AA 545 – Kinetic Modeling: Vlasov-Poisson PIC

Due 23 Apr 2013

Computer Project 1.2 – Vlasov-Poisson Code Development

Complete the development of your electrostatic PIC code by implementing a Leap-Frog time integration method to advance the particles' velocities and positions in phase space, (x, v_x), for each particle i.

$$\frac{v_i^{n+1/2} - v_i^{n-1/2}}{\Delta t} = E_i^n$$

$$\frac{x_i^{n+1} - x_i^n}{\Delta t} = v_i^{n+1/2}$$

Solve the Poisson equation for the electrostatic potential and then the electric field.

$$\frac{\phi_{j+1}^n - 2\phi_j^n + \phi_{j-1}^n}{\Delta x^2} = -\rho_j^n$$

$$E_j^n = -\frac{\phi_{j+1}^n - \phi_{j-1}^n}{2\Delta x}$$

Assume a uniform, static background species that is equal and opposite to the charge of the particles, such that the plasma is net charge neutral, i.e. $\oint E dx = 0$. Note the above expression for the Vlasov-Poisson equation has normalized units. Determine the normalization being used. Allow the user to select either 0^{th} or 1^{st} order weighting for particles and forces, in addition to the number of particles and grid points. Implement a diagnostic that computes the time history of the total electric field energy.

Investigate Langmuir oscillations and the Leap-Frog instability.

1. Initialize two particles with no velocity on a spatially periodic domain, $x = [-\pi, \pi]$, such that the phase space positions for the two particles are $(-\pi/4, 0)_1$ and $(\pi/4, 0)_2$. This produces a displacement perturbation. Observe the simple harmonic motion. Plot the field energy history and measure the oscillation frequency. Compare to the theoretically expected value. Remember the field energy is $\int E^2 dx$.

2. Initialize two particles that are evenly distributed on a spatially periodic domain, $x = [-\pi, \pi]$, with a sinusoidal velocity, i.e. the phase space position for the two particles are $(-\pi/2, -v')_1$ and $(\pi/2, v')_2$. Adjust v' to generate simple harmonic motion. Plot the kinetic energy history and measure the oscillation frequency. Compare to the theoretically expected value. Increase v' so the particles' trajectories cross.

3. Initialize 64 cold, static, evenly-distributed particles onto a grid with 32 grid points. Initialize a small velocity perturbation that varies sinusoidally with a single period in x. The velocity perturbation should be small enough to maintain harmonic oscillations. (The particle trajectories should not cross.) Measure the oscillation frequency from the field energy history plot and compare to the plasma frequency. Repeat for several plasma frequency values and plot the deviation of the measured frequency from the plasma frequency. Compare with the theoretical result for the Leap-Frog instability to demonstrate phase error and instability.