Computer-generated proof of affine involution covering property

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1 Setup

Let n be a positive integer. For the definition of the affine symmetric group $\tilde{S}_n$, see [2]. Fix an affine permutation $w \in \tilde{S}_n$ and an involution $y = y^{-1} \in \tilde{S}_n$. We set $y_a = y(a)$ for $a \in \mathbb{Z}$ and define

$$\mathrm{Cyc}(y) = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a \leq b = y_a\}.$$

As a shorthand, we write $w^{-1} = \text{—}a\text{—}b\text{—}c\text{—}\cdots\text{—}d\text{—}$ to mean that $w_a < w_b < w_c < \cdots < w_d$.

Lemma 1. One has $w \in \mathcal{A}(y)$ if and only if for all $(a, b), (a', b') \in \mathrm{Cyc}(y)$, the following properties hold:

(Y1) If $a < b$ then $w^{-1} = \text{—}b\text{—}a\text{—}$.

(Y2) If $a < a' \leq b' < b$ then $w^{-1} \neq \text{—}b\text{—}a'\text{—}a\text{—}$ and $w^{-1} \neq \text{—}b\text{—}b'\text{—}a\text{—}$.

(Y3) If $a < a'$ and $b < b'$ then $w^{-1} = \text{—}a\text{—}b'\text{—}$.

Proof. This is equivalent to [2, Theorem 7.6]. □

Fix integers $i < j$ that are not congruent modulo n. Let $t = t_{ij} \in \tilde{S}_n$ denote the reflection that interchanges i and j while fixing all integers not congruent to i or j modulo n. Write $\lessdot$ for the covering relation in the Bruhat order on $\tilde{S}_n$.

Lemma 2. One has $w \lessdot wt$ if and only if the following property holds:

(T) $w^{-1} = \text{—}i\text{—}j\text{—}$ but if $i < e < j$ then $w^{-1} \neq \text{—}i\text{—}e\text{—}j\text{—}$.

Moreover, if i' and j' are integers with $i - i' = j - j' \in n\mathbb{Z}$, then property (T) is equivalent to the following:

(U) $w^{-1} = \text{—}i'\text{—}j'\text{—}$ but if $i' < e < j'$ then $w^{-1} \neq \text{—}i'\text{—}e\text{—}j'\text{—}$.

Proof. This is equivalent to [1, Proposition 8.3.6]. □

Recall the definition of the operator τ_{ij}^n from [2, §8] and let $z = z^{-1} = \tau_{ij}^n(y) \in \tilde{S}_n$.

Theorem. Assume $\{i, y_i\} + n\mathbb{Z}$ and $\{j, y_j\} + n\mathbb{Z}$ are disjoint. If $y \neq z$, $w \in \mathcal{A}(y)$, and $w \lessdot wt$, then $wt \in \mathcal{A}(z)$.

The proof of this statement occupies the rest of this computer-generated document.

Proof. Assume that $\{i, y_i\} + n\mathbb{Z}$ and $\{j, y_j\} + n\mathbb{Z}$ are disjoint and that $y \neq z$ and $w \in \mathcal{A}(y)$ and $w \lessdot wt$. Observe that if $i \neq y_i$ then the sets $i + n\mathbb{Z}$ and $y_i + n\mathbb{Z}$ are disjoint, and that if $j \neq y_j$ then the sets $j + n\mathbb{Z}$ and $y_j + n\mathbb{Z}$ are disjoint.

To show that $wt \in \mathcal{A}(z)$, it suffices by Lemma 1 to check that if $(a, b), (a', b') \in \mathrm{Cyc}(z)$ then the following properties hold:

(Z1) If $a < b$ then $(wt)^{-1} = \text{—}b\text{—}a\text{—}$.

(Z2) If $a < a' \leq b' < b$ then $(wt)^{-1} \neq \text{—}b\text{—}a'\text{—}a\text{—}$ and $(wt)^{-1} \neq \text{—}b\text{—}b'\text{—}a$.

(Z3) If $a < a'$ and $b < b'$ then $(wt)^{-1} = \text{—}a\text{—}b'\text{—}$.

Let $E = \{i, j, y_i, y_j\}$. Then $w_a = (wt)_a$ and $y_a = z_a$ for all integers $a \notin E + n\mathbb{Z}$, and we may decompose $\mathrm{Cyc}(z)$ as a disjoint union of four subsets $\mathrm{Cyc}(z) = \mathrm{Cyc}^1(z) \sqcup \mathrm{Cyc}^2(z) \sqcup \mathrm{Cyc}^3(z)$ where

$$\begin{aligned}
\mathrm{Cyc}^1(z) &= \{(a,b) \in \mathrm{Cyc}(z) : a, b \in E\},\\
\mathrm{Cyc}^2(z) &= \{(a,b) \in \mathrm{Cyc}(y) : a, b \notin E + n\mathbb{Z}\} = \{(a,b) \in \mathrm{Cyc}(z) : a, b \notin E + n\mathbb{Z}\},\\
\mathrm{Cyc}^3(z) &= \{(a + mn, b + mn) : 0 \neq m \in \mathbb{Z} \text{ and } (a,b) \in \mathrm{Cyc}^1(z)\}.
\end{aligned}$$

When $(a,b), (a',b') \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, properties (Z1)-(Z3) are equivalent to (Y1)-(Y3) and therefore hold since $w \in \mathcal{A}(y)$. It remains to check properties (Z1)-(Z3) in the following cases:

(i) When $(a,b), (a',b') \in \mathrm{Cyc}^1(z)$.

(ii) When one of the pairs (a,b), (a',b') belongs to $\mathrm{Cyc}^1(z)$ while the other belongs to $\mathrm{Cyc}^2(z)$.

(iii) When $(a,b) \in \mathrm{Cyc}^1(z)$ and $(a',b') \in \mathrm{Cyc}^3(z)$, or $(a',b') \in \mathrm{Cyc}^1(z)$ and $(a,b) \in \mathrm{Cyc}^3(z)$.

Since we assume $z = \tau_{ij}^n(y) \neq y$, there are twelve possibilities for the relative orders of i, j, y_i, and y_j. We examine each of these possibilities in turn and check directly that properties (Z1)-(Z3) hold in cases (i), (ii), and (iii).

2 Case: $y_i = i < j = y_j$

Suppose y is such that $y_i = i < j = y_j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(i,j)\}$.

2.1 Subcase (i)

We must have $w^{-1} = \text{—}i\text{—}j\text{—}$ since no other ordering is possible:

1. If $w^{-1} = \text{—}j\text{—}i\text{—}$ then (T) fails.

Hence if $y_i = i < j = y_j$ then

$(wt)^{-1} = \text{—}j\text{—}i\text{—}$.

When $(a,b), (a',b') \in \mathrm{Cyc}^1(z) = \{(i,j)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in case (i) and $y_i = i < j = y_j$.

2.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i,j\} + n\mathbb{Z}$.

1. Suppose $R < i < j$.

   (a) If $w^{-1} = \text{—}i\text{—}R\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,i)$.
   (b) If $w^{-1} = \text{—}i\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,i)$.

   Thus if $R < i < j$ then one of the following holds:

   - $w^{-1} = \text{—}R\text{—}i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}j\text{—}i\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -j-i-$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = -R-j-$.

2. Suppose $i < j < R$.

   (a) If $w^{-1} = -R-i-j-$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (R,R)$.

   (b) If $w^{-1} = -i-R-j-$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.

   Thus if $i < j < R$ then one of the following holds:

   - $w^{-1} = -i-j-R-$ and $(wt)^{-1} = -j-i-R-$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = -j-i-$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = -i-R-$.

3. It cannot happen that $i < R < j$ since:

   (a) If $w^{-1} = -i-R-j-$ then (T) fails.

   (b) If $w^{-1} = -i-j-R-$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,j)$.

   (c) If $w^{-1} = -R-i-j-$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (R,R)$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i,j\} + n\mathbb{Z}$.

1. Suppose $i < P < j < Q$.

   (a) If $w^{-1} = -i-Q-P-j-$ then (T) fails.

   (b) If $w^{-1} = -Q-i-P-j-$ then (T) fails.

   (c) If $w^{-1} = -i-Q-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.

   (d) If $w^{-1} = -Q-i-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.

   (e) If $w^{-1} = -Q-P-i-j-$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

   Thus if $i < P < j < Q$ then one of the following holds:

   - $w^{-1} = -i-j-Q-P-$ and $(wt)^{-1} = -j-i-Q-P-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-i-$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = -i-Q-$.

2. Suppose $P < Q < i < j$.

   (a) If $w^{-1} = -i-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.

   (b) If $w^{-1} = -i-Q-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.

   (c) If $w^{-1} = -Q-i-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.

   (d) If $w^{-1} = -Q-i-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.

   (e) If $w^{-1} = -i-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.

   Thus if $P < Q < i < j$ then one of the following holds:

   - $w^{-1} = -Q-P-i-j-$ and $(wt)^{-1} = -Q-P-j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —P—j—$.

3. Suppose $P < i < Q < j$.

   (a) If $w^{-1} = —i—Q—j—P—$ then (T) fails.

   (b) If $w^{-1} = —i—Q—P—j—$ then (T) fails.

   (c) If $w^{-1} = —Q—i—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.

   (d) If $w^{-1} = —Q—i—P—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.

   (e) If $w^{-1} = —i—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.

   Thus if $P < i < Q < j$ then one of the following holds:

   - $w^{-1} = —Q—P—i—j—$ and $(wt)^{-1} = —Q—P—j—i—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = —P—j—$.

4. Suppose $i < j < P < Q$.

   (a) If $w^{-1} = —Q—P—i—j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

   (b) If $w^{-1} = —Q—i—j—P—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

   (c) If $w^{-1} = —Q—i—P—j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

   (d) If $w^{-1} = —i—Q—j—P—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (P,Q)$.

   (e) If $w^{-1} = —i—Q—P—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (P,Q)$.

   Thus if $i < j < P < Q$ then one of the following holds:

   - $w^{-1} = —i—j—Q—P—$ and $(wt)^{-1} = —j—i—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$.

5. Suppose $P < i < j < Q$.

   (a) If $w^{-1} = —Q—i—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.

   (b) If $w^{-1} = —Q—i—P—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.

   (c) If $w^{-1} = —i—Q—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.

   Thus if $P < i < j < Q$ then one of the following holds:

   - $w^{-1} = —i—j—Q—P—$ and $(wt)^{-1} = —j—i—Q—P—$.
   - $w^{-1} = —Q—P—i—j—$ and $(wt)^{-1} = —Q—P—j—i—$.
   - $w^{-1} = —i—Q—P—j—$ and $(wt)^{-1} = —j—Q—P—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—}$.
(Z3) $\Leftrightarrow$ (no condition).

6. It cannot happen that $i < P < Q < j$ since:
   (a) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.
   (e) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_i = i < j = y_j$.

2.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < j' < i < j$.
   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',j')$ and $(a',b') = (i,i)$.

   Thus if $i' < j' < i < j$ then one of the following holds:
   - $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}i'\text{—}j\text{—}i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(i,j)\}$ and $(a',b') \in \{(i',j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}i'\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i'\text{—}j\text{—}$.

2. It cannot happen that $i' < i < j' < j$ since:
   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (j',j')$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b),(a',b') \in \mathrm{Cyc}(z)$ when $y_i = i < j = y_j$.

3 Case: $y_i = i < j < y_j$

Suppose y is such that $y_i = i < j < y_j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$.

3.1 Subcase (i)

We must have $w^{-1} = \text{—}i\text{—}y_j\text{—}j\text{—}$ since no other ordering is possible:

1. If $w^{-1} = \text{—}j\text{—}i\text{—}y_j\text{—}$ then (T) fails.
2. If $w^{-1} = \text{—}j\text{—}y_j\text{—}i\text{—}$ then (T) fails.
3. If $w^{-1} = \text{—}y_j\text{—}j\text{—}i\text{—}$ then (T) fails.
4. If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y1) fails for $(a,b) = (j,y_j)$.

5. If $w^{-1} = —y_j—i—j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (j,y_j)$.

Hence if $y_i = i < j < y_j$ then

$(wt)^{-1} = —j—y_j—i—$.

When $(a,b),(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow (wt)^{-1} = —y_j—i—$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq —y_j—j—i—$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in case (i) and $y_i = i < j < y_j$.

3.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i,j,y_j\} + n\mathbb{Z}$.

1. Suppose $i < j < y_j < R$.

   (a) If $w^{-1} = —R—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —i—R—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —i—y_j—R—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.

   Thus if $i < j < y_j < R$ then one of the following holds:

   - $w^{-1} = —i—y_j—j—R—$ and $(wt)^{-1} = —j—y_j—i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —y_j—i—$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow (wt)^{-1} = —i—R—$ and $(wt)^{-1} = —j—R—$.

2. Suppose $i < j < R < y_j$.

   (a) If $w^{-1} = —i—y_j—R—j—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —R—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (R,R)$.

   Thus if $i < j < R < y_j$ then one of the following holds:

   - $w^{-1} = —i—y_j—j—R—$ and $(wt)^{-1} = —j—y_j—i—R—$.
   - $w^{-1} = —i—R—y_j—j—$ and $(wt)^{-1} = —j—R—y_j—i—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —y_j—i—$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq —y_j—R—i—$.
   (Z3) $\Leftrightarrow (wt)^{-1} = —j—R—$.

3. Suppose $R < i < j < y_j$.

   (a) If $w^{-1} = —i—y_j—j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,i)$.
   (b) If $w^{-1} = —i—R—y_j—j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,i)$.
   (c) If $w^{-1} = —i—y_j—R—j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,i)$.

   Thus if $R < i < j < y_j$ then one of the following holds:

- $w^{-1} = -R-i-y_j-j-$ and $(wt)^{-1} = -R-j-y_j-i-$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -y_j-i-$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = -R-j-$ and $(wt)^{-1} = -R-y_j-$.

4. It cannot happen that $i < R < j < y_j$ since:
   (a) If $w^{-1} = -i-R-y_j-j-$ then (T) fails.
   (b) If $w^{-1} = -i-y_j-R-j-$ then (T) fails.
   (c) If $w^{-1} = -i-y_j-j-R-$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,y_j)$.
   (d) If $w^{-1} = -R-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (R,R)$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i,j,y_j\} + n\mathbb{Z}$.

1. Suppose $P < i < j < Q < y_j$.
   (a) If $w^{-1} = -Q-i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.
   (b) If $w^{-1} = -i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (c) If $w^{-1} = -i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (d) If $w^{-1} = -i-y_j-Q-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (e) If $w^{-1} = -i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (f) If $w^{-1} = -i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (g) If $w^{-1} = -Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (h) If $w^{-1} = -Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

   Thus if $P < i < j < Q < y_j$ then one of the following holds:

   - $w^{-1} = -i-Q-P-y_j-j-$ and $(wt)^{-1} = -j-Q-P-y_j-i-$.
   - $w^{-1} = -Q-P-i-y_j-j-$ and $(wt)^{-1} = -Q-P-j-y_j-i-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -y_j-i-$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq -Q-j-P-$.
   (Z3) $\Leftrightarrow (wt)^{-1} = -P-y_j-$.

2. Suppose $P < i < Q < j < y_j$.
   (a) If $w^{-1} = -i-y_j-Q-j-P-$ then (T) fails.
   (b) If $w^{-1} = -i-y_j-Q-P-j-$ then (T) fails.
   (c) If $w^{-1} = -i-Q-y_j-P-j-$ then (T) fails.
   (d) If $w^{-1} = -i-Q-P-y_j-j-$ then (T) fails.
   (e) If $w^{-1} = -i-Q-y_j-j-P-$ then (T) fails.
   (f) If $w^{-1} = -Q-i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.
   (g) If $w^{-1} = -i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (h) If $w^{-1} = -Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (i) If $w^{-1} = -Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

   Thus if $P < i < Q < j < y_j$ then one of the following holds:

- $w^{-1} = —Q—P—i—y_j—j—$ and $(wt)^{-1} = —Q—P—j—y_j—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_j—i—$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_j—$.

3. Suppose $i < j < y_j < P < Q$.

(a) If $w^{-1} = —i—y_j—Q—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = —i—y_j—Q—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = —Q—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = —i—Q—y_j—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = —i—Q—P—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = —Q—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = —i—Q—y_j—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = —Q—i—y_j—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = —Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Thus if $i < j < y_j < P < Q$ then one of the following holds:

- $w^{-1} = —i—y_j—j—Q—P—$ and $(wt)^{-1} = —j—y_j—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_j—i—$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —j—Q—$.

4. Suppose $i < j < P < y_j < Q$.

(a) If $w^{-1} = —i—y_j—Q—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = —i—y_j—Q—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = —Q—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = —i—Q—y_j—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = —i—Q—P—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = —Q—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = —i—Q—y_j—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = —Q—i—y_j—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = —Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Thus if $i < j < P < y_j < Q$ then one of the following holds:

- $w^{-1} = —i—y_j—j—Q—P—$ and $(wt)^{-1} = —j—y_j—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_j—i—$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —j—Q—$.

5. Suppose $P < Q < i < j < y_j$.

(a) If $w^{-1} = -Q-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.
(b) If $w^{-1} = -i-Q-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.
(c) If $w^{-1} = -i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(d) If $w^{-1} = -i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(e) If $w^{-1} = -i-y_j-Q-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(f) If $w^{-1} = -i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(g) If $w^{-1} = -i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(h) If $w^{-1} = -Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = -Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Thus if $P < Q < i < j < y_j$ then one of the following holds:

- $w^{-1} = -Q-P-i-y_j-j-$ and $(wt)^{-1} = -Q-P-j-y_j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -y_j-i-$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = -P-j-$ and $(wt)^{-1} = -P-y_j-$.

6. Suppose $i < P < j < y_j < Q$.

(a) If $w^{-1} = -i-y_j-Q-P-j-$ then (T) fails.
(b) If $w^{-1} = -Q-i-P-y_j-j-$ then (T) fails.
(c) If $w^{-1} = -i-Q-y_j-P-j-$ then (T) fails.
(d) If $w^{-1} = -i-Q-P-y_j-j-$ then (T) fails.
(e) If $w^{-1} = -Q-i-y_j-P-j-$ then (T) fails.
(f) If $w^{-1} = -i-y_j-Q-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(g) If $w^{-1} = -i-Q-y_j-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(h) If $w^{-1} = -Q-i-y_j-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = -Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

Thus if $i < P < j < y_j < Q$ then one of the following holds:

- $w^{-1} = -i-y_j-j-Q-P-$ and $(wt)^{-1} = -j-y_j-i-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -y_j-i-$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq -Q-j-P-$.
(Z3) $\Leftrightarrow (wt)^{-1} = -i-Q-$.

7. Suppose $P < i < j < y_j < Q$.

(a) If $w^{-1} = -Q-i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.
(b) If $w^{-1} = -i-y_j-Q-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(c) If $w^{-1} = -i-Q-y_j-P-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(d) If $w^{-1} = -i-Q-y_j-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(e) If $w^{-1} = -Q-i-y_j-P-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(f) If $w^{-1} = -Q-i-y_j-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Thus if $P < i < j < y_j < Q$ then one of the following holds:

- $w^{-1} = —i—y_j—j—Q—P—$ and $(wt)^{-1} = —j—y_j—i—Q—P—$.
- $w^{-1} = —i—y_j—Q—P—j—$ and $(wt)^{-1} = —j—y_j—Q—P—i—$.
- $w^{-1} = —i—Q—P—y_j—j—$ and $(wt)^{-1} = —j—Q—P—y_j—i—$.
- $w^{-1} = —Q—P—i—y_j—j—$ and $(wt)^{-1} = —Q—P—j—y_j—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_j—i—$.

(Z2) $\Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and } (wt)^{-1} \neq —Q—y_j—P— \text{ and} \\ (wt)^{-1} \neq —Q—j—P—. \end{cases}$

(Z3) $\Leftrightarrow$ (no condition).

8. Suppose $i < j < P < Q < y_j$.

   (a) If $w^{-1} = —i—y_j—Q—j—P—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = —i—y_j—Q—P—j—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = —i—Q—y_j—P—j—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = —Q—i—y_j—P—j—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = —Q—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = —Q—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

   Thus if $i < j < P < Q < y_j$ then one of the following holds:

   - $w^{-1} = —i—Q—y_j—j—P—$ and $(wt)^{-1} = —j—Q—y_j—i—P—$.
   - $w^{-1} = —i—y_j—j—Q—P—$ and $(wt)^{-1} = —j—y_j—i—Q—P—$.
   - $w^{-1} = —i—Q—P—y_j—j—$ and $(wt)^{-1} = —j—Q—P—y_j—i—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_j—i—$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq —y_j—P—i—$ and $(wt)^{-1} \neq —y_j—Q—i—$.
   (Z3) $\Leftrightarrow (wt)^{-1} = —j—Q—$.

9. It cannot happen that $i < P < j < Q < y_j$ since:

   (a) If $w^{-1} = —i—y_j—Q—P—j—$ then (T) fails.
   (b) If $w^{-1} = —Q—i—P—y_j—j—$ then (T) fails.
   (c) If $w^{-1} = —i—Q—y_j—P—j—$ then (T) fails.
   (d) If $w^{-1} = —i—Q—P—y_j—j—$ then (T) fails.
   (e) If $w^{-1} = —Q—i—y_j—P—j—$ then (T) fails.
   (f) If $w^{-1} = —i—y_j—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (g) If $w^{-1} = —i—y_j—Q—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (h) If $w^{-1} = —i—Q—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (i) If $w^{-1} = —Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (j) If $w^{-1} = —Q—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

10. It cannot happen that $i < P < Q < j < y_j$ since:

    (a) If $w^{-1} = —i—y_j—Q—j—P—$ then (T) fails.
    (b) If $w^{-1} = —i—y_j—Q—P—j—$ then (T) fails.

(c) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_i = i < j < y_j$.

3.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < j' < i < j < y_{j'} < y_j$.

   (a) If $w^{-1} = \text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,i)$.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (j,y_j)$.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (j,y_j)$.

   Thus if $i' < j' < i < j < y_{j'} < y_j$ then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}j\text{—}y_j\text{—}i\text{—}$.
   - $w^{-1} = \text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}j\text{—}y_{j'}\text{—}i'\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ and $(a',b') \in \{(i',y_{j'}),(j',j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}y_j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_{j'}\text{—}j\text{—}i'\text{—}$.
   (Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_j\text{—}. \end{cases}$

2. Suppose $i' < j' < y_{j'} < i < j < y_j$.

   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,i)$.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,i)$.
   (c) If $w^{-1} = \text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (j,y_j)$.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (j,y_j)$.

   Thus if $i' < j' < y_{j'} < i < j < y_j$ then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}j\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(i,y_j),(j,j)\}$ and $(a',b') \in \{(i',y_{j'}),(j',j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}y_j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_j\text{—}. \end{cases}$

3. Suppose $i' < j' < i < y_{j'} < j < y_j$.

   (a) If $w^{-1} = —i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (c) If $w^{-1} = —i'—y_{j'}—i—j'—y_j—j—$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i,i)$.
   (d) If $w^{-1} = —i'—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

   Thus if $i' < j' < i < y_{j'} < j < y_j$ then one of the following holds:

   - $w^{-1} = —i'—y_{j'}—j'—i—y_j—j—$ and $(wt)^{-1} = —j'—y_{j'}—i'—j—y_j—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(i, y_j), (j,j)\}$ and $(a',b') \in \{(i', y_{j'}), (j', j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —y_j—i—$ and $(wt)^{-1} = —y_{j'}—i'—$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow$ $\begin{cases} (wt)^{-1} = —i'—j— \text{ and} \\ (wt)^{-1} = —i'—y_j— \text{ and} \\ (wt)^{-1} = —j'—j— \text{ and} \\ (wt)^{-1} = —j'—y_j—. \end{cases}$

4. It cannot happen that $i' < i < j' < y_{j'} < j < y_j$ since:

   (a) If $w^{-1} = —i'—y_{j'}—i—y_j—j'—j—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (c) If $w^{-1} = —i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (d) If $w^{-1} = —i'—y_{j'}—i—j'—y_j—j—$ then (T) fails.
   (e) If $w^{-1} = —i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (j', y_{j'})$.

5. It cannot happen that $i' < i < j' < j < y_{j'} < y_j$ since:

   (a) If $w^{-1} = —i'—y_{j'}—i—y_j—j'—j—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (c) If $w^{-1} = —i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (d) If $w^{-1} = —i'—y_{j'}—i—j'—y_j—j—$ then (T) fails.
   (e) If $w^{-1} = —i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (j', y_{j'})$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(z)$ when $y_i = i < j < y_j$.

4 Case: $y_i = i < y_j < j$

Suppose y is such that $y_i = i < y_j < j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(i,j), (y_j, y_j)\}$.

4.1 Subcase (i)

We must have $w^{-1} = —i—j—y_j—$ since no other ordering is possible:

1. If $w^{-1} = —i—y_j—j—$ then (T) fails.
2. If $w^{-1} = —y_j—j—i—$ then (T) fails.
3. If $w^{-1} = —j—i—y_j—$ then (T) fails.
4. If $w^{-1} = —j—y_j—i—$ then (T) fails.
5. If $w^{-1} = —y_j—i—j—$ then (Y1) fails for $(a,b) = (y_j, j)$.

Hence if $y_i = i < y_j < j$ then

$(wt)^{-1} = \text{—}j\text{—}i\text{—}y_j\text{—}$.

When $(a,b),(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}y_j\text{—}i\text{—}$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in case (i) and $y_i = i < y_j < j$.

4.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i,j,y_j\} + n\mathbb{Z}$.

1. Suppose $i < y_j < j < R$.
   (a) If $w^{-1} = \text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

   Thus if $i < y_j < j < R$ then one of the following holds:

   - $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}y_j\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}R\text{—}$.

2. Suppose $i < y_j < R < j$.
   (a) If $w^{-1} = \text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (R,R)$.

   Thus if $i < y_j < R < j$ then one of the following holds:

   - $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}y_j\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}R\text{—}i\text{—}$.
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_j\text{—}R\text{—}$.

3. Suppose $R < i < y_j < j$.
   (a) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,i)$.
   (b) If $w^{-1} = \text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,i)$.
   (c) If $w^{-1} = \text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,i)$.

   Thus if $R < i < y_j < j$ then one of the following holds:

   - $w^{-1} = \text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}j\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}R\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_j\text{—}$.

4. It cannot happen that $i < R < y_j < j$ since:
   (a) If $w^{-1} = \text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j,j)$.
   (c) If $w^{-1} = \text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j,j)$.
   (d) If $w^{-1} = \text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (R,R)$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i,j,y_j\} + n\mathbb{Z}$.

1. Suppose $P < i < y_j < Q < j$.
   (a) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.
   (e) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (f) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (g) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (h) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (i) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

   Thus if $P < i < y_j < Q < j$ then one of the following holds:

   - $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}$.
   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}j\text{—}$.

2. Suppose $P < i < Q < y_j < j$.
   (a) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.
   (e) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (f) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (g) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (h) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (i) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

   Thus if $P < i < Q < y_j < j$ then one of the following holds:

   - $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_j—$.

3. Suppose $i < y_j < j < P < Q$.

   (a) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = —Q—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = —i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = —i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = —Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   Thus if $i < y_j < j < P < Q$ then one of the following holds:

   - $w^{-1} = —i—j—y_j—Q—P—$ and $(wt)^{-1} = —j—i—y_j—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

4. Suppose $i < y_j < P < j < Q$.

   (a) If $w^{-1} = —Q—i—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —i—Q—P—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = —i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = —Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   Thus if $i < y_j < P < j < Q$ then one of the following holds:

   - $w^{-1} = —i—j—y_j—Q—P—$ and $(wt)^{-1} = —j—i—y_j—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

5. Suppose $P < Q < i < y_j < j$.

   (a) If $w^{-1} = —Q—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.

(b) If $w^{-1} = -i-Q-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.
(c) If $w^{-1} = -i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(d) If $w^{-1} = -i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(e) If $w^{-1} = -i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(f) If $w^{-1} = -i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(g) If $w^{-1} = -i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(h) If $w^{-1} = -Q-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(i) If $w^{-1} = -Q-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

Thus if $P < Q < i < y_j < j$ then one of the following holds:

- $w^{-1} = -Q-P-i-j-y_j-$ and $(wt)^{-1} = -Q-P-j-i-y_j-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-i-$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = -P-j-$ and $(wt)^{-1} = -P-y_j-$.

6. Suppose $i < P < y_j < j < Q$.

(a) If $w^{-1} = -Q-i-P-j-y_j-$ then (T) fails.
(b) If $w^{-1} = -i-Q-P-j-y_j-$ then (T) fails.
(c) If $w^{-1} = -i-j-Q-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(d) If $w^{-1} = -i-Q-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(e) If $w^{-1} = -i-Q-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(f) If $w^{-1} = -Q-i-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(g) If $w^{-1} = -Q-i-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(h) If $w^{-1} = -Q-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

Thus if $i < P < y_j < j < Q$ then one of the following holds:

- $w^{-1} = -i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-i-y_j-Q-P-$.
- $w^{-1} = -i-j-Q-P-y_j-$ and $(wt)^{-1} = -j-i-Q-P-y_j-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-i-$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq -Q-y_j-P-$.
(Z3) $\Leftrightarrow (wt)^{-1} = -i-Q-$.

7. Suppose $P < i < y_j < j < Q$.

(a) If $w^{-1} = -Q-i-P-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.
(b) If $w^{-1} = -i-j-Q-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(c) If $w^{-1} = -i-Q-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(d) If $w^{-1} = -i-Q-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(e) If $w^{-1} = -Q-i-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(f) If $w^{-1} = -Q-i-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

Thus if $P < i < y_j < j < Q$ then one of the following holds:

- $w^{-1} = -i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-i-y_j-Q-P-$.

- $w^{-1} = —i—j—Q—P—y_j—$ and $(wt)^{-1} = —j—i—Q—P—y_j—$.
- $w^{-1} = —i—Q—P—j—y_j—$ and $(wt)^{-1} = —j—Q—P—i—y_j—$.
- $w^{-1} = —Q—P—i—j—y_j—$ and $(wt)^{-1} = —Q—P—j—i—y_j—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.

(Z2) $\Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and } (wt)^{-1} \neq —Q—j—P— \text{ and} \\ (wt)^{-1} \neq —Q—y_j—P—. \end{cases}$

(Z3) $\Leftrightarrow$ (no condition).

8. Suppose $i < y_j < P < Q < j$.
   (a) If $w^{-1} = —Q—i—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —i—Q—j—P—y_j—$ then (T) fails.
   (c) If $w^{-1} = —i—Q—P—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —i—Q—j—y_j—P—$ then (T) fails.
   (e) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

   Thus if $i < y_j < P < Q < j$ then one of the following holds:

   - $w^{-1} = —i—j—y_j—Q—P—$ and $(wt)^{-1} = —j—i—y_j—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq —j—P—i—$ and $(wt)^{-1} \neq —j—Q—i—$.
   (Z3) $\Leftrightarrow (wt)^{-1} = —y_j—Q—$.

9. It cannot happen that $i < P < y_j < Q < j$ since:
   (a) If $w^{-1} = —Q—i—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —i—Q—j—P—y_j—$ then (T) fails.
   (c) If $w^{-1} = —i—Q—P—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —i—Q—j—y_j—P—$ then (T) fails.
   (e) If $w^{-1} = —i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (f) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (g) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (h) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (i) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (j) If $w^{-1} = —Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

10. It cannot happen that $i < P < Q < y_j < j$ since:
    (a) If $w^{-1} = —Q—i—P—j—y_j—$ then (T) fails.
    (b) If $w^{-1} = —i—Q—j—P—y_j—$ then (T) fails.
    (c) If $w^{-1} = —i—Q—P—j—y_j—$ then (T) fails.

(d) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(f) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(g) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(h) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(i) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(j) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_i = i < y_j < j$.

4.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < y_{j'} < i < y_j < j' < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.
   (d) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Thus if $i' < y_{j'} < i < y_j < j' < j$ then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}i'\text{—}y_{j'}\text{—}j\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ and $(a',b') \in \{(i',j'),(y_{j'},y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}i'\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j'\text{—}y_j\text{—}i'\text{—}$.
   (Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}. \end{cases}$

2. Suppose $i' < y_{j'} < j' < i < y_j < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.
   (b) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.
   (c) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Thus if $i' < y_{j'} < j' < i < y_j < j$ then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}i'\text{—}y_{j'}\text{—}j\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ and $(a',b') \in \{(i',j'),(y_{j'},y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}i'\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}. \end{cases}$

3. Suppose $i' < y_{j'} < i < j' < y_j < j$.

(a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(c) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.
(d) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Thus if $i' < y_{j'} < i < j' < y_j < j$ then one of the following holds:

- $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—$ and $(wt)^{-1} = —j'—i'—y_{j'}—j—i—y_j—$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(i,j),(y_j,y_j)\}$ and $(a',b') \in \{(i',j'),(y_{j'},y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —j—i—$ and $(wt)^{-1} = —j'—i'—$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = —i'—j— \text{ and} \\ (wt)^{-1} = —i'—y_j— \text{ and} \\ (wt)^{-1} = —y_{j'}—j— \text{ and} \\ (wt)^{-1} = —y_{j'}—y_j—. \end{cases}$

4. It cannot happen that $i' < i < y_{j'} < j' < y_j < j$ since:
   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (c) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (y_{j'}, j')$.
   (e) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
5. It cannot happen that $i' < i < y_{j'} < y_j < j' < j$ since:
   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (c) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (y_{j'}, j')$.
   (e) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b),(a',b') \in \mathrm{Cyc}(z)$ when $y_i = i < y_j < j$.

5 Case: $i < y_i < j = y_j$

Suppose y is such that $i < y_i < j = y_j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$.

5.1 Subcase (i)

We must have $w^{-1} = —y_i—i—j—$ since no other ordering is possible:

1. If $w^{-1} = —i—y_i—j—$ then (T) fails.
2. If $w^{-1} = —y_i—j—i—$ then (T) fails.
3. If $w^{-1} = —j—i—y_i—$ then (T) fails.
4. If $w^{-1} = —j—y_i—i—$ then (T) fails.
5. If $w^{-1} = —i—j—y_i—$ then (Y1) fails for $(a,b) = (i,y_i)$.

Hence if $i < y_i < j = y_j$ then

$(wt)^{-1} = —y_i—j—i—.$

When $(a,b),(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow (wt)^{-1} = —j—i—.$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —j—y_i—i—.$

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in case (i) and $i < y_i < j = y_j$.

5.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i,j,y_i\} + n\mathbb{Z}$.

1. Suppose $i < y_i < j < R$.

   (a) If $w^{-1} = —y_i—R—i—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —R—y_i—i—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —y_i—i—R—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.

   Thus if $i < y_i < j < R$ then one of the following holds:

   - $w^{-1} = —y_i—i—j—R—$ and $(wt)^{-1} = —y_i—j—i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —j—i—.$
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow (wt)^{-1} = —i—R—$ and $(wt)^{-1} = —y_i—R—.$

2. Suppose $i < R < y_i < j$.

   (a) If $w^{-1} = —y_i—i—R—j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—R—i—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —y_i—i—j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,j)$.

   Thus if $i < R < y_i < j$ then one of the following holds:

   - $w^{-1} = —R—y_i—i—j—$ and $(wt)^{-1} = —R—y_i—j—i—.$

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —j—i—.$
   (Z2) $\Leftrightarrow (wt)^{-1} \neq —j—R—i—.$
   (Z3) $\Leftrightarrow (wt)^{-1} = —R—y_i—.$

3. Suppose $R < i < y_i < j$.

   (a) If $w^{-1} = —y_i—R—i—j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.
   (b) If $w^{-1} = —y_i—i—R—j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.
   (c) If $w^{-1} = —y_i—i—j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,j)$.

   Thus if $R < i < y_i < j$ then one of the following holds:

   - $w^{-1} = —R—y_i—i—j—$ and $(wt)^{-1} = —R—y_i—j—i—.$

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}R\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_i\text{—}$.

4. It cannot happen that $i < y_i < R < j$ since:
   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,j)$.
   (c) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i,j,y_i\} + n\mathbb{Z}$.

1. Suppose $P < i < y_i < Q < j$.
   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (d) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.

   Thus if $P < i < y_i < Q < j$ then one of the following holds:
   - $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}i\text{—}$.
   - $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}i\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—}$.
   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}j\text{—}$.

2. Suppose $P < i < Q < y_i < j$.
   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (d) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (h) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

   Thus if $P < i < Q < y_i < j$ then one of the following holds:
   - $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}y_i\text{—}$.

3. Suppose $i < y_i < j < P < Q$.

   (a) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (P,Q)$.

   Thus if $i < y_i < j < P < Q$ then one of the following holds:

   - $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}Q\text{—}P\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$.

4. Suppose $i < y_i < P < j < Q$.

   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.
   (e) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

   Thus if $i < y_i < P < j < Q$ then one of the following holds:

   - $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}Q\text{—}P\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$.

5. Suppose $i < P < Q < y_i < j$.

   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.

(b) If $w^{-1} = —y_i—Q—i—P—j—$ then (T) fails.
(c) If $w^{-1} = —Q—y_i—i—P—j—$ then (T) fails.
(d) If $w^{-1} = —y_i—i—Q—j—P—$ then (T) fails.
(e) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = —y_i—i—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.
(i) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.

Thus if $i < P < Q < y_i < j$ then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—$ and $(wt)^{-1} = —Q—P—y_i—j—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq —j—P—i—$ and $(wt)^{-1} \neq —j—Q—i—$.
(Z3) $\Leftrightarrow (wt)^{-1} = —P—y_i—$.

6. Suppose $P < Q < i < y_i < j$.

(a) If $w^{-1} = —y_i—i—Q—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = —y_i—Q—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = —y_i—i—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = —Q—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = —y_i—i—Q—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Thus if $P < Q < i < y_i < j$ then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—$ and $(wt)^{-1} = —Q—P—y_i—j—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_i—$.

7. Suppose $i < P < y_i < j < Q$.

(a) If $w^{-1} = —y_i—i—Q—P—j—$ then (T) fails.
(b) If $w^{-1} = —y_i—Q—i—P—j—$ then (T) fails.
(c) If $w^{-1} = —Q—y_i—i—P—j—$ then (T) fails.
(d) If $w^{-1} = —y_i—i—Q—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.
(e) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = —Q—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

Thus if $i < P < y_i < j < Q$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}Q\text{—}P\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$.

8. Suppose $P < i < y_i < j < Q$.

(a) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.

Thus if $P < i < y_i < j < Q$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}i\text{—}$.
- $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}i\text{—}$.
- $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}i\text{—}$.
- $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}Q\text{—}P\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow \begin{cases} (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—} \text{ and } (wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—} \text{ and} \\ (wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—}. \end{cases}$
(Z3) $\Leftrightarrow$ (no condition).

9. It cannot happen that $i < P < y_i < Q < j$ since:

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.
(f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

10. It cannot happen that $i < y_i < P < Q < j$ since:

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.

(c) If $w^{-1} = -Q-y_i-i-P-j-$ then (T) fails.
(d) If $w^{-1} = -y_i-i-Q-j-P-$ then (T) fails.
(e) If $w^{-1} = -y_i-i-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.
(f) If $w^{-1} = -y_i-Q-P-i-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = -y_i-Q-i-j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = -Q-y_i-P-i-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = -Q-P-y_i-i-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = -Q-y_i-i-j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $i < y_i < j = y_j$.

5.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < y_{i'} < j' < i < y_i < j$.

   (a) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (b) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (c) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-$ then (Y3) fails for $(a,b) = (j',j')$ and $(a',b') = (i,y_i)$.
   (d) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-$ then (Y3) fails for $(a,b) = (j',j')$ and $(a',b') = (i,y_i)$.

   Thus if $i' < y_{i'} < j' < i < y_i < j$ then one of the following holds:

   - $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-$ and $(wt)^{-1} = -y_{i'}-j'-i'-y_i-j-i-$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ and $(a',b') \in \{(i',j'),(y_{i'},y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = -j-i-$ and $(wt)^{-1} = -j'-i'-$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_i-. \end{cases}$

2. Suppose $i' < i < y_{i'} < j' < y_i < j$.

   (a) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-$ then (T) fails.
   (b) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-$ then (T) fails.
   (c) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (j',j')$.
   (d) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.

   Thus if $i' < i < y_{i'} < j' < y_i < j$ then one of the following holds:

   - $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-$ and $(wt)^{-1} = -y_{i'}-j'-i'-y_i-j-i-$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(i,j),(y_i,y_i)\}$ and $(a',b') \in \{(i',j'),(y_{i'},y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = -j-i-$ and $(wt)^{-1} = -j'-i'-$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq -j-y_{i'}-i-$.
   (Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_i-. \end{cases}$

3. Suppose $i' < y_{i'} < i < j' < y_i < j$.

   (a) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', j')$.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   Thus if $i' < y_{i'} < i < j' < y_i < j$ then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—$ and $(wt)^{-1} = —y_{i'}—j'—i'—y_i—j—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(i,j), (y_i, y_i)\}$ and $(a', b') \in \{(i', j'), (y_{i'}, y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —j—i—$ and $(wt)^{-1} = —j'—i'—$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = —i'—j— \text{ and} \\ (wt)^{-1} = —i'—y_i— \text{ and} \\ (wt)^{-1} = —y_{i'}—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—y_i—. \end{cases}$

4. It cannot happen that $i' < y_{i'} < i < y_i < j' < j$ since:

   (a) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—$ then (U) fails.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (e) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', j')$.

5. It cannot happen that $i' < i < y_{i'} < y_i < j' < j$ since:

   (a) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—$ then (U) fails.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (e) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', j')$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b), (a', b') \in \mathrm{Cyc}(z)$ when $i < y_i < j = y_j$.

6 Case: $y_i < i < j = y_j$

Suppose y is such that $y_i < i < j = y_j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(y_i, j), (i, i)\}$.

6.1 Subcase (i)

We must have $w^{-1} = —i—y_i—j—$ since no other ordering is possible:

1. If $w^{-1} = —y_i—j—i—$ then (T) fails.
2. If $w^{-1} = —j—y_i—i—$ then (T) fails.
3. If $w^{-1} = —j—i—y_i—$ then (T) fails.
4. If $w^{-1} = —y_i—i—j—$ then (Y1) fails for $(a,b) = (y_i, i)$.
5. If $w^{-1} = —i—j—y_i—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (j, j)$.

Hence if $y_i < i < j = y_j$ then

$(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b),(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}i\text{—}y_i\text{—}$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in case (i) and $y_i < i < j = y_j$.

6.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i,j,y_i\} + n\mathbb{Z}$.

1. Suppose $y_i < i < j < R$.

   (a) If $w^{-1} = \text{—}i\text{—}R\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}R\text{—}i\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}R\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.

   Thus if $y_i < i < j < R$ then one of the following holds:

   - $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}R\text{—}$.

2. Suppose $y_i < R < i < j$.

   (a) If $w^{-1} = \text{—}i\text{—}R\text{—}y_i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,j)$.

   Thus if $y_i < R < i < j$ then one of the following holds:

   - $w^{-1} = \text{—}R\text{—}i\text{—}y_i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}j\text{—}y_i\text{—}i\text{—}$.
   - $w^{-1} = \text{—}i\text{—}y_i\text{—}R\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}R\text{—}i\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}R\text{—}y_i\text{—}$.
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}R\text{—}i\text{—}$.

3. Suppose $R < y_i < i < j$.

   (a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,j)$.
   (b) If $w^{-1} = \text{—}i\text{—}R\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}R\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.

   Thus if $R < y_i < i < j$ then one of the following holds:

   - $w^{-1} = \text{—}R\text{—}i\text{—}y_i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}R\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}j\text{—}$.

4. It cannot happen that $y_i < i < R < j$ since:
   (a) If $w^{-1} = \text{—}i\text{—}R\text{—}y_i\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}R\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,j)$.
   (d) If $w^{-1} = \text{—}R\text{—}i\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i,j,y_i\} + n\mathbb{Z}$.

1. Suppose $P < y_i < i < Q < j$.
   (a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (g) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (h) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.

   Thus if $P < y_i < i < Q < j$ then one of the following holds:

   - $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—}$.
   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}j\text{—}$.

2. Suppose $P < y_i < Q < i < j$.
   (a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (e) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (f) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (g) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (h) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

   Thus if $P < y_i < Q < i < j$ then one of the following holds:

   - $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_i—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —P—i—$ and $(wt)^{-1} = —P—j—$.

3. Suppose $y_i < i < j < P < Q$.

   (a) If $w^{-1} = —i—y_i—Q—P—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = —i—y_i—Q—j—P—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = —i—Q—y_i—P—j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = —i—Q—P—y_i—j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = —i—Q—y_i—j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = —Q—i—y_i—P—j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —Q—i—P—y_i—j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —Q—P—i—y_i—j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —Q—i—y_i—j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   Thus if $y_i < i < j < P < Q$ then one of the following holds:

   - $w^{-1} = —i—y_i—j—Q—P—$ and $(wt)^{-1} = —j—y_i—i—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_i—$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_i—Q—$.

4. Suppose $y_i < i < P < j < Q$.

   (a) If $w^{-1} = —i—y_i—Q—P—j—$ then (T) fails.
   (b) If $w^{-1} = —i—Q—y_i—P—j—$ then (T) fails.
   (c) If $w^{-1} = —i—Q—P—y_i—j—$ then (T) fails.
   (d) If $w^{-1} = —Q—i—y_i—P—j—$ then (T) fails.
   (e) If $w^{-1} = —Q—i—P—y_i—j—$ then (T) fails.
   (f) If $w^{-1} = —i—y_i—Q—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.
   (g) If $w^{-1} = —i—Q—y_i—j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —Q—P—i—y_i—j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —Q—i—y_i—j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   Thus if $y_i < i < P < j < Q$ then one of the following holds:

   - $w^{-1} = —i—y_i—j—Q—P—$ and $(wt)^{-1} = —j—y_i—i—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_i—$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_i—Q—$.

5. Suppose $y_i < P < Q < i < j$.

   (a) If $w^{-1} = —i—Q—y_i—P—j—$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

(b) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(c) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(d) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (j, j)$.
(f) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (j, j)$.
(g) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (j, j)$.

Thus if $y_i < P < Q < i < j$ then one of the following holds:

- $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}$.
- $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}j\text{—}y_i\text{—}P\text{—}i\text{—}$.
- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}P\text{—}y_i\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}Q\text{—}y_i\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}i\text{—}$.

6. Suppose $P < Q < y_i < i < j$.

(a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(b) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(c) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(d) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(e) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(f) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(g) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(h) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(i) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.

Thus if $P < Q < y_i < i < j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}j\text{—}$.

7. Suppose $y_i < P < i < j < Q$.

(a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (j, j)$.
(b) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(c) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(d) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(e) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.

Thus if $y_i < P < i < j < Q$ then one of the following holds:

- $w^{-1} = —i—y_i—Q—P—j—$ and $(wt)^{-1} = —j—y_i—Q—P—i—$.
- $w^{-1} = —i—y_i—j—Q—P—$ and $(wt)^{-1} = —j—y_i—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_i—$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq —Q—i—P—$.
(Z3) $\Leftrightarrow (wt)^{-1} = —y_i—Q—$.

8. Suppose $P < y_i < i < j < Q$.

   (a) If $w^{-1} = —i—y_i—Q—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.
   (b) If $w^{-1} = —i—Q—y_i—P—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = —i—Q—y_i—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = —Q—i—y_i—P—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (e) If $w^{-1} = —Q—i—P—y_i—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (f) If $w^{-1} = —Q—i—y_i—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

   Thus if $P < y_i < i < j < Q$ then one of the following holds:

   - $w^{-1} = —i—y_i—Q—P—j—$ and $(wt)^{-1} = —j—y_i—Q—P—i—$.
   - $w^{-1} = —i—Q—P—y_i—j—$ and $(wt)^{-1} = —j—Q—P—y_i—i—$.
   - $w^{-1} = —Q—P—i—y_i—j—$ and $(wt)^{-1} = —Q—P—j—y_i—i—$.
   - $w^{-1} = —i—y_i—j—Q—P—$ and $(wt)^{-1} = —j—y_i—i—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_i—$.
   (Z2) $\Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and} \\ (wt)^{-1} \neq —Q—y_i—P— \text{ and } (wt)^{-1} \neq —Q—j—P—. \end{cases}$
   (Z3) $\Leftrightarrow$ (no condition).

9. It cannot happen that $y_i < P < i < Q < j$ since:

   (a) If $w^{-1} = —i—y_i—Q—P—j—$ then (T) fails.
   (b) If $w^{-1} = —i—Q—y_i—P—j—$ then (T) fails.
   (c) If $w^{-1} = —i—Q—P—y_i—j—$ then (T) fails.
   (d) If $w^{-1} = —i—Q—y_i—j—P—$ then (T) fails.
   (e) If $w^{-1} = —i—y_i—Q—j—P—$ then (T) fails.
   (f) If $w^{-1} = —i—y_i—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.
   (g) If $w^{-1} = —Q—i—y_i—P—j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —Q—i—P—y_i—j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —Q—P—i—y_i—j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = —Q—i—y_i—j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

10. It cannot happen that $y_i < i < P < Q < j$ since:

    (a) If $w^{-1} = —i—y_i—Q—P—j—$ then (T) fails.
    (b) If $w^{-1} = —i—Q—y_i—P—j—$ then (T) fails.
    (c) If $w^{-1} = —i—Q—P—y_i—j—$ then (T) fails.

(d) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.
(i) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_i < i < j = y_j$.

6.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $y_{i'} < i' < j' < y_i < i < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',j')$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',j')$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.

   Thus if $y_{i'} < i' < j' < y_i < i < j$ then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}j\text{—}y_i\text{—}i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ and $(a',b') \in \{(y_{i'},j'),(i',i')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow$ $\begin{cases} (wt)^{-1} = \text{—}i'\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—}. \end{cases}$

2. Suppose $y_{i'} < y_i < i' < j' < i < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (j',j')$.
   (b) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.

   Thus if $y_{i'} < y_i < i' < j' < i < j$ then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}j\text{—}y_i\text{—}i\text{—}$.
   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}j\text{—}y_i\text{—}i'\text{—}i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,i)\}$ and $(a',b') \in \{(y_{i'},j'),(i',i')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}i'\text{—}y_i\text{—}$.
   (Z3) $\Leftrightarrow$ $\begin{cases} (wt)^{-1} = \text{—}i'\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—}. \end{cases}$

3. Suppose $y_{i'} < i' < y_i < j' < i < j$.

(a) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—j—$ then (U) fails.
(b) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—j—$ then (U) fails.
(c) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—j—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', j')$.
(d) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—j—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < i' < y_i < j' < i < j$ then one of the following holds:

- $w^{-1} = —i'—y_{i'}—j'—i—y_i—j—$ and $(wt)^{-1} = —j'—y_{i'}—i'—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, i)\}$ and $(a',b') \in \{(y_{i'}, j'), (i', i')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_i—$ and $(wt)^{-1} = —j'—y_{i'}—$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow$ $\begin{cases} (wt)^{-1} = —i'—i— \text{ and} \\ (wt)^{-1} = —i'—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—j—. \end{cases}$

4. It cannot happen that $y_{i'} < i' < y_i < i < j' < j$ since:
   (a) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—j—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—j—$ then (T) fails.
   (c) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—j—$ then (T) fails.
   (d) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—j—$ then (T) fails.
   (e) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', j')$.
5. It cannot happen that $y_{i'} < y_i < i' < i < j' < j$ since:
   (a) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—j—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—j—$ then (T) fails.
   (c) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—j—$ then (T) fails.
   (d) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—j—$ then (T) fails.
   (e) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', j')$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(z)$ when $y_i < i < j = y_j$.

7 Case: $i < y_i < j < y_j$

Suppose y is such that $i < y_i < j < y_j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(j,j), (i, y_j), (y_i, y_i)\}$.

7.1 Subcase (i)

We must have $w^{-1} = —y_i—i—y_j—j—$ since no other ordering is possible:

1. If $w^{-1} = —i—y_i—j—y_j—$ then (T) fails.
2. If $w^{-1} = —i—y_i—y_j—j—$ then (T) fails.
3. If $w^{-1} = —i—y_j—y_i—j—$ then (T) fails.
4. If $w^{-1} = —y_i—j—i—y_j—$ then (T) fails.
5. If $w^{-1} = —y_i—j—y_j—i—$ then (T) fails.
6. If $w^{-1} = —y_i—y_j—j—i—$ then (T) fails.

7. If $w^{-1} = —j—i—y_i—y_j—$ then (T) fails.
8. If $w^{-1} = —j—i—y_j—y_i—$ then (T) fails.
9. If $w^{-1} = —j—y_i—i—y_j—$ then (T) fails.
10. If $w^{-1} = —j—y_i—y_j—i—$ then (T) fails.
11. If $w^{-1} = —j—y_j—i—y_i—$ then (T) fails.
12. If $w^{-1} = —j—y_j—y_i—i—$ then (T) fails.
13. If $w^{-1} = —y_j—i—y_i—j—$ then (T) fails.
14. If $w^{-1} = —y_j—y_i—j—i—$ then (T) fails.
15. If $w^{-1} = —y_j—j—i—y_i—$ then (T) fails.
16. If $w^{-1} = —y_j—j—y_i—i—$ then (T) fails.
17. If $w^{-1} = —i—j—y_i—y_j—$ then (Y1) fails for $(a, b) = (i, y_i)$.
18. If $w^{-1} = —i—j—y_j—y_i—$ then (Y1) fails for $(a, b) = (i, y_i)$.
19. If $w^{-1} = —i—y_j—j—y_i—$ then (Y1) fails for $(a, b) = (i, y_i)$.
20. If $w^{-1} = —y_j—i—j—y_i—$ then (Y1) fails for $(a, b) = (i, y_i)$.
21. If $w^{-1} = —y_i—i—j—y_j—$ then (Y1) fails for $(a, b) = (j, y_j)$.
22. If $w^{-1} = —y_i—y_j—i—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (j, y_j)$.
23. If $w^{-1} = —y_j—y_i—i—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (j, y_j)$.

Hence if $i < y_i < j < y_j$ then

$$(wt)^{-1} = —y_i—j—y_j—i—.$$

When $(a, b), (a', b') \in \mathrm{Cyc}^1(z) = \{(j, j), (i, y_j), (y_i, y_i)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow (wt)^{-1} = —y_j—i—$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq —y_j—j—i—$ and $(wt)^{-1} \neq —y_j—y_i—i—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —y_i—j—$.

Thus properties (Z1)-(Z3) hold whenever (a, b), (a', b') are as in case (i) and $i < y_i < j < y_j$.

7.2 Subcase (ii)

Suppose R is an integer such that $(R, R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $i < y_i < j < y_j < R$.
   (a) If $w^{-1} = —R—y_i—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (b) If $w^{-1} = —y_i—R—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (c) If $w^{-1} = —y_i—i—y_j—R—j—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (R, R)$.
   (d) If $w^{-1} = —y_i—i—R—y_j—j—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (R, R)$.

   Thus if $i < y_i < j < y_j < R$ then one of the following holds:

   - $w^{-1} = —y_i—i—y_j—j—R—$ and $(wt)^{-1} = —y_i—j—y_j—i—R—$.

   When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(j, j), (i, y_j), (y_i, y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}y_j\text{—}i\text{—}$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}R\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}R\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}R\text{—}. \end{cases}$

2. Suppose $i < y_i < j < R < y_j$.

   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}R\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a',b') = (R,R)$.

   (b) If $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.

   (c) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.

   Thus if $i < y_i < j < R < y_j$ then one of the following holds:

   - $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}R\text{—}$.
   - $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}R\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j), (i,y_j), (y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}y_j\text{—}i\text{—}$.

   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_j\text{—}R\text{—}i\text{—}$.

   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}R\text{—}$.

3. Suppose $i < R < y_i < j < y_j$.

   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}R\text{—}j\text{—}$ then (T) fails.

   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}y_j\text{—}j\text{—}$ then (T) fails.

   (c) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.

   (d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j, y_j)$.

   Thus if $i < R < y_i < j < y_j$ then one of the following holds:

   - $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j), (i,y_j), (y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}y_j\text{—}i\text{—}$.

   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_j\text{—}R\text{—}i\text{—}$.

   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}R\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_i\text{—}$.

4. Suppose $R < i < y_i < j < y_j$.

   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i, y_i)$.

   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}R\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i, y_i)$.

   (c) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i, y_i)$.

   (d) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i, y_i)$.

   Thus if $R < i < y_i < j < y_j$ then one of the following holds:

   - $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j), (i,y_j), (y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}y_j\text{—}i\text{—}$.

   (Z2) $\Leftrightarrow$ (no condition).

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -R-j- \text{ and} \\ (wt)^{-1} = -R-y_i- \text{ and} \\ (wt)^{-1} = -R-y_j-. \end{cases}$$

5. It cannot happen that $i < y_i < R < j < y_j$ since:
   (a) If $w^{-1} = -y_i-i-y_j-R-j-$ then (T) fails.
   (b) If $w^{-1} = -y_i-i-R-y_j-j-$ then (T) fails.
   (c) If $w^{-1} = -y_i-i-y_j-j-R-$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,y_j)$.
   (d) If $w^{-1} = -R-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (e) If $w^{-1} = -y_i-R-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $P < i < y_i < Q < j < y_j$.
   (a) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (T) fails.
   (b) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (T) fails.
   (c) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (T) fails.
   (d) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (T) fails.
   (e) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (T) fails.
   (f) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (h) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (i) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (j) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (k) If $w^{-1} = -y_i-i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (l) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (m) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

   Thus if $P < i < y_i < Q < j < y_j$ then one of the following holds:
   - $w^{-1} = -Q-P-y_i-i-y_j-j-$ and $(wt)^{-1} = -Q-P-y_i-j-y_j-i-$.
   - $w^{-1} = -y_i-Q-P-i-y_j-j-$ and $(wt)^{-1} = -y_i-Q-P-j-y_j-i-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j), (i,y_j), (y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   $(\text{Z1}) \Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -y_j-i-$.
   $(\text{Z2}) \Leftrightarrow (wt)^{-1} \neq -Q-y_i-P-$.
   $(\text{Z3}) \Leftrightarrow (wt)^{-1} = -P-j-$ and $(wt)^{-1} = -P-y_j-$.

2. Suppose $P < i < Q < y_i < j < y_j$.
   (a) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (T) fails.
   (b) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (T) fails.
   (c) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (T) fails.
   (d) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (T) fails.
   (e) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (T) fails.
   (f) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (h) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(i) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(j) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(k) If $w^{-1} = —y_i—i—y_j—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(l) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(m) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(n) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Thus if $P < i < Q < y_i < j < y_j$ then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—y_j—j—$ and $(wt)^{-1} = —Q—P—y_i—j—y_j—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_j—i—$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = —P—j— \text{ and} \\ (wt)^{-1} = —P—y_i— \text{ and} \\ (wt)^{-1} = —P—y_j—. \end{cases}$

3. Suppose $i < y_i < j < P < y_j < Q$.

(a) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = —Q—P—y_i—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Thus if $i < y_i < j < P < y_j < Q$ then one of the following holds:

- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $(wt)^{-1} = —y_i—j—y_j—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_j—i—$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = —i—Q— \text{ and} \\ (wt)^{-1} = —j—Q— \text{ and} \\ (wt)^{-1} = —y_i—Q—. \end{cases}$

4. Suppose $P < i < y_i < j < Q < y_j$.

(a) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(b) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(e) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(f) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(g) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(h) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = -y_i-i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(k) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(l) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Thus if $P < i < y_i < j < Q < y_j$ then one of the following holds:

- $w^{-1} = -Q-P-y_i-i-y_j-j-$ and $(wt)^{-1} = -Q-P-y_i-j-y_j-i-$.
- $w^{-1} = -y_i-Q-P-i-y_j-j-$ and $(wt)^{-1} = -y_i-Q-P-j-y_j-i-$.
- $w^{-1} = -y_i-i-Q-P-y_j-j-$ and $(wt)^{-1} = -y_i-j-Q-P-y_j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -y_j-i-$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq -Q-j-P-$ and $(wt)^{-1} \neq -Q-y_i-P-$.
(Z3) $\Leftrightarrow (wt)^{-1} = -P-y_j-$.

5. Suppose $P < Q < i < y_i < j < y_j$.

(a) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(j) If $w^{-1} = -y_i-i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(k) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(l) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(m) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(n) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Thus if $P < Q < i < y_i < j < y_j$ then one of the following holds:

- $w^{-1} = -Q-P-y_i-i-y_j-j-$ and $(wt)^{-1} = -Q-P-y_i-j-y_j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -y_j-i-$.
(Z2) $\Leftrightarrow$ (no condition).

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -P-j- \text{ and} \\ (wt)^{-1} = -P-y_i- \text{ and} \\ (wt)^{-1} = -P-y_j-. \end{cases}$$

6. Suppose $i < y_i < j < P < Q < y_j$.
   (a) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = -Q-P-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

   Thus if $i < y_i < j < P < Q < y_j$ then one of the following holds:

   - $w^{-1} = -y_i-i-Q-y_j-j-P-$ and $(wt)^{-1} = -y_i-j-Q-y_j-i-P-$.
   - $w^{-1} = -y_i-i-y_j-j-Q-P-$ and $(wt)^{-1} = -y_i-j-y_j-i-Q-P-$.
   - $w^{-1} = -y_i-i-Q-P-y_j-j-$ and $(wt)^{-1} = -y_i-j-Q-P-y_j-i-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j), (i,y_j), (y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   $(\text{Z1}) \Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -y_j-i-$.
   $(\text{Z2}) \Leftrightarrow (wt)^{-1} \neq -y_j-P-i-$ and $(wt)^{-1} \neq -y_j-Q-i-$.
   $(\text{Z3}) \Leftrightarrow (wt)^{-1} = -j-Q-$ and $(wt)^{-1} = -y_i-Q-$.

7. Suppose $i < y_i < P < j < y_j < Q$.
   (a) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (T) fails.
   (b) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (T) fails.
   (c) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (T) fails.
   (d) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (T) fails.
   (e) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (T) fails.
   (f) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (T) fails.
   (g) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (T) fails.
   (h) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j, y_j)$.
   (i) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j, y_j)$.
   (j) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = -Q-P-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

   Thus if $i < y_i < P < j < y_j < Q$ then one of the following holds:

   - $w^{-1} = -y_i-i-y_j-j-Q-P-$ and $(wt)^{-1} = -y_i-j-y_j-i-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$.

8. Suppose $i < P < y_i < j < y_j < Q$.
   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (j) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

Thus if $i < P < y_i < j < y_j < Q$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}Q\text{—}P\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$.

9. Suppose $i < y_i < j < y_j < P < Q$.
   (a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Thus if $i < y_i < j < y_j < P < Q$ then one of the following holds:

- $w^{-1} = -y_i-i-y_j-j-Q-P-$ and $(wt)^{-1} = -y_i-j-y_j-i-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -y_j-i-$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = -i-Q- \text{ and} \\ (wt)^{-1} = -j-Q- \text{ and} \\ (wt)^{-1} = -y_i-Q-. \end{cases}$

10. Suppose $P < i < y_i < j < y_j < Q$.

(a) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Thus if $P < i < y_i < j < y_j < Q$ then one of the following holds:

- $w^{-1} = -y_i-i-y_j-j-Q-P-$ and $(wt)^{-1} = -y_i-j-y_j-i-Q-P-$.
- $w^{-1} = -y_i-Q-P-i-y_j-j-$ and $(wt)^{-1} = -y_i-Q-P-j-y_j-i-$.
- $w^{-1} = -y_i-i-y_j-Q-P-j-$ and $(wt)^{-1} = -y_i-j-y_j-Q-P-i-$.
- $w^{-1} = -Q-P-y_i-i-y_j-j-$ and $(wt)^{-1} = -Q-P-y_i-j-y_j-i-$.
- $w^{-1} = -y_i-i-Q-P-y_j-j-$ and $(wt)^{-1} = -y_i-j-Q-P-y_j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -y_j-i-$.

(Z2) $\Leftrightarrow \begin{cases} (wt)^{-1} \neq -Q-i-P- \text{ and } (wt)^{-1} \neq -Q-y_j-P- \text{ and} \\ (wt)^{-1} \neq -Q-j-P- \text{ and} \\ (wt)^{-1} \neq -Q-y_i-P-. \end{cases}$

(Z3) $\Leftrightarrow$ (no condition).

11. Suppose $i < P < Q < y_i < j < y_j$.

(a) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (T) fails.
(b) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (T) fails.
(c) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (T) fails.
(d) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (T) fails.
(e) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (T) fails.
(f) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (T) fails.

(h) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (T) fails.
(i) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (T) fails.
(j) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(m) If $w^{-1} = -y_i-i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(n) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Thus if $i < P < Q < y_i < j < y_j$ then one of the following holds:

- $w^{-1} = -Q-P-y_i-i-y_j-j-$ and $(wt)^{-1} = -Q-P-y_i-j-y_j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -y_j-i-$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq -y_j-P-i-$ and $(wt)^{-1} \neq -y_j-Q-i-$.
(Z3) $\Leftrightarrow (wt)^{-1} = -P-j-$ and $(wt)^{-1} = -P-y_i-$.

12. It cannot happen that $i < y_i < P < j < Q < y_j$ since:

(a) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (T) fails.
(b) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (T) fails.
(c) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (T) fails.
(d) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (T) fails.
(e) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (T) fails.
(f) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (T) fails.
(g) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (T) fails.
(h) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = -y_i-i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(k) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = -Q-P-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

13. It cannot happen that $i < P < y_i < j < Q < y_j$ since:

(a) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (T) fails.
(b) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (T) fails.
(c) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (T) fails.
(d) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (T) fails.
(e) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (T) fails.
(f) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (T) fails.
(g) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (T) fails.
(h) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = -y_i-i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(k) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(l) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = -Q-P-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(o) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.

14. It cannot happen that $i < y_i < P < Q < j < y_j$ since:

(a) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (T) fails.
(b) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (T) fails.
(c) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (T) fails.
(d) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (T) fails.
(e) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (T) fails.
(f) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (T) fails.
(i) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (T) fails.
(j) If $w^{-1} = -y_i-i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(k) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = -Q-P-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(o) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.

15. It cannot happen that $i < P < y_i < Q < j < y_j$ since:

(a) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (T) fails.
(b) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (T) fails.
(c) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (T) fails.
(d) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (T) fails.
(e) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (T) fails.
(f) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (T) fails.
(i) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (T) fails.
(j) If $w^{-1} = -y_i-i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(k) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = -Q-P-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(o) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a', b') are as in cases (i) or (ii) and $i < y_i < j < y_j$.

7.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < y_{i'} < j' < i < y_{j'} < y_i < j < y_j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

   Thus if $i' < y_{i'} < j' < i < y_{j'} < y_i < j < y_j$ then one of the following holds:

   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,j), (i,y_j), (y_i,y_i)\}$ and $(a',b') \in \{(j',j'), (i',y_{j'}), (y_{i'},y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$
   $$\begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—}. \end{cases}$$

2. Suppose $i' < y_{i'} < j' < y_{j'} < i < y_i < j < y_j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(k) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

Thus if $i' < y_{i'} < j' < y_{j'} < i < y_i < j < y_j$ then one of the following holds:

- $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $(wt)^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,j), (i,y_j), (y_i,y_i)\}$ and $(a',b') \in \{(j',j'), (i',y_{j'}), (y_{i'},y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —y_j—i—$ and $(wt)^{-1} = —y_{j'}—i'—$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = —i'—j— \text{ and} \\ (wt)^{-1} = —i'—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_j— \text{ and} \\ (wt)^{-1} = —j'—j— \text{ and} \\ (wt)^{-1} = —j'—y_i— \text{ and} \\ (wt)^{-1} = —j'—y_j— \text{ and} \\ (wt)^{-1} = —y_{i'}—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—y_i— \text{ and} \\ (wt)^{-1} = —y_{i'}—y_j—. \end{cases}$$

3. Suppose $i' < y_{i'} < j' < i < y_i < j < y_{j'} < y_j$.

(a) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(b) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(c) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(d) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(e) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(f) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(g) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(h) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(i) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
(j) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
(k) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Thus if $i' < y_{i'} < j' < i < y_i < j < y_{j'} < y_j$ then one of the following holds:

- $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $(wt)^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.
- $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ and $(wt)^{-1} = —y_{i'}—j'—y_i—y_{j'}—i'—j—y_j—i—$.
- $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ and $(wt)^{-1} = —y_{i'}—j'—y_i—j—y_{j'}—i'—y_j—i—$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,j), (i,y_j), (y_i,y_i)\}$ and $(a',b') \in \{(j',j'), (i',y_{j'}), (y_{i'},y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —y_j—i—$ and $(wt)^{-1} = —y_{j'}—i'—$.
(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_{j'}—j—i'—$ and $(wt)^{-1} \neq —y_{j'}—y_i—i'—$.
(Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = —i'—y_j— \text{ and} \\ (wt)^{-1} = —j'—j— \text{ and} \\ (wt)^{-1} = —j'—y_i— \text{ and} \\ (wt)^{-1} = —j'—y_j— \text{ and} \\ (wt)^{-1} = —y_{i'}—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—y_i— \text{ and} \\ (wt)^{-1} = —y_{i'}—y_j—. \end{cases}$$

4. Suppose $i' < y_{i'} < j' < i < y_i < y_{j'} < j < y_j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

   Thus if $i' < y_{i'} < j' < i < y_i < y_{j'} < j < y_j$ then one of the following holds:

   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i'\text{—}j\text{—}y_j\text{—}i\text{—}$.
   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,j), (i,y_j), (y_i,y_i)\}$ and $(a',b') \in \{(j',j'), (i',y_{j'}), (y_{i'},y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}y_j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}$.

   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_{j'}\text{—}y_i\text{—}i'\text{—}$.

   (Z3) $\Leftrightarrow$
   $$\begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—}. \end{cases}$$

5. Suppose $i' < i < y_{i'} < j' < y_{j'} < y_i < j < y_j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.
   (m) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

Thus if $i' < i < y_{i'} < j' < y_{j'} < y_i < j < y_j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-y_{j'}-j'-y_i-i-y_j-j-$ and $(wt)^{-1} = -y_{i'}-j'-y_{j'}-i'-y_i-j-y_j-i-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ and $(a',b') \in \{(j',j'),(i',y_{j'}),(y_{i'},y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -y_j-i-$ and $(wt)^{-1} = -y_{j'}-i'-$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq -y_j-j'-i-$ and $(wt)^{-1} \neq -y_j-y_{i'}-i-$.

(Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -j'-j- \text{ and} \\ (wt)^{-1} = -j'-y_i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_i-. \end{cases}$$

6. Suppose $i' < y_{i'} < i < j' < y_{j'} < y_i < j < y_j$.
   (a) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-y_j-j'-j-$ then (T) fails.
   (b) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-y_j-j'-j-$ then (T) fails.
   (c) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-y_j-j'-j-$ then (T) fails.
   (d) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-y_j-j'-j-$ then (T) fails.
   (e) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-y_j-j'-j-$ then (T) fails.
   (f) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-j'-y_j-j-$ then (T) fails.
   (g) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-j'-y_j-j-$ then (T) fails.
   (h) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-j'-y_j-j-$ then (T) fails.
   (i) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-j'-y_j-j-$ then (T) fails.
   (j) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-j'-y_j-j-$ then (T) fails.
   (k) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-j'-i-y_j-j-$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (j',y_{j'})$.
   (l) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-j'-i-y_j-j-$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (j',y_{j'})$.
   (m) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.

   Thus if $i' < y_{i'} < i < j' < y_{j'} < y_i < j < y_j$ then one of the following holds:

   - $w^{-1} = -y_{i'}-i'-y_{j'}-j'-y_i-i-y_j-j-$ and $(wt)^{-1} = -y_{i'}-j'-y_{j'}-i'-y_i-j-y_j-i-$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,j),(i,y_j),(y_i,y_i)\}$ and $(a',b') \in \{(j',j'),(i',y_{j'}),(y_{i'},y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = -y_j-i-$ and $(wt)^{-1} = -y_{j'}-i'-$.

   (Z2) $\Leftrightarrow (wt)^{-1} \neq -y_j-j'-i-$.

   (Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -j'-j- \text{ and} \\ (wt)^{-1} = -j'-y_i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_i- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_j-. \end{cases}$$

7. It cannot happen that $i' < i < y_{i'} < y_i < j' < j < y_{j'} < y_j$ since:

(a) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-y_j-j'-j-$ then (T) fails.
(b) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-y_j-j'-j-$ then (T) fails.
(c) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-y_j-j'-j-$ then (T) fails.
(d) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-y_j-j'-j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-y_j-j'-j-$ then (T) fails.
(f) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-j'-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-j'-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-j'-y_j-j-$ then (T) fails.
(i) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-j'-y_j-j-$ then (T) fails.
(j) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-j'-y_j-j-$ then (T) fails.
(k) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-j'-i-y_j-j-$ then (U) fails.
(l) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-j'-i-y_j-j-$ then (U) fails.
(m) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(n) If $w^{-1} = -y_{i'}-i'-y_{j'}-j'-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.

8. It cannot happen that $i' < y_{i'} < i < j' < y_i < j < y_{j'} < y_j$ since:

(a) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-y_j-j'-j-$ then (T) fails.
(b) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-y_j-j'-j-$ then (T) fails.
(c) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-y_j-j'-j-$ then (T) fails.
(d) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-y_j-j'-j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-y_j-j'-j-$ then (T) fails.
(f) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-j'-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-j'-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-j'-y_j-j-$ then (T) fails.
(i) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-j'-y_j-j-$ then (T) fails.
(j) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-j'-y_j-j-$ then (T) fails.
(k) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(m) If $w^{-1} = -y_{i'}-i'-y_{j'}-j'-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(n) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.

9. It cannot happen that $i' < i < y_{i'} < y_i < j' < y_{j'} < j < y_j$ since:

(a) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-y_j-j'-j-$ then (T) fails.
(b) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-y_j-j'-j-$ then (T) fails.
(c) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-y_j-j'-j-$ then (T) fails.
(d) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-y_j-j'-j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-y_j-j'-j-$ then (T) fails.
(f) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-j'-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-j'-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-j'-y_j-j-$ then (T) fails.
(i) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-j'-y_j-j-$ then (T) fails.
(j) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-j'-y_j-j-$ then (T) fails.
(k) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-j'-i-y_j-j-$ then (U) fails.
(l) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-j'-i-y_j-j-$ then (U) fails.
(m) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(n) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.

10. It cannot happen that $i' < y_{i'} < i < j' < y_i < y_{j'} < j < y_j$ since:

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.
(n) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.

11. It cannot happen that $i' < i < y_{i'} < j' < y_i < j < y_{j'} < y_j$ since:

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.
(n) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.

12. It cannot happen that $i' < y_{i'} < i < y_i < j' < j < y_{j'} < y_j$ since:

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.

(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (U) fails.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (U) fails.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(n) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.

13. It cannot happen that $i' < y_{i'} < i < y_i < j' < y_{j'} < j < y_j$ since:

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (U) fails.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (U) fails.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(n) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.

14. It cannot happen that $i' < i < y_{i'} < j' < y_i < y_{j'} < j < y_j$ since:

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(n) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(z)$ when $i < y_i < j < y_j$.

8 Case: $i < y_i < y_j < j$

Suppose y is such that $i < y_i < y_j < j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(y_j, y_j), (i,j), (y_i, y_i)\}$.

8.1 Subcase (i)

We must have $w^{-1} = —y_i—i—j—y_j—$ since no other ordering is possible:

1. If $w^{-1} = —i—y_i—y_j—j—$ then (T) fails.
2. If $w^{-1} = —i—y_i—j—y_j—$ then (T) fails.
3. If $w^{-1} = —i—y_j—y_i—j—$ then (T) fails.
4. If $w^{-1} = —i—y_j—j—y_i—$ then (T) fails.
5. If $w^{-1} = —y_i—i—y_j—j—$ then (T) fails.
6. If $w^{-1} = —y_i—y_j—j—i—$ then (T) fails.
7. If $w^{-1} = —y_i—j—i—y_j—$ then (T) fails.
8. If $w^{-1} = —y_i—j—y_j—i—$ then (T) fails.
9. If $w^{-1} = —y_j—i—y_i—j—$ then (T) fails.
10. If $w^{-1} = —y_j—y_i—j—i—$ then (T) fails.
11. If $w^{-1} = —y_j—j—i—y_i—$ then (T) fails.
12. If $w^{-1} = —y_j—j—y_i—i—$ then (T) fails.
13. If $w^{-1} = —j—i—y_i—y_j—$ then (T) fails.
14. If $w^{-1} = —j—i—y_j—y_i—$ then (T) fails.
15. If $w^{-1} = —j—y_i—i—y_j—$ then (T) fails.
16. If $w^{-1} = —j—y_i—y_j—i—$ then (T) fails.
17. If $w^{-1} = —j—y_j—i—y_i—$ then (T) fails.
18. If $w^{-1} = —j—y_j—y_i—i—$ then (T) fails.
19. If $w^{-1} = —i—j—y_i—y_j—$ then (Y1) fails for $(a, b) = (i, y_i)$.
20. If $w^{-1} = —i—j—y_j—y_i—$ then (Y1) fails for $(a, b) = (i, y_i)$.
21. If $w^{-1} = —y_i—y_j—i—j—$ then (Y1) fails for $(a, b) = (y_j, j)$.
22. If $w^{-1} = —y_j—i—j—y_i—$ then (Y1) fails for $(a, b) = (y_j, j)$.
23. If $w^{-1} = —y_j—y_i—i—j—$ then (Y1) fails for $(a, b) = (y_j, j)$.

Hence if $i < y_i < y_j < j$ then

$$(wt)^{-1} = —y_i—j—i—y_j—.$$

When $(a, b), (a', b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (i, j), (y_i, y_i)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

$$\text{(Z1)} \Leftrightarrow (wt)^{-1} = —j—i—.$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq —j—y_i—i— \text{ and } (wt)^{-1} \neq —j—y_j—i—.$$

$$\text{(Z3)} \Leftrightarrow (wt)^{-1} = —y_i—y_j—.$$

Thus properties (Z1)-(Z3) hold whenever (a, b), (a', b') are as in case (i) and $i < y_i < y_j < j$.

8.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $i < y_i < y_j < j < R$.
   (a) If $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (b) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (c) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (R, R)$.
   (d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (R, R)$.

   Thus if $i < y_i < y_j < j < R$ then one of the following holds:
   - $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (i, j), (y_i, y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}R\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}R\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}R\text{—}. \end{cases}$

2. Suppose $i < y_i < y_j < R < j$.
   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (R, R)$.
   (c) If $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (d) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (R, R)$.

   Thus if $i < y_i < y_j < R < j$ then one of the following holds:
   - $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (i, j), (y_i, y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}R\text{—}i\text{—}$.
   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}y_i\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}R\text{—}$.

3. Suppose $i < R < y_i < y_j < j$.
   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (c) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (y_j, j)$.
   (d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (y_j, j)$.

   Thus if $i < R < y_i < y_j < j$ then one of the following holds:
   - $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) = (R,R)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (i, j), (y_i, y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}R\text{—}i\text{—}$.
   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}R\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_j\text{—}$.

4. Suppose $R < i < y_i < y_j < j$.

   (a) If $w^{-1} = —y_i—i—j—y_j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.
   (b) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.
   (c) If $w^{-1} = —y_i—i—R—j—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.
   (d) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.

   Thus if $R < i < y_i < y_j < j$ then one of the following holds:

   - $w^{-1} = —R—y_i—i—j—y_j—$ and $(wt)^{-1} = —R—y_i—j—i—y_j—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —j—i—$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = —R—j— \text{ and} \\ (wt)^{-1} = —R—y_i— \text{ and} \\ (wt)^{-1} = —R—y_j—. \end{cases}$

5. It cannot happen that $i < y_i < R < y_j < j$ since:

   (a) If $w^{-1} = —y_i—i—R—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—i—j—y_j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j,j)$.
   (c) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j,j)$.
   (d) If $w^{-1} = —R—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (e) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i,j,y_i,y_j\} + n\mathbb{Z}$.

1. Suppose $P < i < y_i < Q < y_j < j$.

   (a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
   (c) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
   (d) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (h) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (i) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (j) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (k) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (l) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (m) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

   Thus if $P < i < y_i < Q < y_j < j$ then one of the following holds:

   - $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $(wt)^{-1} = —Q—P—y_i—j—i—y_j—$.
   - $w^{-1} = —y_i—Q—P—i—j—y_j—$ and $(wt)^{-1} = —y_i—Q—P—j—i—y_j—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—i—$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq -Q-y_i-P-$.
(Z3) $\Leftrightarrow (wt)^{-1} = -P-j-$ and $(wt)^{-1} = -P-y_j-$.

2. Suppose $P < i < Q < y_i < y_j < j$.
   (a) If $w^{-1} = -y_i-i-Q-P-j-y_j-$ then (T) fails.
   (b) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (T) fails.
   (c) If $w^{-1} = -y_i-i-Q-j-P-y_j-$ then (T) fails.
   (d) If $w^{-1} = -Q-y_i-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = -y_i-Q-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = -y_i-i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (h) If $w^{-1} = -Q-y_i-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (i) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (j) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (k) If $w^{-1} = -y_i-i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (l) If $w^{-1} = -y_i-Q-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (m) If $w^{-1} = -Q-y_i-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (n) If $w^{-1} = -y_i-Q-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

   Thus if $P < i < Q < y_i < y_j < j$ then one of the following holds:

   - $w^{-1} = -Q-P-y_i-i-j-y_j-$ and $(wt)^{-1} = -Q-P-y_i-j-i-y_j-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-i-$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = -P-j- \text{ and} \\ (wt)^{-1} = -P-y_i- \text{ and} \\ (wt)^{-1} = -P-y_j-. \end{cases}$

3. Suppose $i < y_i < y_j < P < j < Q$.
   (a) If $w^{-1} = -y_i-i-Q-P-j-y_j-$ then (T) fails.
   (b) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (T) fails.
   (c) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (T) fails.
   (d) If $w^{-1} = -Q-y_i-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = -y_i-Q-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = -Q-y_i-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = -Q-P-y_i-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = -y_i-Q-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = -Q-y_i-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = -y_i-Q-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = -y_i-i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = -y_i-i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   Thus if $i < y_i < y_j < P < j < Q$ then one of the following holds:

- $w^{-1} = -y_i-i-j-y_j-Q-P-$ and $(wt)^{-1} = -y_i-j-i-y_j-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-i-$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = -i-Q- \text{ and} \\ (wt)^{-1} = -y_i-Q- \text{ and} \\ (wt)^{-1} = -y_j-Q-. \end{cases}$

4. Suppose $P < i < y_i < y_j < Q < j$.

(a) If $w^{-1} = -y_i-i-Q-P-j-y_j-$ then (T) fails.
(b) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (T) fails.
(c) If $w^{-1} = -y_i-i-Q-j-P-y_j-$ then (T) fails.
(d) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = -Q-y_i-P-i-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = -Q-y_i-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(h) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(i) If $w^{-1} = -y_i-i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(j) If $w^{-1} = -Q-y_i-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(k) If $w^{-1} = -y_i-i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(l) If $w^{-1} = -y_i-Q-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(m) If $w^{-1} = -y_i-Q-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

Thus if $P < i < y_i < y_j < Q < j$ then one of the following holds:

- $w^{-1} = -Q-P-y_i-i-j-y_j-$ and $(wt)^{-1} = -Q-P-y_i-j-i-y_j-$.
- $w^{-1} = -y_i-Q-P-i-j-y_j-$ and $(wt)^{-1} = -y_i-Q-P-j-i-y_j-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-i-$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq -Q-y_i-P-$ and $(wt)^{-1} \neq -Q-y_j-P-$.

(Z3) $\Leftrightarrow (wt)^{-1} = -P-j-$.

5. Suppose $P < Q < i < y_i < y_j < j$.

(a) If $w^{-1} = -Q-y_i-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = -y_i-i-Q-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = -y_i-Q-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = -y_i-i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = -Q-y_i-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(j) If $w^{-1} = -y_i-i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(k) If $w^{-1} = -y_i-Q-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(l) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(n) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Thus if $P < Q < i < y_i < y_j < j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}P\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}P\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}P\text{—}y_j\text{—}. \end{cases}$

6. Suppose $i < y_i < y_j < P < Q < j$.

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

Thus if $i < y_i < y_j < P < Q < j$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}P\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}Q\text{—}i\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}Q\text{—}$.

7. Suppose $i < y_i < P < y_j < j < Q$.

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

(g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

Thus if $i < y_i < P < y_j < j < Q$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}$.
- $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$.

8. Suppose $i < P < y_i < y_j < j < Q$.

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

Thus if $i < P < y_i < y_j < j < Q$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}$.
- $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$.

9. Suppose $i < y_i < y_j < j < P < Q$.

(a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(d) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = -Q-P-y_i-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = -y_i-Q-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = -Q-y_i-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = -y_i-Q-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = -y_i-i-Q-P-j-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = -y_i-i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = -y_i-i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

Thus if $i < y_i < y_j < j < P < Q$ then one of the following holds:

- $w^{-1} = -y_i-i-j-y_j-Q-P-$ and $(wt)^{-1} = -y_i-j-i-y_j-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-i-$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow$ $\begin{cases} (wt)^{-1} = -i-Q- \text{ and} \\ (wt)^{-1} = -y_i-Q- \text{ and} \\ (wt)^{-1} = -y_j-Q-. \end{cases}$

10. Suppose $P < i < y_i < y_j < j < Q$.

(a) If $w^{-1} = -Q-y_i-i-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = -Q-y_i-i-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = -y_i-Q-i-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = -Q-y_i-P-i-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = -y_i-Q-i-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(i) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(j) If $w^{-1} = -y_i-i-Q-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

Thus if $P < i < y_i < y_j < j < Q$ then one of the following holds:

- $w^{-1} = -y_i-i-j-y_j-Q-P-$ and $(wt)^{-1} = -y_i-j-i-y_j-Q-P-$.
- $w^{-1} = -y_i-Q-P-i-j-y_j-$ and $(wt)^{-1} = -y_i-Q-P-j-i-y_j-$.
- $w^{-1} = -y_i-i-j-Q-P-y_j-$ and $(wt)^{-1} = -y_i-j-i-Q-P-y_j-$.
- $w^{-1} = -Q-P-y_i-i-j-y_j-$ and $(wt)^{-1} = -Q-P-y_i-j-i-y_j-$.
- $w^{-1} = -y_i-i-Q-P-j-y_j-$ and $(wt)^{-1} = -y_i-j-Q-P-i-y_j-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-i-$.

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—} \text{ and } (wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—} \text{ and} \\ (wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—} \text{ and} \\ (wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

11. Suppose $i < P < Q < y_i < y_j < j$.

    (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
    (c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (d) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
    (f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (h) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (k) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (l) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (n) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

    Thus if $i < P < Q < y_i < y_j < j$ then one of the following holds:

    - $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

    When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

    (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}i\text{—}$.
    (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}P\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}Q\text{—}i\text{—}$.
    (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}y_j\text{—}$.

12. It cannot happen that $i < y_i < P < y_j < Q < j$ since:

    (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
    (c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (d) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
    (f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (j) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (k) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (l) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (n) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (o) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

13. It cannot happen that $i < P < y_i < y_j < Q < j$ since:

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

14. It cannot happen that $i < y_i < P < Q < y_j < j$ since:

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

15. It cannot happen that $i < P < y_i < Q < y_j < j$ since:

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(j) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(o) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $i < y_i < y_j < j$.

8.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < y_{i'} < y_{j'} < i < j' < y_i < y_j < j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

   Thus if $i' < y_{i'} < y_{j'} < i < j' < y_i < y_j < j$ then one of the following holds:

   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (i,j), (y_i, y_i)\}$ and $(a',b') \in \{(y_{j'}, y_{j'}), (i', j'), (y_{i'}, y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}i'\text{—}$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$
   $$\begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}. \end{cases}$$

2. Suppose $i' < y_{i'} < y_{j'} < j' < i < y_i < y_j < j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(d) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(e) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(f) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(g) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(h) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(i) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(j) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(m) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

Thus if $i' < y_{i'} < y_{j'} < j' < i < y_i < y_j < j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ and $(wt)^{-1} = -y_{i'}-j'-i'-y_{j'}-y_i-j-i-y_j-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (i,j), (y_i, y_i)\}$ and $(a',b') \in \{(y_{j'}, y_{j'}), (i', j'), (y_{i'}, y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = -j-i-$ and $(wt)^{-1} = -j'-i'-$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_i- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_j- \text{ and} \\ (wt)^{-1} = -y_{j'}-j- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_i- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_j-. \end{cases}$$

3. Suppose $i' < y_{i'} < y_{j'} < i < y_i < y_j < j' < j$.

(a) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-y_{j'}-y_j-$ then (T) fails.
(b) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
(c) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(d) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (U) fails.
(f) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (U) fails.
(g) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (U) fails.
(h) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(i) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(j) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(m) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Thus if $i' < y_{i'} < y_{j'} < i < y_i < y_j < j' < j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ and $(wt)^{-1} = -y_{i'}-j'-i'-y_{j'}-y_i-j-i-y_j-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ and $(a',b') \in \{(y_{j'},y_{j'}),(i',j'),(y_{i'},y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}i'\text{—}$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j'\text{—}y_i\text{—}i'\text{—}$ and $(wt)^{-1} \neq \text{—}j'\text{—}y_j\text{—}i'\text{—}$.

(Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}. \end{cases}$$

4. Suppose $i' < y_{i'} < y_{j'} < i < y_i < j' < y_j < j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (U) fails.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (i,y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (i,y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.

   Thus if $i' < y_{i'} < y_{j'} < i < y_i < j' < y_j < j$ then one of the following holds:

   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(i,j),(y_i,y_i)\}$ and $(a',b') \in \{(y_{j'},y_{j'}),(i',j'),(y_{i'},y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}i'\text{—}$.

   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j'\text{—}y_i\text{—}i'\text{—}$.

   (Z3) $\Leftrightarrow$
   $$\begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}. \end{cases}$$

5. Suppose $i' < i < y_{i'} < y_{j'} < j' < y_i < y_j < j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.

(d) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (T) fails.
(g) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (T) fails.
(h) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(i) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(j) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.
(m) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

Thus if $i' < i < y_{i'} < y_{j'} < j' < y_i < y_j < j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ and $(wt)^{-1} = -y_{i'}-j'-i'-y_{j'}-y_i-j-i-y_j-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (i,j), (y_i, y_i)\}$ and $(a', b') \in \{(y_{j'}, y_{j'}), (i', j'), (y_{i'}, y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -j-i-$ and $(wt)^{-1} = -j'-i'-$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq -j-y_{i'}-i-$ and $(wt)^{-1} \neq -j-y_{j'}-i-$.
(Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_i- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_j- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_i- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_j-. \end{cases}$$

6. Suppose $i' < y_{i'} < i < y_{j'} < j' < y_i < y_j < j$.

(a) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-y_{j'}-y_j-$ then (T) fails.
(b) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
(c) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (T) fails.
(d) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (T) fails.
(g) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (T) fails.
(h) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(i) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(j) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.
(m) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

Thus if $i' < y_{i'} < i < y_{j'} < j' < y_i < y_j < j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ and $(wt)^{-1} = -y_{i'}-j'-i'-y_{j'}-y_i-j-i-y_j-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (i,j), (y_i, y_i)\}$ and $(a', b') \in \{(y_{j'}, y_{j'}), (i', j'), (y_{i'}, y_{i'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -j-i-$ and $(wt)^{-1} = -j'-i'-$.

$$(\text{Z2}) \Leftrightarrow (wt)^{-1} \neq -j-y_{j'}-i-.$$

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_i- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_j- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_i- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_j-. \end{cases}$$

7. It cannot happen that $i' < i < y_{i'} < y_i < y_{j'} < y_j < j' < j$ since:
   (a) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-y_{j'}-y_j-$ then (T) fails.
   (b) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
   (c) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (T) fails.
   (d) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
   (e) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
   (f) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (T) fails.
   (g) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (T) fails.
   (h) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (U) fails.
   (i) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (U) fails.
   (j) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
   (m) If $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
   (n) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.

8. It cannot happen that $i' < y_{i'} < i < y_{j'} < y_i < y_j < j' < j$ since:
   (a) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-y_{j'}-y_j-$ then (T) fails.
   (b) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
   (c) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (T) fails.
   (d) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
   (e) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
   (f) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (T) fails.
   (g) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (T) fails.
   (h) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (U) fails.
   (i) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (U) fails.
   (j) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
   (m) If $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
   (n) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.

9. It cannot happen that $i' < i < y_{i'} < y_i < y_{j'} < j' < y_j < j$ since:
   (a) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-y_{j'}-y_j-$ then (T) fails.
   (b) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
   (c) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (T) fails.

(d) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (T) fails.
(g) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (T) fails.
(h) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (U) fails.
(i) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (U) fails.
(j) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(m) If $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(n) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.

10. It cannot happen that $i' < y_{i'} < i < y_{j'} < y_i < j' < y_j < j$ since:

(a) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-y_{j'}-y_j-$ then (T) fails.
(b) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
(c) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (T) fails.
(d) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (T) fails.
(g) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (T) fails.
(h) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (U) fails.
(i) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (U) fails.
(j) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(m) If $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(n) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.

11. It cannot happen that $i' < i < y_{i'} < y_{j'} < y_i < y_j < j' < j$ since:

(a) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-y_{j'}-y_j-$ then (T) fails.
(b) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
(c) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (T) fails.
(d) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (T) fails.
(g) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (T) fails.
(h) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (U) fails.
(i) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (U) fails.
(j) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(m) If $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(n) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.

12. It cannot happen that $i' < y_{i'} < i < y_i < y_{j'} < y_j < j' < j$ since:

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (U) fails.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(n) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.

13. It cannot happen that $i' < y_{i'} < i < y_i < y_{j'} < j' < y_j < j$ since:

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (U) fails.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(n) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.

14. It cannot happen that $i' < i < y_{i'} < y_{j'} < y_i < j' < y_j < j$ since:

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (U) fails.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(n) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(z)$ when $i < y_i < y_j < j$.

9 Case: $y_i < i < j < y_j$

Suppose y is such that $y_i < i < j < y_j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$.

9.1 Subcase (i)

We must have $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ since no other ordering is possible:

1. If $w^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$ then (T) fails.
2. If $w^{-1} = \text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$ then (T) fails.
3. If $w^{-1} = \text{—}y_i\text{—}y_j\text{—}j\text{—}i\text{—}$ then (T) fails.
4. If $w^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$ then (T) fails.
5. If $w^{-1} = \text{—}j\text{—}y_i\text{—}y_j\text{—}i\text{—}$ then (T) fails.
6. If $w^{-1} = \text{—}j\text{—}i\text{—}y_i\text{—}y_j\text{—}$ then (T) fails.
7. If $w^{-1} = \text{—}j\text{—}i\text{—}y_j\text{—}y_i\text{—}$ then (T) fails.
8. If $w^{-1} = \text{—}j\text{—}y_j\text{—}y_i\text{—}i\text{—}$ then (T) fails.
9. If $w^{-1} = \text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$ then (T) fails.
10. If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}j\text{—}i\text{—}$ then (T) fails.
11. If $w^{-1} = \text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$ then (T) fails.
12. If $w^{-1} = \text{—}y_j\text{—}j\text{—}i\text{—}y_i\text{—}$ then (T) fails.
13. If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y1) fails for $(a, b) = (j, y_j)$.
14. If $w^{-1} = \text{—}i\text{—}j\text{—}y_i\text{—}y_j\text{—}$ then (Y1) fails for $(a, b) = (j, y_j)$.
15. If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y1) fails for $(a, b) = (j, y_j)$.
16. If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y1) fails for $(a, b) = (y_i, i)$.
17. If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y1) fails for $(a, b) = (y_i, i)$.
18. If $w^{-1} = \text{—}y_i\text{—}y_j\text{—}i\text{—}j\text{—}$ then (Y1) fails for $(a, b) = (y_i, i)$.
19. If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y1) fails for $(a, b) = (y_i, i)$.
20. If $w^{-1} = \text{—}i\text{—}y_j\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (j, y_j)$.
21. If $w^{-1} = \text{—}i\text{—}y_j\text{—}j\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (j, y_j)$.
22. If $w^{-1} = \text{—}y_j\text{—}i\text{—}y_i\text{—}j\text{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (j, y_j)$.
23. If $w^{-1} = \text{—}y_j\text{—}i\text{—}j\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (j, y_j)$.

Hence if $y_i < i < j < y_j$ then

$$(wt)^{-1} = \text{—}j\text{—}y_i\text{—}y_j\text{—}i\text{—}.$$

When $(a, b), (a', b') \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}i\text{—}$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_i\text{—}y_j\text{—}$.

Thus properties (Z1)-(Z3) hold whenever (a, b), (a', b') are as in case (i) and $y_i < i < j < y_j$.

9.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $y_i < i < j < y_j < R$.

   (a) If $w^{-1} = —i—y_i—y_j—R—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —i—y_i—R—y_j—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —R—i—y_i—y_j—j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = —i—R—y_i—y_j—j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (R,R)$.

   Thus if $y_i < i < j < y_j < R$ then one of the following holds:

   - $w^{-1} = —i—y_i—y_j—j—R—$ and $(wt)^{-1} = —j—y_i—y_j—i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —j—y_i—$ and $(wt)^{-1} = —y_j—i—$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow (wt)^{-1} = —i—R—$ and $(wt)^{-1} = —y_i—R—$.

2. Suppose $y_i < i < j < R < y_j$.

   (a) If $w^{-1} = —i—y_i—y_j—R—j—$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —R—i—y_i—y_j—j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —i—R—y_i—y_j—j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (R,R)$.

   Thus if $y_i < i < j < R < y_j$ then one of the following holds:

   - $w^{-1} = —i—y_i—y_j—j—R—$ and $(wt)^{-1} = —j—y_i—y_j—i—R—$.
   - $w^{-1} = —i—y_i—R—y_j—j—$ and $(wt)^{-1} = —j—y_i—R—y_j—i—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —j—y_i—$ and $(wt)^{-1} = —y_j—i—$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq —y_j—R—i—$.
   (Z3) $\Leftrightarrow (wt)^{-1} = —y_i—R—$.

3. Suppose $y_i < R < i < j < y_j$.

   (a) If $w^{-1} = —i—R—y_i—y_j—j—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —i—y_i—y_j—j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j, y_j)$.
   (c) If $w^{-1} = —i—y_i—y_j—R—j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j, y_j)$.

   Thus if $y_i < R < i < j < y_j$ then one of the following holds:

   - $w^{-1} = —i—y_i—R—y_j—j—$ and $(wt)^{-1} = —j—y_i—R—y_j—i—$.
   - $w^{-1} = —R—i—y_i—y_j—j—$ and $(wt)^{-1} = —R—j—y_i—y_j—i—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —j—y_i—$ and $(wt)^{-1} = —y_j—i—$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq —j—R—y_i—$.
   (Z3) $\Leftrightarrow (wt)^{-1} = —R—y_j—$.

4. Suppose $R < y_i < i < j < y_j$.

(a) If $w^{-1} = -i-y_i-y_j-j-R-$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.
(b) If $w^{-1} = -i-y_i-y_j-R-j-$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.
(c) If $w^{-1} = -i-y_i-R-y_j-j-$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.
(d) If $w^{-1} = -i-R-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.

Thus if $R < y_i < i < j < y_j$ then one of the following holds:

- $w^{-1} = -R-i-y_i-y_j-j-$ and $(wt)^{-1} = -R-j-y_i-y_j-i-$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = -j-y_i-$ and $(wt)^{-1} = -y_j-i-$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow$ $(wt)^{-1} = -R-j-$ and $(wt)^{-1} = -R-y_j-$.

5. It cannot happen that $y_i < i < R < j < y_j$ since:

(a) If $w^{-1} = -i-y_i-y_j-R-j-$ then (T) fails.
(b) If $w^{-1} = -i-y_i-R-y_j-j-$ then (T) fails.
(c) If $w^{-1} = -i-R-y_i-y_j-j-$ then (T) fails.
(d) If $w^{-1} = -i-y_i-y_j-j-R-$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,y_j)$.
(e) If $w^{-1} = -R-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i,j,y_i,y_j\} + n\mathbb{Z}$.

1. Suppose $P < y_i < i < Q < j < y_j$.

(a) If $w^{-1} = -i-y_i-Q-P-y_j-j-$ then (T) fails.
(b) If $w^{-1} = -i-y_i-Q-y_j-j-P-$ then (T) fails.
(c) If $w^{-1} = -i-y_i-y_j-Q-j-P-$ then (T) fails.
(d) If $w^{-1} = -i-Q-P-y_i-y_j-j-$ then (T) fails.
(e) If $w^{-1} = -i-y_i-y_j-Q-P-j-$ then (T) fails.
(f) If $w^{-1} = -i-Q-y_i-P-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -i-Q-y_i-y_j-j-P-$ then (T) fails.
(h) If $w^{-1} = -i-Q-y_i-y_j-P-j-$ then (T) fails.
(i) If $w^{-1} = -i-y_i-Q-y_j-P-j-$ then (T) fails.
(j) If $w^{-1} = -Q-i-y_i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(k) If $w^{-1} = -Q-i-P-y_i-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(l) If $w^{-1} = -Q-i-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(m) If $w^{-1} = -Q-i-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(n) If $w^{-1} = -i-y_i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Thus if $P < y_i < i < Q < j < y_j$ then one of the following holds:

- $w^{-1} = -Q-P-i-y_i-y_j-j-$ and $(wt)^{-1} = -Q-P-j-y_i-y_j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = -P-j-$ and $(wt)^{-1} = -P-y_j-$.

2. Suppose $P < y_i < Q < i < j < y_j$.

   (a) If $w^{-1} = -Q-i-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = -i-y_i-Q-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = -i-y_i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = -i-y_i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (e) If $w^{-1} = -i-Q-P-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (f) If $w^{-1} = -i-y_i-y_j-Q-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (g) If $w^{-1} = -Q-i-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (h) If $w^{-1} = -Q-i-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = -i-Q-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (j) If $w^{-1} = -i-y_i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (k) If $w^{-1} = -i-Q-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (l) If $w^{-1} = -Q-i-P-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (m) If $w^{-1} = -i-Q-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (n) If $w^{-1} = -i-y_i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

   Thus if $P < y_i < Q < i < j < y_j$ then one of the following holds:

   - $w^{-1} = -Q-P-i-y_i-y_j-j-$ and $(wt)^{-1} = -Q-P-j-y_i-y_j-i-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = -P-j-$ and $(wt)^{-1} = -P-y_j-$.

3. Suppose $y_i < i < j < P < y_j < Q$.

   (a) If $w^{-1} = -i-y_i-Q-P-y_j-j-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = -i-y_i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = -i-y_i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = -i-y_i-y_j-Q-P-j-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = -i-y_i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = -Q-i-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = -i-Q-P-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = -Q-i-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = -Q-i-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = -i-Q-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = -Q-P-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = -i-Q-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = -Q-i-P-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = -i-Q-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   Thus if $y_i < i < j < P < y_j < Q$ then one of the following holds:

- $w^{-1} = -i-y_i-y_j-j-Q-P-$ and $(wt)^{-1} = -j-y_i-y_j-i-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = -i-Q-$ and $(wt)^{-1} = -y_i-Q-$.

4. Suppose $P < y_i < i < j < Q < y_j$.

   (a) If $w^{-1} = -Q-i-y_i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = -i-Q-y_i-P-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = -Q-i-P-y_i-y_j-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = -Q-i-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (e) If $w^{-1} = -i-y_i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (f) If $w^{-1} = -i-y_i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (g) If $w^{-1} = -i-y_i-y_j-Q-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (h) If $w^{-1} = -Q-i-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (i) If $w^{-1} = -i-y_i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (j) If $w^{-1} = -i-Q-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (k) If $w^{-1} = -i-Q-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (l) If $w^{-1} = -i-y_i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

   Thus if $P < y_i < i < j < Q < y_j$ then one of the following holds:

   - $w^{-1} = -Q-P-i-y_i-y_j-j-$ and $(wt)^{-1} = -Q-P-j-y_i-y_j-i-$.
   - $w^{-1} = -i-Q-P-y_i-y_j-j-$ and $(wt)^{-1} = -j-Q-P-y_i-y_j-i-$.
   - $w^{-1} = -i-y_i-Q-P-y_j-j-$ and $(wt)^{-1} = -j-y_i-Q-P-y_j-i-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$$

   (Z2) $\Leftrightarrow (wt)^{-1} \neq -Q-y_i-P-$ and $(wt)^{-1} \neq -Q-j-P-$.

   (Z3) $\Leftrightarrow (wt)^{-1} = -P-y_j-$.

5. Suppose $y_i < P < i < j < Q < y_j$.

   (a) If $w^{-1} = -i-y_i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (b) If $w^{-1} = -i-y_i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (c) If $w^{-1} = -i-y_i-y_j-Q-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (d) If $w^{-1} = -i-y_i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (e) If $w^{-1} = -i-y_i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (f) If $w^{-1} = -Q-i-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = -i-Q-P-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = -Q-i-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = -Q-i-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

(j) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.

Thus if $y_i < P < i < j < Q < y_j$ then one of the following holds:

- $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}Q\text{—}P\text{—}y_j\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$.

6. Suppose $P < Q < y_i < i < j < y_j$.

(a) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(d) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(e) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(f) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(g) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(h) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(i) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(j) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(k) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(l) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(m) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(n) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.

Thus if $P < Q < y_i < i < j < y_j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}y_j\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}y_j\text{—}$.

7. Suppose $y_i < i < j < P < Q < y_j$.

(a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.
(b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.

(c) If $w^{-1} = -i-y_i-Q-y_j-P-j-$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = -Q-i-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = -i-Q-P-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = -Q-i-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = -Q-i-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = -i-Q-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = -Q-P-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = -i-Q-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = -Q-i-P-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = -i-Q-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

Thus if $y_i < i < j < P < Q < y_j$ then one of the following holds:

- $w^{-1} = -i-y_i-Q-y_j-j-P-$ and $(wt)^{-1} = -j-y_i-Q-y_j-i-P-$.
- $w^{-1} = -i-y_i-y_j-j-Q-P-$ and $(wt)^{-1} = -j-y_i-y_j-i-Q-P-$.
- $w^{-1} = -i-y_i-Q-P-y_j-j-$ and $(wt)^{-1} = -j-y_i-Q-P-y_j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq -y_j-P-i-$ and $(wt)^{-1} \neq -y_j-Q-i-$.
(Z3) $\Leftrightarrow (wt)^{-1} = -y_i-Q-$.

8. Suppose $y_i < i < P < j < y_j < Q$.

(a) If $w^{-1} = -i-y_i-Q-P-y_j-j-$ then (T) fails.
(b) If $w^{-1} = -i-Q-P-y_i-y_j-j-$ then (T) fails.
(c) If $w^{-1} = -i-y_i-y_j-Q-P-j-$ then (T) fails.
(d) If $w^{-1} = -Q-i-y_i-y_j-P-j-$ then (T) fails.
(e) If $w^{-1} = -Q-i-y_i-P-y_j-j-$ then (T) fails.
(f) If $w^{-1} = -i-Q-y_i-P-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -Q-i-P-y_i-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -i-Q-y_i-y_j-P-j-$ then (T) fails.
(i) If $w^{-1} = -i-y_i-Q-y_j-P-j-$ then (T) fails.
(j) If $w^{-1} = -i-y_i-Q-y_j-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(k) If $w^{-1} = -i-y_i-y_j-Q-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(l) If $w^{-1} = -Q-i-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = -Q-P-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = -i-Q-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

Thus if $y_i < i < P < j < y_j < Q$ then one of the following holds:

- $w^{-1} = -i-y_i-y_j-j-Q-P-$ and $(wt)^{-1} = -j-y_i-y_j-i-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = -i-Q-$ and $(wt)^{-1} = -y_i-Q-$.

9. Suppose $y_i < P < i < j < y_j < Q$.

   (a) If $w^{-1} = -i-y_i-Q-y_j-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (b) If $w^{-1} = -i-y_i-y_j-Q-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (c) If $w^{-1} = -i-y_i-Q-y_j-P-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (d) If $w^{-1} = -Q-i-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = -i-Q-P-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = -Q-i-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = -Q-i-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = -i-Q-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = -Q-P-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = -i-Q-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = -Q-i-P-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = -i-Q-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   Thus if $y_i < P < i < j < y_j < Q$ then one of the following holds:

   - $w^{-1} = -i-y_i-y_j-Q-P-j-$ and $(wt)^{-1} = -j-y_i-y_j-Q-P-i-$.
   - $w^{-1} = -i-y_i-y_j-j-Q-P-$ and $(wt)^{-1} = -j-y_i-y_j-i-Q-P-$.
   - $w^{-1} = -i-y_i-Q-P-y_j-j-$ and $(wt)^{-1} = -j-y_i-Q-P-y_j-i-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$

   (Z2) $\Leftrightarrow (wt)^{-1} \neq -Q-i-P-$ and $(wt)^{-1} \neq -Q-y_j-P-$.

   (Z3) $\Leftrightarrow (wt)^{-1} = -y_i-Q-$.

10. Suppose $y_i < i < j < y_j < P < Q$.

    (a) If $w^{-1} = -i-y_i-Q-P-y_j-j-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (b) If $w^{-1} = -i-y_i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (c) If $w^{-1} = -i-y_i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (d) If $w^{-1} = -i-y_i-y_j-Q-P-j-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (e) If $w^{-1} = -i-y_i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (f) If $w^{-1} = -Q-i-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (g) If $w^{-1} = -i-Q-P-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (h) If $w^{-1} = -Q-i-y_i-y_j-P-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (i) If $w^{-1} = -Q-i-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (j) If $w^{-1} = -i-Q-y_i-P-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (k) If $w^{-1} = -Q-P-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (l) If $w^{-1} = -i-Q-y_i-y_j-j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

(m) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(n) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.

Thus if $y_i < i < j < y_j < P < Q$ then one of the following holds:

- $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}y_j\text{—}i\text{—}Q\text{—}P\text{—}$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$.

11. Suppose $P < y_i < i < j < y_j < Q$.

(a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (j, y_j)$.

(b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (j, y_j)$.

(c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (j, y_j)$.

(d) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.

(e) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.

(f) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.

(g) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.

(h) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.

(i) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.

(j) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.

Thus if $P < y_i < i < j < y_j < Q$ then one of the following holds:

- $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}y_j\text{—}i\text{—}Q\text{—}P\text{—}$.
- $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}Q\text{—}P\text{—}y_i\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}y_j\text{—}Q\text{—}P\text{—}i\text{—}$.
- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}Q\text{—}P\text{—}y_j\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—}. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—} \text{ and } (wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—} \text{ and} \\ (wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—} \text{ and } (wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—}. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

12. Suppose $y_i < P < Q < i < j < y_j$.

(a) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(b) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(c) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(d) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (j, y_j)$.

(e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(f) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(g) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(h) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(k) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(l) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Thus if $y_i < P < Q < i < j < y_j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}j\text{—}y_i\text{—}P\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}Q\text{—}P\text{—}y_j\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_i,j),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}P\text{—}y_i\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}Q\text{—}y_i\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}y_j\text{—}$.

13. It cannot happen that $y_i < i < P < j < Q < y_j$ since:

(a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(k) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(l) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(m) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

14. It cannot happen that $y_i < i < P < Q < j < y_j$ since:

(a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.

(g) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(l) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(m) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(n) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

15. It cannot happen that $y_i < P < i < Q < j < y_j$ since:

(a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(k) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_i < i < j < y_j$.

9.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $y_{i'} < i' < j' < y_i < y_{j'} < i < j < y_j$.

(a) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (y_i,i)$.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (y_i,i)$.
(c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (y_i,i)$.
(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (y_i,i)$.
(e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (y_i,i)$.
(f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (y_i,i)$.
(g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (y_i,i)$.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (y_i,i)$.
(i) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
(j) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
(k) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
(l) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.

(m) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < i' < j' < y_i < y_{j'} < i < j < y_j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}y_{j'}\text{—}i'\text{—}j\text{—}y_i\text{—}y_j\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ and $(a',b') \in \{(y_{i'}, j'), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—}. \end{cases}$$

2. Suppose $y_{i'} < y_i < i' < j' < i < j < y_{j'} < y_j$.

(a) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(i) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(k) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(l) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(m) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < y_i < i' < j' < i < j < y_{j'} < y_j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}j\text{—}y_i\text{—}y_{j'}\text{—}i'\text{—}y_j\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ and $(a',b') \in \{(y_{i'}, j'), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—}. \end{cases}$$

3. Suppose $y_{i'} < i' < j' < y_{j'} < y_i < i < j < y_j$.

(a) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(k) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(l) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(m) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < i' < j' < y_{j'} < y_i < i < j < y_j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}y_{j'}\text{—}i'\text{—}j\text{—}y_i\text{—}y_j\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ and $(a',b') \in \{(y_{i'}, j'), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—}. \end{cases}$$

4. Suppose $y_{i'} < i' < j' < y_i < i < j < y_{j'} < y_j$.

(a) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (y_i, i)$.
(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
(e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
(f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
(g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(h) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(k) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < i' < j' < y_i < i < j < y_{j'} < y_j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}y_{j'}\text{—}i'\text{—}j\text{—}y_i\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}j\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}j\text{—}y_i\text{—}y_{j'}\text{—}i'\text{—}y_j\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ and $(a', b') \in \{(y_{i'}, j'), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_{i'}- \text{ and} \\ (wt)^{-1} = -y_j-i- \text{ and} \\ (wt)^{-1} = -y_{j'}-i'-. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq -y_{j'}-y_i-i'- \text{ and } (wt)^{-1} \neq -y_{j'}-j-i'-.$$

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_j-. \end{cases}$$

5. Suppose $y_{i'} < i' < j' < y_i < i < y_{j'} < j < y_j$.
   (a) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-y_j-j'-j-$ then (T) fails.
   (b) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-y_j-j'-j-$ then (T) fails.
   (c) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-y_j-j'-j-$ then (T) fails.
   (d) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-y_j-j'-j-$ then (T) fails.
   (e) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
   (f) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
   (g) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
   (h) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
   (i) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
   (j) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
   (k) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-j'-y_j-j-$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (y_i, i)$.
   (l) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-j'-y_i-y_j-j-$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (y_i, i)$.
   (m) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.

   Thus if $y_{i'} < i' < j' < y_i < i < y_{j'} < j < y_j$ then one of the following holds:

   - $w^{-1} = -i'-y_{i'}-y_{j'}-j'-i-y_i-y_j-j-$ and $(wt)^{-1} = -j'-y_{i'}-y_{j'}-i'-j-y_i-y_j-i-$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ and $(a', b') \in \{(y_{i'}, j'), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_{i'}- \text{ and} \\ (wt)^{-1} = -y_j-i- \text{ and} \\ (wt)^{-1} = -y_{j'}-i'-. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_j-. \end{cases}$$

6. Suppose $y_{i'} < y_i < i' < j' < y_{j'} < i < j < y_j$.
   (a) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-j'-y_j-j-$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (j', y_{j'})$.
   (b) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-j'-y_i-y_j-j-$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (j', y_{j'})$.
   (c) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-j'-y_i-y_j-j-$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (j', y_{j'})$.
   (d) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.

(e) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
(f) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
(g) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-y_j-j'-j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(h) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-j'-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(k) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-j'-y_j-j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < y_i < i' < j' < y_{j'} < i < j < y_j$ then one of the following holds:

- $w^{-1} = -i'-y_{i'}-y_{j'}-j'-i-y_i-y_j-j-$ and $(wt)^{-1} = -j'-y_{i'}-y_{j'}-i'-j-y_i-y_j-i-$.
- $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-j'-y_j-j-$ and $(wt)^{-1} = -j'-y_{i'}-y_{j'}-j-y_i-i'-y_j-i-$.
- $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-j'-y_j-j-$ and $(wt)^{-1} = -j'-y_{i'}-j-y_i-y_{j'}-i'-y_j-i-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ and $(a',b') \in \{(y_{i'}, j'), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_{i'}- \text{ and} \\ (wt)^{-1} = -y_j-i- \text{ and} \\ (wt)^{-1} = -y_{j'}-i'-. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq -j-i'-y_i- \text{ and } (wt)^{-1} \neq -j-y_{j'}-y_i-.$$

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_j-. \end{cases}$$

7. Suppose $y_{i'} < i' < y_i < j' < y_{j'} < i < j < y_j$.

(a) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-y_j-j'-j-$ then (U) fails.
(b) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-y_j-j'-j-$ then (U) fails.
(c) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-y_j-j'-j-$ then (U) fails.
(d) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-y_j-j'-j-$ then (U) fails.
(e) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-y_j-j'-j-$ then (U) fails.
(f) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-j'-y_j-j-$ then (U) fails.
(g) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-j'-y_j-j-$ then (U) fails.
(h) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-j'-y_j-j-$ then (U) fails.
(i) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-j'-y_j-j-$ then (U) fails.
(j) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-j'-y_j-j-$ then (U) fails.
(k) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-j'-y_i-y_j-j-$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(l) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-j'-y_i-y_j-j-$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(m) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-j'-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < i' < y_i < j' < y_{j'} < i < j < y_j$ then one of the following holds:

- $w^{-1} = -i'-y_{i'}-y_{j'}-j'-i-y_i-y_j-j-$ and $(wt)^{-1} = -j'-y_{i'}-y_{j'}-i'-j-y_i-y_j-i-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_i, j), (i, y_j)\}$ and $(a',b') \in \{(y_{i'}, j'), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_{i'}- \text{ and} \\ (wt)^{-1} = -y_j-i- \text{ and} \\ (wt)^{-1} = -y_{j'}-i'-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—}. \end{cases}$$

8. It cannot happen that $y_{i'} < y_i < i' < i < j' < j < y_{j'} < y_j$ since:
   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (n) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.

9. It cannot happen that $y_{i'} < i' < y_i < j' < i < j < y_{j'} < y_j$ since:
   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (U) fails.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (U) fails.
   (c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (U) fails.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (U) fails.
   (e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (U) fails.
   (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (U) fails.
   (g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (U) fails.
   (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (U) fails.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (U) fails.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (U) fails.
   (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
   (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
   (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
   (n) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

10. It cannot happen that $y_{i'} < y_i < i' < i < j' < y_{j'} < j < y_j$ since:
    (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.

(g) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
(i) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
(j) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
(k) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
(l) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-j'-y_j-j-$ then (T) fails.
(m) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-j'-y_i-y_j-j-$ then (T) fails.
(n) If $w^{-1} = -i'-y_{i'}-y_{j'}-j'-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.

11. It cannot happen that $y_{i'} < i' < y_i < j' < i < y_{j'} < j < y_j$ since:

(a) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-y_j-j'-j-$ then (T) fails.
(b) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-y_j-j'-j-$ then (T) fails.
(c) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-y_j-j'-j-$ then (T) fails.
(d) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-y_j-j'-j-$ then (T) fails.
(e) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
(f) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
(i) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
(j) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
(k) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-y_j-j'-j-$ then (U) fails.
(l) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-j'-y_j-j-$ then (U) fails.
(m) If $w^{-1} = -i'-y_{i'}-y_{j'}-j'-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(n) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-j'-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.

12. It cannot happen that $y_{i'} < i' < y_i < i < j' < j < y_{j'} < y_j$ since:

(a) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-y_j-j'-j-$ then (T) fails.
(b) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-y_j-j'-j-$ then (T) fails.
(c) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-y_j-j'-j-$ then (T) fails.
(d) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-y_j-j'-j-$ then (T) fails.
(e) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-y_j-j'-j-$ then (T) fails.
(f) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
(i) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
(j) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
(k) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
(l) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-j'-y_j-j-$ then (T) fails.
(m) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-j'-y_i-y_j-j-$ then (T) fails.
(n) If $w^{-1} = -i'-y_{i'}-y_{j'}-j'-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.

13. It cannot happen that $y_{i'} < i' < y_i < i < j' < y_{j'} < j < y_j$ since:

(a) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-y_j-j'-j-$ then (T) fails.
(b) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-y_j-j'-j-$ then (T) fails.
(c) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-y_j-j'-j-$ then (T) fails.

(d) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-y_j-j'-j-$ then (T) fails.
(e) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-y_j-j'-j-$ then (T) fails.
(f) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
(i) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
(j) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
(k) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
(l) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-j'-y_j-j-$ then (T) fails.
(m) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-j'-y_i-y_j-j-$ then (T) fails.
(n) If $w^{-1} = -i'-y_{i'}-y_{j'}-j'-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.

14. It cannot happen that $y_{i'} < y_i < i' < j' < i < y_{j'} < j < y_j$ since:

(a) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-y_j-j'-j-$ then (T) fails.
(b) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-y_j-j'-j-$ then (T) fails.
(c) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-y_j-j'-j-$ then (T) fails.
(d) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-y_j-j'-j-$ then (T) fails.
(e) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
(f) If $w^{-1} = -i'-i-y_{i'}-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -i'-i-y_{i'}-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
(h) If $w^{-1} = -i'-y_{i'}-i-y_i-y_{j'}-j'-y_j-j-$ then (T) fails.
(i) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-y_i-j'-y_j-j-$ then (T) fails.
(j) If $w^{-1} = -i'-y_{i'}-i-y_{j'}-j'-y_i-y_j-j-$ then (T) fails.
(k) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(l) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-y_i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(m) If $w^{-1} = -i'-y_{i'}-y_{j'}-j'-i-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.
(n) If $w^{-1} = -i'-y_{i'}-y_{j'}-i-j'-y_i-y_j-j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (j', y_{j'})$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(z)$ when $y_i < i < j < y_j$.

10 Case: $y_i < i < y_j < j$

Suppose y is such that $y_i < i < y_j < j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(y_j, y_j), (y_i, j), (i, i)\}$.

10.1 Subcase (i)

We must have $w^{-1} = -i-y_i-j-y_j-$ since no other ordering is possible:

1. If $w^{-1} = -y_i-i-y_j-j-$ then (T) fails.
2. If $w^{-1} = -y_i-y_j-j-i-$ then (T) fails.
3. If $w^{-1} = -y_i-j-i-y_j-$ then (T) fails.
4. If $w^{-1} = -y_i-j-y_j-i-$ then (T) fails.
5. If $w^{-1} = -i-y_i-y_j-j-$ then (T) fails.
6. If $w^{-1} = -i-y_j-y_i-j-$ then (T) fails.
7. If $w^{-1} = -i-y_j-j-y_i-$ then (T) fails.

8. If $w^{-1} = —y_j—y_i—j—i—$ then (T) fails.
9. If $w^{-1} = —y_j—j—y_i—i—$ then (T) fails.
10. If $w^{-1} = —y_j—j—i—y_i—$ then (T) fails.
11. If $w^{-1} = —j—y_i—i—y_j—$ then (T) fails.
12. If $w^{-1} = —j—y_i—y_j—i—$ then (T) fails.
13. If $w^{-1} = —j—i—y_i—y_j—$ then (T) fails.
14. If $w^{-1} = —j—i—y_j—y_i—$ then (T) fails.
15. If $w^{-1} = —j—y_j—y_i—i—$ then (T) fails.
16. If $w^{-1} = —j—y_j—i—y_i—$ then (T) fails.
17. If $w^{-1} = —y_i—i—j—y_j—$ then (Y1) fails for $(a,b) = (y_i, i)$.
18. If $w^{-1} = —y_i—y_j—i—j—$ then (Y1) fails for $(a,b) = (y_i, i)$.
19. If $w^{-1} = —y_j—y_i—i—j—$ then (Y1) fails for $(a,b) = (y_j, j)$.
20. If $w^{-1} = —y_j—i—y_i—j—$ then (Y1) fails for $(a,b) = (y_j, j)$.
21. If $w^{-1} = —y_j—i—j—y_i—$ then (Y1) fails for $(a,b) = (y_j, j)$.
22. If $w^{-1} = —i—j—y_i—y_j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_j, j)$.
23. If $w^{-1} = —i—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_j, j)$.

Hence if $y_i < i < y_j < j$ then

$$(wt)^{-1} = —j—y_i—i—y_j—.$$

When $(a,b), (a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (y_i, j), (i, i)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow (wt)^{-1} = —j—y_i—.$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —j—i—y_i—$ and $(wt)^{-1} \neq —j—y_j—y_i—.$

(Z3) $\Leftrightarrow (wt)^{-1} = —i—y_j—.$

Thus properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in case (i) and $y_i < i < y_j < j$.

10.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $y_i < i < y_j < j < R$.
   (a) If $w^{-1} = —R—i—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (R, R)$.
   (b) If $w^{-1} = —i—R—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (R, R)$.
   (c) If $w^{-1} = —i—y_i—j—R—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (R, R)$.
   (d) If $w^{-1} = —i—y_i—R—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (R, R)$.

   Thus if $y_i < i < y_j < j < R$ then one of the following holds:

   - $w^{-1} = —i—y_i—j—y_j—R—$ and $(wt)^{-1} = —j—y_i—i—y_j—R—.$

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (y_i, j), (i, i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —j—y_i—.$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}R\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}R\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}R\text{—}. \end{cases}$

2. Suppose $y_i < i < y_j < R < j$.

   (a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (T) fails.

   (b) If $w^{-1} = \text{—}i\text{—}R\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.

   (c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (R,R)$.

   (d) If $w^{-1} = \text{—}R\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (R,R)$.

   Thus if $y_i < i < y_j < R < j$ then one of the following holds:

   - $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.

   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}R\text{—}y_i\text{—}$.

   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}R\text{—}$.

3. Suppose $y_i < R < i < y_j < j$.

   (a) If $w^{-1} = \text{—}i\text{—}R\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (R,R)$.

   (b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j, j)$.

   (c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j, j)$.

   Thus if $y_i < R < i < y_j < j$ then one of the following holds:

   - $w^{-1} = \text{—}i\text{—}y_i\text{—}R\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}R\text{—}i\text{—}y_j\text{—}$.
   - $w^{-1} = \text{—}R\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.

   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}R\text{—}y_i\text{—}$.

   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}R\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_j\text{—}$.

4. Suppose $R < y_i < i < y_j < j$.

   (a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

   (b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

   (c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

   (d) If $w^{-1} = \text{—}i\text{—}R\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

   Thus if $R < y_i < i < y_j < j$ then one of the following holds:

   - $w^{-1} = \text{—}R\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.

   (Z2) $\Leftrightarrow$ (no condition).

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -R-i- \text{ and} \\ (wt)^{-1} = -R-j- \text{ and} \\ (wt)^{-1} = -R-y_j-. \end{cases}$$

5. It cannot happen that $y_i < i < R < y_j < j$ since:
   (a) If $w^{-1} = -i-y_i-R-j-y_j-$ then (T) fails.
   (b) If $w^{-1} = -i-R-y_i-j-y_j-$ then (T) fails.
   (c) If $w^{-1} = -i-y_i-j-y_j-R-$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j, j)$.
   (d) If $w^{-1} = -i-y_i-j-R-y_j-$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j, j)$.
   (e) If $w^{-1} = -R-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (R,R)$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $P < y_i < i < Q < y_j < j$.
   (a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
   (b) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (T) fails.
   (c) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
   (d) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
   (e) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (T) fails.
   (f) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (T) fails.
   (g) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (T) fails.
   (h) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (j) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (k) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (l) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (m) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (n) If $w^{-1} = -i-y_i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.

   Thus if $P < y_i < i < Q < y_j < j$ then one of the following holds:

   - $w^{-1} = -Q-P-i-y_i-j-y_j-$ and $(wt)^{-1} = -Q-P-j-y_i-i-y_j-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (y_i, j), (i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   $(\text{Z1}) \Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-y_i-$.
   $(\text{Z2}) \Leftrightarrow (wt)^{-1} \neq -Q-i-P-$.
   $(\text{Z3}) \Leftrightarrow (wt)^{-1} = -P-j-$ and $(wt)^{-1} = -P-y_j-$.

2. Suppose $P < y_i < Q < i < y_j < j$.
   (a) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (b) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (c) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (d) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (e) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (f) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(i) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(j) If $w^{-1} = -i-y_i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(k) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(l) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(m) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(n) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

Thus if $P < y_i < Q < i < y_j < j$ then one of the following holds:

- $w^{-1} = -Q-P-i-y_i-j-y_j-$ and $(wt)^{-1} = -Q-P-j-y_i-i-y_j-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-y_i-$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = -P-i- \text{ and} \\ (wt)^{-1} = -P-j- \text{ and} \\ (wt)^{-1} = -P-y_j-. \end{cases}$

3. Suppose $y_i < i < y_j < P < j < Q$.

(a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
(b) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
(c) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (T) fails.
(d) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

Thus if $y_i < i < y_j < P < j < Q$ then one of the following holds:

- $w^{-1} = -i-y_i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-y_i-i-y_j-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-y_i-$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = -i-Q- \text{ and} \\ (wt)^{-1} = -y_i-Q- \text{ and} \\ (wt)^{-1} = -y_j-Q-. \end{cases}$

4. Suppose $P < y_i < i < y_j < Q < j$.

(a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.

(b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(i) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(j) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(k) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(l) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(m) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(n) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

Thus if $P < y_i < i < y_j < Q < j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}j\text{—}$.

5. Suppose $P < Q < y_i < i < y_j < j$.

(a) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(d) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(e) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(f) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(g) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(h) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(i) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(j) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(k) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(l) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(m) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(n) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

Thus if $P < Q < y_i < i < y_j < j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
(Z2) $\Leftrightarrow$ (no condition).

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -P-i- \text{ and} \\ (wt)^{-1} = -P-j- \text{ and} \\ (wt)^{-1} = -P-y_j-. \end{cases}$$

6. Suppose $y_i < i < y_j < P < Q < j$.

   (a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
   (b) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (T) fails.
   (c) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
   (d) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (T) fails.
   (e) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
   (f) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (T) fails.
   (g) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (T) fails.
   (h) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (T) fails.
   (i) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (T) fails.
   (j) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (k) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (l) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (m) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (n) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.

   Thus if $y_i < i < y_j < P < Q < j$ then one of the following holds:

   - $w^{-1} = -i-y_i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-y_i-i-y_j-Q-P-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (y_i, j), (i, i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   $(\text{Z1}) \Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-y_i-$.
   $(\text{Z2}) \Leftrightarrow (wt)^{-1} \neq -j-P-y_i-$ and $(wt)^{-1} \neq -j-Q-y_i-$.
   $(\text{Z3}) \Leftrightarrow (wt)^{-1} = -i-Q-$ and $(wt)^{-1} = -y_j-Q-$.

7. Suppose $y_i < i < P < y_j < j < Q$.

   (a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
   (b) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
   (c) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (T) fails.
   (d) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
   (e) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (T) fails.
   (f) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (g) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (h) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (i) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (j) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (k) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (l) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (m) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.

   Thus if $y_i < i < P < y_j < j < Q$ then one of the following holds:

   - $w^{-1} = -i-y_i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-y_i-i-y_j-Q-P-$.
   - $w^{-1} = -i-y_i-j-Q-P-y_j-$ and $(wt)^{-1} = -j-y_i-i-Q-P-y_j-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$.

8. Suppose $y_i < P < i < y_j < j < Q$.
   (a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (d) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   Thus if $y_i < P < i < y_j < j < Q$ then one of the following holds:

   - $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}$.
   - $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}$.
   - $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}$.
   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$.

9. Suppose $y_i < i < y_j < j < P < Q$.
   (a) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

Thus if $y_i < i < y_j < j < P < Q$ then one of the following holds:

- $w^{-1} = -i-y_i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-y_i-i-y_j-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-y_i-$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = -i-Q- \text{ and} \\ (wt)^{-1} = -y_i-Q- \text{ and} \\ (wt)^{-1} = -y_j-Q-. \end{cases}$

10. Suppose $P < y_i < i < y_j < j < Q$.

(a) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(b) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(c) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(d) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(e) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(f) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(g) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(h) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(i) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(j) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

Thus if $P < y_i < i < y_j < j < Q$ then one of the following holds:

- $w^{-1} = -i-y_i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-y_i-i-y_j-Q-P-$.
- $w^{-1} = -i-Q-P-y_i-j-y_j-$ and $(wt)^{-1} = -j-Q-P-y_i-i-y_j-$.
- $w^{-1} = -i-y_i-j-Q-P-y_j-$ and $(wt)^{-1} = -j-y_i-i-Q-P-y_j-$.
- $w^{-1} = -Q-P-i-y_i-j-y_j-$ and $(wt)^{-1} = -Q-P-j-y_i-i-y_j-$.
- $w^{-1} = -i-y_i-Q-P-j-y_j-$ and $(wt)^{-1} = -j-y_i-Q-P-i-y_j-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-y_i-$.

(Z2) $\Leftrightarrow \begin{cases} (wt)^{-1} \neq -Q-i-P- \text{ and} \\ (wt)^{-1} \neq -Q-y_i-P- \text{ and } (wt)^{-1} \neq -Q-j-P- \text{ and} \\ (wt)^{-1} \neq -Q-y_j-P-. \end{cases}$

(Z3) $\Leftrightarrow$ (no condition).

11. Suppose $y_i < P < Q < i < y_j < j$.

(a) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(e) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(f) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(g) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

(h) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(i) If $w^{-1} = -i-y_i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(j) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(l) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.

Thus if $y_i < P < Q < i < y_j < j$ then one of the following holds:

- $w^{-1} = -Q-P-i-y_i-j-y_j-$ and $(wt)^{-1} = -Q-P-j-y_i-i-y_j-$.
- $w^{-1} = -Q-i-y_i-P-j-y_j-$ and $(wt)^{-1} = -Q-j-y_i-P-i-y_j-$.
- $w^{-1} = -i-y_i-Q-P-j-y_j-$ and $(wt)^{-1} = -j-y_i-Q-P-i-y_j-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (y_i, j), (i,i)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -Q-P-$ and $(wt)^{-1} = -j-y_i-$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq -j-P-y_i-$ and $(wt)^{-1} \neq -j-Q-y_i-$.
(Z3) $\Leftrightarrow (wt)^{-1} = -P-i-$ and $(wt)^{-1} = -P-y_j-$.

12. It cannot happen that $y_i < i < P < y_j < Q < j$ since:

(a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
(b) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (T) fails.
(c) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
(d) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (T) fails.
(g) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (T) fails.
(h) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (T) fails.
(i) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (T) fails.
(j) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(l) If $w^{-1} = -i-y_i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(m) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.

13. It cannot happen that $y_i < P < i < y_j < Q < j$ since:

(a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
(b) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (T) fails.
(c) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
(d) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (T) fails.
(f) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (T) fails.
(g) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (T) fails.
(h) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(i) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(j) If $w^{-1} = -i-y_i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.

(l) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.

14. It cannot happen that $y_i < i < P < Q < y_j < j$ since:

(a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
(b) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (T) fails.
(c) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
(d) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (T) fails.
(g) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (T) fails.
(h) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (T) fails.
(i) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (T) fails.
(j) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(l) If $w^{-1} = -i-y_i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(m) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.

15. It cannot happen that $y_i < P < i < Q < y_j < j$ since:

(a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
(b) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (T) fails.
(c) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
(d) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
(e) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (T) fails.
(f) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (T) fails.
(g) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (T) fails.
(h) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(i) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(j) If $w^{-1} = -i-y_i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_i < i < y_j < j$.

10.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $y_{i'} < i' < y_{j'} < y_i < j' < i < y_j < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.
   (d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.
   (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

   Thus if $y_{i'} < i' < y_{j'} < y_i < j' < i < y_j < j$ then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (y_i, j), (i, i)\}$ and $(a',b') \in \{(y_{j'}, y_{j'}), (y_{i'}, j'), (i', i')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$
   $$\begin{cases} (wt)^{-1} = \text{—}i'\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}. \end{cases}$$

2. Suppose $y_{i'} < i' < y_{j'} < j' < y_i < i < y_j < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (b) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < i' < y_{j'} < j' < y_i < i < y_j < j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (y_i, j), (i, i)\}$ and $(a',b') \in \{(y_{j'}, y_{j'}), (y_{i'}, j'), (i', i')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = \text{—}i'\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}. \end{cases}$$

3. Suppose $y_{i'} < i' < y_{j'} < y_i < i < y_j < j' < j$.

(a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Thus if $y_{i'} < i' < y_{j'} < y_i < i < y_j < j' < j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (y_i, j), (i, i)\}$ and $(a',b') \in \{(y_{j'}, y_{j'}), (y_{i'}, j'), (i', i')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}$.
(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j'\text{—}i\text{—}y_{i'}\text{—}$ and $(wt)^{-1} \neq \text{—}j'\text{—}y_j\text{—}y_{i'}\text{—}$.
(Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = \text{—}i'\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}. \end{cases}$$

4. Suppose $y_{i'} < i' < y_{j'} < y_i < i < j' < y_j < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Thus if $y_{i'} < i' < y_{j'} < y_i < i < j' < y_j < j$ then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_j), (y_i, j), (i, i)\}$ and $(a',b') \in \{(y_{j'}, y_{j'}), (y_{i'}, j'), (i', i')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j'\text{—}i\text{—}y_{i'}\text{—}$.
   (Z3) $\Leftrightarrow$
   $$\begin{cases} (wt)^{-1} = \text{—}i'\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}. \end{cases}$$

5. Suppose $y_{i'} < y_i < i' < y_{j'} < j' < i < y_j < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
   (c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
   (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Thus if $y_{i'} < y_i < i' < y_{j'} < j' < i < y_j < j$ then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

- $w^{-1} = -i'-y_{i'}-j'-i-y_i-y_{j'}-j-y_j-$ and $(wt)^{-1} = -j'-y_{i'}-i'-j-y_i-y_{j'}-i-y_j-$.
- $w^{-1} = -i'-y_{i'}-i-y_i-j'-y_{j'}-j-y_j-$ and $(wt)^{-1} = -j'-y_{i'}-j-y_i-i'-y_{j'}-i-y_j-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ and $(a',b') \in \{(y_{j'},y_{j'}),(y_{i'},j'),(i',i')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = -j-y_i-$ and $(wt)^{-1} = -j'-y_{i'}-$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq -j-i'-y_i-$ and $(wt)^{-1} \neq -j-y_{j'}-y_i-$.

(Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = -i'-i- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_j- \text{ and} \\ (wt)^{-1} = -y_{j'}-i- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_j-. \end{cases}$$

6. Suppose $y_{i'} < i' < y_i < y_{j'} < j' < i < y_j < j$.
   (a) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-j-y_{j'}-y_j-$ then (U) fails.
   (b) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-j-y_{j'}-y_j-$ then (U) fails.
   (c) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-y_{j'}-j-y_j-$ then (U) fails.
   (d) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-y_{j'}-j-y_j-$ then (U) fails.
   (e) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-y_{j'}-j-y_j-$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (y_{j'},j')$.
   (f) If $w^{-1} = -i'-y_{i'}-i-j'-y_{j'}-y_i-j-y_j-$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (y_{j'},j')$.
   (g) If $w^{-1} = -i'-y_{i'}-j'-i-y_{j'}-y_i-j-y_j-$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (y_{j'},j')$.
   (h) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (j) If $w^{-1} = -i'-i-y_{i'}-j'-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (k) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.
   (l) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.

   Thus if $y_{i'} < i' < y_i < y_{j'} < j' < i < y_j < j$ then one of the following holds:

   - $w^{-1} = -i'-y_{i'}-j'-i-y_i-y_{j'}-j-y_j-$ and $(wt)^{-1} = -j'-y_{i'}-i'-j-y_i-y_{j'}-i-y_j-$.
   - $w^{-1} = -i'-y_{i'}-j'-y_{j'}-i-y_i-j-y_j-$ and $(wt)^{-1} = -j'-y_{i'}-i'-y_{j'}-j-y_i-i-y_j-$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j,y_j),(y_i,j),(i,i)\}$ and $(a',b') \in \{(y_{j'},y_{j'}),(y_{i'},j'),(i',i')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = -j-y_i-$ and $(wt)^{-1} = -j'-y_{i'}-$.

   (Z2) $\Leftrightarrow (wt)^{-1} \neq -j-y_{j'}-y_i-$.

   (Z3) $\Leftrightarrow$
$$\begin{cases} (wt)^{-1} = -i'-i- \text{ and} \\ (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_j- \text{ and} \\ (wt)^{-1} = -y_{j'}-i- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_j-. \end{cases}$$

7. It cannot happen that $y_{i'} < y_i < i' < i < y_{j'} < y_j < j' < j$ since:

(a) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-j-y_{j'}-y_j-$ then (T) fails.
(b) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-j-y_{j'}-y_j-$ then (T) fails.
(c) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-j-y_{j'}-y_j-$ then (T) fails.
(d) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-j-y_{j'}-y_j-$ then (T) fails.
(e) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-y_{j'}-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -i'-i-y_{i'}-j'-y_{j'}-y_i-j-y_j-$ then (T) fails.
(g) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-y_{j'}-j-y_j-$ then (T) fails.
(h) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-y_{j'}-j-y_j-$ then (T) fails.
(i) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-y_{j'}-j-y_j-$ then (T) fails.
(j) If $w^{-1} = -i'-y_{i'}-i-j'-y_{j'}-y_i-j-y_j-$ then (T) fails.
(k) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-y_{j'}-j-y_j-$ then (T) fails.
(l) If $w^{-1} = -i'-y_{i'}-j'-i-y_{j'}-y_i-j-y_j-$ then (T) fails.
(m) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(n) If $w^{-1} = -i'-y_{i'}-j'-y_{j'}-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.

8. It cannot happen that $y_{i'} < i' < y_i < y_{j'} < i < y_j < j' < j$ since:

(a) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-j-y_{j'}-y_j-$ then (T) fails.
(b) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-j-y_{j'}-y_j-$ then (T) fails.
(c) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-j-y_{j'}-y_j-$ then (T) fails.
(d) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-j-y_{j'}-y_j-$ then (T) fails.
(e) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-y_{j'}-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -i'-i-y_{i'}-j'-y_{j'}-y_i-j-y_j-$ then (T) fails.
(g) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-y_{j'}-j-y_j-$ then (T) fails.
(h) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-y_{j'}-j-y_j-$ then (T) fails.
(i) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-y_{j'}-j-y_j-$ then (T) fails.
(j) If $w^{-1} = -i'-y_{i'}-i-j'-y_{j'}-y_i-j-y_j-$ then (T) fails.
(k) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(l) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(m) If $w^{-1} = -i'-y_{i'}-j'-y_{j'}-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(n) If $w^{-1} = -i'-y_{i'}-j'-i-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.

9. It cannot happen that $y_{i'} < y_i < i' < i < y_{j'} < j' < y_j < j$ since:

(a) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-j-y_{j'}-y_j-$ then (T) fails.
(b) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-j-y_{j'}-y_j-$ then (T) fails.
(c) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-j-y_{j'}-y_j-$ then (T) fails.
(d) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-j-y_{j'}-y_j-$ then (T) fails.
(e) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-y_{j'}-j-y_j-$ then (T) fails.
(f) If $w^{-1} = -i'-i-y_{i'}-j'-y_{j'}-y_i-j-y_j-$ then (T) fails.
(g) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-y_{j'}-j-y_j-$ then (T) fails.
(h) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-y_{j'}-j-y_j-$ then (T) fails.
(i) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-y_{j'}-j-y_j-$ then (T) fails.
(j) If $w^{-1} = -i'-y_{i'}-i-j'-y_{j'}-y_i-j-y_j-$ then (T) fails.
(k) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-y_{j'}-j-y_j-$ then (T) fails.
(l) If $w^{-1} = -i'-y_{i'}-j'-i-y_{j'}-y_i-j-y_j-$ then (T) fails.
(m) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.

(n) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.

10. It cannot happen that $y_{i'} < i' < y_i < y_{j'} < i < j' < y_j < j$ since:

(a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(n) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.

11. It cannot happen that $y_{i'} < y_i < i' < y_{j'} < i < y_j < j' < j$ since:

(a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(n) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.

12. It cannot happen that $y_{i'} < i' < y_i < i < y_{j'} < y_j < j' < j$ since:

(a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.

(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
(n) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.

13. It cannot happen that $y_{i'} < i' < y_i < i < y_{j'} < j' < y_j < j$ since:

(a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
(n) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.

14. It cannot happen that $y_{i'} < y_i < i' < y_{j'} < i < j' < y_j < j$ since:

(a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
(n) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(z)$ when $y_i < i < y_j < j$.

11 Case: $i < j < y_i < y_j$

Suppose y is such that $i < j < y_i < y_j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$.

11.1 Subcase (i)

We must have $w^{-1} = —y_i—i—y_j—j—$ since no other ordering is possible:

1. If $w^{-1} = —j—i—y_i—y_j—$ then (T) fails.
2. If $w^{-1} = —j—i—y_j—y_i—$ then (T) fails.
3. If $w^{-1} = —j—y_i—i—y_j—$ then (T) fails.
4. If $w^{-1} = —j—y_i—y_j—i—$ then (T) fails.
5. If $w^{-1} = —j—y_j—i—y_i—$ then (T) fails.
6. If $w^{-1} = —j—y_j—y_i—i—$ then (T) fails.
7. If $w^{-1} = —y_i—j—i—y_j—$ then (T) fails.
8. If $w^{-1} = —y_i—j—y_j—i—$ then (T) fails.
9. If $w^{-1} = —y_i—y_j—j—i—$ then (T) fails.
10. If $w^{-1} = —y_j—j—i—y_i—$ then (T) fails.
11. If $w^{-1} = —y_j—j—y_i—i—$ then (T) fails.
12. If $w^{-1} = —y_j—y_i—j—i—$ then (T) fails.
13. If $w^{-1} = —i—j—y_i—y_j—$ then (Y1) fails for $(a, b) = (i, y_i)$.
14. If $w^{-1} = —i—j—y_j—y_i—$ then (Y1) fails for $(a, b) = (i, y_i)$.
15. If $w^{-1} = —i—y_i—j—y_j—$ then (Y1) fails for $(a, b) = (i, y_i)$.
16. If $w^{-1} = —i—y_i—y_j—j—$ then (Y1) fails for $(a, b) = (i, y_i)$.
17. If $w^{-1} = —i—y_j—j—y_i—$ then (Y1) fails for $(a, b) = (i, y_i)$.
18. If $w^{-1} = —i—y_j—y_i—j—$ then (Y1) fails for $(a, b) = (i, y_i)$.
19. If $w^{-1} = —y_j—i—j—y_i—$ then (Y1) fails for $(a, b) = (i, y_i)$.
20. If $w^{-1} = —y_j—i—y_i—j—$ then (Y1) fails for $(a, b) = (i, y_i)$.
21. If $w^{-1} = —y_i—i—j—y_j—$ then (Y1) fails for $(a, b) = (j, y_j)$.
22. If $w^{-1} = —y_i—y_j—i—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (j, y_j)$.
23. If $w^{-1} = —y_j—y_i—i—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (j, y_j)$.

Hence if $i < j < y_i < y_j$ then

$$(wt)^{-1} = —y_i—j—y_j—i—.$$

When $(a, b), (a', b') \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow (wt)^{-1} = —y_i—j—$ and $(wt)^{-1} = —y_j—i—$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq —y_j—j—i—$ and $(wt)^{-1} \neq —y_j—y_i—i—$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (Z1)-(Z3) hold whenever (a, b), (a', b') are as in case (i) and $i < j < y_i < y_j$.

11.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $i < j < y_i < y_j < R$.
   (a) If $w^{-1} = —y_i—R—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —R—y_i—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —y_i—i—R—y_j—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = —y_i—i—y_j—R—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (R,R)$.

   Thus if $i < j < y_i < y_j < R$ then one of the following holds:
   - $w^{-1} = —y_i—i—y_j—j—R—$ and $(wt)^{-1} = —y_i—j—y_j—i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —y_i—j—$ and $(wt)^{-1} = —y_j—i—$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow (wt)^{-1} = —i—R—$ and $(wt)^{-1} = —j—R—$.

2. Suppose $i < j < y_i < R < y_j$.
   (a) If $w^{-1} = —y_i—i—y_j—R—j—$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —y_i—R—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —R—y_i—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.

   Thus if $i < j < y_i < R < y_j$ then one of the following holds:
   - $w^{-1} = —y_i—i—R—y_j—j—$ and $(wt)^{-1} = —y_i—j—R—y_j—i—$.
   - $w^{-1} = —y_i—i—y_j—j—R—$ and $(wt)^{-1} = —y_i—j—y_j—i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —y_i—j—$ and $(wt)^{-1} = —y_j—i—$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq —y_j—R—i—$.
   (Z3) $\Leftrightarrow (wt)^{-1} = —j—R—$.

3. Suppose $i < j < R < y_i < y_j$.
   (a) If $w^{-1} = —y_i—R—i—y_j—j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —y_i—i—y_j—R—j—$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a',b') = (R,R)$.

   Thus if $i < j < R < y_i < y_j$ then one of the following holds:
   - $w^{-1} = —y_i—i—R—y_j—j—$ and $(wt)^{-1} = —y_i—j—R—y_j—i—$.
   - $w^{-1} = —R—y_i—i—y_j—j—$ and $(wt)^{-1} = —R—y_i—j—y_j—i—$.
   - $w^{-1} = —y_i—i—y_j—j—R—$ and $(wt)^{-1} = —y_i—j—y_j—i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —y_i—j—$ and $(wt)^{-1} = —y_j—i—$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq —y_i—R—j—$ and $(wt)^{-1} \neq —y_j—R—i—$.
   (Z3) $\Leftrightarrow$ (no condition).

4. Suppose $i < R < j < y_i < y_j$.

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}R\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (R, R)$.
(d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (j, y_j)$.

Thus if $i < R < j < y_i < y_j$ then one of the following holds:

- $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}y_i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_j\text{—}R\text{—}i\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}R\text{—}y_i\text{—}$.

5. Suppose $R < i < j < y_i < y_j$.

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (i, y_i)$.
(b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (i, y_i)$.
(c) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}R\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (i, y_i)$.
(d) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (i, y_i)$.

Thus if $R < i < j < y_i < y_j$ then one of the following holds:

- $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}y_i\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}i\text{—}$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}R\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_j\text{—}$.

Next suppose $P < Q$ are integers with $(P, Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $P < i < j < Q < y_i < y_j$.

(a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(n) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.

Thus if $P < i < j < Q < y_i < y_j$ then one of the following holds:

- $w^{-1} = -Q-P-y_i-i-y_j-j-$ and $(wt)^{-1} = -Q-P-y_i-j-y_j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -y_i-j- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = -P-y_i-$ and $(wt)^{-1} = -P-y_j-$.

2. Suppose $P < i < Q < j < y_i < y_j$.

(a) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (T) fails.
(b) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (T) fails.
(c) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (T) fails.
(d) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (T) fails.
(e) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (T) fails.
(f) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = -y_i-i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(j) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(k) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(l) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(m) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(n) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Thus if $P < i < Q < j < y_i < y_j$ then one of the following holds:

- $w^{-1} = -Q-P-y_i-i-y_j-j-$ and $(wt)^{-1} = -Q-P-y_i-j-y_j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -y_i-j- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = -P-y_i-$ and $(wt)^{-1} = -P-y_j-$.

3. Suppose $i < j < y_i < P < y_j < Q$.

(a) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = -Q-P-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Thus if $i < j < y_i < P < y_j < Q$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}Q\text{—}P\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}Q\text{—}$.

4. Suppose $P < i < j < y_i < Q < y_j$.

(a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(h) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(k) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(l) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Thus if $P < i < j < y_i < Q < y_j$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}y_j\text{—}$.

5. Suppose $i < j < P < y_i < Q < y_j$.

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

(c) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = -Q-P-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

Thus if $i < j < P < y_i < Q < y_j$ then one of the following holds:

- $w^{-1} = -y_i-i-Q-P-y_j-j-$ and $(wt)^{-1} = -y_i-j-Q-P-y_j-i-$.
- $w^{-1} = -y_i-i-Q-y_j-j-P-$ and $(wt)^{-1} = -y_i-j-Q-y_j-i-P-$.
- $w^{-1} = -y_i-i-y_j-j-Q-P-$ and $(wt)^{-1} = -y_i-j-y_j-i-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -y_i-j- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq -y_j-P-i-$ and $(wt)^{-1} \neq -y_j-Q-i-$.
(Z3) $\Leftrightarrow (wt)^{-1} = -j-Q-$.

6. Suppose $P < Q < i < j < y_i < y_j$.

(a) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = -y_i-i-y_j-j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(j) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(k) If $w^{-1} = -y_i-i-Q-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(l) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(m) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(n) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Thus if $P < Q < i < j < y_i < y_j$ then one of the following holds:

- $w^{-1} = -Q-P-y_i-i-y_j-j-$ and $(wt)^{-1} = -Q-P-y_i-j-y_j-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -y_i-j- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = -P-y_i-$ and $(wt)^{-1} = -P-y_j-$.

7. Suppose $i < j < y_i < P < Q < y_j$.

   (a) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.
   (b) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.
   (c) If $w^{-1} = -y_i-i-y_j-Q-P-j-$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.
   (d) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (e) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (f) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (g) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (h) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (i) If $w^{-1} = -Q-P-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (j) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (k) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (l) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.

   Thus if $i < j < y_i < P < Q < y_j$ then one of the following holds:

   - $w^{-1} = -y_i-i-Q-P-y_j-j-$ and $(wt)^{-1} = -y_i-j-Q-P-y_j-i-$.
   - $w^{-1} = -y_i-i-Q-y_j-j-P-$ and $(wt)^{-1} = -y_i-j-Q-y_j-i-P-$.
   - $w^{-1} = -y_i-i-y_j-j-Q-P-$ and $(wt)^{-1} = -y_i-j-y_j-i-Q-P-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -y_i-j- \text{ and} \\ (wt)^{-1} = -y_j-i-. \end{cases}$

   (Z2) $\Leftrightarrow (wt)^{-1} \neq -y_j-P-i-$ and $(wt)^{-1} \neq -y_j-Q-i-$.

   (Z3) $\Leftrightarrow (wt)^{-1} = -j-Q-$.

8. Suppose $i < j < P < y_i < y_j < Q$.

   (a) If $w^{-1} = -Q-y_i-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (b) If $w^{-1} = -Q-y_i-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (c) If $w^{-1} = -y_i-Q-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (d) If $w^{-1} = -Q-y_i-i-P-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (e) If $w^{-1} = -y_i-Q-P-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (f) If $w^{-1} = -Q-P-y_i-i-y_j-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (g) If $w^{-1} = -Q-y_i-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (h) If $w^{-1} = -y_i-Q-i-y_j-j-P-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (i) If $w^{-1} = -y_i-Q-i-y_j-P-j-$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (j) If $w^{-1} = -y_i-i-Q-P-y_j-j-$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.
   (k) If $w^{-1} = -y_i-i-Q-y_j-P-j-$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.
   (l) If $w^{-1} = -y_i-i-y_j-Q-j-P-$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.

(m) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Thus if $i < j < P < y_i < y_j < Q$ then one of the following holds:

- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $(wt)^{-1} = —y_i—j—y_j—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—j— \text{ and} \\ (wt)^{-1} = —y_j—i—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —j—Q—$.

9. Suppose $i < P < j < y_i < y_j < Q$.

(a) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (T) fails.
(b) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (T) fails.
(c) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (T) fails.
(d) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (T) fails.
(e) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (T) fails.
(f) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (T) fails.
(g) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (T) fails.
(h) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = —Q—P—y_i—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

Thus if $i < P < j < y_i < y_j < Q$ then one of the following holds:

- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $(wt)^{-1} = —y_i—j—y_j—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—j— \text{ and} \\ (wt)^{-1} = —y_j—i—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —Q—j—P—$ and $(wt)^{-1} \neq —Q—y_i—P—$.
(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$.

10. Suppose $i < j < P < Q < y_i < y_j$.

(a) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

(f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Thus if $i < j < P < Q < y_i < y_j$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}i\text{—}P\text{—}$.
- $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}P\text{—}$.
- $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}Q\text{—}P\text{—}$.
- $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—}. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq \text{—}y_i\text{—}P\text{—}j\text{—} \text{ and } (wt)^{-1} \neq \text{—}y_i\text{—}Q\text{—}j\text{—} \text{ and} \\ (wt)^{-1} \neq \text{—}y_j\text{—}P\text{—}i\text{—} \text{ and } (wt)^{-1} \neq \text{—}y_j\text{—}Q\text{—}i\text{—}. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

11. Suppose $i < P < j < Q < y_i < y_j$.

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(k) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(l) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(m) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(n) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Thus if $i < P < j < Q < y_i < y_j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_j\text{—}P\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_j\text{—}Q\text{—}i\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}y_i\text{—}$.

12. Suppose $i < j < y_i < y_j < P < Q$.

(a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Thus if $i < j < y_i < y_j < P < Q$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}Q\text{—}P\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}Q\text{—}$.

13. Suppose $P < i < j < y_i < y_j < Q$.

(a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Thus if $P < i < j < y_i < y_j < Q$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}i\text{—}$.

- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $(wt)^{-1} = —y_i—j—y_j—i—Q—P—$.
- $w^{-1} = —Q—P—y_i—i—y_j—j—$ and $(wt)^{-1} = —Q—P—y_i—j—y_j—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—j— \text{ and} \\ (wt)^{-1} = —y_j—i—. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and } (wt)^{-1} \neq —Q—y_j—P— \text{ and} \\ (wt)^{-1} \neq —Q—j—P— \text{ and } (wt)^{-1} \neq —Q—y_i—P—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

14. Suppose $i < P < Q < j < y_i < y_j$.
    (a) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (T) fails.
    (b) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (T) fails.
    (c) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (T) fails.
    (d) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (T) fails.
    (e) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (T) fails.
    (f) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (T) fails.
    (g) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (T) fails.
    (h) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (T) fails.
    (i) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (T) fails.
    (j) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (k) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (l) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
    (m) If $w^{-1} = —y_i—i—y_j—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
    (n) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

    Thus if $i < P < Q < j < y_i < y_j$ then one of the following holds:

    - $w^{-1} = —Q—P—y_i—i—y_j—j—$ and $(wt)^{-1} = —Q—P—y_i—j—y_j—i—$.

    When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

    $$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—j— \text{ and} \\ (wt)^{-1} = —y_j—i—. \end{cases}$$

    (Z2) $\Leftrightarrow (wt)^{-1} \neq —y_j—P—i—$ and $(wt)^{-1} \neq —y_j—Q—i—$.
    (Z3) $\Leftrightarrow (wt)^{-1} = —P—y_i—$.

15. It cannot happen that $i < P < j < y_i < Q < y_j$ since:
    (a) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (T) fails.
    (b) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (T) fails.
    (c) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (T) fails.
    (d) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (T) fails.
    (e) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (T) fails.
    (f) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (T) fails.

(g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(k) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $i < j < y_i < y_j$.

11.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < j' < i < y_{i'} < j < y_i < y_{j'} < y_j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (j,y_j)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (j,y_j)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (j,y_j)$.

   Thus if $i' < j' < i < y_{i'} < j < y_i < y_{j'} < y_j$ then one of the following holds:

   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i'\text{—}j\text{—}y_j\text{—}i\text{—}$.
   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.
   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}i'\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ and $(a',b') \in \{(j',y_{i'}),(i',y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   $$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}. \end{cases}$$

   $$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq \text{—}y_{j'}\text{—}j\text{—}i'\text{—} \text{ and } (wt)^{-1} \neq \text{—}y_{j'}\text{—}y_i\text{—}i'\text{—}.$$

   $$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_j\text{—}. \end{cases}$$

2. Suppose $i' < j' < y_{i'} < i < y_{j'} < j < y_i < y_j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.

(c) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-y_j-j'-j-$ then (T) fails.
(d) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-y_j-j'-j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

Thus if $i' < j' < y_{i'} < i < y_{j'} < j < y_i < y_j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-y_{j'}-j'-y_i-i-y_j-j-$ and $(wt)^{-1} = -y_{i'}-j'-y_{j'}-i'-y_i-j-y_j-i-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ and $(a', b') \in \{(j', y_{i'}), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -y_i-j- \text{ and} \\ (wt)^{-1} = -y_j-i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j'- \text{ and} \\ (wt)^{-1} = -y_{j'}-i'-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -j'-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_j-. \end{cases}$$

3. Suppose $i' < i < j' < j < y_{i'} < y_{j'} < y_i < y_j$.

(a) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-j'-y_j-j-$ then (T) fails.
(b) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-j'-y_j-j-$ then (T) fails.
(c) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-y_j-j'-j-$ then (T) fails.
(d) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-j'-y_j-j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-y_j-j'-j-$ then (T) fails.
(f) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-j'-y_j-j-$ then (T) fails.
(g) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-y_j-j'-j-$ then (T) fails.
(h) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-j'-y_j-j-$ then (T) fails.
(i) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-y_j-j'-j-$ then (T) fails.
(j) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-y_j-j'-j-$ then (T) fails.
(k) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-j'-i-y_j-j-$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(l) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-j'-i-y_j-j-$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(m) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

Thus if $i' < i < j' < j < y_{i'} < y_{j'} < y_i < y_j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-y_{j'}-j'-y_i-i-y_j-j-$ and $(wt)^{-1} = -y_{i'}-j'-y_{j'}-i'-y_i-j-y_j-i-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ and $(a', b') \in \{(j', y_{i'}), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -y_i - j - \text{ and} \\ (wt)^{-1} = -y_j - i - \text{ and} \\ (wt)^{-1} = -y_{i'} - j' - \text{ and} \\ (wt)^{-1} = -y_{j'} - i' -. \end{cases}$$

$$(\text{Z2}) \Leftrightarrow (wt)^{-1} \neq -y_j - j' - i - \text{ and } (wt)^{-1} \neq -y_j - y_{i'} - i -.$$

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -i' - y_i - \text{ and} \\ (wt)^{-1} = -i' - y_j - \text{ and} \\ (wt)^{-1} = -j' - y_i -. \end{cases}$$

4. Suppose $i' < j' < i < y_{i'} < j < y_{j'} < y_i < y_j$.

   (a) If $w^{-1} = -y_{i'} - y_i - i' - y_{j'} - j' - i - y_j - j -$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (b) If $w^{-1} = -y_{i'} - y_i - i' - i - y_{j'} - j' - y_j - j -$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (c) If $w^{-1} = -y_{i'} - y_i - i' - y_{j'} - i - j' - y_j - j -$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (d) If $w^{-1} = -y_{i'} - y_i - i' - i - y_{j'} - y_j - j' - j -$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (e) If $w^{-1} = -y_{i'} - y_i - i' - y_{j'} - i - y_j - j' - j -$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = -y_{i'} - i' - y_i - i - y_{j'} - j' - y_j - j -$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = -y_{i'} - i' - y_i - y_{j'} - i - j' - y_j - j -$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = -y_{i'} - i' - y_{j'} - y_i - i - y_j - j' - j -$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = -y_{i'} - i' - y_i - i - y_{j'} - y_j - j' - j -$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (j) If $w^{-1} = -y_{i'} - i' - y_i - y_{j'} - i - y_j - j' - j -$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = -y_{i'} - i' - y_{j'} - y_i - i - j' - y_j - j -$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = -y_{i'} - i' - y_{j'} - y_i - j' - i - y_j - j -$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (m) If $w^{-1} = -y_{i'} - i' - y_i - y_{j'} - j' - i - y_j - j -$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

   Thus if $i' < j' < i < y_{i'} < j < y_{j'} < y_i < y_j$ then one of the following holds:

   - $w^{-1} = -y_{i'} - i' - y_{j'} - j' - y_i - i - y_j - j -$ and $(wt)^{-1} = -y_{i'} - j' - y_{j'} - i' - y_i - j - y_j - i -$.

   When $(a,b) \in \text{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ and $(a', b') \in \{(j', y_{i'}), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -y_i - j - \text{ and} \\ (wt)^{-1} = -y_j - i - \text{ and} \\ (wt)^{-1} = -y_{i'} - j' - \text{ and} \\ (wt)^{-1} = -y_{j'} - i' -. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -i' - y_i - \text{ and} \\ (wt)^{-1} = -i' - y_j - \text{ and} \\ (wt)^{-1} = -j' - y_i - \text{ and} \\ (wt)^{-1} = -j' - y_j -. \end{cases}$$

5. Suppose $i' < j' < i < j < y_{i'} < y_i < y_{j'} < y_j$.

   (a) If $w^{-1} = -y_{i'} - i' - y_i - y_{j'} - i - j' - y_j - j -$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (b) If $w^{-1} = -y_{i'} - i' - y_{j'} - y_i - i - j' - y_j - j -$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (c) If $w^{-1} = -y_{i'} - i' - y_{j'} - y_i - j' - i - y_j - j -$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
   (d) If $w^{-1} = -y_{i'} - y_i - i' - y_{j'} - j' - i - y_j - j -$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (e) If $w^{-1} = -y_{i'} - y_i - i' - i - y_{j'} - j' - y_j - j -$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = -y_{i'} - y_i - i' - y_{j'} - i - j' - y_j - j -$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Thus if $i' < j' < i < j < y_{i'} < y_i < y_{j'} < y_j$ then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i'\text{—}j\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.
- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}i'\text{—}y_j\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ and $(a',b') \in \{(j',y_{i'}),(i',y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq \text{—}y_{j'}\text{—}j\text{—}i'\text{—} \text{ and } (wt)^{-1} \neq \text{—}y_{j'}\text{—}y_i\text{—}i'\text{—}.$$

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_j\text{—}. \end{cases}$$

6. Suppose $i' < j' < y_{i'} < y_{j'} < i < j < y_i < y_j$.

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

Thus if $i' < j' < y_{i'} < y_{j'} < i < j < y_i < y_j$ then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ and $(a',b') \in \{(j',y_{i'}),(i',y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -j'-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_j-. \end{cases}$$

7. Suppose $i' < j' < y_{i'} < i < j < y_i < y_{j'} < y_j$.

   (a) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-j'-y_j-j-$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (b) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-j'-y_j-j-$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (c) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-j'-i-y_j-j-$ then (Y2) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (d) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (e) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-j'-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-y_j-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
   (j) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
   (k) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

   Thus if $i' < j' < y_{i'} < i < j < y_i < y_{j'} < y_j$ then one of the following holds:

   - $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-j'-i-y_j-j-$ and $(wt)^{-1} = -y_{i'}-j'-y_i-y_{j'}-i'-j-y_j-i-$.
   - $w^{-1} = -y_{i'}-i'-y_{j'}-j'-y_i-i-y_j-j-$ and $(wt)^{-1} = -y_{i'}-j'-y_{j'}-i'-y_i-j-y_j-i-$.
   - $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-j'-y_j-j-$ and $(wt)^{-1} = -y_{i'}-j'-y_i-j-y_{j'}-i'-y_j-i-$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ and $(a',b') \in \{(j', y_{i'}), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -y_i-j- \text{ and} \\ (wt)^{-1} = -y_j-i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j'- \text{ and} \\ (wt)^{-1} = -y_{j'}-i'-. \end{cases}$$

$$(\text{Z2}) \Leftrightarrow (wt)^{-1} \neq -y_{j'}-j-i'- \text{ and } (wt)^{-1} \neq -y_{j'}-y_i-i'-.$$

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -j'-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_j-. \end{cases}$$

8. Suppose $i' < j' < i < j < y_{i'} < y_{j'} < y_i < y_j$.

   (a) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (b) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-j'-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (c) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (d) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-y_j-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (e) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-j'-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(m) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

Thus if $i' < j' < i < j < y_{i'} < y_{j'} < y_i < y_j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-y_{j'}-j'-y_i-i-y_j-j-$ and $(wt)^{-1} = -y_{i'}-j'-y_{j'}-i'-y_i-j-y_j-i-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ and $(a',b') \in \{(j', y_{i'}), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -y_i-j- \text{ and} \\ (wt)^{-1} = -y_j-i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j'- \text{ and} \\ (wt)^{-1} = -y_{j'}-i'-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -j'-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_j-. \end{cases}$$

9. Suppose $i' < j' < y_{i'} < i < j < y_{j'} < y_i < y_j$.

(a) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(b) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-j'-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(c) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(d) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-y_j-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(e) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(f) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-j'-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(g) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(h) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(i) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(j) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-y_j-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-j'-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(m) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

Thus if $i' < j' < y_{i'} < i < j < y_{j'} < y_i < y_j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-y_{j'}-j'-y_i-i-y_j-j-$ and $(wt)^{-1} = -y_{i'}-j'-y_{j'}-i'-y_i-j-y_j-i-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ and $(a',b') \in \{(j', y_{i'}), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -y_i-j- \text{ and} \\ (wt)^{-1} = -y_j-i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j'- \text{ and} \\ (wt)^{-1} = -y_{j'}-i'-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -i'-y_j- \text{ and} \\ (wt)^{-1} = -j'-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_j-. \end{cases}$$

10. Suppose $i' < i < j' < y_{i'} < y_{j'} < j < y_i < y_j$.

(a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

Thus if $i' < i < j' < y_{i'} < y_{j'} < j < y_i < y_j$ then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j, y_i), (i, y_j)\}$ and $(a', b') \in \{(j', y_{i'}), (i', y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq \text{—}y_j\text{—}j'\text{—}i\text{—} \text{ and } (wt)^{-1} \neq \text{—}y_j\text{—}y_{i'}\text{—}i\text{—}.$$

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_i\text{—}. \end{cases}$$

11. Suppose $i' < j' < i < y_{i'} < y_{j'} < j < y_i < y_j$.

(a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

Thus if $i' < j' < i < y_{i'} < y_{j'} < j < y_i < y_j$ then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ and $(a',b') \in \{(j',y_{i'}),(i',y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_j\text{—}. \end{cases}$$

12. Suppose $i' < i < j' < y_{i'} < j < y_{j'} < y_i < y_j$.
    (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
    (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
    (c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
    (e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
    (g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
    (i) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (j',y_{j'})$.
    (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (j',y_{j'})$.
    (m) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.

    Thus if $i' < i < j' < y_{i'} < j < y_{j'} < y_i < y_j$ then one of the following holds:

    - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j'\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

    When $(a,b) \in \mathrm{Cyc}^1(z) = \{(j,y_i),(i,y_j)\}$ and $(a',b') \in \{(j',y_{i'}),(i',y_{j'})\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i'\text{—}. \end{cases}$$

    (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_j\text{—}j'\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_j\text{—}y_{i'}\text{—}i\text{—}$.

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_i\text{—}. \end{cases}$$

13. It cannot happen that $i' < i < j' < j < y_{i'} < y_i < y_{j'} < y_j$ since:
    (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
    (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.
    (c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}y_j\text{—}j'\text{—}j\text{—}$ then (T) fails.
    (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}y_{j'}\text{—}j'\text{—}y_j\text{—}j\text{—}$ then (T) fails.

(e) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (T) fails.
(f) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (T) fails.
(g) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (T) fails.
(h) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (T) fails.
(i) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
(j) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (T) fails.
(k) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(n) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.

14. It cannot happen that $i' < i < j' < y_{i'} < j < y_i < y_{j'} < y_j$ since:
(a) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (T) fails.
(b) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (T) fails.
(c) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (T) fails.
(d) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
(e) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (T) fails.
(f) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (T) fails.
(g) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (T) fails.
(h) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (T) fails.
(i) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
(j) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (T) fails.
(k) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(n) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(z)$ when $i < j < y_i < y_j$.

12 Case: $i < y_j < y_i < j$

Suppose y is such that $i < y_j < y_i < j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(y_j, y_i), (i, j)\}$.

12.1 Subcase (i)

We must have $w^{-1} = —y_i—i—j—y_j—$ since no other ordering is possible:

1. If $w^{-1} = —i—y_j—y_i—j—$ then (T) fails.
2. If $w^{-1} = —i—y_j—j—y_i—$ then (T) fails.
3. If $w^{-1} = —i—y_i—y_j—j—$ then (T) fails.
4. If $w^{-1} = —i—y_i—j—y_j—$ then (T) fails.
5. If $w^{-1} = —y_j—i—y_i—j—$ then (T) fails.
6. If $w^{-1} = —y_j—y_i—j—i—$ then (T) fails.
7. If $w^{-1} = —y_j—j—i—y_i—$ then (T) fails.
8. If $w^{-1} = —y_j—j—y_i—i—$ then (T) fails.

9. If $w^{-1} = —y_i—i—y_j—j—$ then (T) fails.
10. If $w^{-1} = —y_i—y_j—j—i—$ then (T) fails.
11. If $w^{-1} = —y_i—j—i—y_j—$ then (T) fails.
12. If $w^{-1} = —y_i—j—y_j—i—$ then (T) fails.
13. If $w^{-1} = —j—i—y_j—y_i—$ then (T) fails.
14. If $w^{-1} = —j—i—y_i—y_j—$ then (T) fails.
15. If $w^{-1} = —j—y_j—i—y_i—$ then (T) fails.
16. If $w^{-1} = —j—y_j—y_i—i—$ then (T) fails.
17. If $w^{-1} = —j—y_i—i—y_j—$ then (T) fails.
18. If $w^{-1} = —j—y_i—y_j—i—$ then (T) fails.
19. If $w^{-1} = —i—j—y_j—y_i—$ then (Y1) fails for $(a,b) = (i, y_i)$.
20. If $w^{-1} = —i—j—y_i—y_j—$ then (Y1) fails for $(a,b) = (i, y_i)$.
21. If $w^{-1} = —y_j—i—j—y_i—$ then (Y1) fails for $(a,b) = (y_j, j)$.
22. If $w^{-1} = —y_j—y_i—i—j—$ then (Y1) fails for $(a,b) = (y_j, j)$.
23. If $w^{-1} = —y_i—y_j—i—j—$ then (Y1) fails for $(a,b) = (y_j, j)$.

Hence if $i < y_j < y_i < j$ then

$(wt)^{-1} = —y_i—j—i—y_j—$.

When $(a,b), (a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—i—$ and $(wt)^{-1} = —y_i—y_j—$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—y_j—i—$ and $(wt)^{-1} \neq —j—y_i—i—$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in case (i) and $i < y_j < y_i < j$.

12.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $i < y_j < y_i < j < R$.
   (a) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —R—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —y_i—i—R—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (R,R)$.

   Thus if $i < y_j < y_i < j < R$ then one of the following holds:

   - $w^{-1} = —y_i—i—j—y_j—R—$ and $(wt)^{-1} = —y_i—j—i—y_j—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—i—$ and $(wt)^{-1} = —y_i—y_j—$.
   (Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}R\text{—}$.

2. Suppose $i < y_j < y_i < R < j$.

   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.

   Thus if $i < y_j < y_i < R < j$ then one of the following holds:

   - $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}y_j\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}R\text{—}i\text{—}$.
   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}y_j\text{—}R\text{—}$.

3. Suppose $i < y_j < R < y_i < j$.

   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (R,R)$.

   Thus if $i < y_j < R < y_i < j$ then one of the following holds:

   - $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.
   - $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}y_j\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}R\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}R\text{—}y_j\text{—}$.
   (Z3) $\Leftrightarrow$ (no condition).

4. Suppose $i < R < y_j < y_i < j$.

   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j, j)$.
   (d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j, j)$.

   Thus if $i < R < y_j < y_i < j$ then one of the following holds:

   - $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}y_j\text{—}$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}R\text{—}i\text{—}$.
   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}R\text{—}y_i\text{—}$.

5. Suppose $R < i < y_j < y_i < j$.

   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i, y_i)$.

(b) If $w^{-1} = -y_i - i - R - j - y_j -$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = -y_i - i - j - R - y_j -$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = -y_i - R - i - j - y_j -$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.

Thus if $R < i < y_j < y_i < j$ then one of the following holds:

- $w^{-1} = -R - y_i - i - j - y_j -$ and $(wt)^{-1} = -R - y_i - j - i - y_j -$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_i),(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = -j - i -$ and $(wt)^{-1} = -y_i - y_j -$.
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow$ $(wt)^{-1} = -R - j -$ and $(wt)^{-1} = -R - y_i -$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $P < i < y_j < Q < y_i < j$.

(a) If $w^{-1} = -y_i - i - Q - P - j - y_j -$ then (T) fails.
(b) If $w^{-1} = -y_i - i - Q - j - P - y_j -$ then (T) fails.
(c) If $w^{-1} = -y_i - i - Q - j - y_j - P -$ then (T) fails.
(d) If $w^{-1} = -Q - y_i - P - i - j - y_j -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = -Q - y_i - i - j - y_j - P -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = -y_i - i - j - y_j - Q - P -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = -y_i - Q - i - P - j - y_j -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = -Q - y_i - i - P - j - y_j -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = -y_i - Q - P - i - j - y_j -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(j) If $w^{-1} = -y_i - i - j - Q - y_j - P -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(k) If $w^{-1} = -Q - y_i - i - j - P - y_j -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(l) If $w^{-1} = -y_i - Q - i - j - y_j - P -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(m) If $w^{-1} = -y_i - Q - i - j - P - y_j -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(n) If $w^{-1} = -y_i - i - j - Q - P - y_j -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Thus if $P < i < y_j < Q < y_i < j$ then one of the following holds:

- $w^{-1} = -Q - P - y_i - i - j - y_j -$ and $(wt)^{-1} = -Q - P - y_i - j - i - y_j -$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_i),(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow$ $\begin{cases} (wt)^{-1} = -Q - P - \text{ and} \\ (wt)^{-1} = -j - i - \text{ and} \\ (wt)^{-1} = -y_i - y_j -. \end{cases}$
(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow$ $(wt)^{-1} = -P - j -$ and $(wt)^{-1} = -P - y_i -$.

2. Suppose $P < i < Q < y_j < y_i < j$.

(a) If $w^{-1} = -y_i - i - Q - P - j - y_j -$ then (T) fails.
(b) If $w^{-1} = -y_i - i - Q - j - P - y_j -$ then (T) fails.
(c) If $w^{-1} = -y_i - i - Q - j - y_j - P -$ then (T) fails.
(d) If $w^{-1} = -Q - y_i - P - i - j - y_j -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = -Q - y_i - i - j - y_j - P -$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(k) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(l) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(n) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Thus if $P < i < Q < y_j < y_i < j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_i),(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}y_j\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}j\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}y_i\text{—}$.

3. Suppose $i < y_j < y_i < P < j < Q$.

(a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

Thus if $i < y_j < y_i < P < j < Q$ then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_i),(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}y_j\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}Q\text{—}$.

4. Suppose $P < i < y_j < y_i < Q < j$.

   (a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
   (d) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (h) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (i) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (j) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (k) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (l) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (m) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

   Thus if $P < i < y_j < y_i < Q < j$ then one of the following holds:

   - $w^{-1} = —y_i—Q—P—i—j—y_j—$ and $(wt)^{-1} = —y_i—Q—P—j—i—y_j—$.
   - $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $(wt)^{-1} = —Q—P—y_i—j—i—y_j—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_i),(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   $$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —y_i—y_j—. \end{cases}$$

   $$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq —Q—y_j—P— \text{ and } (wt)^{-1} \neq —Q—y_i—P—.$$

   $$\text{(Z3)} \Leftrightarrow (wt)^{-1} = —P—j—.$$

5. Suppose $i < y_j < P < y_i < Q < j$.

   (a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
   (e) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
   (f) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

   Thus if $i < y_j < P < y_i < Q < j$ then one of the following holds:

   - $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $(wt)^{-1} = —y_i—j—i—y_j—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-i- \text{ and} \\ (wt)^{-1} = -y_i-y_j-. \end{cases}$$

$(\mathrm{Z2}) \Leftrightarrow (wt)^{-1} \neq -j-P-i-$ and $(wt)^{-1} \neq -j-Q-i-$.

$(\mathrm{Z3}) \Leftrightarrow (wt)^{-1} = -y_j-Q-$.

6. Suppose $P < Q < i < y_j < y_i < j$.

   (a) If $w^{-1} = -Q-y_i-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (b) If $w^{-1} = -y_i-i-Q-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (c) If $w^{-1} = -Q-y_i-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (d) If $w^{-1} = -y_i-i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = -y_i-i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (h) If $w^{-1} = -y_i-Q-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (i) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (j) If $w^{-1} = -Q-y_i-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (k) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (l) If $w^{-1} = -y_i-Q-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (m) If $w^{-1} = -y_i-Q-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (n) If $w^{-1} = -y_i-i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

   Thus if $P < Q < i < y_j < y_i < j$ then one of the following holds:

   - $w^{-1} = -Q-P-y_i-i-j-y_j-$ and $(wt)^{-1} = -Q-P-y_i-j-i-y_j-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-i- \text{ and} \\ (wt)^{-1} = -y_i-y_j-. \end{cases}$$

   $(\mathrm{Z2}) \Leftrightarrow$ (no condition).

   $(\mathrm{Z3}) \Leftrightarrow (wt)^{-1} = -P-j-$ and $(wt)^{-1} = -P-y_i-$.

7. Suppose $i < y_j < y_i < P < Q < j$.

   (a) If $w^{-1} = -y_i-i-Q-P-j-y_j-$ then (T) fails.
   (b) If $w^{-1} = -y_i-i-Q-j-P-y_j-$ then (T) fails.
   (c) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (T) fails.
   (d) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (T) fails.
   (e) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (T) fails.
   (f) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = -y_i-i-j-Q-P-y_j-$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = -Q-y_i-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = -Q-y_i-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = -y_i-Q-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(k) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.

Thus if $i < y_j < y_i < P < Q < j$ then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $(wt)^{-1} = —y_i—j—i—y_j—Q—P—$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i, j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —y_i—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —j—P—i—$ and $(wt)^{-1} \neq —j—Q—i—$.
(Z3) $\Leftrightarrow (wt)^{-1} = —y_j—Q—$.

8. Suppose $i < y_j < P < y_i < j < Q$.

(a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
(c) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(e) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Thus if $i < y_j < P < y_i < j < Q$ then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $(wt)^{-1} = —y_i—j—i—y_j—Q—P—$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i, j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —y_i—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

9. Suppose $i < P < y_j < y_i < j < Q$.

(a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
(c) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.

(d) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(e) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(f) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(g) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

Thus if $i < P < y_j < y_i < j < Q$ then one of the following holds:

- $w^{-1} = —y_i—i—j—Q—P—y_j—$ and $(wt)^{-1} = —y_i—j—i—Q—P—y_j—$.
- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $(wt)^{-1} = —y_i—j—i—y_j—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —y_i—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —Q—y_j—P—$ and $(wt)^{-1} \neq —Q—y_i—P—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$.

10. Suppose $i < y_j < P < Q < y_i < j$.

(a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
(c) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
(e) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
(f) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

Thus if $i < y_j < P < Q < y_i < j$ then one of the following holds:

- $w^{-1} = —Q—y_i—i—j—y_j—P—$ and $(wt)^{-1} = —Q—y_i—j—i—y_j—P—$.
- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $(wt)^{-1} = —y_i—j—i—y_j—Q—P—$.
- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $(wt)^{-1} = —Q—P—y_i—j—i—y_j—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —y_i—y_j—. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq -j-P-i- \text{ and } (wt)^{-1} \neq -j-Q-i- \text{ and} \\ (wt)^{-1} \neq -y_i-P-y_j- \text{ and } (wt)^{-1} \neq -y_i-Q-y_j-. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

11. Suppose $i < P < y_j < Q < y_i < j$.

    (a) If $w^{-1} = -y_i-i-Q-P-j-y_j-$ then (T) fails.
    (b) If $w^{-1} = -y_i-i-Q-j-P-y_j-$ then (T) fails.
    (c) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (T) fails.
    (d) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (T) fails.
    (e) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (T) fails.
    (f) If $w^{-1} = -Q-y_i-P-i-j-y_j-$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (g) If $w^{-1} = -y_i-Q-P-i-j-y_j-$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (h) If $w^{-1} = -Q-y_i-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (i) If $w^{-1} = -y_i-i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (j) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (k) If $w^{-1} = -Q-y_i-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (l) If $w^{-1} = -y_i-Q-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (m) If $w^{-1} = -y_i-Q-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (n) If $w^{-1} = -y_i-i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

    Thus if $i < P < y_j < Q < y_i < j$ then one of the following holds:

    - $w^{-1} = -Q-P-y_i-i-j-y_j-$ and $(wt)^{-1} = -Q-P-y_i-j-i-y_j-$.

    When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_i),(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-i- \text{ and} \\ (wt)^{-1} = -y_i-y_j-. \end{cases}$$

    (Z2) $\Leftrightarrow (wt)^{-1} \neq -j-P-i-$ and $(wt)^{-1} \neq -j-Q-i-$.
    (Z3) $\Leftrightarrow (wt)^{-1} = -P-y_i-$.

12. Suppose $i < y_j < y_i < j < P < Q$.

    (a) If $w^{-1} = -Q-y_i-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (b) If $w^{-1} = -Q-y_i-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (c) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (d) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (e) If $w^{-1} = -y_i-Q-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (f) If $w^{-1} = -Q-P-y_i-i-j-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (g) If $w^{-1} = -Q-y_i-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (h) If $w^{-1} = -y_i-Q-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (i) If $w^{-1} = -y_i-Q-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (j) If $w^{-1} = -y_i-i-Q-P-j-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
    (k) If $w^{-1} = -y_i-i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
    (l) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
    (m) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
    (n) If $w^{-1} = -y_i-i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

Thus if $i < y_j < y_i < j < P < Q$ then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $(wt)^{-1} = —y_i—j—i—y_j—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_i),(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —y_i—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

13. Suppose $P < i < y_j < y_i < j < Q$.

(a) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(i) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(j) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

Thus if $P < i < y_j < y_i < j < Q$ then one of the following holds:

- $w^{-1} = —y_i—i—Q—P—j—y_j—$ and $(wt)^{-1} = —y_i—j—Q—P—i—y_j—$.
- $w^{-1} = —y_i—Q—P—i—j—y_j—$ and $(wt)^{-1} = —y_i—Q—P—j—i—y_j—$.
- $w^{-1} = —y_i—i—j—Q—P—y_j—$ and $(wt)^{-1} = —y_i—j—i—Q—P—y_j—$.
- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $(wt)^{-1} = —y_i—j—i—y_j—Q—P—$.
- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $(wt)^{-1} = —Q—P—y_i—j—i—y_j—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,y_i),(i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —y_i—y_j—. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and } (wt)^{-1} \neq —Q—j—P— \text{ and} \\ (wt)^{-1} \neq —Q—y_j—P— \text{ and } (wt)^{-1} \neq —Q—y_i—P—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

14. Suppose $i < P < Q < y_j < y_i < j$.

(a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
(c) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
(e) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
(f) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(g) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(i) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(j) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(k) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(l) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(m) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(n) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.

Thus if $i < P < Q < y_j < y_i < j$ then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $(wt)^{-1} = —Q—P—y_i—j—i—y_j—$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —y_i—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —j—P—i—$ and $(wt)^{-1} \neq —j—Q—i—$.
(Z3) $\Leftrightarrow (wt)^{-1} = —P—y_i—$.

15. It cannot happen that $i < P < y_j < y_i < Q < j$ since:

(a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
(c) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
(e) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
(f) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(g) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(h) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(i) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(o) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a', b') are as in cases (i) or (ii) and $i < y_j < y_i < j$.

12.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < y_{j'} < i < y_{i'} < y_j < y_i < j' < j$.

(a) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(c) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(d) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.

(e) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (U) fails.
(f) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (U) fails.
(g) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (U) fails.
(h) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

Thus if $i' < y_{j'} < i < y_{i'} < y_j < y_i < j' < j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ and $(wt)^{-1} = -y_{i'}-j'-i'-y_{j'}-y_i-j-i-y_j-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a', b') \in \{(y_{j'}, y_{i'}), (i', j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -j-i- \text{ and} \\ (wt)^{-1} = -j'-i'- \text{ and} \\ (wt)^{-1} = -y_i-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_{j'}-. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq -j'-y_j-i'- \text{ and } (wt)^{-1} \neq -j'-y_i-i'-.$$

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -y_{j'}-j- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_i-. \end{cases}$$

2. Suppose $i' < y_{j'} < y_{i'} < i < j' < y_j < y_i < j$.

(a) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(b) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
(c) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
(d) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-y_{j'}-y_j-$ then (T) fails.
(e) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

Thus if $i' < y_{j'} < y_{i'} < i < j' < y_j < y_i < j$ then one of the following holds:

- $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ and $(wt)^{-1} = -y_{i'}-j'-i'-y_{j'}-y_i-j-i-y_j-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a', b') \in \{(y_{j'}, y_{i'}), (i', j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -j-i- \text{ and} \\ (wt)^{-1} = -j'-i'- \text{ and} \\ (wt)^{-1} = -y_i-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_{j'}-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_i\text{—}. \end{cases}$$

3. Suppose $i' < i < y_{j'} < y_j < y_{i'} < j' < y_i < j$.
   (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.
   (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

   Thus if $i' < i < y_{j'} < y_j < y_{i'} < j' < y_i < j$ then one of the following holds:
   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a', b') \in \{(y_{j'}, y_{i'}), (i', j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_{j'}\text{—}. \end{cases}$$

   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}y_{j'}\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}y_{i'}\text{—}i\text{—}$.

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_i\text{—}. \end{cases}$$

4. Suppose $i' < y_{j'} < i < y_{i'} < y_j < j' < y_i < j$.
   (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

Thus if $i' < y_{j'} < i < y_{i'} < y_j < j' < y_i < j$ then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a',b') \in \{(y_{j'}, y_{i'}), (i',j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_{j'}\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_i\text{—}. \end{cases}$$

5. Suppose $i' < y_{j'} < i < y_j < y_{i'} < y_i < j' < j$.

(a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.

(b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.

(c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.

(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.

(e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.

(f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.

(g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (U) fails.

(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Thus if $i' < y_{j'} < i < y_j < y_{i'} < y_i < j' < j$ then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a',b') \in \{(y_{j'}, y_{i'}), (i',j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_{j'}\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j'\text{—}y_j\text{—}i'\text{—}$ and $(wt)^{-1} \neq \text{—}j'\text{—}y_i\text{—}i'\text{—}$.

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_i\text{—}. \end{cases}$$

6. Suppose $i' < y_{j'} < y_{i'} < j' < i < y_j < y_i < j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

   Thus if $i' < y_{j'} < y_{i'} < j' < i < y_j < y_i < j$ then one of the following holds:

   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a',b') \in \{(y_{j'}, y_{i'}), (i', j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   $$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_{j'}\text{—}. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

   $$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_i\text{—}. \end{cases}$$

7. Suppose $i' < y_{j'} < y_{i'} < i < y_j < y_i < j' < j$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (U) fails.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Thus if $i' < y_{j'} < y_{i'} < i < y_j < y_i < j' < j$ then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $(wt)^{-1} = —y_{i'}—j'—i'—y_{j'}—y_i—j—i—y_j—$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a',b') \in \{(y_{j'}, y_{i'}), (i',j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —j'—i'— \text{ and} \\ (wt)^{-1} = —y_i—y_j— \text{ and} \\ (wt)^{-1} = —y_{i'}—y_{j'}—. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq —j'—y_j—i'— \text{ and } (wt)^{-1} \neq —j'—y_i—i'—.$$

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i'—j— \text{ and} \\ (wt)^{-1} = —y_{j'}—j— \text{ and} \\ (wt)^{-1} = —y_{j'}—y_i—. \end{cases}$$

8. Suppose $i' < y_{j'} < i < y_j < y_{i'} < j' < y_i < j$.
   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (e) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

   Thus if $i' < y_{j'} < i < y_j < y_{i'} < j' < y_i < j$ then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $(wt)^{-1} = —y_{i'}—j'—i'—y_{j'}—y_i—j—i—y_j—$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a',b') \in \{(y_{j'}, y_{i'}), (i',j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —j'—i'— \text{ and} \\ (wt)^{-1} = —y_i—y_j— \text{ and} \\ (wt)^{-1} = —y_{i'}—y_{j'}—. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i'—j— \text{ and} \\ (wt)^{-1} = —i'—y_i— \text{ and} \\ (wt)^{-1} = —y_{j'}—j— \text{ and} \\ (wt)^{-1} = —y_{j'}—y_i—. \end{cases}$$

9. Suppose $i' < y_{j'} < y_{i'} < i < y_j < j' < y_i < j$.
   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.

(d) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(e) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(f) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(g) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(h) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(i) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(j) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(m) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

Thus if $i' < y_{j'} < y_{i'} < i < y_j < j' < y_i < j$ then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $(wt)^{-1} = —y_{i'}—j'—i'—y_{j'}—y_i—j—i—y_j—$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a',b') \in \{(y_{j'}, y_{i'}), (i', j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —j—i— \text{ and} \\ (wt)^{-1} = —j'—i'— \text{ and} \\ (wt)^{-1} = —y_i—y_j— \text{ and} \\ (wt)^{-1} = —y_{i'}—y_{j'}—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i'—j— \text{ and} \\ (wt)^{-1} = —i'—y_i— \text{ and} \\ (wt)^{-1} = —y_{j'}—j— \text{ and} \\ (wt)^{-1} = —y_{j'}—y_i—. \end{cases}$$

10. Suppose $i' < i < y_{j'} < y_{i'} < j' < y_j < y_i < j$.

(a) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (T) fails.
(c) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(e) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (T) fails.
(f) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (T) fails.
(g) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(h) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.
(i) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.
(j) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(m) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Thus if $i' < i < y_{j'} < y_{i'} < j' < y_j < y_i < j$ then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $(wt)^{-1} = —y_{i'}—j'—i'—y_{j'}—y_i—j—i—y_j—$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a',b') \in \{(y_{j'}, y_{i'}), (i', j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -j-i- \text{ and} \\ (wt)^{-1} = -j'-i'- \text{ and} \\ (wt)^{-1} = -y_i-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_{j'}-. \end{cases}$$

$$(\text{Z2}) \Leftrightarrow (wt)^{-1} \neq -j-y_{j'}-i- \text{ and } (wt)^{-1} \neq -j-y_{i'}-i-.$$

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_i-. \end{cases}$$

11. Suppose $i' < y_{j'} < i < y_{i'} < j' < y_j < y_i < j$.

    (a) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
    (b) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
    (c) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
    (d) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-y_{j'}-y_j-$ then (T) fails.
    (e) If $w^{-1} = -y_{i'}-y_i-i'-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
    (f) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
    (g) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
    (h) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (i) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (j) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (k) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (l) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (m) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

    Thus if $i' < y_{j'} < i < y_{i'} < j' < y_j < y_i < j$ then one of the following holds:

    - $w^{-1} = -y_{i'}-i'-j'-y_{j'}-y_i-i-j-y_j-$ and $(wt)^{-1} = -y_{i'}-j'-i'-y_{j'}-y_i-j-i-y_j-$.

    When $(a,b) \in \text{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a',b') \in \{(y_{j'}, y_{i'}), (i', j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -j-i- \text{ and} \\ (wt)^{-1} = -j'-i'- \text{ and} \\ (wt)^{-1} = -y_i-y_j- \text{ and} \\ (wt)^{-1} = -y_{i'}-y_{j'}-. \end{cases}$$

    (Z2) $\Leftrightarrow$ (no condition).

$$(\text{Z3}) \Leftrightarrow \begin{cases} (wt)^{-1} = -i'-j- \text{ and} \\ (wt)^{-1} = -i'-y_i- \text{ and} \\ (wt)^{-1} = -y_{j'}-j- \text{ and} \\ (wt)^{-1} = -y_{j'}-y_i-. \end{cases}$$

12. Suppose $i' < i < y_{j'} < y_{i'} < y_j < j' < y_i < j$.

    (a) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
    (b) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (T) fails.
    (c) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
    (d) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
    (e) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-y_{j'}-j-y_j-$ then (T) fails.
    (f) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (T) fails.

(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

Thus if $i' < i < y_{j'} < y_{i'} < y_j < j' < y_i < j$ then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_{i'}\text{—}j'\text{—}i'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, y_i), (i,j)\}$ and $(a', b') \in \{(y_{j'}, y_{i'}), (i', j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}y_{j'}\text{—}. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}y_{j'}\text{—}i\text{—} \text{ and } (wt)^{-1} \neq \text{—}j\text{—}y_{i'}\text{—}i\text{—}.$$

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i'\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}y_i\text{—}. \end{cases}$$

13. It cannot happen that $i' < i < y_{j'} < y_j < y_{i'} < y_i < j' < j$ since:

(a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (U) fails.
(j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(n) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.

14. It cannot happen that $i' < i < y_{j'} < y_{i'} < y_j < y_i < j' < j$ since:

(a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.

(h) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (U) fails.
(i) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (U) fails.
(j) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.
(m) If $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.
(n) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(z)$ when $i < y_j < y_i < j$.

13 Case: $y_i < y_j < i < j$

Suppose y is such that $y_i < y_j < i < j$. Then $z = \tau_{ij}^n(y)$ has $\mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$.

13.1 Subcase (i)

We must have $w^{-1} = —i—y_i—j—y_j—$ since no other ordering is possible:

1. If $w^{-1} = —y_i—y_j—j—i—$ then (T) fails.
2. If $w^{-1} = —y_i—j—y_j—i—$ then (T) fails.
3. If $w^{-1} = —y_i—j—i—y_j—$ then (T) fails.
4. If $w^{-1} = —y_j—y_i—j—i—$ then (T) fails.
5. If $w^{-1} = —y_j—j—y_i—i—$ then (T) fails.
6. If $w^{-1} = —y_j—j—i—y_i—$ then (T) fails.
7. If $w^{-1} = —j—y_i—y_j—i—$ then (T) fails.
8. If $w^{-1} = —j—y_i—i—y_j—$ then (T) fails.
9. If $w^{-1} = —j—y_j—y_i—i—$ then (T) fails.
10. If $w^{-1} = —j—y_j—i—y_i—$ then (T) fails.
11. If $w^{-1} = —j—i—y_i—y_j—$ then (T) fails.
12. If $w^{-1} = —j—i—y_j—y_i—$ then (T) fails.
13. If $w^{-1} = —y_i—y_j—i—j—$ then (Y1) fails for $(a,b) = (y_i, i)$.
14. If $w^{-1} = —y_i—i—y_j—j—$ then (Y1) fails for $(a,b) = (y_i, i)$.
15. If $w^{-1} = —y_i—i—j—y_j—$ then (Y1) fails for $(a,b) = (y_i, i)$.
16. If $w^{-1} = —y_j—y_i—i—j—$ then (Y1) fails for $(a,b) = (y_j, j)$.
17. If $w^{-1} = —y_j—i—y_i—j—$ then (Y1) fails for $(a,b) = (y_j, j)$.
18. If $w^{-1} = —y_j—i—j—y_i—$ then (Y1) fails for $(a,b) = (y_j, j)$.
19. If $w^{-1} = —i—y_i—y_j—j—$ then (Y1) fails for $(a,b) = (y_j, j)$.
20. If $w^{-1} = —i—y_j—y_i—j—$ then (Y1) fails for $(a,b) = (y_j, j)$.
21. If $w^{-1} = —i—y_j—j—y_i—$ then (Y1) fails for $(a,b) = (y_j, j)$.
22. If $w^{-1} = —i—j—y_i—y_j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_j, j)$.
23. If $w^{-1} = —i—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_j, j)$.

Hence if $y_i < y_j < i < j$ then

$(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b),(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$, properties (Z1)-(Z3) are equivalent to the following conditions which all hold in this case:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}y_j\text{—}y_i\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}i\text{—}y_i\text{—}$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in case (i) and $y_i < y_j < i < j$.

13.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $R = y_R = z_R \notin \{i,j,y_i,y_j\} + n\mathbb{Z}$.

1. Suppose $y_i < y_j < i < j < R$.
   (a) If $w^{-1} = \text{—}i\text{—}R\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}R\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

   Thus if $y_i < y_j < i < j < R$ then one of the following holds:
   - $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_i\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}R\text{—}$.

2. Suppose $y_i < y_j < i < R < j$.
   (a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i\text{—}R\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = \text{—}R\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.

   Thus if $y_i < y_j < i < R < j$ then one of the following holds:
   - $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}$.
   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}R\text{—}y_i\text{—}$.
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_j\text{—}R\text{—}$.

3. Suppose $y_i < y_j < R < i < j$.
   (a) If $w^{-1} = \text{—}i\text{—}R\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

   Thus if $y_i < y_j < R < i < j$ then one of the following holds:

- $w^{-1} = —i—y_i—R—j—y_j—$ and $(wt)^{-1} = —j—y_i—R—i—y_j—$.
- $w^{-1} = —R—i—y_i—j—y_j—$ and $(wt)^{-1} = —R—j—y_i—i—y_j—$.
- $w^{-1} = —i—y_i—j—y_j—R—$ and $(wt)^{-1} = —j—y_i—i—y_j—R—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

(Z1) $\Leftrightarrow (wt)^{-1} = —i—y_j—$ and $(wt)^{-1} = —j—y_i—$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq —i—R—y_j—$ and $(wt)^{-1} \neq —j—R—y_i—$.
(Z3) $\Leftrightarrow$ (no condition).

4. Suppose $y_i < R < y_j < i < j$.

   (a) If $w^{-1} = —i—R—y_i—j—y_j—$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —i—y_i—j—y_j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j,j)$.
   (c) If $w^{-1} = —i—y_i—j—R—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j,j)$.

   Thus if $y_i < R < y_j < i < j$ then one of the following holds:

   - $w^{-1} = —i—y_i—R—j—y_j—$ and $(wt)^{-1} = —j—y_i—R—i—y_j—$.
   - $w^{-1} = —R—i—y_i—j—y_j—$ and $(wt)^{-1} = —R—j—y_i—i—y_j—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —i—y_j—$ and $(wt)^{-1} = —j—y_i—$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq —j—R—y_i—$.
   (Z3) $\Leftrightarrow (wt)^{-1} = —R—i—$.

5. Suppose $R < y_i < y_j < i < j$.

   (a) If $w^{-1} = —i—y_i—j—y_j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = —i—y_i—R—j—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = —i—y_i—j—R—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = —i—R—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.

   Thus if $R < y_i < y_j < i < j$ then one of the following holds:

   - $w^{-1} = —R—i—y_i—j—y_j—$ and $(wt)^{-1} = —R—j—y_i—i—y_j—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   (Z1) $\Leftrightarrow (wt)^{-1} = —i—y_j—$ and $(wt)^{-1} = —j—y_i—$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow (wt)^{-1} = —R—i—$ and $(wt)^{-1} = —R—j—$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(z) \subset \mathrm{Cyc}(y)$, so that $Q = y_P = z_P$ and $P, Q \notin \{i,j,y_i,y_j\} + n\mathbb{Z}$.

1. Suppose $P < y_i < y_j < Q < i < j$.

   (a) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = —i—y_i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (e) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (f) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

(g) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(h) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(j) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(k) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(l) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(m) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(n) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

Thus if $P < y_i < y_j < Q < i < j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}j\text{—}$.

2. Suppose $P < y_i < Q < y_j < i < j$.

(a) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(c) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(d) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(f) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(g) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(h) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(j) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(k) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(l) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(m) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(n) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

Thus if $P < y_i < Q < y_j < i < j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}j\text{—}$.

3. Suppose $y_i < y_j < i < P < j < Q$.

   (a) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (T) fails.
   (b) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
   (c) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
   (d) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (T) fails.
   (e) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
   (f) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   Thus if $y_i < y_j < i < P < j < Q$ then one of the following holds:

   - $w^{-1} = -i-y_i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-y_i-i-y_j-Q-P-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

   $$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -i-y_j- \text{ and} \\ (wt)^{-1} = -j-y_i-. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = -y_i-Q-$ and $(wt)^{-1} = -y_j-Q-$.

4. Suppose $P < y_i < y_j < i < Q < j$.

   (a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
   (b) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (T) fails.
   (c) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
   (d) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
   (e) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (T) fails.
   (f) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (T) fails.
   (g) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (T) fails.
   (h) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (j) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (k) If $w^{-1} = -i-y_i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (l) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (m) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (n) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

   Thus if $P < y_i < y_j < i < Q < j$ then one of the following holds:

   - $w^{-1} = -Q-P-i-y_i-j-y_j-$ and $(wt)^{-1} = -Q-P-j-y_i-i-y_j-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -i-y_j- \text{ and} \\ (wt)^{-1} = -j-y_i-. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq -Q-y_j-P- \text{ and } (wt)^{-1} \neq -Q-i-P-.$$

$$\text{(Z3)} \Leftrightarrow (wt)^{-1} = -P-j-.$$

5. Suppose $y_i < y_j < P < i < Q < j$.

   (a) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (T) fails.
   (b) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (T) fails.
   (c) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (T) fails.
   (d) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (T) fails.
   (e) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (T) fails.
   (f) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (T) fails.
   (g) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (T) fails.
   (h) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   Thus if $y_i < y_j < P < i < Q < j$ then one of the following holds:

   - $w^{-1} = -i-y_i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-y_i-i-y_j-Q-P-$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -i-y_j- \text{ and} \\ (wt)^{-1} = -j-y_i-. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq -j-P-y_i- \text{ and } (wt)^{-1} \neq -j-Q-y_i-.$$

$$\text{(Z3)} \Leftrightarrow (wt)^{-1} = -y_j-Q-.$$

6. Suppose $P < Q < y_i < y_j < i < j$.

   (a) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = -i-y_i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (e) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (f) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (g) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (h) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (j) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

(k) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(l) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(m) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(n) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

Thus if $P < Q < y_i < y_j < i < j$ then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}j\text{—}$.

7. Suppose $y_i < y_j < i < P < Q < j$.

(a) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

Thus if $y_i < y_j < i < P < Q < j$ then one of the following holds:

- $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}P\text{—}y_i\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}Q\text{—}y_i\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}y_j\text{—}Q\text{—}$.

8. Suppose $y_i < y_j < P < i < j < Q$.

(a) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

(d) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = -i-y_i-Q-P-j-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = -i-y_i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

Thus if $y_i < y_j < P < i < j < Q$ then one of the following holds:

- $w^{-1} = -i-y_i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-y_i-i-y_j-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -i-y_j- \text{ and} \\ (wt)^{-1} = -j-y_i-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).
(Z3) $\Leftrightarrow (wt)^{-1} = -y_i-Q-$ and $(wt)^{-1} = -y_j-Q-$.

9. Suppose $y_i < P < y_j < i < j < Q$.

(a) If $w^{-1} = -i-y_i-Q-j-P-y_j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(b) If $w^{-1} = -i-y_i-j-Q-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(c) If $w^{-1} = -i-y_i-Q-j-y_j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
(d) If $w^{-1} = -Q-i-P-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = -Q-i-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = -i-Q-y_i-P-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = -Q-i-y_i-P-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = -i-Q-P-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = -Q-P-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = -Q-i-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = -i-Q-y_i-j-y_j-P-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = -i-Q-y_i-j-P-y_j-$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

Thus if $y_i < P < y_j < i < j < Q$ then one of the following holds:

- $w^{-1} = -i-y_i-Q-P-j-y_j-$ and $(wt)^{-1} = -j-y_i-Q-P-i-y_j-$.
- $w^{-1} = -i-y_i-j-Q-P-y_j-$ and $(wt)^{-1} = -j-y_i-i-Q-P-y_j-$.
- $w^{-1} = -i-y_i-j-y_j-Q-P-$ and $(wt)^{-1} = -j-y_i-i-y_j-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -i-y_j- \text{ and} \\ (wt)^{-1} = -j-y_i-. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$.

10. Suppose $y_i < y_j < P < Q < i < j$.

(a) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.

Thus if $y_i < y_j < P < Q < i < j$ then one of the following holds:

- $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}$.
- $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}$.
- $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}j\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}$.
- $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}$.
- $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}$.
- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—}. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq \text{—}i\text{—}P\text{—}y_j\text{—} \text{ and } (wt)^{-1} \neq \text{—}i\text{—}Q\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} \neq \text{—}j\text{—}P\text{—}y_i\text{—} \text{ and } (wt)^{-1} \neq \text{—}j\text{—}Q\text{—}y_i\text{—}. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

11. Suppose $y_i < P < y_j < Q < i < j$.

(a) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(f) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(g) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(h) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(j) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(l) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.

Thus if $y_i < P < y_j < Q < i < j$ then one of the following holds:

- $w^{-1} = —i—y_i—Q—P—j—y_j—$ and $(wt)^{-1} = —j—y_i—Q—P—i—y_j—$.
- $w^{-1} = —Q—P—i—y_i—j—y_j—$ and $(wt)^{-1} = —Q—P—j—y_i—i—y_j—$.
- $w^{-1} = —Q—i—y_i—P—j—y_j—$ and $(wt)^{-1} = —Q—j—y_i—P—i—y_j—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_j— \text{ and} \\ (wt)^{-1} = —j—y_i—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —j—P—y_i—$ and $(wt)^{-1} \neq —j—Q—y_i—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —P—i—$.

12. Suppose $y_i < y_j < i < j < P < Q$.

(a) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = —Q—P—i—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

Thus if $y_i < y_j < i < j < P < Q$ then one of the following holds:

- $w^{-1} = —i—y_i—j—y_j—Q—P—$ and $(wt)^{-1} = —j—y_i—i—y_j—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_j— \text{ and} \\ (wt)^{-1} = —j—y_i—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —y_i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

13. Suppose $P < y_i < y_j < i < j < Q$.

(a) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(b) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(c) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(d) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(e) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(f) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(g) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

(h) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(j) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.

Thus if $P < y_i < y_j < i < j < Q$ then one of the following holds:

- $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}$.
- $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}$.
- $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}$.
- $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}$.
- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—}. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—} \text{ and } (wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—} \text{ and} \\ (wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—} \text{ and } (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—}. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

14. Suppose $y_i < P < Q < y_j < i < j$.

(a) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(f) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(g) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(h) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(j) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(l) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.

Thus if $y_i < P < Q < y_j < i < j$ then one of the following holds:

- $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}$.
- $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.
- $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}Q\text{—}j\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ or vice versa, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}P\text{—}y_i\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}Q\text{—}y_i\text{—}$.
(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}i\text{—}$.

15. It cannot happen that $y_i < P < y_j < i < Q < j$ since:

(a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(i) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(j) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(o) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.

We conclude that properties (Z1)-(Z3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_i < y_j < i < j$.

13.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $y_{i'} < y_{j'} < y_i < i' < y_j < i < j' < j$.

(a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(h) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(i) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Thus if $y_{i'} < y_{j'} < y_i < i' < y_j < i < j' < j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$ and $(a',b') \in \{(y_{j'}, i'), (y_{i'}, j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq -j'-y_j-y_{i'}-$ and $(wt)^{-1} \neq -j'-i-y_{i'}-$.

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{j'}-i- \text{ and} \\ (wt)^{-1} = -y_{j'}-j-. \end{cases}$$

2. Suppose $y_{i'} < y_{j'} < i' < y_i < j' < y_j < i < j$.

   (a) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-y_{j'}-j-y_j-$ then (U) fails.
   (b) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-y_{j'}-j-y_j-$ then (U) fails.
   (c) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-j-y_{j'}-y_j-$ then (U) fails.
   (d) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-j-y_{j'}-y_j-$ then (U) fails.
   (e) If $w^{-1} = -i'-i-y_{i'}-j'-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (f) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (j) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (k) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (l) If $w^{-1} = -i'-y_{i'}-j'-i-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (m) If $w^{-1} = -i'-y_{i'}-i-j'-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

   Thus if $y_{i'} < y_{j'} < i' < y_i < j' < y_j < i < j$ then one of the following holds:

   - $w^{-1} = -i'-y_{i'}-j'-y_{j'}-i-y_i-j-y_j-$ and $(wt)^{-1} = -j'-y_{i'}-i'-y_{j'}-j-y_i-i-y_j-$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$ and $(a',b') \in \{(y_{j'}, i'), (y_{i'}, j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -i-y_j- \text{ and} \\ (wt)^{-1} = -i'-y_{j'}- \text{ and} \\ (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_{i'}-. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -y_{i'}-i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{j'}-i- \text{ and} \\ (wt)^{-1} = -y_{j'}-j-. \end{cases}$$

3. Suppose $y_{i'} < y_i < y_{j'} < y_j < i' < j' < i < j$.

   (a) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-y_{j'}-j-y_j-$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
   (b) If $w^{-1} = -i'-y_{i'}-j'-i-y_{j'}-y_i-j-y_j-$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
   (c) If $w^{-1} = -i'-y_{i'}-i-j'-y_{j'}-y_i-j-y_j-$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
   (d) If $w^{-1} = -i'-i-y_{i'}-j'-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (e) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (f) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
   (j) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

(k) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Thus if $y_{i'} < y_i < y_{j'} < y_j < i' < j' < i < j$ then one of the following holds:

- $w^{-1} = -i'-y_{i'}-j'-y_{j'}-i-y_i-j-y_j-$ and $(wt)^{-1} = -j'-y_{i'}-i'-y_{j'}-j-y_i-i-y_j-$.
- $w^{-1} = -i'-y_{i'}-j'-i-y_i-y_{j'}-j-y_j-$ and $(wt)^{-1} = -j'-y_{i'}-i'-j-y_i-y_{j'}-i-y_j-$.
- $w^{-1} = -i'-y_{i'}-i-y_i-j'-y_{j'}-j-y_j-$ and $(wt)^{-1} = -j'-y_{i'}-j-y_i-i'-y_{j'}-i-y_j-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$ and $(a',b') \in \{(y_{j'}, i'), (y_{i'}, j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -i-y_j- \text{ and} \\ (wt)^{-1} = -i'-y_{j'}- \text{ and} \\ (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_{i'}-. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq -j-y_{j'}-y_i- \text{ and } (wt)^{-1} \neq -j-i'-y_i-.$$

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -y_{i'}-i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{j'}-i-. \end{cases}$$

4. Suppose $y_{i'} < y_{j'} < y_i < i' < y_j < j' < i < j$.

(a) If $w^{-1} = -i'-i-y_{i'}-j'-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(b) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(c) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(d) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(e) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(f) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(g) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(h) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(k) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(l) If $w^{-1} = -i'-y_{i'}-j'-i-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(m) If $w^{-1} = -i'-y_{i'}-i-j'-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < y_{j'} < y_i < i' < y_j < j' < i < j$ then one of the following holds:

- $w^{-1} = -i'-y_{i'}-j'-y_{j'}-i-y_i-j-y_j-$ and $(wt)^{-1} = -j'-y_{i'}-i'-y_{j'}-j-y_i-i-y_j-$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$ and $(a',b') \in \{(y_{j'}, i'), (y_{i'}, j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -i-y_j- \text{ and} \\ (wt)^{-1} = -i'-y_{j'}- \text{ and} \\ (wt)^{-1} = -j-y_i- \text{ and} \\ (wt)^{-1} = -j'-y_{i'}-. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \text{(no condition)}.$$

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = -y_{i'}-i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{j'}-i- \text{ and} \\ (wt)^{-1} = -y_{j'}-j-. \end{cases}$$

5. Suppose $y_{i'} < y_{j'} < y_i < y_j < i' < i < j' < j$.
   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (i) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Thus if $y_{i'} < y_{j'} < y_i < y_j < i' < i < j' < j$ then one of the following holds:
   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$ and $(a',b') \in \{(y_{j'}, i'), (y_{i'}, j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow (wt)^{-1} \neq \text{—}j'\text{—}y_j\text{—}y_{i'}\text{—} \text{ and } (wt)^{-1} \neq \text{—}j'\text{—}i\text{—}y_{i'}\text{—}.$$

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—}. \end{cases}$$

6. Suppose $y_{i'} < y_{j'} < i' < j' < y_i < y_j < i < j$.
   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (b) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

   Thus if $y_{i'} < y_{j'} < i' < j' < y_i < y_j < i < j$ then one of the following holds:
   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ and $(a',b') \in \{(y_{j'},i'),(y_{i'},j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_{i'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—}. \end{cases}$$

7. Suppose $y_{i'} < y_{j'} < i' < y_i < y_j < i < j' < j$.
   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (i) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.

   Thus if $y_{i'} < y_{j'} < i' < y_i < y_j < i < j' < j$ then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j,i),(y_i,j)\}$ and $(a',b') \in \{(y_{j'},i'),(y_{i'},j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}. \end{cases}$$

   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j'\text{—}y_j\text{—}y_{i'}\text{—}$ and $(wt)^{-1} \neq \text{—}j'\text{—}i\text{—}y_{i'}\text{—}$.

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—}. \end{cases}$$

8. Suppose $y_{i'} < y_{j'} < y_i < y_j < i' < j' < i < j$.
   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.

(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < y_{j'} < y_i < y_j < i' < j' < i < j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$ and $(a',b') \in \{(y_{j'}, i'), (y_{i'}, j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_{i'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—}. \end{cases}$$

9. Suppose $y_{i'} < y_{j'} < i' < y_i < y_j < j' < i < j$.

(a) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.
(b) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.
(c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
(d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < y_{j'} < i' < y_i < y_j < j' < i < j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$ and $(a',b') \in \{(y_{j'}, i'), (y_{i'}, j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_{i'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—}. \end{cases}$$

10. Suppose $y_{i'} < y_i < y_{j'} < i' < j' < y_j < i < j$.

    (a) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
    (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
    (c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
    (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
    (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
    (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
    (g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
    (h) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
    (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
    (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
    (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

    Thus if $y_{i'} < y_i < y_{j'} < i' < j' < y_j < i < j$ then one of the following holds:

    - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.
    - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}j\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}$.
    - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}j\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}$.

    When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$ and $(a',b') \in \{(y_{j'}, i'), (y_{i'}, j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}. \end{cases}$$

    (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}y_{j'}\text{—}y_i\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}i'\text{—}y_i\text{—}$.

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_{i'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—}. \end{cases}$$

11. Suppose $y_{i'} < y_{j'} < y_i < i' < j' < y_j < i < j$.

    (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
    (b) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
    (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
    (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
    (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
    (f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

Thus if $y_{i'} < y_{j'} < y_i < i' < j' < y_j < i < j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$ and $(a',b') \in \{(y_{j'}, i'), (y_{i'}, j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

$$\text{(Z3)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_{i'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j\text{—}. \end{cases}$$

12. Suppose $y_{i'} < y_i < y_{j'} < i' < y_j < j' < i < j$.

(a) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(c) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a',b') = (y_{j'}, j')$.
(d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(h) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Thus if $y_{i'} < y_i < y_{j'} < i' < y_j < j' < i < j$ then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.
- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}j\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}$.
- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}j\text{—}y_i\text{—}i'\text{—}y_{j'}\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(z) = \{(y_j, i), (y_i, j)\}$ and $(a',b') \in \{(y_{j'}, i'), (y_{i'}, j')\}$, properties (Z1)-(Z3) correspond to the following conditions which hold in each of the available cases for wt:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{i'}\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq -j-y_{j'}-y_i-$ and $(wt)^{-1} \neq -j-i'-y_i-$.

(Z3) $\Leftrightarrow \begin{cases} (wt)^{-1} = -y_{i'}-i- \text{ and} \\ (wt)^{-1} = -y_{i'}-j- \text{ and} \\ (wt)^{-1} = -y_{j'}-i-. \end{cases}$

13. It cannot happen that $y_{i'} < y_i < y_{j'} < y_j < i' < i < j' < j$ since:
    (a) If $w^{-1} = -i'-i-y_{i'}-j'-y_{j'}-y_i-j-y_j-$ then (T) fails.
    (b) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-y_{j'}-j-y_j-$ then (T) fails.
    (c) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-y_{j'}-j-y_j-$ then (T) fails.
    (d) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-y_{j'}-j-y_j-$ then (T) fails.
    (e) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-j-y_{j'}-y_j-$ then (T) fails.
    (f) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-y_{j'}-j-y_j-$ then (T) fails.
    (g) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-j-y_{j'}-y_j-$ then (T) fails.
    (h) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-j-y_{j'}-y_j-$ then (T) fails.
    (i) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-j-y_{j'}-y_j-$ then (T) fails.
    (j) If $w^{-1} = -i'-y_{i'}-i-j'-y_{j'}-y_i-j-y_j-$ then (T) fails.
    (k) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
    (l) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
    (m) If $w^{-1} = -i'-y_{i'}-j'-y_{j'}-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
    (n) If $w^{-1} = -i'-y_{i'}-j'-i-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.

14. It cannot happen that $y_{i'} < y_i < y_{j'} < i' < y_j < i < j' < j$ since:
    (a) If $w^{-1} = -i'-i-y_{i'}-j'-y_{j'}-y_i-j-y_j-$ then (T) fails.
    (b) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-y_{j'}-j-y_j-$ then (T) fails.
    (c) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-y_{j'}-j-y_j-$ then (T) fails.
    (d) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-y_{j'}-j-y_j-$ then (T) fails.
    (e) If $w^{-1} = -i'-y_{i'}-i-y_i-j'-j-y_{j'}-y_j-$ then (T) fails.
    (f) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-y_{j'}-j-y_j-$ then (T) fails.
    (g) If $w^{-1} = -i'-y_{i'}-i-j'-y_i-j-y_{j'}-y_j-$ then (T) fails.
    (h) If $w^{-1} = -i'-i-y_{i'}-y_i-j'-j-y_{j'}-y_j-$ then (T) fails.
    (i) If $w^{-1} = -i'-i-y_{i'}-j'-y_i-j-y_{j'}-y_j-$ then (T) fails.
    (j) If $w^{-1} = -i'-y_{i'}-i-j'-y_{j'}-y_i-j-y_j-$ then (T) fails.
    (k) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
    (l) If $w^{-1} = -i'-y_{i'}-j'-i-y_i-y_{j'}-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
    (m) If $w^{-1} = -i'-y_{i'}-j'-y_{j'}-i-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.
    (n) If $w^{-1} = -i'-y_{i'}-j'-i-y_{j'}-y_i-j-y_j-$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a', b') = (y_{j'}, j')$.

We conclude that properties (Z1)-(Z3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(z)$ when $y_i < y_j < i < j$.

14 Conclusion

It follows from this exhaustive case analysis that properties (Z1)-(Z3) hold for all pairs $(a,b), (a',b') \in \mathrm{Cyc}(z)$. We conclude by Lemma 1 that $wt \in \mathcal{A}(z)$. This completes the proof of the theorem. □

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