Computer-generated proof of affine involution toggling property

Eric MARBERG Yifeng ZHANG

March 23, 2020

Contents

1 Setup

Let n be a positive integer. For the definition of the affine symmetric group $\tilde{S}_n$, see [2]. Fix an affine permutation $w \in \tilde{S}_n$ and an involution $y = y^{-1} \in \tilde{S}_n$. We set $y_a = y(a)$ for $a \in \mathbb{Z}$ and define

$$\mathrm{Cyc}(y) = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : a \leq b = y_a\}.$$

As a shorthand, we write $w^{-1} = —a—b—c— \cdots —d—$ to mean that $w_a < w_b < w_c < \cdots < w_d$.

Lemma 1. One has $w \in \mathcal{A}(y)$ if and only if for all $(a,b), (a',b') \in \mathrm{Cyc}(y)$, the following properties hold:

(Y1) If $a < b$ then $w^{-1} = —b—a—$.

(Y2) If $a < a' \leq b' < b$ then $w^{-1} \neq —b—a'—a—$ and $w^{-1} \neq —b—b'—a—$.

(Y3) If $a < a'$ and $b < b'$ then $w^{-1} = —a—b'—$.

Proof. This is equivalent to [2, Theorem 7.6]. □

Fix integers $i < j$ that are not congruent modulo n. Let $t_{ij} \in \tilde{S}_n$ be the reflection that interchanges i and j while fixing all integers not congruent to i or j modulo n. Write $\lessdot$ for the covering relation in the Bruhat order on $\tilde{S}_n$.

Lemma 2. One has $w \lessdot wt_{ij}$ if and only if the following property holds:

(T) $w^{-1} = —i—j—$ but if $i < e < j$ then $w^{-1} \neq —i—e—j—$.

Moreover, if i' and j' are integers with $i - i' = j - j' \in n\mathbb{Z}$, then property (T) is equivalent to the following:

(U) $w^{-1} = —i'—j'—$ but if $i' < e < j'$ then $w^{-1} \neq —i'—e—j'—$.

Proof. This is equivalent to [1, Proposition 8.3.6]. □

Recall the definition of the operator τ^n_{ij} from [2, §8] and let $z = z^{-1} = \tau^n_{ij}(y) \in \tilde{S}_n$.

Lemma 3. Suppose $w \in \mathcal{A}(y)$ and $w \lessdot wt_{ij}$ and $y = \tau^n_{ij}(y)$. Then one of the following cases occurs:

(A1) $i < j = y_j < y_i$ and $w^{-1} = —y_i—i—j—$.

(A2) $i < j < y_j < y_i$ and $w^{-1} = —y_j—y_i—i—j—$.

(A3) $i < y_j < j < y_i$ and $w^{-1} = —y_i—i—j—y_j—$.

(B1) $y_j < i = y_i < j$ and $w^{-1} = —i—j—y_j—$.

(B2) $y_j < i < y_i < j$ and $w^{-1} = —y_i—i—j—y_j—$.

(B3) $y_j < y_i < i < j$ and $w^{-1} = —i—j—y_j—y_i—$.

(C1) $i < j < y_j < y_i$ and $w^{-1} = —y_i—i—y_j—j—$.

(C2) $y_j < y_i < i < j$ and $w^{-1} = —i—y_i—j—y_j—$.

Proof. By definition, the only way one can have $y = \tau^n_{ij}(y)$ outside the given cases is if $y_j = i < j = y_i$ or $y_j < i < j < y_i$, but then any element $w \in \mathcal{A}(y)$ has $w^{-1} = —j—i—$ by Lemma 1 so it cannot hold that $w \lessdot wt_{ij}$. When y corresponds to the one of the given cases, the possibilities for $w \in \mathcal{A}(y)$ with $w \lessdot wt_{ij}$ are completely determined by Lemmas 1 and 2. □

Theorem. Suppose $w \in \mathcal{A}(y)$ and $w \lessdot wt_{ij}$ and $y = \tau^n_{ij}(y)$. Define

$$k = \begin{cases} j & \text{in cases (A1)-(A3)} \\ y_j & \text{in cases (B1)-(B3) and (C1)-(C2)} \end{cases} \quad \text{and} \quad l = \begin{cases} y_i & \text{in cases (A1)-(A3) and (C1)-(C2)} \\ i & \text{in cases (B1)-(B3).} \end{cases}$$

Then $k < l \notin k + n\mathbb{Z}$ and $w \neq wt_{ij}t_{kl} \in \mathcal{A}(y)$.

The proof of this statement occupies the rest of this computer-generated document.

Proof. By hypothesis, we are in one of the eight cases in Lemma 3. The sets $\{i, y_i\}+n\mathbb{Z}$ and $\{j, y_j\}+n\mathbb{Z}$ are therefore disjoint. Note that if $i \neq y_i$ then the sets $i+n\mathbb{Z}$ and $y_i+n\mathbb{Z}$ are disjoint, and that if $j \neq y_j$ then the sets $j+n\mathbb{Z}$ and $y_j+n\mathbb{Z}$ are disjoint.

Let $v = wt_{ij}t_{kl}$. To show that $v \in \mathcal{A}(y)$, it suffices by Lemma 1 to check that if $(a,b),(a',b') \in \mathrm{Cyc}(y)$ then the following properties hold:

(V1) If $a < b$ then $v^{-1} = —b—a—$.

(V2) If $a < a' \leq b' < b$ then $v^{-1} \neq —b—a'—a—$ and $v^{-1} \neq —b—b'—a—$.

(V3) If $a < a'$ and $b < b'$ then $v^{-1} = —a—b'—$.

Let $E = \{i, j, y_i, y_j\}$. Then $v_a = w_a$ for all integers $a \notin E + n\mathbb{Z}$, and $\mathrm{Cyc}(y) = \mathrm{Cyc}^1(y) \sqcup \mathrm{Cyc}^2(y) \sqcup \mathrm{Cyc}^3(y)$ where

$$\begin{aligned}
\mathrm{Cyc}^1(y) &= \{(a,b) \in \mathrm{Cyc}(y) : a, b \in E\},\\
\mathrm{Cyc}^2(y) &= \{(a,b) \in \mathrm{Cyc}(y) : a, b \notin E + n\mathbb{Z}\},\\
\mathrm{Cyc}^3(y) &= \{(a+mn, b+mn) : 0 \neq m \in \mathbb{Z} \text{ and } (a,b) \in \mathrm{Cyc}^1(y)\}.
\end{aligned}$$

When $(a,b),(a',b') \in \mathrm{Cyc}^2(y)$, properties (V1)-(V3) are equivalent to (Y1)-(Y3) and therefore hold since $w \in \mathcal{A}(y)$. It remains to check properties (V1)-(V3) in the following cases:

(i) When $(a,b),(a',b') \in \mathrm{Cyc}^1(y)$.

(ii) When one of the pairs (a,b), (a',b') belongs to $\mathrm{Cyc}^1(y)$ while the other belongs to $\mathrm{Cyc}^2(y)$.

(iii) When $(a,b) \in \mathrm{Cyc}^1(y)$ and $(a',b') \in \mathrm{Cyc}^3(y)$.

We check directly that properties (V1)-(V3) hold in cases (i), (ii), and (iii) for each of the eight cases in Lemma 3.

2 Case A1

Suppose $i < j = y_j < y_i$ and $w^{-1} = —y_i—i—j—$ so that $k = j < y_i = l$.

2.1 Subcase (i)

In this case $v = wt_{ij}t_{kl}$ is such that

$$v^{-1} = —j—y_i—i—.$$

When $(a,b),(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(j,j)\}$, properties (V1)-(V3) are equivalent to the following conditions which evidently hold:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —y_i—i—$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—j—i—$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (V1)-(V3) hold whenever (a,b), (a',b') are as in case (i) and $i < j = y_j < y_i$.

2.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(y)$, so that $R = y_R \notin \{i,j,y_i\} + n\mathbb{Z}$.

1. Suppose $i < j < y_i < R$.
   (a) If $w^{-1} = —y_i—R—i—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —y_i—i—R—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —R—y_i—i—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (j,y_i)$. We conclude that if $i < j < y_i < R$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—R—$ and $v^{-1} = —j—y_i—i—R—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow (wt)^{-1} = —y_i—i—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —i—R—$ and $(wt)^{-1} = —j—R—$.

2. Suppose $i < R < j < y_i$.

   (a) If $w^{-1} = —y_i—i—R—j—$ then (T) fails.

   (b) If $w^{-1} = —y_i—R—i—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.

   (c) If $w^{-1} = —y_i—i—j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,j)$.

   Recall that $(k,l) = (j,y_i)$. We conclude that if $i < R < j < y_i$ and then one of the following holds:

   - $w^{-1} = —R—y_i—i—j—$ and $v^{-1} = —R—j—y_i—i—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow (wt)^{-1} = —y_i—i—$.

   (Z2) $\Leftrightarrow (wt)^{-1} \neq —y_i—R—i—$.

   (Z3) $\Leftrightarrow (wt)^{-1} = —R—j—$.

3. Suppose $i < j < R < y_i$.

   (a) If $w^{-1} = —y_i—R—i—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.

   (b) If $w^{-1} = —y_i—i—R—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.

   (c) If $w^{-1} = —R—y_i—i—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (j,y_i)$. We conclude that if $i < j < R < y_i$ and then one of the following holds:

   - $w^{-1} = —y_i—i—j—R—$ and $v^{-1} = —j—y_i—i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow (wt)^{-1} = —y_i—i—$.

   (Z2) $\Leftrightarrow (wt)^{-1} \neq —y_i—R—i—$.

   (Z3) $\Leftrightarrow (wt)^{-1} = —j—R—$.

4. Suppose $R < i < j < y_i$.

   (a) If $w^{-1} = —y_i—R—i—j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.

   (b) If $w^{-1} = —y_i—i—R—j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.

   (c) If $w^{-1} = —y_i—i—j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,j)$.

   Recall that $(k,l) = (j,y_i)$. We conclude that if $R < i < j < y_i$ and then one of the following holds:

   - $w^{-1} = —R—y_i—i—j—$ and $v^{-1} = —R—j—y_i—i—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow (wt)^{-1} = —y_i—i—$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = —R—j—$ and $(wt)^{-1} = —R—y_i—$.

Next suppose $P < Q$ are integers with $(P, Q) \in \mathrm{Cyc}^2(y)$, so that $Q = y_P$ and $P, Q \notin \{i, j, y_i\} + n\mathbb{Z}$.

1. Suppose $P < i < j < Q < y_i$.
   (a) If $w^{-1} = —y_i—i—Q—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (b) If $w^{-1} = —y_i—i—j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (c) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (d) If $w^{-1} = —y_i—i—Q—P—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (e) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = —Q—y_i—i—P—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = —y_i—Q—i—P—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $P < i < j < Q < y_i$ and then one of the following holds:

   - $w^{-1} = —Q—P—y_i—i—j—$ and $v^{-1} = —Q—P—j—y_i—i—$.

   When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_i—i—$.
   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —Q—j—P—$.
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—y_i—$.

2. Suppose $P < i < Q < j < y_i$.
   (a) If $w^{-1} = —y_i—i—Q—j—P—$ then (T) fails.
   (b) If $w^{-1} = —y_i—i—Q—P—j—$ then (T) fails.
   (c) If $w^{-1} = —y_i—i—j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (d) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (e) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = —Q—y_i—i—P—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = —y_i—Q—i—P—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $P < i < Q < j < y_i$ and then one of the following holds:

   - $w^{-1} = —Q—P—y_i—i—j—$ and $v^{-1} = —Q—P—j—y_i—i—$.

   When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_i—i—$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_i—$.

3. Suppose $i < j < y_i < P < Q$.
   (a) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (b) If $w^{-1} = —Q—P—y_i—i—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (c) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (d) If $w^{-1} = —Q—y_i—i—P—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (e) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(f) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(g) If $w^{-1} = —y_i—Q—i—P—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(h) If $w^{-1} = —y_i—i—Q—j—P—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (P,Q)$.

(i) If $w^{-1} = —y_i—i—Q—P—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (j,y_i)$. We conclude that if $i < j < y_i < P < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—Q—P—$ and $v^{-1} = —j—y_i—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_i—i—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —j—Q—$.

4. Suppose $i < j < P < y_i < Q$.

(a) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(b) If $w^{-1} = —Q—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(c) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(d) If $w^{-1} = —Q—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(e) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(f) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(g) If $w^{-1} = —y_i—Q—i—P—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(h) If $w^{-1} = —y_i—i—Q—j—P—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (P,Q)$.

(i) If $w^{-1} = —y_i—i—Q—P—j—$ then (Y3) fails for $(a,b) = (j,j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (j,y_i)$. We conclude that if $i < j < P < y_i < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—Q—P—$ and $v^{-1} = —j—y_i—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_i—i—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —j—Q—$.

5. Suppose $i < P < j < Q < y_i$.

(a) If $w^{-1} = —y_i—i—Q—P—j—$ then (T) fails.

(b) If $w^{-1} = —Q—y_i—i—P—j—$ then (T) fails.

(c) If $w^{-1} = —y_i—Q—i—P—j—$ then (T) fails.

(d) If $w^{-1} = —y_i—i—Q—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.

(e) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,j)$.

(f) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(g) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(h) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (j,y_i)$. We conclude that if $i < P < j < Q < y_i$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—Q—P—$ and $v^{-1} = —j—y_i—i—Q—P—$.
- $w^{-1} = —Q—P—y_i—i—j—$ and $v^{-1} = —Q—P—j—y_i—i—$.

When $(a,b)=(P,Q)$ and $(a',b')\in \mathrm{Cyc}^1(y)=\{(i,y_i),(j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_i—i—$.

(Z2) $\Leftrightarrow$ $\begin{cases} (wt)^{-1} \neq —Q—j—P— \text{ and} \\ (wt)^{-1} \neq —y_i—P—i— \text{ and } (wt)^{-1} \neq —y_i—Q—i—. \end{cases}$

(Z3) $\Leftrightarrow$ (no condition).

6. Suppose $i < P < Q < j < y_i$.

   (a) If $w^{-1} = —y_i—i—Q—j—P—$ then (T) fails.
   (b) If $w^{-1} = —y_i—i—Q—P—j—$ then (T) fails.
   (c) If $w^{-1} = —Q—y_i—i—P—j—$ then (T) fails.
   (d) If $w^{-1} = —y_i—Q—i—P—j—$ then (T) fails.
   (e) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y2) fails for $(a,b)=(i,y_i)$ and $(a',b')=(P,Q)$.
   (f) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y2) fails for $(a,b)=(i,y_i)$ and $(a',b')=(P,Q)$.
   (g) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y2) fails for $(a,b)=(i,y_i)$ and $(a',b')=(P,Q)$.
   (h) If $w^{-1} = —y_i—i—j—Q—P—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(j,j)$.
   (i) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(j,j)$.

   Recall that $(k,l)=(j,y_i)$. We conclude that if $i < P < Q < j < y_i$ and then one of the following holds:

   - $w^{-1} = —Q—P—y_i—i—j—$ and $v^{-1} = —Q—P—j—y_i—i—$.

   When $(a,b)=(P,Q)$ and $(a',b')\in \mathrm{Cyc}^1(y)=\{(i,y_i),(j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_i—i—$.

   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—j—$.

7. Suppose $P < Q < i < j < y_i$.

   (a) If $w^{-1} = —y_i—i—Q—j—P—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (b) If $w^{-1} = —y_i—i—j—Q—P—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (c) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (d) If $w^{-1} = —y_i—i—Q—P—j—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (e) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (f) If $w^{-1} = —Q—y_i—i—P—j—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (g) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (h) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (i) If $w^{-1} = —y_i—Q—i—P—j—$ then (Y3) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.

   Recall that $(k,l)=(j,y_i)$. We conclude that if $P < Q < i < j < y_i$ and then one of the following holds:

   - $w^{-1} = —Q—P—y_i—i—j—$ and $v^{-1} = —Q—P—j—y_i—i—$.

   When $(a,b)=(P,Q)$ and $(a',b')\in \mathrm{Cyc}^1(y)=\{(i,y_i),(j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_i—i—$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_i—$.

8. Suppose $i < P < j < y_i < Q$.

(a) If $w^{-1} = —y_i—i—Q—P—j—$ then (T) fails.
(b) If $w^{-1} = —Q—y_i—i—P—j—$ then (T) fails.
(c) If $w^{-1} = —y_i—Q—i—P—j—$ then (T) fails.
(d) If $w^{-1} = —y_i—i—Q—j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, j)$.
(e) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = —Q—P—y_i—i—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < P < j < y_i < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—Q—P—$ and $v^{-1} = —j—y_i—i—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_i—i—$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq —Q—j—P—$.
(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$.

9. Suppose $P < i < j < y_i < Q$.

(a) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(b) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(c) If $w^{-1} = —Q—y_i—i—P—j—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(d) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(e) If $w^{-1} = —y_i—Q—i—P—j—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = —y_i—i—Q—j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, j)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $P < i < j < y_i < Q$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—$ and $v^{-1} = —Q—P—j—y_i—i—$.
- $w^{-1} = —y_i—i—j—Q—P—$ and $v^{-1} = —j—y_i—i—Q—P—$.
- $w^{-1} = —y_i—Q—P—i—j—$ and $v^{-1} = —j—Q—P—y_i—i—$.
- $w^{-1} = —y_i—i—Q—P—j—$ and $v^{-1} = —j—y_i—Q—P—i—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_i—i—$.
(Z2) $\Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and } (wt)^{-1} \neq —Q—y_i—P— \text{ and} \\ (wt)^{-1} \neq —Q—j—P—. \end{cases}$
(Z3) $\Leftrightarrow$ (no condition).

10. Suppose $i < j < P < Q < y_i$.

(a) If $w^{-1} = —y_i—Q—i—j—P—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(b) If $w^{-1} = —Q—y_i—P—i—j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(c) If $w^{-1} = —y_i—Q—P—i—j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(d) If $w^{-1} = —y_i—Q—i—P—j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(e) If $w^{-1} = —y_i—i—Q—j—P—$ then (Y3) fails for $(a, b) = (j, j)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = —y_i—i—Q—P—j—$ then (Y3) fails for $(a, b) = (j, j)$ and $(a', b') = (P, Q)$.

(g) If $w^{-1} = —Q—P—y_i—i—j—$ then (Y3) fails for $(a, b) = (j, j)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —Q—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (j, j)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —Q—y_i—i—P—j—$ then (Y3) fails for $(a, b) = (j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < j < P < Q < y_i$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—Q—P—$ and $v^{-1} = —j—y_i—i—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —y_i—i—$.
(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.
(Z3) $\Leftrightarrow$ $(wt)^{-1} = —j—Q—$.

We conclude that properties (V1)-(V3) hold whenever (a, b), (a', b') are as in cases (i) or (ii) and $i < j = y_j < y_i$.

2.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < j' < i < j < y_{i'} < y_i$.

   (a) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—$ then (Y3) fails for $(a, b) = (j', j')$ and $(a', b') = (i, y_i)$.
   (d) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—$ then (Y3) fails for $(a, b) = (j', j')$ and $(a', b') = (i, y_i)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $i' < j' < i < j < y_{i'} < y_i$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—$ and $v^{-1} = —j'—y_{i'}—i'—j—y_i—i—$.

   When $(a, b) \in \mathrm{Cyc}^1(y) = \{(i, y_i), (j, j)\}$ and $(a', b') \in \{(i', y_{i'}), (j', j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (V1) $\Leftrightarrow$ $(wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_{i'}—i'—$.
   (V2) $\Leftrightarrow$ (no condition).
   (V3) $\Leftrightarrow$ (no condition).

2. Suppose $i' < j' < y_{i'} < i < j < y_i$.

   (a) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—$ then (Y3) fails for $(a, b) = (j', j')$ and $(a', b') = (i, y_i)$.
   (d) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—$ then (Y3) fails for $(a, b) = (j', j')$ and $(a', b') = (i, y_i)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $i' < j' < y_{i'} < i < j < y_i$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—$ and $v^{-1} = —j'—y_{i'}—i'—j—y_i—i—$.

   When $(a, b) \in \mathrm{Cyc}^1(y) = \{(i, y_i), (j, j)\}$ and $(a', b') \in \{(i', y_{i'}), (j', j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (V1) $\Leftrightarrow$ $(wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_{i'}—i'—$.
   (V2) $\Leftrightarrow$ (no condition).
   (V3) $\Leftrightarrow$ (no condition).

3. Suppose $i' < i < j' < y_{i'} < j < y_i$.

   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—$ then (T) fails.

(b) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—$ then (T) fails.

(c) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (j', j')$.

(d) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i' < i < j' < y_{i'} < j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—$ and $v^{-1} = —j'—y_{i'}—i'—j—y_i—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(i, y_i), (j, j)\}$ and $(a', b') \in \{(i', y_{i'}), (j', j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(V1) $\Leftrightarrow$ $(wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_{i'}—i'—$.

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—j'—i—$.

(V3) $\Leftrightarrow$ (no condition).

4. Suppose $i' < j' < i < y_{i'} < j < y_i$.

(a) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(b) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—$ then (Y3) fails for $(a, b) = (j', j')$ and $(a', b') = (i, y_i)$.

(d) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—$ then (Y3) fails for $(a, b) = (j', j')$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i' < j' < i < y_{i'} < j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—$ and $v^{-1} = —j'—y_{i'}—i'—j—y_i—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(i, y_i), (j, j)\}$ and $(a', b') \in \{(i', y_{i'}), (j', j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(V1) $\Leftrightarrow$ $(wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_{i'}—i'—$.

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

5. Suppose $i' < i < j' < j < y_{i'} < y_i$.

(a) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—$ then (T) fails.

(b) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—$ then (T) fails.

(c) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (j', j')$.

(d) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i' < i < j' < j < y_{i'} < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—$ and $v^{-1} = —j'—y_{i'}—i'—j—y_i—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(i, y_i), (j, j)\}$ and $(a', b') \in \{(i', y_{i'}), (j', j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(V1) $\Leftrightarrow$ $(wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_{i'}—i'—$.

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—j'—i—$.

(V3) $\Leftrightarrow$ (no condition).

We conclude that properties (V1)-(V3) hold for all $(a, b), (a', b') \in \mathrm{Cyc}(y)$ when $i < j = y_j < y_i$.

3 Case A2

Suppose $i < j < y_j < y_i$ and $w^{-1} = —y_j—y_i—i—j—$ so that $k = j < y_i = l$.

3.1 Subcase (i)

In this case $v = wt_{ij}t_{kl}$ is such that

$$v^{-1} = \text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}.$$

When $(a,b),(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$, properties (V1)-(V3) are equivalent to the following conditions which evidently hold:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_i\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}j\text{—}$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}y_i\text{—}j\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}y_j\text{—}i\text{—}$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (V1)-(V3) hold whenever (a,b), (a',b') are as in case (i) and $i < j < y_j < y_i$.

3.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(y)$, so that $R = y_R \notin \{i,j,y_i,y_j\} + n\mathbb{Z}$.

1. Suppose $i < j < y_j < y_i < R$.
   (a) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}R\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}y_j\text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (j,y_i)$. We conclude that if $i < j < y_j < y_i < R$ and then one of the following holds:

   - $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}$ and $v^{-1} = \text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_i\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}j\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}R\text{—}$.

2. Suppose $i < j < y_j < R < y_i$.
   (a) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}R\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = \text{—}y_j\text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (j,y_i)$. We conclude that if $i < j < y_j < R < y_i$ and then one of the following holds:

   - $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}$ and $v^{-1} = \text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_i\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}j\text{—}$.
   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}y_i\text{—}R\text{—}i\text{—}$.
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}j\text{—}R\text{—}$.

3. Suppose $i < j < R < y_j < y_i$.
   (a) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.

(c) If $w^{-1} = \text{—}y_j\text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j, y_j)$ and $(a', b') = (R, R)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i < j < R < y_j < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}$ and $v^{-1} = \text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}R\text{—}$.
- $w^{-1} = \text{—}R\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ and $v^{-1} = \text{—}R\text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_i\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}j\text{—}$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}y_i\text{—}R\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_j\text{—}R\text{—}j\text{—}$.

(Z3) $\Leftrightarrow$ (no condition).

4. Suppose $i < R < j < y_j < y_i$.

(a) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}$ then (T) fails.

(b) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (R, R)$.

(c) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (j, y_j)$.

(d) If $w^{-1} = \text{—}y_j\text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i < R < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}R\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ and $v^{-1} = \text{—}R\text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_i\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}j\text{—}$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}y_i\text{—}R\text{—}i\text{—}$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}R\text{—}y_j\text{—}$.

5. Suppose $R < i < j < y_j < y_i$.

(a) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (i, y_i)$.

(b) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (i, y_i)$.

(d) If $w^{-1} = \text{—}y_j\text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (R, R)$ and $(a', b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $R < i < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}R\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ and $v^{-1} = \text{—}R\text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_i\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}j\text{—}$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}R\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}R\text{—}y_j\text{—}$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(y)$, so that $Q = y_P$ and $P, Q \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $P < i < j < Q < y_j < y_i$.

(a) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(b) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(d) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(e) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(f) If $w^{-1} = —Q—y_j—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(g) If $w^{-1} = —y_j—y_i—i—Q—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(h) If $w^{-1} = —y_j—y_i—i—Q—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(i) If $w^{-1} = —Q—y_j—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(j) If $w^{-1} = —y_j—y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(k) If $w^{-1} = —y_j—y_i—Q—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(l) If $w^{-1} = —y_j—Q—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(m) If $w^{-1} = —y_j—Q—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

(n) If $w^{-1} = —Q—y_j—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Recall that $(k,l) = (j,y_i)$. We conclude that if $P < i < j < Q < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_j—y_i—i—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—y_i—$ and $(wt)^{-1} = —P—y_j—$.

2. Suppose $P < i < Q < j < y_j < y_i$.

(a) If $w^{-1} = —y_j—y_i—i—Q—j—P—$ then (T) fails.

(b) If $w^{-1} = —y_j—y_i—i—Q—P—j—$ then (T) fails.

(c) If $w^{-1} = —y_j—y_i—i—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(d) If $w^{-1} = —Q—y_j—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(e) If $w^{-1} = —y_j—y_i—Q—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(f) If $w^{-1} = —y_j—Q—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(g) If $w^{-1} = —y_j—Q—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(h) If $w^{-1} = —Q—y_j—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(i) If $w^{-1} = —Q—y_j—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(j) If $w^{-1} = —y_j—y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(k) If $w^{-1} = —y_j—y_i—Q—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(l) If $w^{-1} = —y_j—Q—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

(m) If $w^{-1} = —y_j—Q—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

(n) If $w^{-1} = —Q—y_j—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Recall that $(k,l) = (j,y_i)$. We conclude that if $P < i < Q < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_j—y_i—i—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—y_i—$ and $(wt)^{-1} = —P—y_j—$.

3. Suppose $i < j < y_j < P < y_i < Q$.

   (a) If $w^{-1} = —y_j—Q—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = —Q—y_j—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = —y_j—y_i—Q—i—j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = —y_j—Q—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = —y_j—Q—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = —Q—y_j—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —Q—y_j—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —Q—P—y_j—y_i—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —Q—y_j—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = —y_j—y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = —y_j—y_i—Q—i—P—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = —y_j—Q—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = —y_j—y_i—i—Q—j—P—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = —y_j—y_i—i—Q—P—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

   Recall that $(k,l) = (j, y_i)$. We conclude that if $i < j < y_j < P < y_i < Q$ and then one of the following holds:

   - $w^{-1} = —y_j—y_i—i—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —j—Q—$.

4. Suppose $P < i < j < y_j < Q < y_i$.

   (a) If $w^{-1} = —Q—y_j—P—y_i—i—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j, y_j)$.
   (b) If $w^{-1} = —y_j—y_i—i—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (c) If $w^{-1} = —Q—y_j—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (d) If $w^{-1} = —y_j—y_i—Q—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (e) If $w^{-1} = —y_j—Q—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = —y_j—Q—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = —Q—y_j—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = —y_j—y_i—i—Q—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = —y_j—y_i—i—Q—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = —Q—y_j—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = —y_j—y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = —y_j—y_i—Q—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = —y_j—Q—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.

   Recall that $(k,l) = (j, y_i)$. We conclude that if $P < i < j < y_j < Q < y_i$ and then one of the following holds:

   - $w^{-1} = —Q—P—y_j—y_i—i—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.
   - $w^{-1} = —y_j—Q—P—y_i—i—j—$ and $v^{-1} = —y_j—Q—P—j—y_i—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}$.

(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}y_i\text{—}$.

5. Suppose $i < j < P < y_j < Q < y_i$.
   (a) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

   Recall that $(k,l) = (j,y_i)$. We conclude that if $i < j < P < y_j < Q < y_i$ and then one of the following holds:

   - $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ and $v^{-1} = \text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—}. \end{cases}$$

   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_i\text{—}P\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}Q\text{—}i\text{—}$.

   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}Q\text{—}$.

6. Suppose $i < P < j < y_j < Q < y_i$.
   (a) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (f) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (g) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (h) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (i) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
   (j) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(k) If $w^{-1} = —y_j—y_i—Q—P—i—j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —y_j—Q—y_i—P—i—j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i < P < j < y_j < Q < y_i$ and then one of the following holds:

- $w^{-1} = —y_j—y_i—i—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.
- $w^{-1} = —Q—P—y_j—y_i—i—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.
- $w^{-1} = —y_j—Q—P—y_i—i—j—$ and $v^{-1} = —y_j—Q—P—j—y_i—i—$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—j—P— \text{ and } (wt)^{-1} \neq —Q—y_j—P— \text{ and} \\ (wt)^{-1} \neq —y_i—P—i— \text{ and } (wt)^{-1} \neq —y_i—Q—i—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

7. Suppose $P < Q < i < j < y_j < y_i$.

(a) If $w^{-1} = —y_j—y_i—i—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(b) If $w^{-1} = —Q—y_j—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(c) If $w^{-1} = —y_j—y_i—Q—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(d) If $w^{-1} = —y_j—Q—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(e) If $w^{-1} = —y_j—Q—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = —Q—y_j—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = —y_j—y_i—i—Q—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = —y_j—y_i—i—Q—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = —Q—y_j—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = —y_j—y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = —y_j—y_i—Q—i—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_j—Q—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = —y_j—Q—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (j, y_j)$.
(n) If $w^{-1} = —Q—y_j—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $P < Q < i < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_j—y_i—i—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —P—y_i—$ and $(wt)^{-1} = —P—y_j—$.

8. Suppose $i < j < y_j < P < Q < y_i$.

(a) If $w^{-1} = —y_j—Q—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.
(b) If $w^{-1} = —Q—y_j—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.

(c) If $w^{-1} = —y_j—y_i—Q—i—j—P—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(d) If $w^{-1} = —y_j—Q—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(e) If $w^{-1} = —y_j—Q—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(f) If $w^{-1} = —Q—y_j—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(g) If $w^{-1} = —y_j—y_i—i—Q—j—P—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(h) If $w^{-1} = —y_j—y_i—i—Q—P—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(i) If $w^{-1} = —Q—y_j—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(j) If $w^{-1} = —Q—P—y_j—y_i—i—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(k) If $w^{-1} = —Q—y_j—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(l) If $w^{-1} = —y_j—y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(m) If $w^{-1} = —y_j—y_i—Q—i—P—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(n) If $w^{-1} = —y_j—Q—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i < j < y_j < P < Q < y_i$ and then one of the following holds:

- $w^{-1} = —y_j—y_i—i—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

$(\mathrm{Z2}) \Leftrightarrow (wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.

$(\mathrm{Z3}) \Leftrightarrow (wt)^{-1} = —j—Q—$.

9. Suppose $i < j < P < y_j < y_i < Q$.

(a) If $w^{-1} = —y_j—Q—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(b) If $w^{-1} = —Q—y_j—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(c) If $w^{-1} = —y_j—y_i—Q—i—j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(d) If $w^{-1} = —y_j—Q—y_i—i—j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(e) If $w^{-1} = —y_j—Q—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(f) If $w^{-1} = —Q—y_j—y_i—i—P—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(g) If $w^{-1} = —Q—y_j—P—y_i—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(h) If $w^{-1} = —Q—P—y_j—y_i—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(i) If $w^{-1} = —Q—y_j—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(j) If $w^{-1} = —y_j—y_i—Q—P—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(k) If $w^{-1} = —y_j—y_i—Q—i—P—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(l) If $w^{-1} = —y_j—Q—y_i—P—i—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (P,Q)$.

(m) If $w^{-1} = —y_j—y_i—i—Q—j—P—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

(n) If $w^{-1} = —y_j—y_i—i—Q—P—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i < j < P < y_j < y_i < Q$ and then one of the following holds:

- $w^{-1} = —y_j—y_i—i—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{— and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{— and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}Q\text{—}$.

10. Suppose $i < P < j < y_j < y_i < Q$.
    (a) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
    (b) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
    (c) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (T) fails.
    (d) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (T) fails.
    (e) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
    (f) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (g) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (h) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (i) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (j) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (k) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (l) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (m) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (n) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

    Recall that $(k,l) = (j,y_i)$. We conclude that if $i < P < j < y_j < y_i < Q$ and then one of the following holds:

    - $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}$ and $v^{-1} = \text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}$.

    When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{— and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{— and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—}. \end{cases}$$

    (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}$.

    (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}Q\text{—}$.

11. Suppose $i < j < P < Q < y_j < y_i$.
    (a) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (b) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (c) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (d) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (e) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (f) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (g) If $w^{-1} = \text{—}y_j\text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (h) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (i) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (j) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (k) If $w^{-1} = \text{—}Q\text{—}y_j\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (l) If $w^{-1} = \text{—}y_j\text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < j < P < Q < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_j—y_i—i—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.
- $w^{-1} = —Q—P—y_j—y_i—i—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.
- $w^{-1} = —Q—y_j—y_i—i—j—P—$ and $v^{-1} = —Q—y_j—j—y_i—i—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

$$(\text{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —y_i—P—i— \text{ and } (wt)^{-1} \neq —y_i—Q—i— \text{ and} \\ (wt)^{-1} \neq —y_j—P—j— \text{ and } (wt)^{-1} \neq —y_j—Q—j—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

12. Suppose $i < P < j < Q < y_j < y_i$.

(a) If $w^{-1} = —y_j—Q—y_i—i—P—j—$ then (T) fails.
(b) If $w^{-1} = —Q—y_j—y_i—i—P—j—$ then (T) fails.
(c) If $w^{-1} = —y_j—y_i—i—Q—P—j—$ then (T) fails.
(d) If $w^{-1} = —y_j—y_i—Q—i—P—j—$ then (T) fails.
(e) If $w^{-1} = —y_j—y_i—i—j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(f) If $w^{-1} = —y_j—Q—P—y_i—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(g) If $w^{-1} = —Q—y_j—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(h) If $w^{-1} = —y_j—y_i—Q—i—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(i) If $w^{-1} = —y_j—Q—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(j) If $w^{-1} = —y_j—y_i—i—Q—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(k) If $w^{-1} = —Q—y_j—P—y_i—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(l) If $w^{-1} = —Q—y_j—y_i—P—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(m) If $w^{-1} = —y_j—y_i—Q—P—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
(n) If $w^{-1} = —y_j—Q—y_i—P—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < P < j < Q < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_j—y_i—i—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—y_j—$.

13. Suppose $i < j < y_j < y_i < P < Q$.

(a) If $w^{-1} = —y_j—Q—P—y_i—i—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(b) If $w^{-1} = —Q—y_j—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(c) If $w^{-1} = —y_j—y_i—Q—i—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(d) If $w^{-1} = —y_j—Q—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(e) If $w^{-1} = -y_j-Q-y_i-i-P-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = -Q-y_j-y_i-i-P-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = -Q-y_j-P-y_i-i-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = -Q-P-y_j-y_i-i-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = -Q-y_j-y_i-P-i-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = -y_j-y_i-Q-P-i-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = -y_j-y_i-Q-i-P-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = -y_j-Q-y_i-P-i-j-$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = -y_j-y_i-i-Q-j-P-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = -y_j-y_i-i-Q-P-j-$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (j,y_i)$. We conclude that if $i < j < y_j < y_i < P < Q$ and then one of the following holds:

- $w^{-1} = -y_j-y_i-i-j-Q-P-$ and $v^{-1} = -y_j-j-y_i-i-Q-P-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -y_i-i- \text{ and} \\ (wt)^{-1} = -y_j-j-. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = -i-Q-$ and $(wt)^{-1} = -j-Q-$.

14. Suppose $P < i < j < y_j < y_i < Q$.

(a) If $w^{-1} = -Q-y_j-y_i-i-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = -y_j-y_i-Q-i-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = -y_j-Q-y_i-i-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = -y_j-Q-y_i-i-P-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = -Q-y_j-y_i-i-P-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = -Q-y_j-y_i-P-i-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = -y_j-y_i-Q-i-P-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = -y_j-Q-y_i-P-i-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = -y_j-y_i-i-Q-j-P-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = -Q-y_j-P-y_i-i-j-$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Recall that $(k,l) = (j,y_i)$. We conclude that if $P < i < j < y_j < y_i < Q$ and then one of the following holds:

- $w^{-1} = -y_j-y_i-i-j-Q-P-$ and $v^{-1} = -y_j-j-y_i-i-Q-P-$.
- $w^{-1} = -y_j-y_i-i-Q-P-j-$ and $v^{-1} = -y_j-j-y_i-Q-P-i-$.
- $w^{-1} = -Q-P-y_j-y_i-i-j-$ and $v^{-1} = -Q-P-y_j-j-y_i-i-$.
- $w^{-1} = -y_j-Q-P-y_i-i-j-$ and $v^{-1} = -y_j-Q-P-j-y_i-i-$.
- $w^{-1} = -y_j-y_i-Q-P-i-j-$ and $v^{-1} = -y_j-j-Q-P-y_i-i-$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -y_i-i- \text{ and} \\ (wt)^{-1} = -y_j-j-. \end{cases}$$

$$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and } (wt)^{-1} \neq —Q—y_i—P— \text{ and} \\ (wt)^{-1} \neq —Q—j—P— \text{ and } (wt)^{-1} \neq —Q—y_j—P—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

15. Suppose $i < P < Q < j < y_j < y_i$.
    (a) If $w^{-1} = —y_j—Q—y_i—i—P—j—$ then (T) fails.
    (b) If $w^{-1} = —Q—y_j—y_i—i—P—j—$ then (T) fails.
    (c) If $w^{-1} = —y_j—y_i—i—Q—j—P—$ then (T) fails.
    (d) If $w^{-1} = —y_j—y_i—i—Q—P—j—$ then (T) fails.
    (e) If $w^{-1} = —y_j—y_i—Q—i—P—j—$ then (T) fails.
    (f) If $w^{-1} = —y_j—y_i—i—j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
    (g) If $w^{-1} = —y_j—Q—P—y_i—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
    (h) If $w^{-1} = —Q—y_j—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
    (i) If $w^{-1} = —y_j—y_i—Q—i—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
    (j) If $w^{-1} = —y_j—Q—y_i—i—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
    (k) If $w^{-1} = —Q—y_j—P—y_i—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
    (l) If $w^{-1} = —Q—y_j—y_i—P—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
    (m) If $w^{-1} = —y_j—y_i—Q—P—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
    (n) If $w^{-1} = —y_j—Q—y_i—P—i—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.

    Recall that $(k, l) = (j, y_i)$. We conclude that if $i < P < Q < j < y_j < y_i$ and then one of the following holds:

    - $w^{-1} = —Q—P—y_j—y_i—i—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.

    When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—y_j—$.

We conclude that properties (V1)-(V3) hold whenever (a, b), (a', b') are as in cases (i) or (ii) and $i < j < y_j < y_i$.

3.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < i < j' < j < y_{j'} < y_j < y_{i'} < y_i$.
    (a) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—i—j'—j—$ then (T) fails.
    (b) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—i—j'—j—$ then (T) fails.
    (c) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—i—j'—j—$ then (T) fails.
    (d) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—i—j'—j—$ then (T) fails.
    (e) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—i—j'—j—$ then (T) fails.
    (f) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
    (g) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—j'—i—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
    (h) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—j'—i—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.
    (i) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—j'—y_i—i—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.
    (j) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—j'—y_i—i—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.

(k) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(l) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(m) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < i < j' < j < y_{j'} < y_j < y_{i'} < y_i$ and then one of the following holds:

- $w^{-1} = —y_{j'}—y_{i'}—i'—j'—y_j—y_i—i—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—j'—i—$ and $(wt)^{-1} \neq —y_i—y_{j'}—i—$.

(V3) $\Leftrightarrow$ (no condition).

2. Suppose $i' < j' < i < y_{j'} < j < y_j < y_{i'} < y_i$.

(a) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(b) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(c) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(d) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(e) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(f) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(g) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(h) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(i) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(j) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(l) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(m) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < j' < i < y_{j'} < j < y_j < y_{i'} < y_i$ and then one of the following holds:

- $w^{-1} = —y_{j'}—y_{i'}—i'—j'—y_j—y_i—i—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

3. Suppose $i' < j' < y_{j'} < i < y_{i'} < j < y_j < y_i$.

(a) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(b) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(c) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(d) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(e) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(f) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(g) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(h) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(i) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(j) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(k) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < j' < y_{j'} < i < y_{i'} < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{j'}—y_{i'}—i'—j'—y_j—y_i—i—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a',b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

4. Suppose $i' < i < j' < j < y_{j'} < y_{i'} < y_j < y_i$.

(a) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—i—j'—j—$ then (T) fails.

(b) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—i—j'—j—$ then (T) fails.

(c) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—i—j'—j—$ then (T) fails.

(d) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—i—j'—j—$ then (T) fails.

(e) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—i—j'—j—$ then (T) fails.

(f) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(g) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(h) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(i) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(j) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(k) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(l) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(m) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < i < j' < j < y_{j'} < y_{i'} < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{j'}—y_{i'}—i'—j'—y_j—y_i—i—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a',b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}y_i\text{—}j'\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}y_{j'}\text{—}i\text{—}$.

(V3) $\Leftrightarrow$ (no condition).

5. Suppose $i' < j' < i < y_{j'} < j < y_{i'} < y_j < y_i$.

   (a) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (b) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (c) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (d) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (e) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
   (f) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
   (g) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
   (h) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
   (i) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
   (j) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
   (k) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

   Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < j' < i < y_{j'} < j < y_{i'} < y_j < y_i$ and then one of the following holds:

   - $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ and $v^{-1} = \text{—}y_{j'}\text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j'\text{—}. \end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

6. Suppose $i' < i < j' < y_{j'} < j < y_j < y_{i'} < y_i$.

   (a) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
   (i) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.
   (j) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(k) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(l) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(m) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < i < j' < y_{j'} < j < y_j < y_{i'} < y_i$ and then one of the following holds:

- $w^{-1} = —y_{j'}—y_{i'}—i'—j'—y_j—y_i—i—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—j'—i—$ and $(wt)^{-1} \neq —y_i—y_{j'}—i—$.

(V3) $\Leftrightarrow$ (no condition).

7. Suppose $i' < j' < i < j < y_{j'} < y_j < y_{i'} < y_i$.

(a) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(b) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(c) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(d) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(e) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(f) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(g) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(h) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(i) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(j) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(l) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(m) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < j' < i < j < y_{j'} < y_j < y_{i'} < y_i$ and then one of the following holds:

- $w^{-1} = —y_{j'}—y_{i'}—i'—j'—y_j—y_i—i—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

8. Suppose $i' < j' < y_{j'} < y_{i'} < i < j < y_j < y_i$.

(a) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(b) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(c) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(d) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(e) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(f) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(g) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(h) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(i) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(j) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(k) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < j' < y_{j'} < y_{i'} < i < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ and $v^{-1} = \text{—}y_{j'}\text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a',b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

9. Suppose $i' < j' < y_{j'} < i < j < y_j < y_{i'} < y_i$.

(a) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(b) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(c) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(d) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(e) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(f) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(g) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(h) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(i) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(j) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(l) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

(m) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < j' < y_{j'} < i < j < y_j < y_{i'} < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ and $v^{-1} = \text{—}y_{j'}\text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a',b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -y_i - i - \text{ and} \\ (wt)^{-1} = -y_j - j - \text{ and} \\ (wt)^{-1} = -y_{i'} - i' - \text{ and} \\ (wt)^{-1} = -y_{j'} - j' - . \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

10. Suppose $i' < j' < i < j < y_{j'} < y_{i'} < y_j < y_i$.

(a) If $w^{-1} = -y_{j'}-y_j-y_{i'}-y_i-i'-j'-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(b) If $w^{-1} = -y_{j'}-y_j-y_{i'}-y_i-i'-i-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(c) If $w^{-1} = -y_{j'}-y_{i'}-y_j-y_i-i'-j'-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(d) If $w^{-1} = -y_{j'}-y_{i'}-y_j-y_i-i'-i-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(e) If $w^{-1} = -y_{j'}-y_j-y_{i'}-i'-y_i-j'-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(f) If $w^{-1} = -y_{j'}-y_j-y_{i'}-i'-y_i-i-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(g) If $w^{-1} = -y_{j'}-y_j-y_{i'}-i'-j'-y_i-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(h) If $w^{-1} = -y_{j'}-y_{i'}-y_j-i'-y_i-j'-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(i) If $w^{-1} = -y_{j'}-y_{i'}-y_j-i'-y_i-i-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(j) If $w^{-1} = -y_{j'}-y_{i'}-y_j-i'-j'-y_i-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(k) If $w^{-1} = -y_{j'}-y_{i'}-i'-y_j-y_i-j'-i-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = -y_{j'}-y_{i'}-i'-y_j-y_i-i-j'-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
(m) If $w^{-1} = -y_{j'}-y_{i'}-i'-y_j-j'-y_i-i-j-$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < j' < i < j < y_{j'} < y_{i'} < y_j < y_i$ and then one of the following holds:

- $w^{-1} = -y_{j'}-y_{i'}-i'-j'-y_j-y_i-i-j-$ and $v^{-1} = -y_{j'}-j'-y_{i'}-i'-y_j-j-y_i-i-$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -y_i - i - \text{ and} \\ (wt)^{-1} = -y_j - j - \text{ and} \\ (wt)^{-1} = -y_{i'} - i' - \text{ and} \\ (wt)^{-1} = -y_{j'} - j' - . \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

11. Suppose $i' < j' < y_{j'} < i < j < y_{i'} < y_j < y_i$.

(a) If $w^{-1} = -y_{j'}-y_j-y_{i'}-y_i-i'-j'-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(b) If $w^{-1} = -y_{j'}-y_j-y_{i'}-y_i-i'-i-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(c) If $w^{-1} = -y_{j'}-y_{i'}-y_j-y_i-i'-j'-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(d) If $w^{-1} = -y_{j'}-y_{i'}-y_j-y_i-i'-i-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(e) If $w^{-1} = -y_{j'}-y_j-y_{i'}-i'-y_i-j'-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(f) If $w^{-1} = -y_{j'}-y_j-y_{i'}-i'-y_i-i-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(g) If $w^{-1} = -y_{j'}-y_j-y_{i'}-i'-j'-y_i-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(h) If $w^{-1} = -y_{j'}-y_{i'}-y_j-i'-y_i-j'-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(i) If $w^{-1} = -y_{j'}-y_{i'}-y_j-i'-y_i-i-j'-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.
(j) If $w^{-1} = -y_{j'}-y_{i'}-y_j-i'-j'-y_i-i-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (j, y_j)$.

(k) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(l) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(m) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < j' < y_{j'} < i < j < y_{i'} < y_j < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ and $v^{-1} = \text{—}y_{j'}\text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

12. Suppose $i' < i < j' < y_{j'} < y_{i'} < j < y_j < y_i$.

(a) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.

(b) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.

(c) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.

(d) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.

(e) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}$ then (T) fails.

(f) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(g) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(h) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(i) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(j) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(k) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(l) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.

(m) If $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < i < j' < y_{j'} < y_{i'} < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}y_{j'}\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_j\text{—}y_i\text{—}i\text{—}j\text{—}$ and $v^{-1} = \text{—}y_{j'}\text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}y_i\text{—}j'\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}y_{j'}\text{—}i\text{—}$.

(V3) $\Leftrightarrow$ (no condition).

13. Suppose $i' < j' < i < y_{j'} < y_{i'} < j < y_j < y_i$.

(a) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(b) If $w^{-1} = \text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(d) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(e) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(f) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(g) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(h) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(i) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(j) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(k) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(l) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—i—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(m) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i' < j' < i < y_{j'} < y_{i'} < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{j'}—y_{i'}—i'—j'—y_j—y_i—i—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a', b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

14. Suppose $i' < i < j' < y_{j'} < j < y_{i'} < y_j < y_i$.

(a) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—i—j'—j—$ then (T) fails.

(b) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—i—j'—j—$ then (T) fails.

(c) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—i—j'—j—$ then (T) fails.

(d) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—i—j'—j—$ then (T) fails.

(e) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—i—j'—j—$ then (T) fails.

(f) If $w^{-1} = —y_{j'}—y_j—y_{i'}—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(g) If $w^{-1} = —y_{j'}—y_{i'}—y_j—y_i—i'—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(h) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(i) If $w^{-1} = —y_{j'}—y_j—y_{i'}—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(j) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(k) If $w^{-1} = —y_{j'}—y_{i'}—y_j—i'—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (j, y_j)$.

(l) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—j'—y_i—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.

(m) If $w^{-1} = —y_{j'}—y_{i'}—i'—y_j—y_i—j'—i—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a', b') = (j, y_j)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i' < i < j' < y_{j'} < j < y_{i'} < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{j'}—y_{i'}—i'—j'—y_j—y_i—i—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a', b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{j'}\text{—}j'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_i\text{—}j'\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}y_{j'}\text{—}i\text{—}$.

(V3) $\Leftrightarrow$ (no condition).

We conclude that properties (V1)-(V3) hold for all $(a,b),(a',b') \in \mathrm{Cyc}(y)$ when $i < j < y_j < y_i$.

4 Case A3

Suppose $i < y_j < j < y_i$ and $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ so that $k = j < y_i = l$.

4.1 Subcase (i)

In this case $v = wt_{ij}t_{kl}$ is such that

$$v^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}.$$

When $(a,b),(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j,j),(i,y_i)\}$, properties (V1)-(V3) are equivalent to the following conditions which evidently hold:

(Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}i\text{—}$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_i\text{—}y_j\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}j\text{—}i\text{—}$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (V1)-(V3) hold whenever (a,b), (a',b') are as in case (i) and $i < y_j < j < y_i$.

4.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(y)$, so that $R = y_R \notin \{i,j,y_i,y_j\} + n\mathbb{Z}$.

1. Suppose $i < y_j < j < y_i < R$.
   (a) If $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (j,y_i)$. We conclude that if $i < y_j < j < y_i < R$ and then one of the following holds:

   - $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}$ and $v^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j,j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow (wt)^{-1} = \text{—}j\text{—}y_j\text{—}$ and $(wt)^{-1} = \text{—}y_i\text{—}i\text{—}$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}R\text{—}$.

2. Suppose $i < y_j < j < R < y_i$.
   (a) If $w^{-1} = \text{—}y_i\text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}y_i\text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = \text{—}R\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < y_j < j < R < y_i$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—R—$ and $v^{-1} = —j—y_i—i—y_j—R—$.

When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —y_i—i—$.
(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—R—i—$.
(Z3) $\Leftrightarrow$ $(wt)^{-1} = —y_j—R—$.

3. Suppose $i < y_j < R < j < y_i$.
   (a) If $w^{-1} = —y_i—i—R—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (c) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (R, R)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $i < y_j < R < j < y_i$ and then one of the following holds:

   - $w^{-1} = —R—y_i—i—j—y_j—$ and $v^{-1} = —R—j—y_i—i—y_j—$.
   - $w^{-1} = —y_i—i—j—y_j—R—$ and $v^{-1} = —j—y_i—i—y_j—R—$.

   When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —y_i—i—$.
   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—R—y_j—$ and $(wt)^{-1} \neq —y_i—R—i—$.
   (Z3) $\Leftrightarrow$ (no condition).

4. Suppose $i < R < y_j < j < y_i$.
   (a) If $w^{-1} = —y_i—i—R—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (c) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (y_j, j)$.
   (d) If $w^{-1} = —y_i—i—j—y_j—R—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (y_j, j)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $i < R < y_j < j < y_i$ and then one of the following holds:

   - $w^{-1} = —R—y_i—i—j—y_j—$ and $v^{-1} = —R—j—y_i—i—y_j—$.

   When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —y_i—i—$.
   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—R—i—$.
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —R—j—$.

5. Suppose $R < i < y_j < j < y_i$.
   (a) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.
   (b) If $w^{-1} = —y_i—i—R—j—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.
   (c) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.
   (d) If $w^{-1} = —y_i—i—j—y_j—R—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $R < i < y_j < j < y_i$ and then one of the following holds:

   - $w^{-1} = —R—y_i—i—j—y_j—$ and $v^{-1} = —R—j—y_i—i—y_j—$.

   When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —y_i—i—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —R—j—$ and $(wt)^{-1} = —R—y_i—$.

Next suppose $P < Q$ are integers with $(P, Q) \in \mathrm{Cyc}^2(y)$, so that $Q = y_P$ and $P, Q \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $P < i < y_j < Q < j < y_i$.
   (a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
   (d) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (e) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (j) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (m) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (n) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $P < i < y_j < Q < j < y_i$ and then one of the following holds:

   - $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_i—i—y_j—$.

   When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $\begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_i—$.

2. Suppose $P < i < Q < y_j < j < y_i$.
   (a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
   (d) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (e) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (j) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (m) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(n) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $P < i < Q < y_j < j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_i—i—y_j—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_i—$.

3. Suppose $i < y_j < j < P < y_i < Q$.

(a) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(b) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(c) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(d) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(e) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < y_j < j < P < y_i < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —j—y_i—i—y_j—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

4. Suppose $P < i < y_j < j < Q < y_i$.

(a) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(b) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(c) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(d) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(e) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(g) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(h) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(i) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(j) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(k) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(l) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(m) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(n) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $P < i < y_j < j < Q < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_i—i—y_j—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —Q—y_j—P—$ and $(wt)^{-1} \neq —Q—j—P—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —P—y_i—$.

5. Suppose $i < y_j < P < j < Q < y_i$.

(a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.

(b) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.

(c) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.

(d) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(e) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(f) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(g) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(h) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(i) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(j) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(k) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(l) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(m) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(n) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < y_j < P < j < Q < y_i$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —j—y_i—i—y_j—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —y_j—Q—$.

6. Suppose $i < P < y_j < j < Q < y_i$.

   (a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (e) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (f) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (g) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (h) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (i) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (j) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (k) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (l) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $i < P < y_j < j < Q < y_i$ and then one of the following holds:

   - $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —j—y_i—i—y_j—Q—P—$.
   - $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_i—i—y_j—$.
   - $w^{-1} = —y_i—i—j—Q—P—y_j—$ and $v^{-1} = —j—y_i—i—Q—P—y_j—$.

   When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

   $$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—y_j—P— \text{ and } (wt)^{-1} \neq —Q—j—P— \text{ and} \\ (wt)^{-1} \neq —y_i—P—i— \text{ and } (wt)^{-1} \neq —y_i—Q—i—. \end{cases}$$

   (Z3) $\Leftrightarrow$ (no condition).

7. Suppose $P < Q < i < y_j < j < y_i$.

   (a) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (b) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (c) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (d) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (e) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (j) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (m) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
   (n) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $P < Q < i < y_j < j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_i—i—y_j—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j,j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_i—$.

8. Suppose $i < y_j < j < P < Q < y_i$.
   (a) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   Recall that $(k,l) = (j,y_i)$. We conclude that if $i < y_j < j < P < Q < y_i$ and then one of the following holds:

   - $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —j—y_i—i—y_j—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j,j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

   (Z2) $\Leftrightarrow (wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.

   (Z3) $\Leftrightarrow (wt)^{-1} = —y_j—Q—$.

9. Suppose $i < y_j < P < j < y_i < Q$.
   (a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(j) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < y_j < P < j < y_i < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —j—y_i—i—y_j—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

10. Suppose $i < P < y_j < j < y_i < Q$.

(a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
(c) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(e) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(f) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(g) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < P < y_j < j < y_i < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —j—y_i—i—y_j—Q—P—$.
- $w^{-1} = —y_i—i—j—Q—P—y_j—$ and $v^{-1} = —j—y_i—i—Q—P—y_j—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —Q—y_j—P—$ and $(wt)^{-1} \neq —Q—j—P—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$.

11. Suppose $i < y_j < P < Q < j < y_i$.

(a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.

(c) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
(e) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
(f) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < y_j < P < Q < j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—y_i—i—j—y_j—P—$ and $v^{-1} = —Q—j—y_i—i—y_j—P—$.
- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_i—i—y_j—$.
- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —j—y_i—i—y_j—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq —j—P—y_j— \text{ and } (wt)^{-1} \neq —j—Q—y_j— \text{ and} \\ (wt)^{-1} \neq —y_i—P—i— \text{ and } (wt)^{-1} \neq —y_i—Q—i—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

12. Suppose $i < P < y_j < Q < j < y_i$.

(a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
(c) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
(e) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
(f) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(i) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(j) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(k) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(l) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(m) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(n) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i < P < y_j < Q < j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_i—i—y_j—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}y_i\text{—}P\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}Q\text{—}i\text{—}$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}P\text{—}j\text{—}$.

13. Suppose $i < y_j < j < y_i < P < Q$.

    (a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (b) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (c) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (e) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (g) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (k) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (l) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (m) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (n) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

    Recall that $(k,l) = (j, y_i)$. We conclude that if $i < y_j < j < y_i < P < Q$ and then one of the following holds:

    - $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $v^{-1} = \text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}$.

    When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

    $$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—}. \end{cases}$$

    (Z2) $\Leftrightarrow$ (no condition).

    (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}Q\text{—}$.

14. Suppose $P < i < y_j < j < y_i < Q$.

    (a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (b) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (c) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (d) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (e) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
    (i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
    (j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.

    Recall that $(k,l) = (j, y_i)$. We conclude that if $P < i < y_j < j < y_i < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—Q—P—j—y_j—$ and $v^{-1} = —j—y_i—Q—P—i—y_j—$.
- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_i—i—y_j—$.
- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —j—y_i—i—y_j—Q—P—$.
- $w^{-1} = —y_i—i—j—Q—P—y_j—$ and $v^{-1} = —j—y_i—i—Q—P—y_j—$.
- $w^{-1} = —y_i—Q—P—i—j—y_j—$ and $v^{-1} = —j—Q—P—y_i—i—y_j—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

$$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and } (wt)^{-1} \neq —Q—y_i—P— \text{ and} \\ (wt)^{-1} \neq —Q—y_j—P— \text{ and } (wt)^{-1} \neq —Q—j—P—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

15. Suppose $i < P < Q < y_j < j < y_i$.

    (a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
    (b) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
    (c) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
    (d) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
    (e) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
    (f) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (g) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (h) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (i) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (j) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (k) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (l) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (m) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (n) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

    Recall that $(k, l) = (j, y_i)$. We conclude that if $i < P < Q < y_j < j < y_i$ and then one of the following holds:

    - $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_i—i—y_j—$.

    When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

    $$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

    (Z2) $\Leftrightarrow (wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.
    (Z3) $\Leftrightarrow (wt)^{-1} = —P—j—$.

We conclude that properties (V1)-(V3) hold whenever (a, b), (a', b') are as in cases (i) or (ii) and $i < y_j < j < y_i$.

4.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $i' < i < y_{j'} < y_j < j' < j < y_{i'} < y_i$.

   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (T) fails.
   (e) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (T) fails.
   (f) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (g) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (h) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
   (i) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
   (j) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.
   (m) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

   Recall that $(k, l) = (j, y_i)$. We conclude that if $i' < i < y_{j'} < y_j < j' < j < y_{i'} < y_i$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —j'—y_{i'}—i'—y_{j'}—j—y_i—i—y_j—$.

   When $(a, b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a', b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

   (V2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—y_{j'}—i—$ and $(wt)^{-1} \neq —y_i—j'—i—$.

   (V3) $\Leftrightarrow$ (no condition).

2. Suppose $i' < y_{j'} < i < j' < y_j < j < y_{i'} < y_i$.

   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (e) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (f) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (g) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
   (h) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (i) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (j) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (k) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (l) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
   (m) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

Recall that $(k,l)=(j,y_i)$. We conclude that if $i'<y_{j'}<i<j'<y_j<j<y_{i'}<y_i$ and then one of the following holds:

- $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1}=\text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b)\in\mathrm{Cyc}^1(y)=\{(y_j,j),(i,y_i)\}$ and $(a',b')\in\{(y_{j'},j'),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)}\Leftrightarrow\begin{cases}(wt)^{-1}=\text{—}j\text{—}y_j\text{—}\text{ and}\\(wt)^{-1}=\text{—}j'\text{—}y_{j'}\text{—}\text{ and}\\(wt)^{-1}=\text{—}y_i\text{—}i\text{—}\text{ and}\\(wt)^{-1}=\text{—}y_{i'}\text{—}i'\text{—}.\end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

3. Suppose $i'<y_{j'}<j'<i<y_{i'}<y_j<j<y_i$.
   (a) If $w^{-1}=\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(i',y_{i'})$ and $(a',b')=(i,y_i)$.
   (b) If $w^{-1}=\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(i',y_{i'})$ and $(a',b')=(i,y_i)$.
   (c) If $w^{-1}=\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(i',y_{i'})$ and $(a',b')=(i,y_i)$.
   (d) If $w^{-1}=\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(i',y_{i'})$ and $(a',b')=(i,y_i)$.
   (e) If $w^{-1}=\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(i',y_{i'})$ and $(a',b')=(i,y_i)$.
   (f) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(y_{j'},j')$ and $(a',b')=(i,y_i)$.
   (g) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(y_{j'},j')$ and $(a',b')=(i,y_i)$.
   (h) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(y_{j'},j')$ and $(a',b')=(i,y_i)$.
   (i) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(y_{j'},j')$ and $(a',b')=(i,y_i)$.
   (j) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(y_{j'},j')$ and $(a',b')=(i,y_i)$.
   (k) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(y_{j'},j')$ and $(a',b')=(i,y_i)$.
   (l) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(y_{j'},j')$ and $(a',b')=(i,y_i)$.
   (m) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(y_{j'},j')$ and $(a',b')=(i,y_i)$.

   Recall that $(k,l)=(j,y_i)$. We conclude that if $i'<y_{j'}<j'<i<y_{i'}<y_j<j<y_i$ and then one of the following holds:

   - $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1}=\text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

   When $(a,b)\in\mathrm{Cyc}^1(y)=\{(y_j,j),(i,y_i)\}$ and $(a',b')\in\{(y_{j'},j'),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$\text{(V1)}\Leftrightarrow\begin{cases}(wt)^{-1}=\text{—}j\text{—}y_j\text{—}\text{ and}\\(wt)^{-1}=\text{—}j'\text{—}y_{j'}\text{—}\text{ and}\\(wt)^{-1}=\text{—}y_i\text{—}i\text{—}\text{ and}\\(wt)^{-1}=\text{—}y_{i'}\text{—}i'\text{—}.\end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

4. Suppose $i'<i<y_{j'}<y_j<j'<y_{i'}<j<y_i$.
   (a) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1}=\text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1}=\text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.

(f) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(g) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(h) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(i) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (y_{j'}, j')$.
(j) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.
(m) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i' < i < y_{j'} < y_j < j' < y_{i'} < j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —j'—y_{i'}—i'—y_{j'}—j—y_i—i—y_j—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a', b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—y_{j'}—i—$ and $(wt)^{-1} \neq —y_i—j'—i—$.

(V3) $\Leftrightarrow$ (no condition).

5. Suppose $i' < y_{j'} < i < j' < y_j < y_{i'} < j < y_i$.

(a) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(c) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(d) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(e) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (j, y_i)$. We conclude that if $i' < y_{j'} < i < j' < y_j < y_{i'} < j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —j'—y_{i'}—i'—y_{j'}—j—y_i—i—y_j—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a', b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

6. Suppose $i' < i < y_{j'} < j' < y_j < j < y_{i'} < y_i$.
   (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
   (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < i < y_{j'} < j' < y_j < j < y_{i'} < y_i$ and then one of the following holds:

   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a',b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

   (V2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}y_i\text{—}y_{j'}\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}j'\text{—}i\text{—}$.

   (V3) $\Leftrightarrow$ (no condition).

7. Suppose $i' < y_{j'} < i < y_j < j' < j < y_{i'} < y_i$.
   (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

   Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < y_{j'} < i < y_j < j' < j < y_{i'} < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a', b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

8. Suppose $i' < y_{j'} < j' < y_{i'} < i < y_j < j < y_i$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   (b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   (f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

   (g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

   (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

   (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

   (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

   (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

   Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < y_{j'} < j' < y_{i'} < i < y_j < j < y_i$ and then one of the following holds:

   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a', b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

9. Suppose $i' < y_{j'} < j' < i < y_j < j < y_{i'} < y_i$.

   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   (b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

   (f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

   (g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < y_{j'} < j' < i < y_j < j < y_{i'} < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a',b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

10. Suppose $i' < y_{j'} < i < y_j < j' < y_{i'} < j < y_i$.

(a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.

(b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.

(c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.

(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.

(e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < y_{j'} < i < y_j < j' < y_{i'} < j < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a',b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

11. Suppose $i' < y_{j'} < j' < i < y_j < y_{i'} < j < y_i$.

   (a) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (c) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (e) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

   Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < y_{j'} < j' < i < y_j < y_{i'} < j < y_i$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —j'—y_{i'}—i'—y_{j'}—j—y_i—i—y_j—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a',b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

12. Suppose $i' < i < y_{j'} < j' < y_{i'} < y_j < j < y_i$.

   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (T) fails.
   (e) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (T) fails.
   (f) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (g) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (h) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.
   (i) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.
   (j) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
   (m) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < i < y_{j'} < j' < y_{i'} < y_j < j < y_i$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —j'—y_{i'}—i'—y_{j'}—j—y_i—i—y_j—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a',b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{— and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{— and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{— and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}y_i\text{—}y_{j'}\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}j'\text{—}i\text{—}$.

(V3) $\Leftrightarrow$ (no condition).

13. Suppose $i' < y_{j'} < i < j' < y_{i'} < y_j < j < y_i$.
    (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
    (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
    (d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
    (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
    (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
    (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

    Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < y_{j'} < i < j' < y_{i'} < y_j < j < y_i$ and then one of the following holds:

    - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{i'}\text{—}i'\text{—}y_{j'}\text{—}j\text{—}y_i\text{—}i\text{—}y_j\text{—}$.

    When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a',b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{— and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{— and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{— and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

    (V2) $\Leftrightarrow$ (no condition).

    (V3) $\Leftrightarrow$ (no condition).

14. Suppose $i' < i < y_{j'} < j' < y_j < y_{i'} < j < y_i$.
    (a) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
    (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (c) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
    (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.

(i) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (y_{j'}, j')$.
(j) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(m) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Recall that $(k,l) = (j, y_i)$. We conclude that if $i' < i < y_{j'} < j' < y_j < y_{i'} < j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —j'—y_{i'}—i'—y_{j'}—j—y_i—i—y_j—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, y_i)\}$ and $(a',b') \in \{(y_{j'}, j'), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—y_{j'}—i—$ and $(wt)^{-1} \neq —y_i—j'—i—$.

(V3) $\Leftrightarrow$ (no condition).

We conclude that properties (V1)-(V3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(y)$ when $i < y_j < j < y_i$.

5 Case B1

Suppose $y_j < y_i = i < j$ and $w^{-1} = —i—j—y_j—$ so that $k = y_j < i = l$.

5.1 Subcase (i)

In this case $v = wt_{ij}t_{kl}$ is such that

$$v^{-1} = —j—y_j—i—.$$

When $(a,b), (a',b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, i)\}$, properties (V1)-(V3) are equivalent to the following conditions which evidently hold:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—i—y_j—$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (V1)-(V3) hold whenever (a,b), (a',b') are as in case (i) and $y_j < y_i = i < j$.

5.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(y)$, so that $R = y_R \notin \{i, j, y_j\} + n\mathbb{Z}$.

1. Suppose $y_j < i < j < R$.

   (a) If $w^{-1} = —R—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —i—j—R—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —i—R—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < i < j < R$ and then one of the following holds:

   - $w^{-1} = —i—j—y_j—R—$ and $v^{-1} = —j—y_j—i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—R—$ and $(wt)^{-1} = —y_j—R—$.

2. Suppose $y_j < R < i < j$.
   (a) If $w^{-1} = —i—j—R—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, i)$.
   (b) If $w^{-1} = —i—j—y_j—R—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, i)$.
   (c) If $w^{-1} = —i—R—j—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, i)$.

   Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < R < i < j$ and then one of the following holds:

   - $w^{-1} = —R—i—j—y_j—$ and $v^{-1} = —R—j—y_j—i—$.

   When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$.

   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—R—y_j—$.

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —R—i—$.

3. Suppose $y_j < i < R < j$.
   (a) If $w^{-1} = —i—R—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —i—j—R—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (R, R)$.
   (c) If $w^{-1} = —R—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, i)$ and $(a', b') = (R, R)$.

   Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < i < R < j$ and then one of the following holds:

   - $w^{-1} = —i—j—y_j—R—$ and $v^{-1} = —j—y_j—i—R—$.

   When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$.

   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—R—y_j—$.

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—R—$.

4. Suppose $R < y_j < i < j$.
   (a) If $w^{-1} = —i—j—R—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, i)$.
   (b) If $w^{-1} = —i—j—y_j—R—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, i)$.
   (c) If $w^{-1} = —i—R—j—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, i)$.

   Recall that $(k, l) = (y_j, i)$. We conclude that if $R < y_j < i < j$ and then one of the following holds:

   - $w^{-1} = —R—i—j—y_j—$ and $v^{-1} = —R—j—y_j—i—$.

   When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —R—i—$ and $(wt)^{-1} = —R—j—$.

Next suppose $P < Q$ are integers with $(P, Q) \in \mathrm{Cyc}^2(y)$, so that $Q = y_P$ and $P, Q \notin \{i, j, y_j\} + n\mathbb{Z}$.

1. Suppose $P < y_j < i < Q < j$.
   (a) If $w^{-1} = —i—Q—P—j—y_j—$ then (T) fails.

(b) If $w^{-1} = —i—Q—j—y_j—P—$ then (T) fails.

(c) If $w^{-1} = —i—Q—j—P—y_j—$ then (T) fails.

(d) If $w^{-1} = —Q—i—P—j—y_j—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, i)$.

(e) If $w^{-1} = —i—j—y_j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(f) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(g) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(h) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(i) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $P < y_j < i < Q < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_j—$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —Q—i—P—$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—j—$.

2. Suppose $P < y_j < Q < i < j$.

(a) If $w^{-1} = —i—Q—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, i)$.

(b) If $w^{-1} = —Q—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, i)$.

(c) If $w^{-1} = —i—j—y_j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(d) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(e) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(f) If $w^{-1} = —i—Q—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(g) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(h) If $w^{-1} = —i—Q—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(i) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $P < y_j < Q < i < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i, i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_j—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—i—$ and $(wt)^{-1} = —P—j—$.

3. Suppose $y_j < i < j < P < Q$.

(a) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(b) If $w^{-1} = —i—Q—P—j—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(c) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(d) If $w^{-1} = —Q—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(e) If $w^{-1} = —Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(f) If $w^{-1} = —i—Q—j—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(g) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(h) If $w^{-1} = —i—Q—j—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(i) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < i < j < P < Q$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j,j),(i,i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_j—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

4. Suppose $y_j < i < P < j < Q$.

(a) If $w^{-1} = —i—Q—P—j—y_j—$ then (T) fails.

(b) If $w^{-1} = —Q—i—P—j—y_j—$ then (T) fails.

(c) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(d) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(e) If $w^{-1} = —Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(f) If $w^{-1} = —i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(g) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(h) If $w^{-1} = —i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(i) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < i < P < j < Q$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j,j),(i,i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_j—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

5. Suppose $y_j < P < i < Q < j$.

(a) If $w^{-1} = —i—Q—P—j—y_j—$ then (T) fails.

(b) If $w^{-1} = —i—Q—j—y_j—P—$ then (T) fails.

(c) If $w^{-1} = —i—Q—j—P—y_j—$ then (T) fails.

(d) If $w^{-1} = —Q—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, i)$.

(e) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, i)$.

(f) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(g) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(h) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < P < i < Q < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—$.
- $w^{-1} = —i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j,j),(i,i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_j—$.

$$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—} \text{ and} \\ (wt)^{-1} \neq \text{—}j\text{—}P\text{—}y_j\text{—} \text{ and } (wt)^{-1} \neq \text{—}j\text{—}Q\text{—}y_j\text{—}. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

6. Suppose $y_j < P < Q < i < j$.

   (a) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (b) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (c) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (d) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (e) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, i)$.
   (f) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, i)$.
   (g) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, i)$.
   (h) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, i)$.
   (i) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, i)$.

   Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < P < Q < i < j$ and then one of the following holds:

   - $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i,i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_j\text{—}$.
   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}P\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}Q\text{—}y_j\text{—}$.
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}P\text{—}i\text{—}$.

7. Suppose $P < Q < y_j < i < j$.

   (a) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, i)$.
   (b) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, i)$.
   (c) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (d) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (e) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (f) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (g) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (h) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
   (i) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.

   Recall that $(k, l) = (y_j, i)$. We conclude that if $P < Q < y_j < i < j$ and then one of the following holds:

   - $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i,i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}Q\text{—}P\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_j\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}P\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}j\text{—}$.

8. Suppose $y_j < P < i < j < Q$.

   (a) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (b) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (c) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(d) If $w^{-1} = —Q—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = —Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = —i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = —i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < P < i < j < Q$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i,i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_j—$.
(Z2) $\Leftrightarrow (wt)^{-1} \neq —Q—i—P—$.
(Z3) $\Leftrightarrow (wt)^{-1} = —y_j—Q—$.

9. Suppose $P < y_j < i < j < Q$.

(a) If $w^{-1} = —Q—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,i)$.
(b) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(c) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(d) If $w^{-1} = —i—Q—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(e) If $w^{-1} = —i—Q—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(f) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $P < y_j < i < j < Q$ and then one of the following holds:

- $w^{-1} = —i—j—Q—P—y_j—$ and $v^{-1} = —j—y_j—Q—P—i—$.
- $w^{-1} = —i—Q—P—j—y_j—$ and $v^{-1} = —j—Q—P—y_j—i—$.
- $w^{-1} = —i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—Q—P—$.
- $w^{-1} = —Q—P—i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i,i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow (wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_j—$.

(Z2) $\Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and} \\ (wt)^{-1} \neq —Q—y_j—P— \text{ and } (wt)^{-1} \neq —Q—j—P—. \end{cases}$

(Z3) $\Leftrightarrow$ (no condition).

10. Suppose $y_j < i < P < Q < j$.

(a) If $w^{-1} = —i—Q—P—j—y_j—$ then (T) fails.
(b) If $w^{-1} = —Q—i—P—j—y_j—$ then (T) fails.
(c) If $w^{-1} = —i—Q—j—y_j—P—$ then (T) fails.
(d) If $w^{-1} = —i—Q—j—P—y_j—$ then (T) fails.
(e) If $w^{-1} = —i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = —Q—i—j—P—y_j—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = —i—j—Q—P—y_j—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = —Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = —Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,i)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < i < P < Q < j$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_j,j),(i,i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —Q—P—$ and $(wt)^{-1} = —j—y_j—$.
(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—P—y_j—$ and $(wt)^{-1} \neq —j—Q—y_j—$.
(Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—Q—$.

We conclude that properties (V1)-(V3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_j < y_i = i < j$.

5.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $y_{j'} < i' < y_j < i < j' < j$.
   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (c) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.
   (d) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j,j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_j < i < j' < j$ and then one of the following holds:

   - $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—j—y_j—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j,j),(i,i)\}$ and $(a',b') \in \{(y_{j'},j'),(i',i')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (V1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —j'—y_{j'}—$.
   (V2) $\Leftrightarrow$ (no condition).
   (V3) $\Leftrightarrow$ (no condition).

2. Suppose $y_{j'} < i' < j' < y_j < i < j$.
   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.
   (b) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.
   (c) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j,j)$.
   (d) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j,j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < j' < y_j < i < j$ and then one of the following holds:

   - $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—j—y_j—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j,j),(i,i)\}$ and $(a',b') \in \{(y_{j'},j'),(i',i')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (V1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —j'—y_{j'}—$.
   (V2) $\Leftrightarrow$ (no condition).
   (V3) $\Leftrightarrow$ (no condition).

3. Suppose $y_{j'} < y_j < i' < j' < i < j$.
   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.
   (b) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.
   (c) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j,j)$.
   (d) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j,j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < i' < j' < i < j$ and then one of the following holds:

- $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—j—y_j—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i,i)\}$ and $(a',b') \in \{(y_{j'}, j'), (i', i')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(V1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —j'—y_{j'}—$.

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—i'—y_j—$.

(V3) $\Leftrightarrow$ (no condition).

4. Suppose $y_{j'} < i' < y_j < j' < i < j$.

   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.

   (b) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.

   (c) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   (d) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_j < j' < i < j$ and then one of the following holds:

   - $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—j—y_j—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i,i)\}$ and $(a',b') \in \{(y_{j'}, j'), (i', i')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (V1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —j'—y_{j'}—$.

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

5. Suppose $y_{j'} < y_j < i' < i < j' < j$.

   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.

   (b) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.

   (c) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i,i)$.

   (d) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < i' < i < j' < j$ and then one of the following holds:

   - $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—j—y_j—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_j, j), (i,i)\}$ and $(a',b') \in \{(y_{j'}, j'), (i', i')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (V1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —j'—y_{j'}—$.

   (V2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—i'—y_j—$.

   (V3) $\Leftrightarrow$ (no condition).

We conclude that properties (V1)-(V3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(y)$ when $y_j < y_i = i < j$.

6 Case B2

Suppose $y_j < i < y_i < j$ and $w^{-1} = —y_i—i—j—y_j—$ so that $k = y_j < i = l$.

6.1 Subcase (i)

In this case $v = wt_{ij}t_{kl}$ is such that

$v^{-1} = —y_i—j—y_j—i—.$

When $(a, b), (a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$, properties (V1)-(V3) are equivalent to the following conditions which evidently hold:

(Z1) $\Leftrightarrow (wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —y_i—i—$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq —j—i—y_j—$ and $(wt)^{-1} \neq —j—y_i—y_j—$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (V1)-(V3) hold whenever (a, b), (a', b') are as in case (i) and $y_j < i < y_i < j$.

6.2 Subcase (ii)

Suppose R is an integer such that $(R, R) \in \mathrm{Cyc}^2(y)$, so that $R = y_R \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $y_j < i < y_i < j < R$.
   (a) If $w^{-1} = —R—y_i—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (b) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (c) If $w^{-1} = —y_i—i—R—j—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (R, R)$.
   (d) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (R, R)$.

   Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < i < y_i < j < R$ and then one of the following holds:

   - $w^{-1} = —y_i—i—j—y_j—R—$ and $v^{-1} = —y_i—j—y_j—i—R—$.

   When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow (wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —y_i—i—$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow (wt)^{-1} = —i—R—$ and $(wt)^{-1} = —y_j—R—$.

2. Suppose $y_j < i < y_i < R < j$.
   (a) If $w^{-1} = —y_i—i—R—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (R, R)$.
   (c) If $w^{-1} = —R—y_i—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (R, R)$.
   (d) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (R, R)$.

   Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < i < y_i < R < j$ and then one of the following holds:

   - $w^{-1} = —y_i—i—j—y_j—R—$ and $v^{-1} = —y_i—j—y_j—i—R—$.

   When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow (wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —y_i—i—$.
   (Z2) $\Leftrightarrow (wt)^{-1} \neq —j—R—y_j—$.
   (Z3) $\Leftrightarrow (wt)^{-1} = —i—R—$.

3. Suppose $y_j < i < R < y_i < j$.
   (a) If $w^{-1} = —y_i—i—R—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (R, R)$.

(c) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (R, R)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < i < R < y_i < j$ and then one of the following holds:

- $w^{-1} = —R—y_i—i—j—y_j—$ and $v^{-1} = —R—y_i—j—y_j—i—$.
- $w^{-1} = —y_i—i—j—y_j—R—$ and $v^{-1} = —y_i—j—y_j—i—R—$.

When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —y_i—i—$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—R—y_j—$ and $(wt)^{-1} \neq —y_i—R—i—$.

(Z3) $\Leftrightarrow$ (no condition).

4. Suppose $y_j < R < i < y_i < j$.

(a) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (R, R)$.

(b) If $w^{-1} = —y_i—i—R—j—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.

(d) If $w^{-1} = —y_i—i—j—y_j—R—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < R < i < y_i < j$ and then one of the following holds:

- $w^{-1} = —R—y_i—i—j—y_j—$ and $v^{-1} = —R—y_i—j—y_j—i—$.

When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —y_i—i—$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—R—y_j—$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —R—y_i—$.

5. Suppose $R < y_j < i < y_i < j$.

(a) If $w^{-1} = —y_i—i—R—j—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.

(b) If $w^{-1} = —y_i—R—i—j—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = —y_i—i—j—y_j—R—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.

(d) If $w^{-1} = —y_i—i—j—R—y_j—$ then (Y3) fails for $(a, b) = (R, R)$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $R < y_j < i < y_i < j$ and then one of the following holds:

- $w^{-1} = —R—y_i—i—j—y_j—$ and $v^{-1} = —R—y_i—j—y_j—i—$.

When $(a, b) = (R, R)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —j—y_j—$ and $(wt)^{-1} = —y_i—i—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —R—j—$ and $(wt)^{-1} = —R—y_i—$.

Next suppose $P < Q$ are integers with $(P, Q) \in \mathrm{Cyc}^2(y)$, so that $Q = y_P$ and $P, Q \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $P < y_j < i < Q < y_i < j$.

(a) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.

(b) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.

(c) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.

(d) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(e) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(f) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(j) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(k) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(l) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(m) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(n) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Recall that $(k,l) = (y_j,i)$. We conclude that if $P < y_j < i < Q < y_i < j$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—y_i—j—y_j—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_i—$.

2. Suppose $P < y_j < Q < i < y_i < j$.

(a) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(j) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(k) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(l) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(m) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(n) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Recall that $(k,l) = (y_j,i)$. We conclude that if $P < y_j < Q < i < y_i < j$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—y_i—j—y_j—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_i—$.

3. Suppose $y_j < i < y_i < P < j < Q$.
   (a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (m) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (n) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   Recall that $(k,l) = (y_j,i)$. We conclude that if $y_j < i < y_i < P < j < Q$ and then one of the following holds:

   - $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —y_i—j—y_j—i—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

4. Suppose $P < y_j < i < y_i < Q < j$.
   (a) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
   (d) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (h) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (i) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (j) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (k) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (l) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (m) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

   Recall that $(k,l) = (y_j,i)$. We conclude that if $P < y_j < i < y_i < Q < j$ and then one of the following holds:

   - $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—y_i—j—y_j—i—$.
   - $w^{-1} = —y_i—Q—P—i—j—y_j—$ and $v^{-1} = —y_i—Q—P—j—y_j—i—$.

When $(a,b)=(P,Q)$ and $(a',b')\in \mathrm{Cyc}^1(y)=\{(i,y_i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—}$.

(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}j\text{—}$.

5. Suppose $y_j < i < P < y_i < Q < j$.
   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b)=(y_j,j)$ and $(a',b')=(P,Q)$.
   (g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b)=(y_j,j)$ and $(a',b')=(P,Q)$.
   (h) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(i,y_i)$ and $(a',b')=(P,Q)$.
   (i) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b)=(i,y_i)$ and $(a',b')=(P,Q)$.
   (j) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(i,y_i)$ and $(a',b')=(P,Q)$.
   (k) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(i,y_i)$ and $(a',b')=(P,Q)$.
   (l) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b)=(i,y_i)$ and $(a',b')=(P,Q)$.
   (m) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(i,y_i)$ and $(a',b')=(P,Q)$.
   (n) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b)=(i,y_i)$ and $(a',b')=(P,Q)$.

   Recall that $(k,l)=(y_j,i)$. We conclude that if $y_j < i < P < y_i < Q < j$ and then one of the following holds:

   - $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $v^{-1} = \text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}Q\text{—}P\text{—}$.

   When $(a,b)=(P,Q)$ and $(a',b')\in \mathrm{Cyc}^1(y)=\{(i,y_i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—}. \end{cases}$$

   (Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}P\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}Q\text{—}y_j\text{—}$.

   (Z3) $\Leftrightarrow (wt)^{-1} = \text{—}i\text{—}Q\text{—}$.

6. Suppose $y_j < P < i < y_i < Q < j$.
   (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (e) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (g) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (i) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.
   (j) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b)=(P,Q)$ and $(a',b')=(i,y_i)$.

(k) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(l) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < P < i < y_i < Q < j$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —y_i—j—y_j—i—Q—P—$.
- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—y_i—j—y_j—i—$.
- $w^{-1} = —y_i—Q—P—i—j—y_j—$ and $v^{-1} = —y_i—Q—P—j—y_j—i—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

$$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and } (wt)^{-1} \neq —Q—y_i—P— \text{ and} \\ (wt)^{-1} \neq —j—P—y_j— \text{ and } (wt)^{-1} \neq —j—Q—y_j—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

7. Suppose $P < Q < y_j < i < y_i < j$.

(a) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(b) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(d) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(e) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(f) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(g) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(h) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(i) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(j) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(k) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(l) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(m) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

(n) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $P < Q < y_j < i < y_i < j$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—y_i—j—y_j—i—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —P—j—$ and $(wt)^{-1} = —P—y_i—$.

8. Suppose $y_j < i < y_i < P < Q < j$.

(a) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.

(b) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.

(c) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.

(d) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.

(e) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.

(f) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(g) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(h) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(i) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(j) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(k) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(l) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(m) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(n) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < i < y_i < P < Q < j$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —y_i—j—y_j—i—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —j—P—y_j—$ and $(wt)^{-1} \neq —j—Q—y_j—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$.

9. Suppose $y_j < i < P < y_i < j < Q$.

(a) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.

(b) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (T) fails.

(c) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (T) fails.

(d) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(e) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(f) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(g) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(h) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(i) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(j) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(k) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(l) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(m) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(n) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < i < P < y_i < j < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —y_i—j—y_j—i—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(Z1) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}Q\text{—}$.

10. Suppose $y_j < P < i < y_i < j < Q$.
    (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (b) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (d) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (f) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (g) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (i) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (j) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (k) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (l) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (m) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (n) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

    Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < P < i < y_i < j < Q$ and then one of the following holds:

    - $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ and $v^{-1} = \text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}Q\text{—}P\text{—}$.

    When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(Z1) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—}$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_j\text{—}Q\text{—}$.

11. Suppose $y_j < i < P < Q < y_i < j$.
    (a) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
    (b) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (c) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
    (e) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
    (f) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (g) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (k) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (l) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < i < P < Q < y_i < j$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —y_i—j—y_j—i—Q—P—$.
- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—y_i—j—y_j—i—$.
- $w^{-1} = —Q—y_i—i—j—y_j—P—$ and $v^{-1} = —Q—y_i—j—y_j—i—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

$$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —j—P—y_j— \text{ and } (wt)^{-1} \neq —j—Q—y_j— \text{ and} \\ (wt)^{-1} \neq —y_i—P—i— \text{ and } (wt)^{-1} \neq —y_i—Q—i—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

12. Suppose $y_j < P < i < Q < y_i < j$.
    (a) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (T) fails.
    (b) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (T) fails.
    (c) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (T) fails.
    (d) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
    (e) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
    (f) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
    (g) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
    (h) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
    (i) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
    (j) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
    (k) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
    (l) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
    (m) If $w^{-1} = —y_i—i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
    (n) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

    Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < P < i < Q < y_i < j$ and then one of the following holds:

    - $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—y_i—j—y_j—i—$.

    When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

    $$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

    (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—P—y_j—$ and $(wt)^{-1} \neq —j—Q—y_j—$.

    (Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—y_i—$.

13. Suppose $y_j < i < y_i < j < P < Q$.
    (a) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (b) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (c) If $w^{-1} = —Q—P—y_i—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (d) If $w^{-1} = —y_i—Q—P—i—j—y_j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(e) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —y_i—i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —y_i—i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < i < y_i < j < P < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —y_i—j—y_j—i—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

14. Suppose $P < y_j < i < y_i < j < Q$.

(a) If $w^{-1} = —Q—y_i—P—i—j—y_j—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(b) If $w^{-1} = —y_i—Q—i—j—y_j—P—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(c) If $w^{-1} = —y_i—Q—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(d) If $w^{-1} = —Q—y_i—i—j—y_j—P—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(e) If $w^{-1} = —y_i—Q—i—j—P—y_j—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = —Q—y_i—i—j—P—y_j—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = —Q—y_i—i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = —y_i—i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(i) If $w^{-1} = —y_i—i—Q—j—P—y_j—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.
(j) If $w^{-1} = —y_i—i—Q—j—y_j—P—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_j, j)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $P < y_j < i < y_i < j < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—j—y_j—Q—P—$ and $v^{-1} = —y_i—j—y_j—i—Q—P—$.
- $w^{-1} = —y_i—i—j—Q—P—y_j—$ and $v^{-1} = —y_i—j—y_j—Q—P—i—$.
- $w^{-1} = —y_i—i—Q—P—j—y_j—$ and $v^{-1} = —y_i—j—Q—P—y_j—i—$.
- $w^{-1} = —Q—P—y_i—i—j—y_j—$ and $v^{-1} = —Q—P—y_i—j—y_j—i—$.
- $w^{-1} = —y_i—Q—P—i—j—y_j—$ and $v^{-1} = —y_i—Q—P—j—y_j—i—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —y_i—i—. \end{cases}$$

$$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq -Q-i-P- \text{ and } (wt)^{-1} \neq -Q-y_i-P- \text{ and} \\ (wt)^{-1} \neq -Q-y_j-P- \text{ and } (wt)^{-1} \neq -Q-j-P-. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

15. Suppose $y_j < P < Q < i < y_i < j$.
    (a) If $w^{-1} = -y_i-i-j-Q-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (b) If $w^{-1} = -Q-y_i-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (c) If $w^{-1} = -y_i-Q-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (d) If $w^{-1} = -y_i-i-j-Q-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (e) If $w^{-1} = -y_i-i-Q-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (f) If $w^{-1} = -y_i-i-Q-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (g) If $w^{-1} = -y_i-Q-P-i-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (h) If $w^{-1} = -y_i-Q-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (i) If $w^{-1} = -Q-y_i-i-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (j) If $w^{-1} = -y_i-Q-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (k) If $w^{-1} = -Q-y_i-i-j-P-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (l) If $w^{-1} = -y_i-i-Q-j-y_j-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (m) If $w^{-1} = -y_i-i-j-y_j-Q-P-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.
    (n) If $w^{-1} = -Q-y_i-i-P-j-y_j-$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i, y_i)$.

    Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < P < Q < i < y_i < j$ and then one of the following holds:
    - $w^{-1} = -Q-P-y_i-i-j-y_j-$ and $v^{-1} = -Q-P-y_i-j-y_j-i-$.

    When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

    $$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -Q-P- \text{ and} \\ (wt)^{-1} = -j-y_j- \text{ and} \\ (wt)^{-1} = -y_i-i-. \end{cases}$$

    (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq -j-P-y_j-$ and $(wt)^{-1} \neq -j-Q-y_j-$.

    (Z3) $\Leftrightarrow$ $(wt)^{-1} = -P-y_i-$.

We conclude that properties (V1)-(V3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_j < i < y_i < j$.

6.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $y_{j'} < y_j < i' < i < y_{i'} < y_i < j' < j$.
    (a) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-j-y_{j'}-y_j-$ then (T) fails.
    (b) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-y_{j'}-j-y_j-$ then (T) fails.
    (c) If $w^{-1} = -y_{i'}-y_i-i'-i-j'-y_{j'}-j-y_j-$ then (T) fails.
    (d) If $w^{-1} = -y_{i'}-i'-y_i-i-j'-j-y_{j'}-y_j-$ then (T) fails.
    (e) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-y_{j'}-j-y_j-$ then (U) fails.
    (f) If $w^{-1} = -y_{i'}-i'-y_i-j'-y_{j'}-i-j-y_j-$ then (U) fails.
    (g) If $w^{-1} = -y_{i'}-i'-y_i-j'-i-j-y_{j'}-y_j-$ then (U) fails.
    (h) If $w^{-1} = -y_{i'}-i'-j'-y_i-y_{j'}-i-j-y_j-$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (i) If $w^{-1} = -y_{i'}-i'-j'-y_i-i-y_{j'}-j-y_j-$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
    (j) If $w^{-1} = -y_{i'}-y_i-i'-j'-i-j-y_{j'}-y_j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(l) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(m) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < i' < i < y_{i'} < y_i < j' < j$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ and $(a', b') \in \{(i', y_{i'}), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—i'—y_j—$ and $(wt)^{-1} \neq —j—y_{i'}—y_j—$.

(V3) $\Leftrightarrow$ (no condition).

2. Suppose $y_{j'} < i' < y_j < y_{i'} < i < y_i < j' < j$.

(a) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.

(b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.

(c) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.

(d) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.

(e) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (U) fails.

(f) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (U) fails.

(g) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (U) fails.

(h) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

(i) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

(j) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(k) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(l) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(m) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_j < y_{i'} < i < y_i < j' < j$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ and $(a', b') \in \{(i', y_{i'}), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

3. Suppose $y_{j'} < i' < y_{i'} < y_j < j' < i < y_i < j$.

(a) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(b) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(d) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(e) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_{i'} < y_j < j' < i < y_i < j$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ and $(a',b') \in \{(i',y_{i'}),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

4. Suppose $y_{j'} < y_j < i' < i < y_{i'} < j' < y_i < j$.

(a) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(c) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(e) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < i' < i < y_{i'} < j' < y_i < j$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ and $(a',b') \in \{(i',y_{i'}),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow (wt)^{-1} \neq \text{—}j\text{—}i'\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}y_{i'}\text{—}y_j\text{—}$.

(V3) $\Leftrightarrow$ (no condition).

5. Suppose $y_{j'} < i' < y_j < y_{i'} < i < j' < y_i < j$.
   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_j < y_{i'} < i < j' < y_i < j$ and then one of the following holds:

   - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ and $(a',b') \in \{(i', y_{i'}), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

6. Suppose $y_{j'} < y_j < i' < y_{i'} < i < y_i < j' < j$.
   (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.
   (f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (U) fails.
   (g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.
   (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < i' < y_{i'} < i < y_i < j' < j$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ and $(a',b') \in \{(i',y_{i'}),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—i'—y_j—$ and $(wt)^{-1} \neq —j—y_{i'}—y_j—$.

(V3) $\Leftrightarrow$ (no condition).

7. Suppose $y_{j'} < i' < y_j < i < y_{i'} < y_i < j' < j$.

   (a) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.

   (b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.

   (c) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.

   (d) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.

   (e) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (U) fails.

   (f) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (U) fails.

   (g) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (U) fails.

   (h) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

   (i) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

   (j) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

   (k) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

   (l) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

   (m) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_j < i < y_{i'} < y_i < j' < j$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ and $(a',b') \in \{(i',y_{i'}),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

8. Suppose $y_{j'} < i' < y_{i'} < j' < y_j < i < y_i < j$.

   (a) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

   (b) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(c) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(d) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(e) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
(f) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(g) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(h) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(i) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(j) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
(m) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_{i'} < j' < y_j < i < y_i < j$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ and $(a',b') \in \{(i',y_{i'}),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

9. Suppose $y_{j'} < i' < y_{i'} < y_j < i < y_i < j' < j$.
   (a) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.
   (e) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (U) fails.
   (f) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (U) fails.
   (g) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (U) fails.
   (h) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_{i'} < y_j < i < y_i < j' < j$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ and $(a',b') \in \{(i',y_{i'}),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

10. Suppose $y_{j'} < i' < y_j < i < y_{i'} < j' < y_i < j$.

    (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.

    (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.

    (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.

    (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.

    (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

    (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

    (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

    (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

    (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

    (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

    (k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

    (l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

    (m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

    Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_j < i < y_{i'} < j' < y_i < j$ and then one of the following holds:

    - $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

    When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ and $(a',b') \in \{(i',y_{i'}),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

    $$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

    (V2) $\Leftrightarrow$ (no condition).

    (V3) $\Leftrightarrow$ (no condition).

11. Suppose $y_{j'} < i' < y_{i'} < y_j < i < j' < y_i < j$.

    (a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.

    (b) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.

    (c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.

    (d) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.

    (e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

    (f) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

    (g) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

    (h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

    (i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

    (j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_{i'} < y_j < i < j' < y_i < j$ and then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ and $(a',b') \in \{(i',y_{i'}),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

12. Suppose $y_{j'} < y_j < i' < y_{i'} < j' < i < y_i < j$.

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(c) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(d) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(e) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(f) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(g) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(h) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(i) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(j) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_i\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}y_i\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (i, y_i)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < i' < y_{i'} < j' < i < y_i < j$ and then one of the following holds:

- $w^{-1} = \text{—}y_{i'}\text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_i\text{—}j\text{—}y_j\text{—}i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(i,y_i),(y_j,j)\}$ and $(a',b') \in \{(i',y_{i'}),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}i'\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}y_{i'}\text{—}y_j\text{—}$.

(V3) $\Leftrightarrow$ (no condition).

13. Suppose $y_{j'} < i' < y_j < y_{i'} < j' < i < y_i < j$.

(a) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(b) If $w^{-1} = \text{—}y_{i'}\text{—}y_i\text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(c) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(d) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(e) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $y_{j'} < i' < y_j < y_{i'} < j' < i < y_i < j$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ and $(a', b') \in \{(i', y_{i'}), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

14. Suppose $y_{j'} < y_j < i' < y_{i'} < i < j' < y_i < j$.

(a) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(b) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(c) If $w^{-1} = —y_{i'}—y_i—i'—i—j'—y_{j'}—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —y_{i'}—i'—y_i—i—j'—j—y_{j'}—y_j—$ then (T) fails.
(e) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = —y_{i'}—y_i—i'—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = —y_{i'}—y_i—i'—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = —y_{i'}—i'—j'—y_i—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—i'—j'—y_i—i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_i—j'—y_{j'}—i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = —y_{i'}—i'—y_i—j'—i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < i' < y_{i'} < i < j' < y_i < j$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—j'—y_{j'}—y_i—i—j—y_j—$ and $v^{-1} = —y_{i'}—j'—y_{j'}—i'—y_i—j—y_j—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(i, y_i), (y_j, j)\}$ and $(a', b') \in \{(i', y_{i'}), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_{i'}\text{—}i'\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}i'\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}y_{i'}\text{—}y_j\text{—}$.

(V3) $\Leftrightarrow$ (no condition).

We conclude that properties (V1)-(V3) hold for all $(a,b),(a',b') \in \mathrm{Cyc}(y)$ when $y_j < i < y_i < j$.

7 Case B3

Suppose $y_j < y_i < i < j$ and $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}$ so that $k = y_j < i = l$.

7.1 Subcase (i)

In this case $v = wt_{ij}t_{kl}$ is such that

$$v^{-1} = \text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}.$$

When $(a,b),(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$, properties (V1)-(V3) are equivalent to the following conditions which evidently hold:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_j\text{—}$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}y_i\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}i\text{—}y_j\text{—}$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (V1)-(V3) hold whenever (a,b), (a',b') are as in case (i) and $y_j < y_i < i < j$.

7.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(y)$, so that $R = y_R \notin \{i,j,y_i,y_j\} + n\mathbb{Z}$.

1. Suppose $y_j < y_i < i < j < R$.
   (a) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = \text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = \text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (y_j,i)$. We conclude that if $y_j < y_i < i < j < R$ and then one of the following holds:

   - $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}R\text{—}$ and $v^{-1} = \text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}R\text{—}$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}y_j\text{—}$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_i\text{—}R\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}R\text{—}$.

2. Suppose $y_j < y_i < i < R < j$.
   (a) If $w^{-1} = \text{—}i\text{—}R\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i\text{—}j\text{—}R\text{—}y_j\text{—}y_i\text{—}$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = \text{—}R\text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}R\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < y_i < i < R < j$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—y_i—R—$ and $v^{-1} = —j—y_j—i—y_i—R—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —i—y_i—$ and $(wt)^{-1} = —j—y_j—$.
(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—R—y_j—$.
(Z3) $\Leftrightarrow$ $(wt)^{-1} = —y_i—R—$.

3. Suppose $y_j < y_i < R < i < j$.
   (a) If $w^{-1} = —i—j—y_j—R—y_i—$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —i—R—j—y_j—y_i—$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —i—j—R—y_j—y_i—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < y_i < R < i < j$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—y_i—R—$ and $v^{-1} = —j—y_j—i—y_i—R—$.
- $w^{-1} = —R—i—j—y_j—y_i—$ and $v^{-1} = —R—j—y_j—i—y_i—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —i—y_i—$ and $(wt)^{-1} = —j—y_j—$.
(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —i—R—y_i—$ and $(wt)^{-1} \neq —j—R—y_j—$.
(Z3) $\Leftrightarrow$ (no condition).

4. Suppose $y_j < R < y_i < i < j$.
   (a) If $w^{-1} = —i—j—R—y_j—y_i—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —i—j—y_j—y_i—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = —i—j—y_j—R—y_i—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = —i—R—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < R < y_i < i < j$ and then one of the following holds:

- $w^{-1} = —R—i—j—y_j—y_i—$ and $v^{-1} = —R—j—y_j—i—y_i—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —i—y_i—$ and $(wt)^{-1} = —j—y_j—$.
(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—R—y_j—$.
(Z3) $\Leftrightarrow$ $(wt)^{-1} = —R—i—$.

5. Suppose $R < y_j < y_i < i < j$.
   (a) If $w^{-1} = —i—R—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = —i—j—y_j—y_i—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j,j)$.
   (c) If $w^{-1} = —i—j—R—y_j—y_i—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j,j)$.
   (d) If $w^{-1} = —i—j—y_j—R—y_i—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_j,j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $R < y_j < y_i < i < j$ and then one of the following holds:

- $w^{-1} = —R—i—j—y_j—y_i—$ and $v^{-1} = —R—j—y_j—i—y_i—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —i—y_i—$ and $(wt)^{-1} = —j—y_j—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —R—i—$ and $(wt)^{-1} = —R—j—$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(y)$, so that $Q = y_P$ and $P, Q \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $P < y_j < y_i < Q < i < j$.

   (a) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (b) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (c) If $w^{-1} = —i—j—y_j—y_i—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (d) If $w^{-1} = —Q—i—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (e) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (f) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (g) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (h) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (i) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (j) If $w^{-1} = —i—j—y_j—Q—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (k) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (l) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (m) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (n) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $P < y_j < y_i < Q < i < j$ and then one of the following holds:

   - $w^{-1} = —Q—P—i—j—y_j—y_i—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—i—$ and $(wt)^{-1} = —P—j—$.

2. Suppose $P < y_j < Q < y_i < i < j$.

   (a) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (b) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
   (c) If $w^{-1} = —i—j—y_j—y_i—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (d) If $w^{-1} = —Q—i—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (e) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (f) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (g) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (h) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (i) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (j) If $w^{-1} = —i—j—y_j—Q—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (k) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (l) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
   (m) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.

(n) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $P < y_j < Q < y_i < i < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—j—y_j—y_i—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(Z1) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—i—$ and $(wt)^{-1} = —P—j—$.

3. Suppose $y_j < y_i < i < P < j < Q$.

(a) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (T) fails.

(b) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (T) fails.

(c) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(d) If $w^{-1} = —i—j—y_j—Q—P—y_i—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(e) If $w^{-1} = —Q—i—j—y_j—y_i—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(f) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(g) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(h) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(i) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(j) If $w^{-1} = —Q—P—i—j—y_j—y_i—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(k) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(l) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(m) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(n) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < y_i < i < P < j < Q$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—y_i—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(Z1) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —y_i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

4. Suppose $P < y_j < y_i < i < Q < j$.

(a) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (T) fails.

(b) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (T) fails.

(c) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (T) fails.

(d) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (T) fails.

(e) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_i, i)$.

(f) If $w^{-1} = —i—j—y_j—y_i—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(g) If $w^{-1} = —Q—i—j—y_j—y_i—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(h) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(i) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(j) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(k) If $w^{-1} = —i—j—y_j—Q—P—y_i—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(l) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(m) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

(n) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $P < y_j < y_i < i < Q < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—j—y_j—y_i—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

$(\text{Z2}) \Leftrightarrow (wt)^{-1} \neq —Q—y_i—P—$ and $(wt)^{-1} \neq —Q—i—P—$.

$(\text{Z3}) \Leftrightarrow (wt)^{-1} = —P—j—$.

5. Suppose $y_j < y_i < P < i < Q < j$.

(a) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (T) fails.

(b) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (T) fails.

(c) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (T) fails.

(d) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (T) fails.

(e) If $w^{-1} = —Q—i—j—y_j—y_i—P—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(f) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(g) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(h) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(i) If $w^{-1} = —i—j—y_j—Q—P—y_i—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(j) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(k) If $w^{-1} = —Q—P—i—j—y_j—y_i—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(l) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(m) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(n) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < y_i < P < i < Q < j$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—y_i—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

$(\text{Z2}) \Leftrightarrow (wt)^{-1} \neq —j—P—y_j—$ and $(wt)^{-1} \neq —j—Q—y_j—$.

$(\text{Z3}) \Leftrightarrow (wt)^{-1} = —y_i—Q—$.

6. Suppose $y_j < P < y_i < i < Q < j$.

   (a) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (T) fails.
   (b) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (T) fails.
   (c) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (T) fails.
   (d) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (T) fails.
   (e) If $w^{-1} = —Q—i—j—y_j—y_i—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (f) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (g) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (h) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (j) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   Recall that $(k,l) = (y_j,i)$. We conclude that if $y_j < P < y_i < i < Q < j$ and then one of the following holds:

   - $w^{-1} = —i—j—y_j—y_i—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.
   - $w^{-1} = —i—j—y_j—Q—P—y_i—$ and $v^{-1} = —j—y_j—i—Q—P—y_i—$.
   - $w^{-1} = —Q—P—i—j—y_j—y_i—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

   $$(\text{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—y_i—P— \text{ and } (wt)^{-1} \neq —Q—i—P— \text{ and} \\ (wt)^{-1} \neq —j—P—y_j— \text{ and } (wt)^{-1} \neq —j—Q—y_j—. \end{cases}$$

   (Z3) $\Leftrightarrow$ (no condition).

7. Suppose $P < Q < y_j < y_i < i < j$.

   (a) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = —i—j—y_j—y_i—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (d) If $w^{-1} = —Q—i—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (e) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (f) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (g) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (h) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (i) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (j) If $w^{-1} = —i—j—y_j—Q—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (k) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (l) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (m) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
   (n) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

   Recall that $(k,l) = (y_j,i)$. We conclude that if $P < Q < y_j < y_i < i < j$ and then one of the following holds:

- $w^{-1} = {—}Q{—}P{—}i{—}j{—}y_j{—}y_i{—}$ and $v^{-1} = {—}Q{—}P{—}j{—}y_j{—}i{—}y_i{—}$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = {—}Q{—}P{—} \text{ and} \\ (wt)^{-1} = {—}i{—}y_i{—} \text{ and} \\ (wt)^{-1} = {—}j{—}y_j{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = {—}P{—}i{—}$ and $(wt)^{-1} = {—}P{—}j{—}$.

8. Suppose $y_j < y_i < i < P < Q < j$.

   (a) If $w^{-1} = {—}i{—}Q{—}P{—}j{—}y_j{—}y_i{—}$ then (T) fails.
   (b) If $w^{-1} = {—}i{—}Q{—}j{—}y_j{—}y_i{—}P{—}$ then (T) fails.
   (c) If $w^{-1} = {—}i{—}Q{—}j{—}y_j{—}P{—}y_i{—}$ then (T) fails.
   (d) If $w^{-1} = {—}Q{—}i{—}P{—}j{—}y_j{—}y_i{—}$ then (T) fails.
   (e) If $w^{-1} = {—}i{—}Q{—}j{—}P{—}y_j{—}y_i{—}$ then (T) fails.
   (f) If $w^{-1} = {—}Q{—}i{—}j{—}y_j{—}y_i{—}P{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (g) If $w^{-1} = {—}i{—}j{—}Q{—}y_j{—}y_i{—}P{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (h) If $w^{-1} = {—}Q{—}i{—}j{—}y_j{—}P{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (i) If $w^{-1} = {—}i{—}j{—}y_j{—}Q{—}y_i{—}P{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (j) If $w^{-1} = {—}i{—}j{—}y_j{—}Q{—}P{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (k) If $w^{-1} = {—}Q{—}P{—}i{—}j{—}y_j{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (l) If $w^{-1} = {—}Q{—}i{—}j{—}P{—}y_j{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (m) If $w^{-1} = {—}i{—}j{—}Q{—}P{—}y_j{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (n) If $w^{-1} = {—}i{—}j{—}Q{—}y_j{—}P{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

   Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < y_i < i < P < Q < j$ and then one of the following holds:

   - $w^{-1} = {—}i{—}j{—}y_j{—}y_i{—}Q{—}P{—}$ and $v^{-1} = {—}j{—}y_j{—}i{—}y_i{—}Q{—}P{—}$.

   When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = {—}Q{—}P{—} \text{ and} \\ (wt)^{-1} = {—}i{—}y_i{—} \text{ and} \\ (wt)^{-1} = {—}j{—}y_j{—}. \end{cases}$$

   (Z2) $\Leftrightarrow (wt)^{-1} \neq {—}j{—}P{—}y_j{—}$ and $(wt)^{-1} \neq {—}j{—}Q{—}y_j{—}$.

   (Z3) $\Leftrightarrow (wt)^{-1} = {—}y_i{—}Q{—}$.

9. Suppose $y_j < y_i < P < i < j < Q$.

   (a) If $w^{-1} = {—}i{—}j{—}y_j{—}Q{—}y_i{—}P{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (b) If $w^{-1} = {—}i{—}j{—}y_j{—}Q{—}P{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
   (c) If $w^{-1} = {—}i{—}Q{—}P{—}j{—}y_j{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (d) If $w^{-1} = {—}Q{—}i{—}j{—}y_j{—}y_i{—}P{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (e) If $w^{-1} = {—}i{—}j{—}Q{—}y_j{—}y_i{—}P{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (f) If $w^{-1} = {—}i{—}Q{—}j{—}y_j{—}y_i{—}P{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (g) If $w^{-1} = {—}i{—}Q{—}j{—}y_j{—}P{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (h) If $w^{-1} = {—}Q{—}i{—}j{—}y_j{—}P{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
   (i) If $w^{-1} = {—}Q{—}i{—}P{—}j{—}y_j{—}y_i{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

(j) If $w^{-1} = —Q—P—i—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < y_i < P < i < j < Q$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—y_i—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —y_i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

10. Suppose $y_j < P < y_i < i < j < Q$.

(a) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y2) fails for $(a,b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(b) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(c) If $w^{-1} = —Q—i—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(d) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(e) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —Q—P—i—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < P < y_i < i < j < Q$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—y_i—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.
- $w^{-1} = —i—j—y_j—Q—P—y_i—$ and $v^{-1} = —j—y_j—i—Q—P—y_i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —Q—y_i—P—$ and $(wt)^{-1} \neq —Q—i—P—$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —y_j—Q—$.

11. Suppose $y_j < y_i < P < Q < i < j$.

(a) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(b) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.

(c) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(d) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(e) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = —i—j—y_j—Q—P—y_i—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < y_i < P < Q < i < j$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—y_i—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.
- $w^{-1} = —Q—P—i—j—y_j—y_i—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.
- $w^{-1} = —Q—i—j—y_j—y_i—P—$ and $v^{-1} = —Q—j—y_j—i—y_i—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq —i—P—y_i— \text{ and } (wt)^{-1} \neq —i—Q—y_i— \text{ and} \\ (wt)^{-1} \neq —j—P—y_j— \text{ and } (wt)^{-1} \neq —j—Q—y_j—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

12. Suppose $y_j < P < y_i < Q < i < j$.

(a) If $w^{-1} = —i—j—y_j—y_i—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(b) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(c) If $w^{-1} = —Q—i—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(d) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(e) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(f) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(g) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(h) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(i) If $w^{-1} = —i—j—y_j—Q—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(j) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(k) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(l) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(m) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.
(n) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (y_i, i)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_j < P < y_i < Q < i < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—j—y_j—y_i—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}P\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}Q\text{—}y_j\text{—}$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}P\text{—}i\text{—}$.

13. Suppose $y_j < y_i < i < j < P < Q$.
    (a) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}y_i\text{—}P\text{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
    (b) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
    (c) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (d) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}P\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (e) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}y_i\text{—}P\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (f) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}y_i\text{—}P\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (g) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (h) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (i) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (j) If $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (k) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (l) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (m) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
    (n) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

    Recall that $(k, l) = (y_j, i)$. We conclude that if $y_j < y_i < i < j < P < Q$ and then one of the following holds:

    - $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}Q\text{—}P\text{—}$ and $v^{-1} = \text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}$.

    When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—}. \end{cases}$$

    (Z2) $\Leftrightarrow$ (no condition).

    (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}y_i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}y_j\text{—}Q\text{—}$.

14. Suppose $P < y_j < y_i < i < j < Q$.
    (a) If $w^{-1} = \text{—}i\text{—}j\text{—}y_j\text{—}Q\text{—}y_i\text{—}P\text{—}$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_i, i)$.
    (b) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_i, i)$.
    (c) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}P\text{—}$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (d) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}y_i\text{—}P\text{—}$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (e) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}y_i\text{—}P\text{—}$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (f) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}y_i\text{—}$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (g) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}y_j\text{—}P\text{—}y_i\text{—}$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (h) If $w^{-1} = \text{—}Q\text{—}i\text{—}j\text{—}P\text{—}y_j\text{—}y_i\text{—}$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (i) If $w^{-1} = \text{—}i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}y_i\text{—}$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.
    (j) If $w^{-1} = \text{—}i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}y_i\text{—}$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_j, j)$.

    Recall that $(k, l) = (y_j, i)$. We conclude that if $P < y_j < y_i < i < j < Q$ and then one of the following holds:

- $w^{-1} = —i—j—y_j—y_i—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.
- $w^{-1} = —i—j—y_j—Q—P—y_i—$ and $v^{-1} = —j—y_j—i—Q—P—y_i—$.
- $w^{-1} = —Q—P—i—j—y_j—y_i—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.
- $w^{-1} = —i—Q—P—j—y_j—y_i—$ and $v^{-1} = —j—Q—P—y_j—i—y_i—$.
- $w^{-1} = —i—j—Q—P—y_j—y_i—$ and $v^{-1} = —j—y_j—Q—P—i—y_i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

$$(\text{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—y_i—P— \text{ and } (wt)^{-1} \neq —Q—i—P— \text{ and} \\ (wt)^{-1} \neq —Q—y_j—P— \text{ and } (wt)^{-1} \neq —Q—j—P—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

15. Suppose $y_j < P < Q < y_i < i < j$.

(a) If $w^{-1} = —i—j—y_j—y_i—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(b) If $w^{-1} = —i—Q—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(c) If $w^{-1} = —Q—i—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(d) If $w^{-1} = —i—j—Q—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(e) If $w^{-1} = —i—Q—j—y_j—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(f) If $w^{-1} = —i—Q—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(g) If $w^{-1} = —Q—i—j—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(h) If $w^{-1} = —i—j—y_j—Q—y_i—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(i) If $w^{-1} = —i—j—y_j—Q—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(j) If $w^{-1} = —Q—i—P—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(k) If $w^{-1} = —Q—i—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(l) If $w^{-1} = —i—j—Q—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(m) If $w^{-1} = —i—j—Q—y_j—P—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(n) If $w^{-1} = —i—Q—j—P—y_j—y_i—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

Recall that $(k,l) = (y_j,i)$. We conclude that if $y_j < P < Q < y_i < i < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—j—y_j—y_i—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —j—P—y_j—$ and $(wt)^{-1} \neq —j—Q—y_j—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —P—i—$.

We conclude that properties (V1)-(V3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_j < y_i < i < j$.

7.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $y_{j'} < y_j < y_{i'} < y_i < i' < i < j' < j$.
   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (T) fails.
   (c) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (T) fails.
   (d) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (T) fails.
   (e) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (T) fails.
   (f) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (j) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (k) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
   (m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < y_{i'} < y_i < i' < i < j' < j$ and then one of the following holds:

   - $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

   (V2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—y_{i'}—y_j—$ and $(wt)^{-1} \neq —j—i'—y_j—$.

   (V3) $\Leftrightarrow$ (no condition).

2. Suppose $y_{j'} < y_{i'} < y_j < i' < y_i < i < j' < j$.
   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (T) fails.
   (c) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (T) fails.
   (d) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (T) fails.
   (e) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (T) fails.
   (f) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
   (i) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
   (j) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
   (k) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
   (l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
   (m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_{i'} < y_j < i' < y_i < i < j' < j$ and then one of the following holds:

- $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

3. Suppose $y_{j'} < y_{i'} < i' < y_j < j' < y_i < i < j$.
   (a) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_j,j)$.
   (c) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_j,j)$.
   (d) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (e) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (f) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (g) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (j) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.
   (k) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.
   (l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.
   (m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_{i'} < i' < y_j < j' < y_i < i < j$ and then one of the following holds:

   - $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

4. Suppose $y_{j'} < y_j < y_{i'} < y_i < i' < j' < i < j$.
   (a) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (e) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.

(f) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(g) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < y_{i'} < y_i < i' < j' < i < j$ and then one of the following holds:

- $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a', b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—y_{i'}—y_j—$ and $(wt)^{-1} \neq —j—i'—y_j—$.

(V3) $\Leftrightarrow$ (no condition).

5. Suppose $y_{j'} < y_{i'} < y_j < i' < y_i < j' < i < j$.

(a) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
(b) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
(c) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
(d) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(e) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(f) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(g) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_{i'} < y_j < i' < y_i < j' < i < j$ and then one of the following holds:

- $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a', b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

6. Suppose $y_{j'} < y_j < y_{i'} < i' < y_i < i < j' < j$.

   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (T) fails.
   (c) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (T) fails.
   (d) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (T) fails.
   (e) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (T) fails.
   (f) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (j) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (k) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
   (m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < y_{i'} < i' < y_i < i < j' < j$ and then one of the following holds:

   - $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

   (V2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—y_{i'}—y_j—$ and $(wt)^{-1} \neq —j—i'—y_j—$.

   (V3) $\Leftrightarrow$ (no condition).

7. Suppose $y_{j'} < y_{i'} < y_j < y_i < i' < i < j' < j$.

   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (T) fails.
   (c) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (T) fails.
   (d) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (T) fails.
   (e) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (T) fails.
   (f) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
   (i) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
   (j) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
   (k) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
   (l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
   (m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_{i'} < y_j < y_i < i' < i < j' < j$ and then one of the following holds:

- $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

8. Suppose $y_{j'} < y_{i'} < i' < j' < y_j < y_i < i < j$.
   (a) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_j,j)$.
   (c) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_j,j)$.
   (d) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (e) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (f) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (g) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
   (j) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.
   (k) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.
   (l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.
   (m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.

   Recall that $(k,l) = (y_j,i)$. We conclude that if $y_{j'} < y_{i'} < i' < j' < y_j < y_i < i < j$ and then one of the following holds:

   - $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

9. Suppose $y_{j'} < y_{i'} < i' < y_j < y_i < i < j' < j$.
   (a) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (T) fails.
   (b) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (T) fails.
   (c) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (T) fails.
   (d) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (T) fails.
   (e) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (T) fails.
   (f) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
   (g) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.

(h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.

(i) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.

(j) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.

(k) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.

(l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

(m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_{i'} < i' < y_j < y_i < i < j' < j$ and then one of the following holds:

- $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

10. Suppose $y_{j'} < y_{i'} < y_j < y_i < i' < j' < i < j$.

(a) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

(b) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.

(c) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.

(d) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(e) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(f) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(g) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(i) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(j) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

(k) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

(l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

(m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_{i'} < y_j < y_i < i' < j' < i < j$ and then one of the following holds:

- $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

11. Suppose $y_{j'} < y_{i'} < i' < y_j < y_i < j' < i < j$.

 (a) If $w^{-1} = \text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
 (b) If $w^{-1} = \text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
 (c) If $w^{-1} = \text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_j, j)$.
 (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (g) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (h) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (i) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (j) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
 (k) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
 (l) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
 (m) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

 Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_{i'} < i' < y_j < y_i < j' < i < j$ and then one of the following holds:

 - $w^{-1} = \text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_{i'}\text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$.

 When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}. \end{cases}$$

 (V2) $\Leftrightarrow$ (no condition).

 (V3) $\Leftrightarrow$ (no condition).

12. Suppose $y_{j'} < y_j < y_{i'} < i' < j' < y_i < i < j$.

 (a) If $w^{-1} = \text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
 (b) If $w^{-1} = \text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
 (c) If $w^{-1} = \text{—}i'\text{—}j'\text{—}y_{j'}\text{—}i\text{—}j\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
 (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (g) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (h) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (i) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
 (j) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
 (k) If $w^{-1} = \text{—}i'\text{—}i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
 (l) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_{i'}\text{—}y_j\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
 (m) If $w^{-1} = \text{—}i'\text{—}j'\text{—}i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}y_{i'}\text{—}y_i\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

 Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < y_{i'} < i' < j' < y_i < i < j$ and then one of the following holds:

 - $w^{-1} = \text{—}i'\text{—}j'\text{—}y_{j'}\text{—}y_{i'}\text{—}i\text{—}j\text{—}y_j\text{—}y_i\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow (wt)^{-1} \neq —j—y_{i'}—y_j—$ and $(wt)^{-1} \neq —j—i'—y_j—$.

(V3) $\Leftrightarrow$ (no condition).

13. Suppose $y_{j'} < y_{i'} < y_j < i' < j' < y_i < i < j$.

    (a) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
    (b) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_j,j)$.
    (c) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_j,j)$.
    (d) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
    (e) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
    (f) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
    (g) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
    (h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
    (i) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
    (j) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.
    (k) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.
    (l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.
    (m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.

    Recall that $(k,l) = (y_j,i)$. We conclude that if $y_{j'} < y_{i'} < y_j < i' < j' < y_i < i < j$ and then one of the following holds:

    - $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

    When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

    $$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

    (V2) $\Leftrightarrow$ (no condition).

    (V3) $\Leftrightarrow$ (no condition).

14. Suppose $y_{j'} < y_j < y_{i'} < i' < y_i < j' < i < j$.

    (a) If $w^{-1} = —i'—j'—y_{j'}—i—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
    (b) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
    (c) If $w^{-1} = —i'—j'—y_{j'}—i—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{i'},i')$ and $(a',b') = (y_i,i)$.
    (d) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
    (e) If $w^{-1} = —i'—i—j'—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
    (f) If $w^{-1} = —i'—i—j'—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
    (g) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
    (h) If $w^{-1} = —i'—j'—i—y_{j'}—j—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.

(i) If $w^{-1} = —i'—j'—i—y_{j'}—y_{i'}—j—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = —i'—i—j'—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(l) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_{i'}—y_j—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.
(m) If $w^{-1} = —i'—j'—i—j—y_{j'}—y_j—y_{i'}—y_i—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

Recall that $(k,l) = (y_j, i)$. We conclude that if $y_{j'} < y_j < y_{i'} < i' < y_i < j' < i < j$ and then one of the following holds:

- $w^{-1} = —i'—j'—y_{j'}—y_{i'}—i—j—y_j—y_i—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a',b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow (wt)^{-1} \neq —j—y_{i'}—y_j—$ and $(wt)^{-1} \neq —j—i'—y_j—$.

(V3) $\Leftrightarrow$ (no condition).

We conclude that properties (V1)-(V3) hold for all $(a,b), (a',b') \in \mathrm{Cyc}(y)$ when $y_j < y_i < i < j$.

8 Case C1

Suppose $i < j < y_j < y_i$ and $w^{-1} = —y_i—i—y_j—j—$ so that $k = y_j < y_i = l$.

8.1 Subcase (i)

In this case $v = wt_{ij}t_{kl}$ is such that

$$v^{-1} = —y_j—j—y_i—i—.$$

When $(a,b), (a',b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$, properties (V1)-(V3) are equivalent to the following conditions which evidently hold:

(Z1) $\Leftrightarrow (wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_j—j—$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq —y_i—j—i—$ and $(wt)^{-1} \neq —y_i—y_j—i—$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (V1)-(V3) hold whenever (a,b), (a',b') are as in case (i) and $i < j < y_j < y_i$.

8.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(y)$, so that $R = y_R \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $i < j < y_j < y_i < R$.

   (a) If $w^{-1} = —R—y_i—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (b) If $w^{-1} = —y_i—R—i—y_j—j—$ then (Y3) fails for $(a,b) = (i, y_i)$ and $(a',b') = (R,R)$.
   (c) If $w^{-1} = —y_i—i—y_j—R—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (R,R)$.
   (d) If $w^{-1} = —y_i—i—R—y_j—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i < j < y_j < y_i < R$ and then one of the following holds:

   - $w^{-1} = —y_i—i—y_j—j—R—$ and $v^{-1} = —y_j—j—y_i—i—R—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_j—j—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—R—$ and $(wt)^{-1} = —j—R—$.

2. Suppose $i < j < y_j < R < y_i$.

   (a) If $w^{-1} = —y_i—R—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.

   (b) If $w^{-1} = —y_i—i—y_j—R—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.

   (c) If $w^{-1} = —y_i—i—R—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.

   (d) If $w^{-1} = —R—y_i—i—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.

Recall that $(k,l) = (y_j,y_i)$. We conclude that if $i < j < y_j < R < y_i$ and then one of the following holds:

- $w^{-1} = —y_i—i—y_j—j—R—$ and $v^{-1} = —y_j—j—y_i—i—R—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_j—j—$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—R—i—$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —j—R—$.

3. Suppose $i < j < R < y_j < y_i$.

   (a) If $w^{-1} = —y_i—R—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.

   (b) If $w^{-1} = —y_i—i—y_j—R—j—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (R,R)$.

Recall that $(k,l) = (y_j,y_i)$. We conclude that if $i < j < R < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —R—y_i—i—y_j—j—$ and $v^{-1} = —R—y_j—j—y_i—i—$.
- $w^{-1} = —y_i—i—R—y_j—j—$ and $v^{-1} = —y_j—j—R—y_i—i—$.
- $w^{-1} = —y_i—i—y_j—j—R—$ and $v^{-1} = —y_j—j—y_i—i—R—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_j—j—$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—R—i—$ and $(wt)^{-1} \neq —y_j—R—j—$.

(Z3) $\Leftrightarrow$ (no condition).

4. Suppose $i < R < j < y_j < y_i$.

   (a) If $w^{-1} = —y_i—i—y_j—R—j—$ then (T) fails.

   (b) If $w^{-1} = —y_i—i—R—y_j—j—$ then (T) fails.

   (c) If $w^{-1} = —y_i—R—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (R,R)$.

   (d) If $w^{-1} = —y_i—i—y_j—j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (j,y_j)$.

Recall that $(k,l) = (y_j,y_i)$. We conclude that if $i < R < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —R—y_i—i—y_j—j—$ and $v^{-1} = —R—y_j—j—y_i—i—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_j—j—$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—R—i—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —R—y_j—$.

5. Suppose $R < i < j < y_j < y_i$.
   (a) If $w^{-1} = —y_i—i—y_j—R—j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.
   (b) If $w^{-1} = —y_i—i—R—y_j—j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.
   (c) If $w^{-1} = —y_i—R—i—y_j—j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.
   (d) If $w^{-1} = —y_i—i—y_j—j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (i,y_i)$.

   Recall that $(k,l) = (y_j, y_i)$. We conclude that if $R < i < j < y_j < y_i$ and then one of the following holds:

   - $w^{-1} = —R—y_i—i—y_j—j—$ and $v^{-1} = —R—y_j—j—y_i—i—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow (wt)^{-1} = —y_i—i—$ and $(wt)^{-1} = —y_j—j—$.
   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow (wt)^{-1} = —R—y_i—$ and $(wt)^{-1} = —R—y_j—$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(y)$, so that $Q = y_P$ and $P, Q \notin \{i,j,y_i,y_j\} + n\mathbb{Z}$.

1. Suppose $P < i < j < Q < y_j < y_i$.
   (a) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (b) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (c) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (d) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (h) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (i) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (j) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (k) If $w^{-1} = —y_i—i—y_j—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (l) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (m) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (n) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

   Recall that $(k,l) = (y_j, y_i)$. We conclude that if $P < i < j < Q < y_j < y_i$ and then one of the following holds:

   - $w^{-1} = —Q—P—y_i—i—y_j—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).
   (Z3) $\Leftrightarrow (wt)^{-1} = —P—y_i—$ and $(wt)^{-1} = —P—y_j—$.

2. Suppose $P < i < Q < j < y_j < y_i$.
   (a) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (T) fails.

(b) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (T) fails.
(c) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (T) fails.
(d) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (T) fails.
(e) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (T) fails.
(f) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_i—i—y_j—j—Q—P—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.
(n) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (Y3) fails for $(a, b) = (P, Q)$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (y_j, y_i)$. We conclude that if $P < i < Q < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—y_j—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—y_i—$ and $(wt)^{-1} = —P—y_j—$.

3. Suppose $i < j < y_j < P < y_i < Q$.

(a) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(b) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(c) If $w^{-1} = —Q—P—y_i—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(d) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(e) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (y_j, y_i)$. We conclude that if $i < j < y_j < P < y_i < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{— and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{— and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}i\text{—}Q\text{—}$ and $(wt)^{-1} = \text{—}j\text{—}Q\text{—}$.

4. Suppose $P < i < j < y_j < Q < y_i$.

   (a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (b) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (c) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (j) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (k) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (l) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (m) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
   (n) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

   Recall that $(k,l) = (y_j, y_i)$. We conclude that if $P < i < j < y_j < Q < y_i$ and then one of the following holds:

   - $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $v^{-1} = \text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{— and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{— and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—}. \end{cases}$$

   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}Q\text{—}j\text{—}P\text{—}$ and $(wt)^{-1} \neq \text{—}Q\text{—}y_j\text{—}P\text{—}$.

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}P\text{—}y_i\text{—}$.

5. Suppose $i < j < P < y_j < Q < y_i$.

   (a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (b) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (c) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (f) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
   (l) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

(m) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.

(n) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (j, y_j)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i < j < P < y_j < Q < y_i$ and then one of the following holds:

- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

$(\mathrm{Z2}) \Leftrightarrow (wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.

$(\mathrm{Z3}) \Leftrightarrow (wt)^{-1} = —j—Q—$.

6. Suppose $i < P < j < y_j < Q < y_i$.

(a) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (T) fails.

(b) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (T) fails.

(c) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (T) fails.

(d) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (T) fails.

(e) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (T) fails.

(f) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (T) fails.

(g) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (T) fails.

(h) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (j, y_j)$.

(i) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (j, y_j)$.

(j) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (j, y_j)$.

(k) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a', b') = (j, y_j)$.

(l) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.

(m) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i < P < j < y_j < Q < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—y_j—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.
- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

$$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—j—P— \text{ and } (wt)^{-1} \neq —Q—y_j—P— \text{ and} \\ (wt)^{-1} \neq —y_i—P—i— \text{ and } (wt)^{-1} \neq —y_i—Q—i—. \end{cases}$$

$(\mathrm{Z3}) \Leftrightarrow$ (no condition).

7. Suppose $P < Q < i < j < y_j < y_i$.

(a) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.

(b) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.

(d) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a', b') = (i, y_i)$.

(e) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(j) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(k) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(l) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(m) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(n) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $P < Q < i < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ and $v^{-1} = \text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow (wt)^{-1} = \text{—}P\text{—}y_i\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}y_j\text{—}$.

8. Suppose $i < j < y_j < P < Q < y_i$.

(a) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(b) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(c) If $w^{-1} = \text{—}Q\text{—}P\text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(d) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(e) If $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}Q\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(f) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(g) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(h) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(i) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(j) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}P\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = \text{—}y_i\text{—}Q\text{—}P\text{—}i\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = \text{—}y_i\text{—}i\text{—}Q\text{—}y_j\text{—}j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = \text{—}Q\text{—}y_i\text{—}i\text{—}P\text{—}y_j\text{—}j\text{—}$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i < j < y_j < P < Q < y_i$ and then one of the following holds:

- $w^{-1} = \text{—}y_i\text{—}i\text{—}y_j\text{—}j\text{—}Q\text{—}P\text{—}$ and $v^{-1} = \text{—}y_j\text{—}j\text{—}y_i\text{—}i\text{—}Q\text{—}P\text{—}$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_i\text{—}i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}y_j\text{—}j\text{—}. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq \text{—}y_i\text{—}P\text{—}i\text{—}$ and $(wt)^{-1} \neq \text{—}y_i\text{—}Q\text{—}i\text{—}$.

(Z3) $\Leftrightarrow (wt)^{-1} = —j—Q—$.

9. Suppose $i < j < P < y_j < y_i < Q$.

   (a) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (b) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (c) If $w^{-1} = —Q—P—y_i—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (d) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (e) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (f) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (g) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (h) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (i) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
   (j) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (P, Q)$.
   (k) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (P, Q)$.
   (l) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (P, Q)$.
   (m) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (P, Q)$.
   (n) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y3) fails for $(a, b) = (j, y_j)$ and $(a', b') = (P, Q)$.

   Recall that $(k, l) = (y_j, y_i)$. We conclude that if $i < j < P < y_j < y_i < Q$ and then one of the following holds:

   - $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.

   When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —j—Q—$.

10. Suppose $i < P < j < y_j < y_i < Q$.

    (a) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (T) fails.
    (b) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (T) fails.
    (c) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (T) fails.
    (d) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (T) fails.
    (e) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (T) fails.
    (f) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (T) fails.
    (g) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (T) fails.
    (h) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
    (i) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (j, y_j)$.
    (j) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (k) If $w^{-1} = —Q—P—y_i—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (l) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (m) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.
    (n) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y3) fails for $(a, b) = (i, y_i)$ and $(a', b') = (P, Q)$.

    Recall that $(k, l) = (y_j, y_i)$. We conclude that if $i < P < j < y_j < y_i < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —Q—j—P—$ and $(wt)^{-1} \neq —Q—y_j—P—$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —i—Q—$.

11. Suppose $i < j < P < Q < y_j < y_i$.

    (a) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (b) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (c) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (d) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
    (e) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (f) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (g) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (h) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.
    (i) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (Y2) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (y_j,y_i)$. We conclude that if $i < j < P < Q < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_i—i—Q—P—y_j—j—$ and $v^{-1} = —y_j—j—Q—P—y_i—i—$.
- $w^{-1} = —Q—y_i—i—y_j—j—P—$ and $v^{-1} = —Q—y_j—j—y_i—i—P—$.
- $w^{-1} = —Q—P—y_i—i—y_j—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.
- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.
- $w^{-1} = —Q—y_i—i—P—y_j—j—$ and $v^{-1} = —Q—y_j—j—P—y_i—i—$.
- $w^{-1} = —y_i—i—Q—y_j—j—P—$ and $v^{-1} = —y_j—j—Q—y_i—i—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq —y_i—P—i— \text{ and } (wt)^{-1} \neq —y_i—Q—i— \text{ and} \\ (wt)^{-1} \neq —y_j—P—j— \text{ and } (wt)^{-1} \neq —y_j—Q—j—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

12. Suppose $i < P < j < Q < y_j < y_i$.

    (a) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (T) fails.
    (b) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (T) fails.
    (c) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (T) fails.
    (d) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (T) fails.
    (e) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (T) fails.
    (f) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (T) fails.
    (g) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (T) fails.

(h) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(i) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(j) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

(k) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

(l) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

(m) If $w^{-1} = —y_i—i—y_j—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

(n) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i < P < j < Q < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—y_j—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

$(\mathrm{Z2}) \Leftrightarrow (wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.

$(\mathrm{Z3}) \Leftrightarrow (wt)^{-1} = —P—y_j—$.

13. Suppose $i < j < y_j < y_i < P < Q$.

(a) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(b) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(c) If $w^{-1} = —Q—P—y_i—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(d) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(e) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(f) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(g) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(h) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(i) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (Y3) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.

(j) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

(k) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

(l) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

(m) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

(n) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y3) fails for $(a,b) = (j,y_j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i < j < y_j < y_i < P < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

$(\mathrm{Z2}) \Leftrightarrow$ (no condition).

$(\mathrm{Z3}) \Leftrightarrow (wt)^{-1} = —i—Q—$ and $(wt)^{-1} = —j—Q—$.

14. Suppose $P < i < j < y_j < y_i < Q$.

(a) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(b) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(c) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(d) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(e) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(f) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(g) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (i,y_i)$.
(h) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(i) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(j) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $P < i < j < y_j < y_i < Q$ and then one of the following holds:

- $w^{-1} = —y_i—i—Q—P—y_j—j—$ and $v^{-1} = —y_j—j—Q—P—y_i—i—$.
- $w^{-1} = —Q—P—y_i—i—y_j—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.
- $w^{-1} = —y_i—i—y_j—j—Q—P—$ and $v^{-1} = —y_j—j—y_i—i—Q—P—$.
- $w^{-1} = —y_i—i—y_j—Q—P—j—$ and $v^{-1} = —y_j—j—y_i—Q—P—i—$.
- $w^{-1} = —y_i—Q—P—i—y_j—j—$ and $v^{-1} = —y_j—Q—P—j—y_i—i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

$$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—i—P— \text{ and } (wt)^{-1} \neq —Q—y_i—P— \text{ and} \\ (wt)^{-1} \neq —Q—j—P— \text{ and } (wt)^{-1} \neq —Q—y_j—P—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

15. Suppose $i < P < Q < j < y_j < y_i$.

(a) If $w^{-1} = —Q—y_i—i—y_j—P—j—$ then (T) fails.
(b) If $w^{-1} = —y_i—i—y_j—Q—P—j—$ then (T) fails.
(c) If $w^{-1} = —y_i—i—y_j—Q—j—P—$ then (T) fails.
(d) If $w^{-1} = —y_i—Q—i—y_j—P—j—$ then (T) fails.
(e) If $w^{-1} = —y_i—i—Q—P—y_j—j—$ then (T) fails.
(f) If $w^{-1} = —y_i—i—Q—y_j—P—j—$ then (T) fails.
(g) If $w^{-1} = —y_i—Q—i—P—y_j—j—$ then (T) fails.
(h) If $w^{-1} = —y_i—i—Q—y_j—j—P—$ then (T) fails.
(i) If $w^{-1} = —Q—y_i—i—P—y_j—j—$ then (T) fails.
(j) If $w^{-1} = —Q—y_i—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = —y_i—Q—P—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = —Q—y_i—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(m) If $w^{-1} = —y_i—Q—i—y_j—j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.
(n) If $w^{-1} = —y_i—i—y_j—j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (j,y_j)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i < P < Q < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —Q—P—y_i—i—y_j—j—$ and $v^{-1} = —Q—P—y_j—j—y_i—i—$.

When $(a,b)=(P,Q)$ and $(a',b')\in \mathrm{Cyc}^1(y)=\{(j,y_j),(i,y_i)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j—. \end{cases}$$

$(\mathrm{Z2}) \Leftrightarrow (wt)^{-1} \neq —y_i—P—i—$ and $(wt)^{-1} \neq —y_i—Q—i—$.

$(\mathrm{Z3}) \Leftrightarrow (wt)^{-1} = —P—y_j—$.

We conclude that properties (V1)-(V3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $i<j<y_j<y_i$.

8.3 Subcase (iii)

Suppose i' and j' are integers such that $0\neq i-i'=j-j'\in n\mathbb{Z}$, so that $w(i)-w(i')=w(j)-w(j')=i-i'$.

1. Suppose $i'<i<j'<j<y_{j'}<y_j<y_{i'}<y_i$.
   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (T) fails.
   (d) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (e) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (T) fails.
   (f) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (T) fails.
   (g) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (T) fails.
   (h) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (T) fails.
   (i) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (j) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (k) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y2) fails for $(a,b)=(i,y_i)$ and $(a',b')=(j',y_{j'})$.
   (l) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y2) fails for $(a,b)=(i,y_i)$ and $(a',b')=(j',y_{j'})$.
   (m) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b)=(i',y_{i'})$ and $(a',b')=(i,y_i)$.

   Recall that $(k,l)=(y_j,y_i)$. We conclude that if $i'<i<j'<j<y_{j'}<y_j<y_{i'}<y_i$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

   When $(a,b)\in \mathrm{Cyc}^1(y)=\{(j,y_j),(i,y_i)\}$ and $(a',b')\in\{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

   $(\mathrm{V2}) \Leftrightarrow (wt)^{-1} \neq —y_i—j'—i—$ and $(wt)^{-1} \neq —y_i—y_{j'}—i—$.

   $(\mathrm{V3}) \Leftrightarrow$ (no condition).

2. Suppose $i'<j'<i<y_{j'}<j<y_j<y_{i'}<y_i$.
   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (e) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b)=(i',y_{i'})$ and $(a',b')=(i,y_i)$.

(f) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(g) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(h) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(i) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(j) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i' < j' < i < y_{j'} < j < y_j < y_{i'} < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a',b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

3. Suppose $i' < j' < y_{j'} < i < y_{i'} < j < y_j < y_i$.

(a) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(b) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(c) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(d) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(e) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

(f) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(g) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(h) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(i) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(j) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(k) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(l) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

(m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i' < j' < y_{j'} < i < y_{i'} < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a',b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

4. Suppose $i' < i < j' < j < y_{j'} < y_{i'} < y_j < y_i$.

   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (T) fails.
   (d) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (e) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (T) fails.
   (f) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (T) fails.
   (g) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (T) fails.
   (h) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (T) fails.
   (i) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (j) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (k) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.
   (l) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.
   (m) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

   Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i' < i < j' < j < y_{j'} < y_{i'} < y_j < y_i$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a',b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

   (V2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—j'—i—$ and $(wt)^{-1} \neq —y_i—y_{j'}—i—$.

   (V3) $\Leftrightarrow$ (no condition).

5. Suppose $i' < j' < i < y_{j'} < j < y_{i'} < y_j < y_i$.

   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (e) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

   Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i' < j' < i < y_{j'} < j < y_{i'} < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

6. Suppose $i' < i < j' < y_{j'} < j < y_j < y_{i'} < y_i$.
   (a) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (T) fails.
   (c) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (T) fails.
   (d) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (e) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (T) fails.
   (f) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (T) fails.
   (g) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (T) fails.
   (h) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (T) fails.
   (i) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
   (j) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
   (k) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (j',y_{j'})$.
   (l) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y2) fails for $(a,b) = (i,y_i)$ and $(a',b') = (j',y_{j'})$.
   (m) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.

   Recall that $(k,l) = (y_j,y_i)$. We conclude that if $i' < i < j' < y_{j'} < j < y_j < y_{i'} < y_i$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

   (V2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—j'—i—$ and $(wt)^{-1} \neq —y_i—y_{j'}—i—$.

   (V3) $\Leftrightarrow$ (no condition).

7. Suppose $i' < j' < i < j < y_{j'} < y_j < y_{i'} < y_i$.
   (a) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (c) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (e) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.
   (f) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.
   (g) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.

(h) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(i) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(j) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(k) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(l) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (y_j, y_i)$. We conclude that if $i' < j' < i < j < y_{j'} < y_j < y_{i'} < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a', b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

8. Suppose $i' < j' < y_{j'} < y_{i'} < i < j < y_j < y_i$.

(a) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(b) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(c) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(d) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(e) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

(f) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(g) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(h) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(i) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(j) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(k) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(l) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

(m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (y_j, y_i)$. We conclude that if $i' < j' < y_{j'} < y_{i'} < i < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a', b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

9. Suppose $i' < j' < y_{j'} < i < j < y_j < y_{i'} < y_i$.

   (a) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (c) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (e) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

   Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i' < j' < y_{j'} < i < j < y_j < y_{i'} < y_i$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

10. Suppose $i' < j' < i < j < y_{j'} < y_{i'} < y_j < y_i$.

   (a) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (b) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (c) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (d) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (e) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.
   (f) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (g) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (h) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (i) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (j) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (k) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (l) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.
   (m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a,b) = (j', y_{j'})$ and $(a',b') = (i, y_i)$.

   Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i' < j' < i < j < y_{j'} < y_{i'} < y_j < y_i$ and then one of the following holds:

   - $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{---}y_i\text{---}i\text{---} \text{ and} \\ (wt)^{-1} = \text{---}y_j\text{---}j\text{---} \text{ and} \\ (wt)^{-1} = \text{---}y_{i'}\text{---}i'\text{---} \text{ and} \\ (wt)^{-1} = \text{---}y_{j'}\text{---}j'\text{---}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

11. Suppose $i' < j' < y_{j'} < i < j < y_{i'} < y_j < y_i$.

    (a) If $w^{-1} = \text{---}y_{i'}\text{---}y_i\text{---}i'\text{---}y_{j'}\text{---}i\text{---}y_j\text{---}j'\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.

    (b) If $w^{-1} = \text{---}y_{i'}\text{---}y_i\text{---}i'\text{---}y_{j'}\text{---}j'\text{---}i\text{---}y_j\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.

    (c) If $w^{-1} = \text{---}y_{i'}\text{---}y_i\text{---}i'\text{---}y_{j'}\text{---}i\text{---}j'\text{---}y_j\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.

    (d) If $w^{-1} = \text{---}y_{i'}\text{---}y_i\text{---}i'\text{---}i\text{---}y_{j'}\text{---}y_j\text{---}j'\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.

    (e) If $w^{-1} = \text{---}y_{i'}\text{---}y_i\text{---}i'\text{---}i\text{---}y_{j'}\text{---}j'\text{---}y_j\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (i',y_{i'})$ and $(a',b') = (i,y_i)$.

    (f) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_i\text{---}i\text{---}y_{j'}\text{---}y_j\text{---}j'\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.

    (g) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_i\text{---}y_{j'}\text{---}i\text{---}y_j\text{---}j'\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.

    (h) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_i\text{---}i\text{---}y_{j'}\text{---}j'\text{---}y_j\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.

    (i) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_i\text{---}y_{j'}\text{---}i\text{---}j'\text{---}y_j\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.

    (j) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_{j'}\text{---}y_i\text{---}i\text{---}y_j\text{---}j'\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.

    (k) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_{j'}\text{---}y_i\text{---}i\text{---}j'\text{---}y_j\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.

    (l) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_i\text{---}y_{j'}\text{---}j'\text{---}i\text{---}y_j\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.

    (m) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_{j'}\text{---}y_i\text{---}j'\text{---}i\text{---}y_j\text{---}j\text{---}$ then (Y3) fails for $(a,b) = (j',y_{j'})$ and $(a',b') = (i,y_i)$.

    Recall that $(k,l) = (y_j,y_i)$. We conclude that if $i' < j' < y_{j'} < i < j < y_{i'} < y_j < y_i$ and then one of the following holds:

    - $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_{j'}\text{---}j'\text{---}y_i\text{---}i\text{---}y_j\text{---}j\text{---}$ and $v^{-1} = \text{---}y_{j'}\text{---}j'\text{---}y_{i'}\text{---}i'\text{---}y_j\text{---}j\text{---}y_i\text{---}i\text{---}$.

    When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

    $$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{---}y_i\text{---}i\text{---} \text{ and} \\ (wt)^{-1} = \text{---}y_j\text{---}j\text{---} \text{ and} \\ (wt)^{-1} = \text{---}y_{i'}\text{---}i'\text{---} \text{ and} \\ (wt)^{-1} = \text{---}y_{j'}\text{---}j'\text{---}. \end{cases}$$

    (V2) $\Leftrightarrow$ (no condition).

    (V3) $\Leftrightarrow$ (no condition).

12. Suppose $i' < i < j' < y_{j'} < y_{i'} < j < y_j < y_i$.

    (a) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_i\text{---}i\text{---}y_{j'}\text{---}y_j\text{---}j'\text{---}j\text{---}$ then (T) fails.

    (b) If $w^{-1} = \text{---}y_{i'}\text{---}y_i\text{---}i'\text{---}y_{j'}\text{---}i\text{---}y_j\text{---}j'\text{---}j\text{---}$ then (T) fails.

    (c) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_i\text{---}y_{j'}\text{---}i\text{---}y_j\text{---}j'\text{---}j\text{---}$ then (T) fails.

    (d) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_i\text{---}i\text{---}y_{j'}\text{---}j'\text{---}y_j\text{---}j\text{---}$ then (T) fails.

    (e) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_i\text{---}y_{j'}\text{---}i\text{---}j'\text{---}y_j\text{---}j\text{---}$ then (T) fails.

    (f) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_{j'}\text{---}y_i\text{---}i\text{---}y_j\text{---}j'\text{---}j\text{---}$ then (T) fails.

    (g) If $w^{-1} = \text{---}y_{i'}\text{---}i'\text{---}y_{j'}\text{---}y_i\text{---}i\text{---}j'\text{---}y_j\text{---}j\text{---}$ then (T) fails.

    (h) If $w^{-1} = \text{---}y_{i'}\text{---}y_i\text{---}i'\text{---}y_{j'}\text{---}i\text{---}j'\text{---}y_j\text{---}j\text{---}$ then (T) fails.

(i) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
(j) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
(k) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(l) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y2) fails for $(a, b) = (i, y_i)$ and $(a', b') = (j', y_{j'})$.
(m) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (y_j, y_i)$. We conclude that if $i' < i < j' < y_{j'} < y_{i'} < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a', b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq —y_i—j'—i—$ and $(wt)^{-1} \neq —y_i—y_{j'}—i—$.

(V3) $\Leftrightarrow$ (no condition).

13. Suppose $i' < j' < i < y_{j'} < y_{i'} < j < y_j < y_i$.

(a) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—y_j—j'—j—$ then (T) fails.
(b) If $w^{-1} = —y_{i'}—i'—y_i—i—y_{j'}—j'—y_j—j—$ then (T) fails.
(c) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—y_j—j'—j—$ then (T) fails.
(d) If $w^{-1} = —y_{i'}—y_i—i'—i—y_{j'}—j'—y_j—j—$ then (T) fails.
(e) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(f) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(g) If $w^{-1} = —y_{i'}—y_i—i'—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a, b) = (i', y_{i'})$ and $(a', b') = (i, y_i)$.
(h) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—y_j—j'—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(i) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—i—j'—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(j) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—y_j—j'—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(k) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—i—j'—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(l) If $w^{-1} = —y_{i'}—i'—y_i—y_{j'}—j'—i—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.
(m) If $w^{-1} = —y_{i'}—i'—y_{j'}—y_i—j'—i—y_j—j—$ then (Y3) fails for $(a, b) = (j', y_{j'})$ and $(a', b') = (i, y_i)$.

Recall that $(k, l) = (y_j, y_i)$. We conclude that if $i' < j' < i < y_{j'} < y_{i'} < j < y_j < y_i$ and then one of the following holds:

- $w^{-1} = —y_{i'}—i'—y_{j'}—j'—y_i—i—y_j—j—$ and $v^{-1} = —y_{j'}—j'—y_{i'}—i'—y_j—j—y_i—i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(j, y_j), (i, y_i)\}$ and $(a', b') \in \{(j', y_{j'}), (i', y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\text{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —y_i—i— \text{ and} \\ (wt)^{-1} = —y_j—j— \text{ and} \\ (wt)^{-1} = —y_{i'}—i'— \text{ and} \\ (wt)^{-1} = —y_{j'}—j'—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

14. Suppose $i' < i < j' < y_{j'} < j < y_{i'} < y_j < y_i$.

(a) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-y_j-j'-j-$ then (T) fails.

(b) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-y_j-j'-j-$ then (T) fails.

(c) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-y_j-j'-j-$ then (T) fails.

(d) If $w^{-1} = -y_{i'}-i'-y_i-i-y_{j'}-j'-y_j-j-$ then (T) fails.

(e) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-i-j'-y_j-j-$ then (T) fails.

(f) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-y_j-j'-j-$ then (T) fails.

(g) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-i-j'-y_j-j-$ then (T) fails.

(h) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-i-j'-y_j-j-$ then (T) fails.

(i) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-y_j-j'-j-$ then (T) fails.

(j) If $w^{-1} = -y_{i'}-y_i-i'-i-y_{j'}-j'-y_j-j-$ then (T) fails.

(k) If $w^{-1} = -y_{i'}-i'-y_i-y_{j'}-j'-i-y_j-j-$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.

(l) If $w^{-1} = -y_{i'}-i'-y_{j'}-y_i-j'-i-y_j-j-$ then (Y2) fails for $(a,b) = (i, y_i)$ and $(a',b') = (j', y_{j'})$.

(m) If $w^{-1} = -y_{i'}-y_i-i'-y_{j'}-j'-i-y_j-j-$ then (Y3) fails for $(a,b) = (i', y_{i'})$ and $(a',b') = (i, y_i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $i' < i < j' < y_{j'} < j < y_{i'} < y_j < y_i$ and then one of the following holds:

- $w^{-1} = -y_{i'}-i'-y_{j'}-j'-y_i-i-y_j-j-$ and $v^{-1} = -y_{j'}-j'-y_{i'}-i'-y_j-j-y_i-i-$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(j,y_j),(i,y_i)\}$ and $(a',b') \in \{(j',y_{j'}),(i',y_{i'})\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = -y_i-i- \text{ and} \\ (wt)^{-1} = -y_j-j- \text{ and} \\ (wt)^{-1} = -y_{i'}-i'- \text{ and} \\ (wt)^{-1} = -y_{j'}-j'-. \end{cases}$$

(V2) $\Leftrightarrow (wt)^{-1} \neq -y_i-j'-i-$ and $(wt)^{-1} \neq -y_i-y_{j'}-i-$.

(V3) $\Leftrightarrow$ (no condition).

We conclude that properties (V1)-(V3) hold for all $(a,b),(a',b') \in \mathrm{Cyc}(y)$ when $i < j < y_j < y_i$.

9 Case C2

Suppose $y_j < y_i < i < j$ and $w^{-1} = -i-y_i-j-y_j-$ so that $k = y_j < y_i = l$.

9.1 Subcase (i)

In this case $v = wt_{ij}t_{kl}$ is such that

$$v^{-1} = -j-y_j-i-y_i-.$$

When $(a,b),(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$, properties (V1)-(V3) are equivalent to the following conditions which evidently hold:

(Z1) $\Leftrightarrow (wt)^{-1} = -i-y_i-$ and $(wt)^{-1} = -j-y_j-$.

(Z2) $\Leftrightarrow (wt)^{-1} \neq -j-y_i-y_j-$ and $(wt)^{-1} \neq -j-i-y_j-$.

(Z3) $\Leftrightarrow$ (no condition).

Thus properties (V1)-(V3) hold whenever (a,b), (a',b') are as in case (i) and $y_j < y_i < i < j$.

9.2 Subcase (ii)

Suppose R is an integer such that $(R,R) \in \mathrm{Cyc}^2(y)$, so that $R = y_R \notin \{i,j,y_i,y_j\} + n\mathbb{Z}$.

1. Suppose $y_j < y_i < i < j < R$.

   (a) If $w^{-1} = —R—i—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.

   (b) If $w^{-1} = —i—R—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.

   (c) If $w^{-1} = —i—y_i—R—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

   (d) If $w^{-1} = —i—y_i—j—R—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (y_j,y_i)$. We conclude that if $y_j < y_i < i < j < R$ and then one of the following holds:

   - $w^{-1} = —i—y_i—j—y_j—R—$ and $v^{-1} = —j—y_j—i—y_i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —i—y_i—$ and $(wt)^{-1} = —j—y_j—$.

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —y_i—R—$ and $(wt)^{-1} = —y_j—R—$.

2. Suppose $y_j < y_i < i < R < j$.

   (a) If $w^{-1} = —i—y_i—R—j—y_j—$ then (T) fails.

   (b) If $w^{-1} = —i—R—y_i—j—y_j—$ then (T) fails.

   (c) If $w^{-1} = —i—y_i—j—R—y_j—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

   (d) If $w^{-1} = —R—i—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (y_j,y_i)$. We conclude that if $y_j < y_i < i < R < j$ and then one of the following holds:

   - $w^{-1} = —i—y_i—j—y_j—R—$ and $v^{-1} = —j—y_j—i—y_i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —i—y_i—$ and $(wt)^{-1} = —j—y_j—$.

   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—R—y_j—$.

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = —y_i—R—$.

3. Suppose $y_j < y_i < R < i < j$.

   (a) If $w^{-1} = —i—R—y_i—j—y_j—$ then (Y2) fails for $(a,b) = (y_i,i)$ and $(a',b') = (R,R)$.

   (b) If $w^{-1} = —i—y_i—j—R—y_j—$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (R,R)$.

   Recall that $(k,l) = (y_j,y_i)$. We conclude that if $y_j < y_i < R < i < j$ and then one of the following holds:

   - $w^{-1} = —i—y_i—R—j—y_j—$ and $v^{-1} = —j—y_j—R—i—y_i—$.
   - $w^{-1} = —R—i—y_i—j—y_j—$ and $v^{-1} = —R—j—y_j—i—y_i—$.
   - $w^{-1} = —i—y_i—j—y_j—R—$ and $v^{-1} = —j—y_j—i—y_i—R—$.

   When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   (Z1) $\Leftrightarrow$ $(wt)^{-1} = —i—y_i—$ and $(wt)^{-1} = —j—y_j—$.

   (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —i—R—y_i—$ and $(wt)^{-1} \neq —j—R—y_j—$.

   (Z3) $\Leftrightarrow$ (no condition).

4. Suppose $y_j < R < y_i < i < j$.

(a) If $w^{-1} = —i—y_i—j—R—y_j—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a',b') = (R,R)$.

(b) If $w^{-1} = —i—y_i—R—j—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

(c) If $w^{-1} = —i—R—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

(d) If $w^{-1} = —i—y_i—j—y_j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_j < R < y_i < i < j$ and then one of the following holds:

- $w^{-1} = —R—i—y_i—j—y_j—$ and $v^{-1} = —R—j—y_j—i—y_i—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —i—y_i—$ and $(wt)^{-1} = —j—y_j—$.

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—R—y_j—$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —R—i—$.

5. Suppose $R < y_j < y_i < i < j$.

(a) If $w^{-1} = —i—y_i—R—j—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

(b) If $w^{-1} = —i—R—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

(c) If $w^{-1} = —i—y_i—j—y_j—R—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

(d) If $w^{-1} = —i—y_i—j—R—y_j—$ then (Y3) fails for $(a,b) = (R,R)$ and $(a',b') = (y_i, i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $R < y_j < y_i < i < j$ and then one of the following holds:

- $w^{-1} = —R—i—y_i—j—y_j—$ and $v^{-1} = —R—j—y_j—i—y_i—$.

When $(a,b) = (R,R)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

(Z1) $\Leftrightarrow$ $(wt)^{-1} = —i—y_i—$ and $(wt)^{-1} = —j—y_j—$.

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —R—i—$ and $(wt)^{-1} = —R—j—$.

Next suppose $P < Q$ are integers with $(P,Q) \in \mathrm{Cyc}^2(y)$, so that $Q = y_P$ and $P, Q \notin \{i, j, y_i, y_j\} + n\mathbb{Z}$.

1. Suppose $P < y_j < y_i < Q < i < j$.

(a) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(b) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(c) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(d) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(e) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(f) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(g) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(h) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(i) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(j) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(k) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(l) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(m) If $w^{-1} = —i—y_i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

(n) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $P < y_j < y_i < Q < i < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—y_i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—i—$ and $(wt)^{-1} = —P—j—$.

2. Suppose $P < y_j < Q < y_i < i < j$.

   (a) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (e) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (f) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (g) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (h) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (j) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (k) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (l) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (m) If $w^{-1} = —i—y_i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (n) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

Recall that $(k,l) = (y_j,y_i)$. We conclude that if $P < y_j < Q < y_i < i < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—y_i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—i—$ and $(wt)^{-1} = —P—j—$.

3. Suppose $y_j < y_i < i < P < j < Q$.

   (a) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (T) fails.
   (b) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (T) fails.
   (c) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (T) fails.
   (d) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (T) fails.
   (e) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (T) fails.
   (f) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (g) If $w^{-1} = —Q—P—i—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (h) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
   (i) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

(j) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_i, i)$ and $(a',b') = (P,Q)$.
(k) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(l) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(m) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.
(n) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_j < y_i < i < P < j < Q$ and then one of the following holds:

- $w^{-1} = —i—y_i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —y_i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

4. Suppose $P < y_j < y_i < i < Q < j$.

(a) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (T) fails.
(b) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (T) fails.
(c) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (T) fails.
(e) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (T) fails.
(f) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (T) fails.
(g) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (T) fails.
(h) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
(i) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i, i)$.
(j) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(k) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(l) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(m) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.
(n) If $w^{-1} = —i—y_i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j, j)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $P < y_j < y_i < i < Q < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—y_i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —Q—y_i—P—$ and $(wt)^{-1} \neq —Q—i—P—$.

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—j—$.

5. Suppose $y_j < y_i < P < i < Q < j$.

(a) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (T) fails.
(b) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (T) fails.

(c) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (T) fails.
(e) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (T) fails.
(f) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (T) fails.
(g) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (T) fails.
(h) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —Q—P—i—y_i—j—y_j—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y3) fails for $(a, b) = (y_i, i)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (y_j, y_i)$. We conclude that if $y_j < y_i < P < i < Q < j$ and then one of the following holds:

- $w^{-1} = —i—y_i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

$(\mathrm{Z2}) \Leftrightarrow (wt)^{-1} \neq —j—P—y_j—$ and $(wt)^{-1} \neq —j—Q—y_j—$.

$(\mathrm{Z3}) \Leftrightarrow (wt)^{-1} = —y_i—Q—$.

6. Suppose $y_j < P < y_i < i < Q < j$.

(a) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (T) fails.
(b) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (T) fails.
(c) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (T) fails.
(d) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (T) fails.
(e) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (T) fails.
(f) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (T) fails.
(g) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (T) fails.
(h) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(i) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(j) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(k) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y2) fails for $(a, b) = (P, Q)$ and $(a', b') = (y_i, i)$.
(l) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y2) fails for $(a, b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k, l) = (y_j, y_i)$. We conclude that if $y_j < P < y_i < i < Q < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—y_i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.
- $w^{-1} = —i—y_i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.

When $(a, b) = (P, Q)$ and $(a', b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—}. \end{cases}$$

$$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq \text{—}Q\text{—}y_i\text{—}P\text{—} \text{ and } (wt)^{-1} \neq \text{—}Q\text{—}i\text{—}P\text{—} \text{ and} \\ (wt)^{-1} \neq \text{—}j\text{—}P\text{—}y_j\text{—} \text{ and } (wt)^{-1} \neq \text{—}j\text{—}Q\text{—}y_j\text{—}. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

7. Suppose $P < Q < y_j < y_i < i < j$.
   (a) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (b) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (c) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (d) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (e) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (f) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (g) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (h) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (i) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (j) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (k) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (l) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (m) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}Q\text{—}P\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
   (n) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

   Recall that $(k,l) = (y_j,y_i)$. We conclude that if $P < Q < y_j < y_i < i < j$ and then one of the following holds:

   - $w^{-1} = \text{—}Q\text{—}P\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}Q\text{—}P\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—}. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow$ $(wt)^{-1} = \text{—}P\text{—}i\text{—}$ and $(wt)^{-1} = \text{—}P\text{—}j\text{—}$.

8. Suppose $y_j < y_i < i < P < Q < j$.
   (a) If $w^{-1} = \text{—}Q\text{—}i\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}i\text{—}Q\text{—}P\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}i\text{—}Q\text{—}y_i\text{—}j\text{—}P\text{—}y_j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}i\text{—}y_i\text{—}Q\text{—}j\text{—}y_j\text{—}P\text{—}$ then (T) fails.
   (i) If $w^{-1} = \text{—}Q\text{—}i\text{—}y_i\text{—}P\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (j) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}y_j\text{—}P\text{—}$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
   (k) If $w^{-1} = \text{—}i\text{—}y_i\text{—}j\text{—}Q\text{—}P\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

(l) If $w^{-1} = —Q—P—i—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

(m) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

(n) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (y_j,y_i)$. We conclude that if $y_j < y_i < i < P < Q < j$ and then one of the following holds:

- $w^{-1} = —i—y_i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —j—P—y_j—$ and $(wt)^{-1} \neq —j—Q—y_j—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —y_i—Q—$.

9. Suppose $y_j < y_i < P < i < j < Q$.

   (a) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   (b) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   (c) If $w^{-1} = —Q—P—i—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   (d) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   (e) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   (f) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   (g) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   (h) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   (i) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.

   (j) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   (k) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   (l) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   (m) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   (n) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

   Recall that $(k,l) = (y_j,y_i)$. We conclude that if $y_j < y_i < P < i < j < Q$ and then one of the following holds:

   - $w^{-1} = —i—y_i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.

   When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

   $$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

   (Z2) $\Leftrightarrow$ (no condition).

   (Z3) $\Leftrightarrow (wt)^{-1} = —y_i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

10. Suppose $y_j < P < y_i < i < j < Q$.

    (a) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

    (b) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

    (c) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

    (d) If $w^{-1} = —Q—P—i—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

(e) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(j) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(k) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(l) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(m) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(n) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_j < P < y_i < i < j < Q$ and then one of the following holds:

- $w^{-1} = —i—y_i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow (wt)^{-1} \neq —Q—y_i—P—$ and $(wt)^{-1} \neq —Q—i—P—$.

(Z3) $\Leftrightarrow (wt)^{-1} = —y_j—Q—$.

11. Suppose $y_j < y_i < P < Q < i < j$.

(a) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(b) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(c) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(d) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y2) fails for $(a,b) = (y_i, i)$ and $(a', b') = (P, Q)$.
(e) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(f) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(g) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(h) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.
(i) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y2) fails for $(a,b) = (y_j, j)$ and $(a', b') = (P, Q)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_j < y_i < P < Q < i < j$ and then one of the following holds:

- $w^{-1} = —i—y_i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.
- $w^{-1} = —Q—i—y_i—j—y_j—P—$ and $v^{-1} = —Q—j—y_j—i—y_i—P—$.
- $w^{-1} = —i—y_i—Q—j—y_j—P—$ and $v^{-1} = —j—y_j—Q—i—y_i—P—$.
- $w^{-1} = —Q—i—y_i—P—j—y_j—$ and $v^{-1} = —Q—j—y_j—P—i—y_i—$.
- $w^{-1} = —Q—P—i—y_i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.
- $w^{-1} = —i—y_i—Q—P—j—y_j—$ and $v^{-1} = —j—y_j—Q—P—i—y_i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

$$(\mathrm{Z2}) \Leftrightarrow \begin{cases} (wt)^{-1} \neq —i—P—y_i— \text{ and } (wt)^{-1} \neq —i—Q—y_i— \text{ and} \\ (wt)^{-1} \neq —j—P—y_j— \text{ and } (wt)^{-1} \neq —j—Q—y_j—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

12. Suppose $y_j < P < y_i < Q < i < j$.
    (a) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (b) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (c) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (d) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (e) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (f) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (g) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (h) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (i) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (j) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (k) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (l) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (m) If $w^{-1} = —i—y_i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (n) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

    Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_j < P < y_i < Q < i < j$ and then one of the following holds:

    - $w^{-1} = —Q—P—i—y_i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

    When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

    $$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

    (Z2) $\Leftrightarrow$ $(wt)^{-1} \neq —j—P—y_j—$ and $(wt)^{-1} \neq —j—Q—y_j—$.

    (Z3) $\Leftrightarrow$ $(wt)^{-1} = —P—i—$.

13. Suppose $y_j < y_i < i < j < P < Q$.
    (a) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (b) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (c) If $w^{-1} = —Q—P—i—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (d) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (e) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (f) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (g) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (h) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (i) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_i,i)$ and $(a',b') = (P,Q)$.
    (j) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
    (k) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
    (l) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
    (m) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.
    (n) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (y_j,j)$ and $(a',b') = (P,Q)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_j < y_i < i < j < P < Q$ and then one of the following holds:

- $w^{-1} = —i—y_i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

(Z2) $\Leftrightarrow$ (no condition).

(Z3) $\Leftrightarrow$ $(wt)^{-1} = —y_i—Q—$ and $(wt)^{-1} = —y_j—Q—$.

14. Suppose $P < y_j < y_i < i < j < Q$.
    (a) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (b) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (c) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (d) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (e) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (f) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (g) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (h) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (i) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.
    (j) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (Y2) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_j,j)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $P < y_j < y_i < i < j < Q$ and then one of the following holds:

- $w^{-1} = —i—y_i—j—y_j—Q—P—$ and $v^{-1} = —j—y_j—i—y_i—Q—P—$.
- $w^{-1} = —i—y_i—j—Q—P—y_j—$ and $v^{-1} = —j—y_j—i—Q—P—y_i—$.
- $w^{-1} = —i—y_i—Q—P—j—y_j—$ and $v^{-1} = —j—y_j—Q—P—i—y_i—$.
- $w^{-1} = —Q—P—i—y_i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.
- $w^{-1} = —i—Q—P—y_i—j—y_j—$ and $v^{-1} = —j—Q—P—y_j—i—y_i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(Z1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

$$\text{(Z2)} \Leftrightarrow \begin{cases} (wt)^{-1} \neq —Q—y_i—P— \text{ and } (wt)^{-1} \neq —Q—i—P— \text{ and} \\ (wt)^{-1} \neq —Q—y_j—P— \text{ and } (wt)^{-1} \neq —Q—j—P—. \end{cases}$$

(Z3) $\Leftrightarrow$ (no condition).

15. Suppose $y_j < P < Q < y_i < i < j$.
    (a) If $w^{-1} = —i—y_i—j—Q—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (b) If $w^{-1} = —Q—i—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (c) If $w^{-1} = —i—Q—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (d) If $w^{-1} = —i—y_i—j—Q—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (e) If $w^{-1} = —i—y_i—Q—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
    (f) If $w^{-1} = —i—y_i—Q—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

(g) If $w^{-1} = —i—Q—P—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(h) If $w^{-1} = —i—Q—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(i) If $w^{-1} = —Q—i—y_i—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(j) If $w^{-1} = —i—Q—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(k) If $w^{-1} = —Q—i—y_i—j—P—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(l) If $w^{-1} = —i—y_i—Q—j—y_j—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(m) If $w^{-1} = —i—y_i—j—y_j—Q—P—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.
(n) If $w^{-1} = —Q—i—y_i—P—j—y_j—$ then (Y3) fails for $(a,b) = (P,Q)$ and $(a',b') = (y_i,i)$.

Recall that $(k,l) = (y_j,y_i)$. We conclude that if $y_j < P < Q < y_i < i < j$ and then one of the following holds:

- $w^{-1} = —Q—P—i—y_i—j—y_j—$ and $v^{-1} = —Q—P—j—y_j—i—y_i—$.

When $(a,b) = (P,Q)$ and $(a',b') \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ or vice versa, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{Z1}) \Leftrightarrow \begin{cases} (wt)^{-1} = —Q—P— \text{ and} \\ (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —j—y_j—. \end{cases}$$

$(\mathrm{Z2}) \Leftrightarrow (wt)^{-1} \neq —j—P—y_j—$ and $(wt)^{-1} \neq —j—Q—y_j—$.

$(\mathrm{Z3}) \Leftrightarrow (wt)^{-1} = —P—i—$.

We conclude that properties (V1)-(V3) hold whenever (a,b), (a',b') are as in cases (i) or (ii) and $y_j < y_i < i < j$.

9.3 Subcase (iii)

Suppose i' and j' are integers such that $0 \neq i - i' = j - j' \in n\mathbb{Z}$, so that $w(i) - w(i') = w(j) - w(j') = i - i'$.

1. Suppose $y_{j'} < y_j < y_{i'} < y_i < i' < i < j' < j$.

(a) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—j—y_{j'}—y_j—$ then (T) fails.
(b) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—y_{j'}—j—y_j—$ then (T) fails.
(c) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—j—y_{j'}—y_j—$ then (T) fails.
(d) If $w^{-1} = —i'—i—y_{i'}—j'—y_{j'}—y_i—j—y_j—$ then (T) fails.
(e) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—y_{j'}—j—y_j—$ then (T) fails.
(f) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—y_{j'}—j—y_j—$ then (T) fails.
(g) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—y_{j'}—j—y_j—$ then (T) fails.
(h) If $w^{-1} = —i'—y_{i'}—i—j'—y_{j'}—y_i—j—y_j—$ then (T) fails.
(i) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—j—y_{j'}—y_j—$ then (T) fails.
(j) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—j—y_{j'}—y_j—$ then (T) fails.
(k) If $w^{-1} = —i'—y_{i'}—j'—i—y_{j'}—y_i—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
(l) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—y_{j'}—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_i,i)$.
(m) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'},j')$ and $(a',b') = (y_j,j)$.

Recall that $(k,l) = (y_j,y_i)$. We conclude that if $y_{j'} < y_j < y_{i'} < y_i < i' < i < j' < j$ and then one of the following holds:

- $w^{-1} = —i'—y_{i'}—j'—y_{j'}—i—y_i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}y_{i'}\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}i'\text{—}y_j\text{—}$.

(V3) $\Leftrightarrow$ (no condition).

2. Suppose $y_{j'} < y_{i'} < y_j < i' < y_i < i < j' < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_{i'} < y_j < i' < y_i < i < j' < j$ and then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a',b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}. \end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

3. Suppose $y_{j'} < y_{i'} < i' < y_j < j' < y_i < i < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (b) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(k) If $w^{-1} = —i'—y_{i'}—i—j'—y_{j'}—y_i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(l) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(m) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

Recall that $(k, l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_{i'} < i' < y_j < j' < y_i < i < j$ and then one of the following holds:

- $w^{-1} = —i'—y_{i'}—j'—y_{j'}—i—y_i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a', b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

4. Suppose $y_{j'} < y_j < y_{i'} < y_i < i' < j' < i < j$.

(a) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.

(b) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.

(c) If $w^{-1} = —i'—i—y_{i'}—j'—y_{j'}—y_i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.

(d) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.

(e) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.

(f) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(g) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(h) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(i) If $w^{-1} = —i'—y_{i'}—j'—i—y_{j'}—y_i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(j) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—y_{j'}—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(k) If $w^{-1} = —i'—y_{i'}—i—j'—y_{j'}—y_i—j—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(l) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(m) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a, b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

Recall that $(k, l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_j < y_{i'} < y_i < i' < j' < i < j$ and then one of the following holds:

- $w^{-1} = —i'—y_{i'}—j'—y_{j'}—i—y_i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a, b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a', b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow (wt)^{-1} \neq —j—y_{i'}—y_j—$ and $(wt)^{-1} \neq —j—i'—y_j—$.

(V3) $\Leftrightarrow$ (no condition).

5. Suppose $y_{j'} < y_{i'} < y_j < i' < y_i < j' < i < j$.

(a) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—j—y_{j'}—y_j—$ then (U) fails.

(b) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—y_{j'}—j—y_j—$ then (U) fails.

(c) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—y_{j'}—j—y_j—$ then (U) fails.

(d) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—j—y_{j'}—y_j—$ then (U) fails.

(e) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.

(f) If $w^{-1} = —i'—i—y_{i'}—j'—y_{j'}—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.

(g) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.

(h) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(i) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(j) If $w^{-1} = —i'—y_{i'}—j'—i—y_{j'}—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(k) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—y_{j'}—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(l) If $w^{-1} = —i'—y_{i'}—i—j'—y_{j'}—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(m) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_{i'} < y_j < i' < y_i < j' < i < j$ and then one of the following holds:

- $w^{-1} = —i'—y_{i'}—j'—y_{j'}—i—y_i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a', b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

6. Suppose $y_{j'} < y_j < y_{i'} < i' < y_i < i < j' < j$.

(a) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—j—y_{j'}—y_j—$ then (T) fails.

(b) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—y_{j'}—j—y_j—$ then (T) fails.

(c) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—j—y_{j'}—y_j—$ then (T) fails.

(d) If $w^{-1} = —i'—i—y_{i'}—j'—y_{j'}—y_i—j—y_j—$ then (T) fails.

(e) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—y_{j'}—j—y_j—$ then (T) fails.

(f) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—y_{j'}—j—y_j—$ then (T) fails.

(g) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—y_{j'}—j—y_j—$ then (T) fails.

(h) If $w^{-1} = —i'—y_{i'}—i—j'—y_{j'}—y_i—j—y_j—$ then (T) fails.

(i) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—j—y_{j'}—y_j—$ then (T) fails.

(j) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—j—y_{j'}—y_j—$ then (T) fails.

(k) If $w^{-1} = —i'—y_{i'}—j'—i—y_{j'}—y_i—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(l) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—y_{j'}—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(m) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_j < y_{i'} < i' < y_i < i < j' < j$ and then one of the following holds:

- $w^{-1} = —i'—y_{i'}—j'—y_{j'}—i—y_i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a', b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}y_{i'}\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}i'\text{—}y_j\text{—}$.

(V3) $\Leftrightarrow$ (no condition).

7. Suppose $y_{j'} < y_{i'} < y_j < y_i < i' < i < j' < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (e) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (T) fails.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (T) fails.
   (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_j, j)$.

   Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_{i'} < y_j < y_i < i' < i < j' < j$ and then one of the following holds:

   - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$.

   When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}. \end{cases}$$

   (V2) $\Leftrightarrow$ (no condition).

   (V3) $\Leftrightarrow$ (no condition).

8. Suppose $y_{j'} < y_{i'} < i' < j' < y_j < y_i < i < j$.

   (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (b) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.
   (f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.
   (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(k) If $w^{-1} = —i'—y_{i'}—i—j'—y_{j'}—y_i—j—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(l) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(m) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_{i'} < i' < j' < y_j < y_i < i < j$ and then one of the following holds:

- $w^{-1} = —i'—y_{i'}—j'—y_{j'}—i—y_i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a', b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

9. Suppose $y_{j'} < y_{i'} < i' < y_j < y_i < i < j' < j$.

(a) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—j—y_{j'}—y_j—$ then (T) fails.

(b) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—y_{j'}—j—y_j—$ then (T) fails.

(c) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—j—y_{j'}—y_j—$ then (T) fails.

(d) If $w^{-1} = —i'—i—y_{i'}—j'—y_{j'}—y_i—j—y_j—$ then (T) fails.

(e) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—y_{j'}—j—y_j—$ then (T) fails.

(f) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—y_{j'}—j—y_j—$ then (T) fails.

(g) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—y_{j'}—j—y_j—$ then (T) fails.

(h) If $w^{-1} = —i'—y_{i'}—i—j'—y_{j'}—y_i—j—y_j—$ then (T) fails.

(i) If $w^{-1} = —i'—y_{i'}—i—y_i—j'—j—y_{j'}—y_j—$ then (T) fails.

(j) If $w^{-1} = —i'—y_{i'}—i—j'—y_i—j—y_{j'}—y_j—$ then (T) fails.

(k) If $w^{-1} = —i'—y_{i'}—j'—i—y_{j'}—y_i—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(l) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—y_{j'}—j—y_j—$ then (Y2) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(m) If $w^{-1} = —i'—y_{i'}—j'—i—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_j, j)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_{i'} < i' < y_j < y_i < i < j' < j$ and then one of the following holds:

- $w^{-1} = —i'—y_{i'}—j'—y_{j'}—i—y_i—j—y_j—$ and $v^{-1} = —j'—y_{j'}—i'—y_{i'}—j—y_j—i—y_i—$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a', b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = —i—y_i— \text{ and} \\ (wt)^{-1} = —i'—y_{i'}— \text{ and} \\ (wt)^{-1} = —j—y_j— \text{ and} \\ (wt)^{-1} = —j'—y_{j'}—. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

10. Suppose $y_{j'} < y_{i'} < y_j < y_i < i' < j' < i < j$.

(a) If $w^{-1} = —i'—i—y_{i'}—y_i—j'—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.

(b) If $w^{-1} = —i'—i—y_{i'}—j'—y_i—j—y_{j'}—y_j—$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.

(c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

(d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

(f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_{i'} < y_j < y_i < i' < j' < i < j$ and then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

11. Suppose $y_{j'} < y_{i'} < i' < y_j < y_i < j' < i < j$.

(a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.

(b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.

(c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.

(d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.

(e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

(f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

(g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

(h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_{i'} < i' < y_j < y_i < j' < i < j$ and then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

12. Suppose $y_{j'} < y_j < y_{i'} < i' < j' < y_i < i < j$.
    (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.
    (b) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.
    (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.
    (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.
    (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.
    (f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

    Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_j < y_{i'} < i' < j' < y_i < i < j$ and then one of the following holds:

    - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$.

    When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i, i), (y_j, j)\}$ and $(a', b') \in \{(y_{i'}, i'), (y_{j'}, j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$(\mathrm{V1}) \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}. \end{cases}$$

    (V2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}y_{i'}\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}i'\text{—}y_j\text{—}$.

    (V3) $\Leftrightarrow$ (no condition).

13. Suppose $y_{j'} < y_{i'} < y_j < i' < j' < y_i < i < j$.
    (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.
    (b) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.
    (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.
    (d) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.
    (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a', b') = (y_i, i)$.
    (f) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (g) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.
    (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a', b') = (y_i, i)$.

(k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

(m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_{i'} < y_j < i' < j' < y_i < i < j$ and then one of the following holds:

- $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$.

When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}. \end{cases}$$

(V2) $\Leftrightarrow$ (no condition).

(V3) $\Leftrightarrow$ (no condition).

14. Suppose $y_{j'} < y_j < y_{i'} < i' < y_i < j' < i < j$.

 (a) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.

 (b) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.

 (c) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}y_i\text{—}j'\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (U) fails.

 (d) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}y_i\text{—}j'\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (U) fails.

 (e) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

 (f) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

 (g) If $w^{-1} = \text{—}i'\text{—}i\text{—}y_{i'}\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{i'}, i')$ and $(a',b') = (y_i, i)$.

 (h) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

 (i) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

 (j) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

 (k) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}i\text{—}y_i\text{—}y_{j'}\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

 (l) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_{j'}\text{—}y_i\text{—}j\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

 (m) If $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}i\text{—}j'\text{—}y_i\text{—}j\text{—}y_{j'}\text{—}y_j\text{—}$ then (Y3) fails for $(a,b) = (y_{j'}, j')$ and $(a',b') = (y_i, i)$.

 Recall that $(k,l) = (y_j, y_i)$. We conclude that if $y_{j'} < y_j < y_{i'} < i' < y_i < j' < i < j$ and then one of the following holds:

 - $w^{-1} = \text{—}i'\text{—}y_{i'}\text{—}j'\text{—}y_{j'}\text{—}i\text{—}y_i\text{—}j\text{—}y_j\text{—}$ and $v^{-1} = \text{—}j'\text{—}y_{j'}\text{—}i'\text{—}y_{i'}\text{—}j\text{—}y_j\text{—}i\text{—}y_i\text{—}$.

 When $(a,b) \in \mathrm{Cyc}^1(y) = \{(y_i,i),(y_j,j)\}$ and $(a',b') \in \{(y_{i'},i'),(y_{j'},j')\}$, properties (V1)-(V3) correspond to the following conditions which hold in each of the available cases for v:

$$\text{(V1)} \Leftrightarrow \begin{cases} (wt)^{-1} = \text{—}i\text{—}y_i\text{—} \text{ and} \\ (wt)^{-1} = \text{—}i'\text{—}y_{i'}\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j\text{—}y_j\text{—} \text{ and} \\ (wt)^{-1} = \text{—}j'\text{—}y_{j'}\text{—}. \end{cases}$$

 (V2) $\Leftrightarrow$ $(wt)^{-1} \neq \text{—}j\text{—}y_{i'}\text{—}y_j\text{—}$ and $(wt)^{-1} \neq \text{—}j\text{—}i'\text{—}y_j\text{—}$.

 (V3) $\Leftrightarrow$ (no condition).

We conclude that properties (V1)-(V3) hold for all $(a,b),(a',b') \in \mathrm{Cyc}(y)$ when $y_j < y_i < i < j$.

10 Conclusion

It follows from this exhaustive case analysis that properties (V1)-(V3) hold for all pairs $(a, b), (a', b') \in \mathrm{Cyc}(y)$. We conclude by Lemma 1 that $wt_{ij}t_{kl} \in \mathcal{A}(y)$. This completes the proof of the theorem. $\square$

References

[1] A. Björner and F. Brenti, *Combinatorics of Coxeter groups*, Graduate Texts in Mathematics 231, Springer, New York, 2005.

[2] E. Marberg, On some actions of the 0-Hecke monoids of affine symmetric groups, *J. Combin. Theory Ser. A* **161** (2019), 178–219.