}} W^{\mathrm{T}} \tag{19a}
$$

$$
\text{subject to}
$$

$$
\sum_{n\in\mathcal{N}}\left(P_{n,\mathrm{p}}+P_{n,\mathrm{c}}\right) \leq P^{\mathrm{T}}, \tag{19b}
$$

$$
P_{n,\mathrm{p}} \geq 0 \tag{19c}
$$

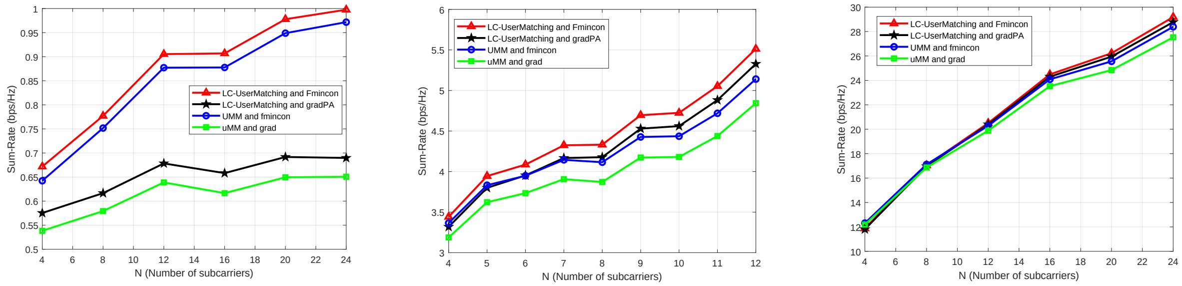
$$
P_{n,\mathrm{c}} \geq 0 \tag{19d}
$$

The objective function in this work was proven to be concave, which ensures that any local maximum is also a global maximum. Given this property, we employed MATLAB's fmincon function, a versatile solver for constrained nonlinear optimization, which is well-suited for handling problems with concave objective functions and constraints.

With the power $P_{n,\mathrm{p}}$ destined to the private streams in a given subcarrier n, the power $P_{n,\mathrm{p}}$ must be divided by $P_{n,1}$ and $P_{n,2}$ that maximize the weighted sum rate of private streams for this, we employed the water-filling solution:

$$
P_{n,k} = \max\left(\left(U_{n,k}\cdot\mu - \frac{1}{||\mathbf{h}_{n,k}||^2\rho_n}\right), 0\right), k = 1, 2, \tag{20}
$$

where we define μ as the water level, which must be chosen to ensure the equality $P_{n,1} + P_{n,2} = P_{n,\mathrm{p}}$, thus $\mu = \frac{P_{n,\mathrm{p}}}{(U_{n,1}+U_{n,2})} + \frac{1}{\rho_n(U_{n,1}+U_{n,2})}\left[\frac{1}{||\mathbf{h}_{n,1}||^2} + \frac{1}{||\mathbf{h}_{n,2}||^2}\right]$.

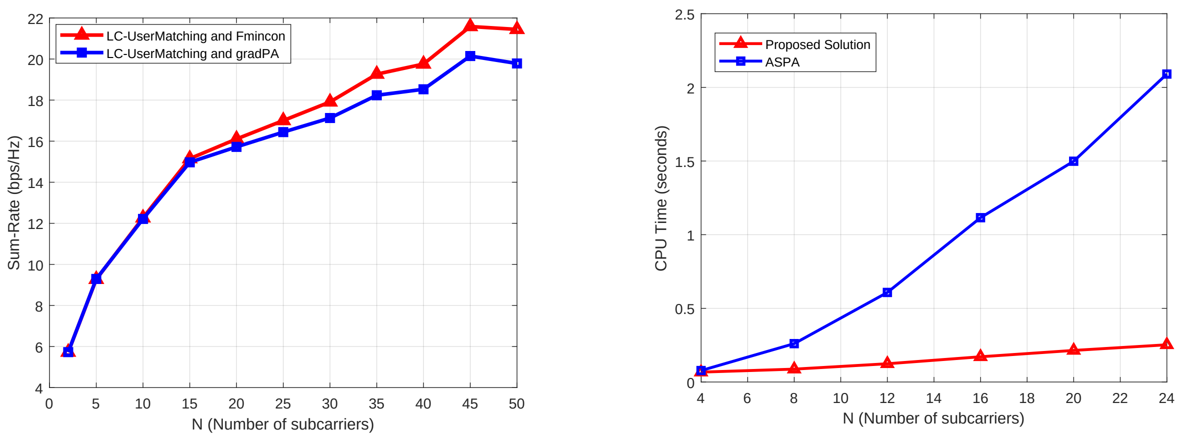


(a) ($P^T = 10$)

(b) ($P^T = 100$)

(c) ($P^T = 1000$)

Figure 1: Sum-rate versus number of subcarriers for 8 users.



(a) Sum-Rate X Number of Subcarriers considering ($P^T = 100$), 50 users and unit weights

(b) CPU Time X Number of Subcarriers - considering 8 users and unit weights

4 Results

In this section, we present simulation results obtained from a simulator where we implemented the main aspects of the presented system model and algorithms. We assume a BS with $N_\mathrm{t} = 4$ antennas, located at the center of a ring-shaped area with an inner radius equal to 1 m and an outer radius equal to 10 m, where the users are uniformly distributed. The entries of channel frequency response vector for user j in subcarrier n, $\mathbf{h}^{'}_{n,j}$, are $\mathcal{CN}(0, \frac{1}{N_t \cdot \Delta_j^{\zeta}})$, where Δ_j is the distance between user j and the BS, and $\zeta = 3$ is the path loss exponent.

In Figure 1, a fixed number of users equal to 8 was considered, and the number of subcarriers was varied. The user weights were also considered to be unitary. To obtain statistical confidence, we performed 1,000 repetitions for each point in the curves.

Na Figura 2a, apresenta-se o tempo de processamento de CPU para a solução proposta neste trabalho, com *low-complexity user matching* e alocação de potência entre os fluxos privados e comum, comparado ao tempo de processamento da referência, que se baseia no *user matching* com matriz totalmente unimodular, resolvido de forma ótima pelo algoritmo de Karmarkar, que possui complexidade polinomial $O(l^{3.5})$. Na Figura 2b, apresenta-se a comparação do algoritmo de alocação de potência proposto, considerando o *low-complexity user matching*, fixando-se o número de 50 usuários e variando o número de subportadoras.

In Figure 3, distinct weights were considered as $u_j = \frac{j}{2}$. In this plot, we disregarded the effect of channel path loss in order to account only for fast fading, to show how the weights can be used to achieve user fairness. We used ($P^T = 20dB$), 4 users, and 4 subcarriers.

In Figure 4, deterministic channels were considered to evaluate the effect of channel strength disparity and the degree of orthogonality, analyzing how RSMA outperforms other multiple access schemes, such as Multicasting, OMA, NOMA, and SDMA. A single-carrier system with unitary weights was considered, where pairs of users $\mathbf{h}_{n,1} = \frac{1}{\sqrt{2}}[1;1]^H$ and $\mathbf{h}_{n,2} = \frac{\gamma}{\sqrt{2}}[1; e^{j\theta}]^H$ are allocated to each

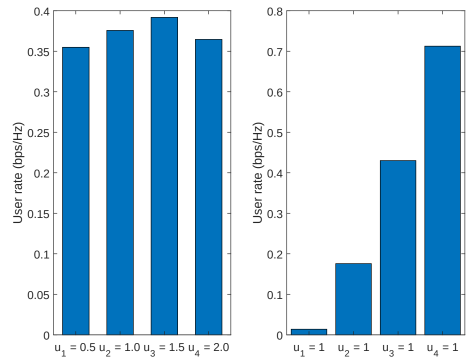


Figure 3: Average user rate distribution for 4 users and 4 subcarriers for 1,000 realizations.

subcarrier. The analysis was conducted as a function of ρ_n (ranging from 0 to 1) and $\gamma_{\mathrm{dB}} = 20\log_{10}(\gamma)$ (ranging from 0 to -20 dB).

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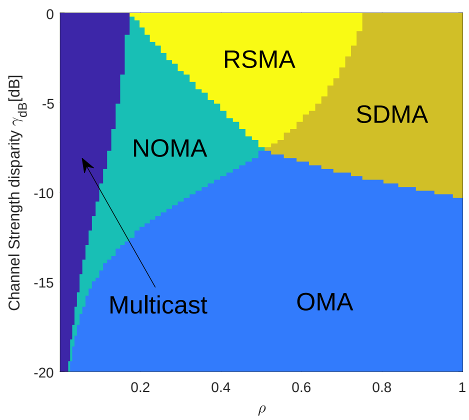
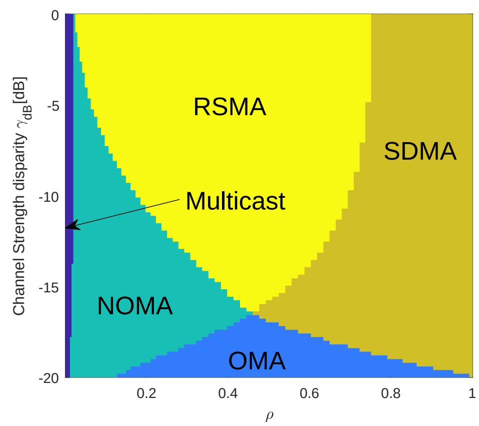
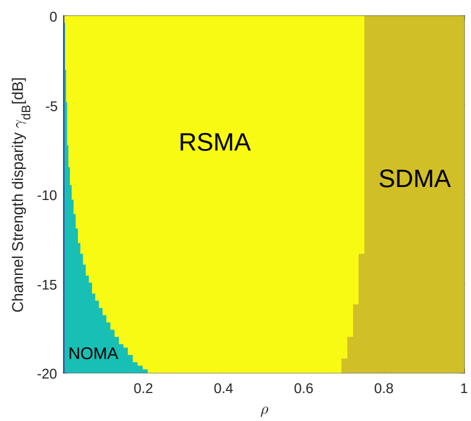


(a) $(P^T = 10)$



(b) $(P^T = 100)$



(c) $(P^T = 1000)$

Figure 4: Regions of operation for RS,SDMA,NOMA,OMA and Multicast for $(P^T = 10, 100, 1000)$(SNR=10dB,20dB and 30dB).