Eqs

July 14, 2021

1 Paper

Eq1

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O \tag{1}$$

Eq2

$$\begin{aligned} &F_i^{cake} \geq F_i^{in} \cdot \eta_i^{\mathrm{j}} - \mathrm{M} \cdot (1 - y^j) \\ &\mathrm{i} \in \{P, N\} \\ &j \in \{\text{filtermedia}\} \end{aligned} \tag{2}$$

Eq3

$$\sum \mathrm{y}^{\mathrm{j}} = 1 \tag{3}$$

Eq4

$$F_i^{\text{liquid effluent}} = F_i^{in} - F_{\mathrm{i}}^{\text{cake}} \tag{4}$$

Eq5

$$F_k^{cake} = F_k^{in};\ \mathrm{k} \in \{TS, C, K\} \tag{5}$$

Eq6

$$F_{Wa}^{cake} = \left(F_{TS}^{cake} + \sum_i F_i^{cake} \right) \cdot \frac{C_{Wa}^{cake}}{1 - C_{Wa}^{cake}} \tag{6}$$

Eq7

$$F_{Wa}^{\text{liquid effluent}} = F_{Wa}^{in} - F_{Wa}^{cake} \tag{7}$$

Eq8

$$F\left(\frac{ft^3}{\text{min}}\right) = \frac{F_{in}}{\rho_{digestate}} \tag{8}$$

Eq9

$$\text{n}_{\text{filters}} \geq \frac{F_{total}^{filter}}{F_{\max}^{filter}} \tag{9}$$

Eq10

$$F_{design}^{filter}\left(\frac{ft^3}{\text{min}}\right) = \min\left(F_{\max}^{filter}, F_{total}^{filter}\right) \tag{10}$$

Eq11

$$FC_{filtration}\,(\$) = 4.7436 \cdot F_{design}^{filter} + 807.6923 \tag{11}$$

Eq12

$$\text{ChemC}_{\text{filtration}}\left(\frac{\text{EUR}}{\text{year}}\right) = \frac{\text{F}_{\text{P}}^{\text{in}} \cdot 3600 \cdot \text{h} \cdot \text{d}}{\frac{\text{kg}_{\text{P}}}{\text{kg}_{\text{filter media}}}} \cdot \text{Price}_{\text{filter media}} \tag{12}$$

Eq13

$$\text{Labour cost}\left(\frac{EUR}{year}\right) = \left(61.33 \cdot F_P^{recovered} \cdot 3.6 \cdot \text{h}^{(-0.82)}\right) \cdot \left(F_P^{recovered} \cdot 3.6 \cdot h \cdot d\right) \cdot \left(\frac{Salary}{\cdot h \cdot d}\right) \cdot n_{OP} \tag{13}$$

Eq14

$$\text{Operatingcost}\left(\frac{EUR}{year}\right) = \frac{\text{ChemC} + 1.5 \cdot \text{Labourcost} + 0.3 \cdot FixedCost \cdot f_i \cdot f_j}{(1 - Utilities)} \tag{14}$$

Eq15

$$Cost_{cake}\left(\frac{EUR}{year}\right) = \left(F_P^{recovered} \cdot \text{Price}_P + F_N^{recovered} \cdot \text{Price}_N + F_K^{recovered} \cdot \text{Price}_K\right) \cdot 3600 \cdot h \cdot d \tag{15}$$

Eq16

$$\text{Benefits}_{Filtration}\left(\frac{EUR}{year}\right) = Cost_{cake} - \text{Operatingcost} \tag{16}$$

Eq17

$$F_j^{\text{coagtank}} \geq \frac{F_P^{\text{in}}}{MW_P} \cdot MeP_{ratio} \cdot \frac{MW_j}{C_{Me}} - M \cdot (1 - y^j);\ \text{j} \in \{\text{coagulantagents}\} \tag{17}$$

Eq18

$$\sum \text{y}^j = 1 \tag{18}$$

Eq19

$$F_j^{\text{coagtank}} = F_j^{\text{floctank}} = F_j^{\text{sedimentator}} = F_j^{\text{centrifuge}} = F_j^{\text{cake}};\ j \in \{coagulants\} \tag{19}$$

Eq20

$$F_i^{in} = F_i^{\text{coagtank}} = F_i^{\text{floctank}} = F_i^{\text{sedimentator}};\ \text{i} \in \{P, N\} \tag{20}$$

Eq21

$$F_i^{cake} = F_i^{centrifuge} = F_i^{\text{sedimentator}} \cdot \eta_i^j \tag{21}$$

Eq22

$$F_i^{\text{sink1}} = F_i^{\text{sedimentator}} - F_i^{\text{cntrifuge}} \tag{22}$$

Eq23

$$F_k^{in} = F_k^{\text{coagtank}} = F_k^{\text{floctank}} = F_k^{\text{sedimentator}} = F_k^{\text{centrifuge}} = \mathrm{F}_{\mathrm{k}}^{\text{cake}};\ \mathrm{k} \in \{TS, C, K\} \tag{23}$$

Eq24

$$F_{Wa}^{in} = F_{Wa}^{\text{coagtank}} = F_{Wa}^{\text{floctank}} = F_{Wa}^{\text{sedimentator}} \tag{24}$$

Eq25

$$\mathrm{F}_{\mathrm{Wa}}^{centrifuge} = \left(\mathrm{F}_{TS}^{centrifuge} + \sum_i \mathrm{F}_i^{centrifuge} + \sum_j \mathrm{F}_j^{centrifuge}\right) \cdot \frac{C_{Wa}^{\text{sedimentato}r}}{1 - C_{Wa}^{centrifuge}} \tag{25}$$

Eq26

$$\mathrm{F}_{\mathrm{Wa}}^{\text{sink1}} = \mathrm{F}_{\mathrm{Wa}}^{\text{sedimentator}} - \mathrm{F}_{\mathrm{Wa}}^{centrifuge} \tag{26}$$

Eq27

$$\mathrm{F}_{\mathrm{Wa}}^{cake} = \left(\mathrm{F}_{TS}^{cake} + \sum_i \mathrm{F}_i^{cake} + \sum_j \mathrm{F}_j^{cake}\right) \cdot \frac{C_{Wa}^{centrifuge}}{1 - C_{Wa}^{centrifuge}} \tag{27}$$

Eq28

$$\mathrm{F}_{\mathrm{Wa}}^{\text{sink2}} = \mathrm{F}_{\mathrm{Wa}}^{centrifuge} - \mathrm{F}_{\mathrm{Wa}}^{cake} \tag{28}$$

Eq29

$$\mathrm{V}_{\text{Coagtank}}\left(m^3\right) = HRT_{\text{Coagtank}} \cdot \frac{F_{digestate}^{in}}{\rho_{digestate}} \tag{29}$$

Eq30

$$D_{\text{Coagtank}}\,(m) = \left(\frac{6 \cdot \text{V}_{\text{Coagtank}}}{7 \cdot \pi}\right)^{1/3} \tag{30}$$

Eq31

$$L_{\text{Coagtank}}\,(m) = 4 \cdot D_{\text{Coagtank}} \tag{31}$$

Eq32

$$e_{\text{Coagtank}}\,(m) = 0.0023 + 0.003 \cdot D_{\text{Coagtank}} \tag{32}$$

Eq33

$$W_{\text{Coagtank}}\,(kg) = \rho_{SS316} \cdot \left[\pi \cdot \left(\left(\frac{D_{\text{Coagtank}}}{2} + e_{\text{Coagtank}}\right)^2 - \left(\frac{D_{\text{Prectank}}}{2}\right)^2\right) \cdot L_{\text{Coagtank}} + \frac{4}{3} \cdot \pi \cdot \left(\left(\frac{D_{\text{Coagtank}}}{2} + e_{\text{Coagtank}}\right)^3 - \left(\frac{D_{\text{Coagtank}}}{2}\right)^3\right)\right] \tag{33}$$

Eq34

$$\text{Cost}_{Vessel} = 6839.8 \cdot \text{V}_{\text{Coagtank}}\left(m^3\right)^{0.65} \tag{34}$$

Eq35

$$P_{agitator}\,(HP) = \text{V}_{\text{Coagtank}}\,(\text{USgallon}) \cdot \frac{\kappa_{agitator}}{1000} \tag{35}$$

Eq36

$$Cost_{\text{agitator1985}}\,(\$) = e^{a + b \cdot \ln(P_{agitator}(HP)) + c \cdot [\ln(P_{agitator}(HP))]^2} \tag{36}$$

Eq37

$$\text{Cost}_{\text{Coagtank}} = \text{Cost}_{Vessel} + Cost_{\text{agitator2016}} \tag{37}$$

Eq38

$$\text{A}_{clarifier} = \frac{A_{specific} \cdot F^{in}_{digestate} \left(\frac{m^3}{day}\right)}{1000} \tag{38}$$

Eq39

$$D_{clarifier} = \left(\frac{4 \cdot A_{clarifier}}{\pi}\right)^{1/2} \tag{39}$$

Eq40

$$\text{n}_{clarifiers} \geq \frac{D^{clarifier}_{total}}{D^{clarifier}_{\max}} \tag{40}$$

Eq41

$$D^{clarifier}_{design} = \min(D^{clarifier}_{\max}, D^{clarifier}_{total}) \tag{41}$$

Eq42

$$D^{clarifier}_{design} = \frac{D^{clarifier}_{\max}}{1 + e^{\left(-F^{in}_{digestate} + 0.342\right) \cdot 2.718}} \tag{42}$$

Eq43

$$Cost_{\text{clarifier1979}} = \left(13060 \cdot D^{clarifier}_{design} - 58763\right) \cdot \text{n}_{clarifiers} \tag{43}$$

Eq44

$$D_{Centrifuge}\,(in) = 0.3308 \cdot \frac{F^{in}_{digestate}}{1000} \cdot 3600 + 9.5092 \tag{44}$$

Eq45

$$\mathrm{n}_{centrifuges} \geq \frac{D_{total}^{centrifuge}}{D_{\max}^{centrifuge}} \tag{45}$$

Eq46

$$D_{design}^{\text{centrifuge}} = \min\left(D_{\max}^{centrifuge}, D_{total}^{centrifuge}\right) \tag{46}$$

Eq47

$$D_{design}^{\text{centrifuge}} = \frac{D_{\max}^{centrifuge}}{1 + e^{\left(-F_{digestate}^{in} + 35.369\right) \cdot 0.0395}} \tag{47}$$

Eq48

$$Cost_{\text{centrifuge2004}}\,(\$) = \left(10272 \cdot D_{design}^{\text{centrifuge}} - 24512\right) \cdot \mathrm{n}_{centrifuges} \tag{48}$$

Eq49

$$\mathrm{FC}_{coagulation}\left(\frac{EUR}{year}\right) = (\mathrm{Cost}_{\text{Coagtank}} + \mathrm{Cost}_{\text{Floctank}} + Cost_{\text{centrifuge2016}}) \cdot f_i \cdot f_j + Cost_{\text{clarifier2016}} \tag{49}$$

Eq50

$$\mathrm{ChemC}_{coagulation}\left(\frac{EUR}{year}\right) = \Big(F_{Fe_2(SO_4)_3}^{in} \cdot \mathrm{Price}_{Fe_2(SO_4)_3} + F_{Al_2(SO_4)_3}^{in} \cdot \mathrm{Price}_{Al_2(SO_4)_3} + F_{FeCl_3}^{in} \cdot \mathrm{Price}_{FeCl_3} + F_{AlCl}^{in} \tag{50}$$

Eq51

$$F_j^{in} = F_{TS}^{in} \cdot \frac{\varphi_{\mathrm{j}}}{\mathrm{C_j}} \mathrm{j} \in \{\text{precipitationagents}\} \tag{51}$$

Eq52

$$F_j^{in} = F_j^{\text{prectank}} = F_j^{\text{centrifuge}} = F_j^{\text{cake}} \tag{52}$$

Eq53

$$F_i^{in} = F_i^{\text{prectank}} = F_i^{\text{centrifuge}} \text{i} \in \{\text{P}, \text{N}\} \tag{53}$$

Eq54

$$F_i^{cake} = F_i^{\text{centrifuge}} \cdot \eta_i \tag{54}$$

Eq55

$$F_i^{\text{liquideffluent}} = F_i^{\text{centrifuge}} - F_i^{cake} \tag{55}$$

Eq56

$$F_k^{in} = F_k^{\text{prectank}} = F_k^{\text{centrifuge}} = F_k^{\text{cake}} \text{k} \in \{TS, C, K\} \tag{56}$$

Eq57

$$F_{Wa}^{in} = F_{Wa}^{\text{prectank}} = F_{Wa}^{\text{centrifuge}} \tag{57}$$

Eq58

$$\text{F}_{\text{Wa}}^{cake} = \left(\text{F}_{TS}^{cake} + \sum_i \text{F}_i^{cake} + \sum_j \text{F}_j^{cake} \right) \cdot \frac{C_{Wa}^{centrifuge}}{1 - C_{Wa}^{centrifuge}} \tag{58}$$

Eq59

$$\text{F}_{\text{Wa}}^{\text{liquideffluent}} = \text{F}_{\text{Wa}}^{centrifuge} - \text{F}_{\text{Wa}}^{cake} \tag{59}$$

Eq60

$$\mathrm{V}_{\mathrm{Prectank}}\left(m^3\right) = HRT_{\mathrm{Prectank}} \cdot \left(\frac{F^{in}_{digestate}}{\rho_{digestate}} + \frac{F^{in}_{FeCl_3}}{\rho_{FeCl_3}}\right) \tag{60}$$

Eq61

$$\mathrm{FC}_{centrifugation}\left(\frac{EUR}{year}\right) = \left(Cost_{\mathrm{centrifuge2016}} + \mathrm{Cost}_{\mathrm{Prectank}}\right) \cdot f_i \cdot f_j \tag{61}$$

Eq62

$$\mathrm{ChemC}_{centrifugation}\left(\frac{EUR}{year}\right) = \left(F^{in}_{CaCO_3} \cdot \mathrm{Price}_{CaCO_3} + F^{in}_{FeCl_3} \cdot \mathrm{Price}_{FeCl_3}\right) \cdot 3600 \cdot h \cdot d \tag{62}$$

Eq63

$$Mg^{2+} + NH_4^+ + H_nPO_4^{3-n} + 6H_2O \Leftrightarrow MgNH_4PO_4 \cdot 6H_2O + nH^+ \tag{63}$$

Eq64

$$K^+ + Mg^{2+} + H_nPO_4^{3-n} + 6H_2O \Leftrightarrow KMgPO_4 \cdot 6H_2O + nH^+ \tag{64}$$

Eq64bis

$$\frac{-dC}{dt} = k\left(C - C_{eq}\right) \tag{65}$$

Eq65

$$\ln\left(C - C_{eq}\right) = -kt + \ln\left(C_0 - C_{eq}\right) \tag{66}$$

Eq66

$$IAP_{eq} = \left(Mg^{2+}\right)\left(NH_4^+\right)\left(PO_4^{3-}\right) = 7.08 \cdot 10^{-14} \tag{67}$$

Eq67

$$u_{\mathrm{mf}} = \frac{\mathrm{Re}_{\mathrm{lmf}} \cdot \mu_{digestate}}{\rho_{digestate} - d_p} \tag{68}$$

Eq68

$$\mathrm{Re}_{\mathrm{lmf}} = \sqrt{33.72 + 0.0404 Ar_l (1 - \alpha_{mf})^3} - 33.7 \tag{69}$$

Eq69

$$Ar_l = \rho_{digestate} \left(\rho_{struvite} - \rho_{digestate}\right) g \frac{d_p^3}{\mu_{digestate}^2} \tag{70}$$

Eq70

$$u_t = \left(\frac{1.78 \cdot 10^{-2} \cdot \eta^2}{\rho_{digestate} \cdot \mu_{digestate}} \right)^{1/3} d_p \tag{71}$$

Eq71

$$\eta = g \left(\rho_{struvite} - \rho_{digestate}\right) \tag{72}$$

Eq72

$$u_{mf} < u_0 < u_t \tag{73}$$

Eq73

$$u_0 = 5 \cdot u_{mf} \tag{74}$$

Eq74

$$A_{FBR} = \frac{F_{digestate}^{in}}{u_0} \tag{75}$$

Eq75

$$D_{FBR} = \sqrt{\frac{4A_{FBR}}{\pi}} \tag{76}$$

Eq76

$$t = \frac{\ln\left(C_0 - C_{eq}\right) - \ln\left(C - C_{eq}\right)}{k} \tag{77}$$

Eq77

$$L_{bed} = \frac{u_0}{t} \tag{78}$$

Eq78

$$L_{FBR} = 1.15 \cdot L_{bed} \tag{79}$$

Eq79

$$\mathrm{D}_{\mathrm{hydrocyclone}}\left(\mathrm{in}\right) = F_{digestate}^{in}\left(\frac{\mathrm{USgallon}}{\mathrm{min}}\right) \cdot \frac{20}{1000} \tag{80}$$

Eq80

$$\mathrm{n}_{\mathrm{hydrocyclone}} \geq \frac{D_{total}^{\mathrm{hydrocyclone}}}{D_{\max}^{\mathrm{hydrocyclone}}} \tag{81}$$

Eq81

$$D_{design}^{\mathrm{hydrocyclone}} = \min(D_{total}^{\mathrm{hydrocyclone}}, D_{\max}^{\mathrm{hydrocyclone}}) \tag{82}$$

Eq82

$$Cost_{\mathrm{hydrocyclone2014}} = \mathrm{n}_{\mathrm{hydrocyclone}} \cdot \left(2953.2 \cdot D_{design}^{\mathrm{hydrocyclone}} - 34131\right) \tag{83}$$

Eq83

$$Cost_{\mathrm{dryer2007}}\,(\$) = 1.15 \cdot \left(6477.1 \cdot \frac{F_{water}^{in}}{e_{capacity}} + 102394\right) \tag{84}$$

Eq84

$$\mathrm{FC}_{struvite}\left(\frac{EUR}{year}\right) = \left(\sum Cost_{\mathrm{equipment}}\right) \cdot f_i \cdot f_j \tag{85}$$

Eq85

$$Cost_{struvite}\left(\frac{EUR}{year}\right) = \left(F_{struvite}^{recovered} \cdot \mathrm{Price}_{struvite}\right) \cdot 3600 \cdot h \cdot d \tag{86}$$

Eq86

$$\mathrm{Operatingcost}\left(\frac{EUR}{year}\right) = 20521 \cdot F_{design}\left(\frac{ft^3}{\mathrm{min}}\right) - 33488 \cdot \mathrm{a_{Filter}} \tag{87}$$

Eq87

$$\mathrm{a_{Filter}} = \frac{1}{1 + e^{(-F_{design}+0.049)\cdot 361}} \tag{88}$$

Eq88

$$\mathrm{Operatingcost}\left(\frac{EUR}{year}\right) = 1019589.91 \cdot F_{digestate}^{in}\left(\frac{kg}{s}\right) - 368838.56 \cdot \mathrm{a_{Coag}} \tag{89}$$

Eq89

$$\mathrm{a_{Coag}} = \frac{1}{1 + e^{\left(-F_{digestate}^{in}+0.068\right)\cdot 863}} \tag{90}$$

Eq90

$$\mathrm{Operatingcost}\left(\frac{EUR}{year}\right) = 458498.29 \cdot F_{digestate}^{in} + 24924.67 \cdot \mathrm{a}_{Centrifugation} \tag{91}$$

Eq91

$$a_{\text{Centrifugation}} = \frac{1}{1 + e^{\left(-F^{in}_{digestate} + 0.068\right) \cdot 863}} \tag{92}$$

Eq92

$$\text{OperatingCost}_{FBR}\left(\frac{EUR}{year}\right) = 245008 \cdot F^{in}_{digestate} + 1 \cdot 10^{6} \cdot a_{FBR} \tag{93}$$

Eq93

$$a_{FBR} = \frac{1}{1 + e^{\left(-F^{in}_{digestate} + 0.06785\right) \cdot 862.9679}} \tag{94}$$

Eq94

$$\text{Operatingcost}_{CSTR}\left(\frac{EUR}{year}\right) = 277051 \cdot F^{in}_{digestate} + 1 \cdot 10^{6} \cdot a_{CSTR} \tag{95}$$

Eq95

$$a_{CSTR} = \frac{1}{1 + e^{\left(-F^{in}_{digestate} + 0.06785\right) \cdot 862.9679}} \tag{96}$$

Eq96

$$\text{Price}\left(\frac{EUR}{year}\right) = \left(F^{\text{recovered}}_{P} \cdot \text{Price}_{P} + F^{\text{recovered}}_{N} \cdot \text{Price}_{N} + F^{\text{recovered}}_{\text{K}} \cdot \text{Price}_{K}\right) \cdot \frac{1}{F^{\text{recovered}}_{total}} \cdot 3600 \cdot h \cdot d \tag{97}$$

Eq97

$$Z = \left[\left(\sum_{i \in turbinebody} \text{W}_{(\text{Turbine})} + W_{(GasTurb)} - \sum_{j \in compressors} W_{(compressors)}\right) \cdot 3600 \cdot h \cdot d \cdot C_{\text{Electricity}}\right]$$
$$+ \text{Benefits}_{\text{Filtration}} + \text{Benefits}_{\text{Centrif}} + \text{Benefits}_{\text{Coagulation}} + \text{Benefits}_{\text{FBR}} + \text{Benefits}_{\text{CSTR}} \tag{98}$$

2 Nomenclature

2.1 Sets

$$\text{i} \in \{P, N\}$$
$$j \in \{\text{filtermedia}\}$$

k $\in \{TS, C, K\}$

$$a' \in \{\text{CH}_4, \text{CO}_2, \text{NH}_3,\text{H}_2\text{S},\text{O}_2 and/or N_2\}$$

$$a \in \{\text{H}_2\text{O}, \text{CH}_4, \text{CO}_2, \text{NH}_3,\text{H}_2\text{S},\text{O}_2\text{and/orN}_2\}$$

$$d \in \{C, Norg, Nam, P, K, H2O and/or Rest\}$$

$$e \in \{C\text{H}_4, \text{NH}_3\text{and/or}H_2S\}$$

$$h \in \{\text{CH}_4, \text{CO}_2,\text{O}_2,\text{N}_2\}, \{\text{O}_2,\text{N}_2\} \text{ or } \{\text{CO}_2,\text{O}_2,\text{N}_2\}$$

2.2 Parameters

$$\text{A}_{\text{specific}}$$

: specific clarifier area (m2 / (ton·day))

$$\text{A(i)}$$

: Antoine A coefficient for vapor pressure of component i

$$\text{B(i)}$$

: Antoine B coefficient for vapor pressure of component i

$$C(i)$$

: Antoine C coefficient for vapor pressure of component i

$$Cp_{sat}$$

: specific heat capacity of flue gas.

$$d$$

: work days per year

$$d_p$$

: particle diameter (m)

$$k$$

: kinetic constant (s-1)

$$IAP_{\mathrm{eq}}$$

: equilibrium ion activity product

$$h$$

: work hours per day HRT_{unit}: hydraulic retention time of unit (s)

$$MW_{\mathrm{component}}$$

: molecular weight of component (kg/kmol)

$$MeP_{\mathrm{ratio}}$$

: metal/phosphorus molar ratio in coagulation process $Price_{\mathrm{component}}$: price of the component

(€/kg)

$$g$$

: gravity acceleration (m2/s)

$$k$$

: polytropic coefficient (1.4)

$$\kappa_{\text{agitator}}$$

: agitators specific power consumed (HP / 1000 USgallon) φ_{j}: precipitation agent j per total solids mass ratio

$$\eta_c$$

: Compressor's efficiency (0.85)

$$\eta_s$$

: Isentropic efficiency (0.9)

$$\eta_{\mathrm{i}}^{\mathrm{j}}$$

: i component separation yield using in the process the element j

$$\mathrm{P}_{atm}$$

: atmospheric pressure (1 bar)

$$\mathrm{T}_{atm}$$

: atmospheric temperature (25 ºC)

$$\mathrm{R}$$

: ideal gas constant (8.314 J/mol·K)

$$\mathrm{Cp}_{H_2O}$$

: specific heat capa

2.3 Variables

$\mathrm{a}_{\mathrm{technology}}$: parameter which takes the value 0 when

$$F_{design}^{\mathrm{technology}}$$

is 0 and 1 if

$$F_{design}^{\mathrm{technology}}$$

is not equal to 0

$$\alpha_{mf}$$

: parameter dependent of the phases number in the FBR

$$Ar_l$$

: Arquimedes number for liquid

$$\mathrm{A}_{unit}$$

: area of unit (m2)

$$\mathrm{Benefits}_{\mathrm{technology}}$$

: benefits or losses obtained with technology

$$\mathrm{C-N}$$

: carbon to nitrogen molar ratio

$$C_{eq}$$

: equilibrium concentration (kmol/m3)

$$C_0$$

: initial concentration (kmol/m3)

$$\mathrm{Cost}_{unit}$$

: cost of unit

$$C_{component}^{unit}$$

: concentration of component in the unit inlet stream (kgcomponent/kgtotal)

$$\mathrm{ChemC}_{\mathrm{technology}}$$

: cost of chemicals for technology

$$D_{\mathrm{unit}}$$

: diameter of unit

$$e_{\mathrm{unit}}$$

: thickness of unit

$$Ec_j\,(T)$$

: equilibrium constant of component j at temperature T.

$$F_{component}^{unit}$$

:mass flow of component in the unit inlet stream (kg/s) $F_{\max}^{unit}$: maximum mass inlet flow admitted by a single unit (kg/s) $F_{\mathrm{design}}^{unit}$: mass inlet flow used in the design of unit (kg/s)

$$\mathrm{FC}_{\mathrm{technology}}$$

: fixed cost of technology

$$F_{total}^{\mathrm{recovered}}$$

: recovered matter total mass flow (kg/s)

$$F_{(unit,unit1)}$$

: mass flow from stream from unit to unit1 (kg/s)

$$fc_{(J,unit,unit1)}$$

: mass flow of component J from unit to unit1 (kg/s)

$$H_{b,(unit,unit1)}$$

: enthalpy of the stream at the state b from the stream from unit to unit1 (kJ/kg).

$$\mathrm{H}_{\mathrm{steam(isoentropy)}}$$

: enthalpy of the stream at the if the expansion is isentropic (kJ/kg).

$$l_{j-i}$$

: molar fraction of component j in the liquid phase of equilibrium system i.

$$K_{index}$$

: Potassium index of fertilizer.

$$L_{\mathrm{unit}}$$

: length of unit

$$N_{am}$$

: nitrogen contained in ammonia.

$$N_{org}$$

: nitrogen contained in organic matter. $\mathrm{n}_{\mathrm{unit}}$: number of units used in the process

$$n_{(unit,unit1)}$$

: total mol flow from stream from unit to unit1 (kmol/s).

$$N_{index}$$

: nitrogen index of fertilizer.

$$P_{in/compressor}$$

: inlet pressure to compressor (bar).

$$P_{out/compressor}$$

: outlet pressure of compressor (bar).

$$P_j * (T)$$

: saturation pressure of pure component j at temperature T (bar).

$$P_v$$

: vapor pressure (bar)

$$P_{index}$$

: phosphorous index of fertilizer.

$$p_{turbi}$$

: inlet pressure to body i in the turbine (bar)

$$P_{unit}$$

: power of unit

$$Q_{(unit)}$$

: heat exchanged in unit (kW).

$$R_{C-N/k}$$

: carbon to nitrogen ratio in k.

$$R_{C-N/fertilizer}$$

: carbon to nitrogen ratio in fertilizer.

$$R_{V/F-i}$$

: rate of evaporation in equilibrium system i.

$$Rest$$

: rest of the elements contained in the biomass.

$$\mathrm{Re}_{\mathrm{lmf}}$$

: Reynolds number for liquid in minimum fluidization conditions

$$s_{b(unit,unit1)}$$

: entropy the stream at the state b for the stream from unit to uni1 kJ/kg.K

$$T_{turbimin}$$

: saturating temperature at exit of body i (ºC)

$$T_{(unit,unit1)}$$

: temperature of the stream from unit to unit 1 (ºC)

$$T_{bubble/i}$$

: bubble point temperature of equilibrium system i (ºC).

$$T_{m/i}$$

: average temperature in equilibrium system i (ºC).

$$T_{in/compressor}$$

: inlet temperature to compressor (ºC).

$$T_{out/compressor}$$

: outlet temperature of compressor (ºC).

$$t$$

: time (s)

$$u_t$$

: terminal velocity (m/s)

$$u_0$$

: fluid velocity (m/s)

$$u_{\mathrm{mf}}$$

: minimum fluidization velocity (m/s)

$$v_{j-i}$$

: molar fraction of component j in the vapor phase of equilibrium system i.

$$V_{\mathrm{biogas,k}}$$

: biogas volume produced per unit of volatile solids (VS) (m3biogás/kgVS/k) associated to k.

$\mathrm{V_{unit}}$: volume of unit

$$W_{\mathrm{unit}}$$

: weight of unit

$$w_{DM/k}$$

: dry mass fraction of k (kgDM/k/kg). : dry mass fraction of volatile solids out of the dry mass of k (kgVS/k/kgDM/k). : dry mass fraction of C in k (kgC/k/kgDM/k). : dry mass fraction of Nam in k (kgNam/k/kgDM/k). : dry mass fraction of Norg in k (kgNorg/k/kgDM/k). : dry mass fraction of P in k (kgP/k/kgDM/k). : dry mass fraction of K in k (kgK/k/kgDM/k). : dry mass fraction of the rest of the elements contained in k (kgK/k/kgMS/k).

$$W_{(unit)}$$

: power produced or consumed in unit (kW).

$$x_{a/biogas}$$

: mass fraction of component a in the biogas

$$y^j$$

: binary variable to evaluate the element j

$$y_{biogas}$$

: specific saturated moisture of biogas

$$Y_{a'biogas-dry}$$

: molar fraction of component a in the dry biogas.

$$\Delta H_{reaction(Bioreactor)}$$

: Heat of the anaerobic digestion's reaction (kW).

$$\Delta H_{comb}\,(k)$$

: heat of combustion of component k (kW).

$$\Delta H_{comb}\,(e)$$

: heat of combustion of component e (kW).

$$\Delta H_{comb}\,(Digestate - dry)$$

: heat of combustion of dry digestate (kW)

$$\Delta H_f(h)_{T(unit,unit1)}$$

: heat of formation of component h at temperature T(unit,unit1) (kW)

$$Z$$

: objective function $\rho_{component}$: component density (kg/m3)

$$\mu_{component}$$

: viscosity of component (kg/(m·s))

3 Supplementary Material

Eq1

$$\mathrm{MW_{dry-biogas}} = \sum_{\mathrm{a'}} \mathrm{Y_{a'/biogas-dry}} \mathrm{MW_{a'}} \tag{99}$$

Eq2

$$\begin{aligned}
&0.7 \leq YCH4 \leq 0.5\\
&0.3 \leq YCO2 \leq 0.5\\
&0.02 \leq YN2 \leq 0.06\\
&0.005 \leq YO2 \leq 0.16\\
&YH2S \leq 0.002\\
&9 \cdot 10^{-5} \leq YNH3 \leq 1 \cdot 10^{-4}
\end{aligned} \tag{100}$$

Eq3

$$\mathrm{y_{biogas}}{=}\frac{\mathrm{MW_{H_2O}}}{\mathrm{MW_{biogas-dry}}}\frac{\mathrm{Pv(T)}}{\mathrm{P-Pv(T)}} \tag{101}$$

Eq4

$$\mathrm{F_{biogas}}{=}\rho_{\mathrm{biogas}}\sum_{\mathrm{waste}}\mathrm{w'_{SV/waste\_i}w_{MS/waste\_i}F_{waste\_i}{\cdot}V_{biogas/waste\_i}} \tag{102}$$

Eq5

$$\mathrm{fc(H_2O)_{biogas}}{=}\mathrm{y_{biogas}}{\cdot}\sum_{\mathrm{a'}}\mathrm{fc(a')_{biogas}} \tag{103}$$

Eq6

$$\mathrm{fc_{(a',Bioreactor,Compres1)}/MW_{a'}}{=}\frac{\mathrm{Y_{a'/biogas-dry}}}{\mathrm{MW_{biogas-dry}}}\left(\mathrm{F_{(Bioreactor,Compres1)}}{-}\mathrm{fc_{(H_2O,Bioreactor,Compres1)}}\right) \tag{104}$$

Eq7

$$\mathrm{MW_{biogas}}\sum_{\mathrm{a}}\frac{\mathrm{x_{a/biogas}}}{\mathrm{MW_a}}{=}\sum_{\mathrm{a}}\mathrm{x_{a/biogas}} \tag{105}$$

Eq8

$$\begin{aligned}
&0.20 \leq Vbiogas, waste \leq 0.50 \\
&0.10 \leq wMS, Waste \leq 0.20 \\
&0.50 \leq wVSB, Waste \leq 0.80
\end{aligned} \tag{106}$$

Eq9

$$\mathrm{w'}_{\mathrm{C/k}} = \mathrm{R}_{\mathrm{C-N/k}} \left(\mathrm{w'}_{\mathrm{Norg/k}} + \mathrm{w'}_{\mathrm{Nam/k}}\right) \tag{107}$$

Eq10

$$\begin{aligned}
&6 \leq R_{C-N/Waste} \leq 20 \\
&0.005 \leq wN/Waste \leq 0.047 \\
&0.005 \leq wNorg/Waste \leq 0.036 \\
&0.008 \leq wP/Waste \leq 0.013 \\
&0.033 \leq wK/Waste \leq 0.1
\end{aligned} \tag{108}$$

Eq11

$$\mathrm{w}_{\mathrm{C/Waste}} + \mathrm{w}_{\mathrm{Norg/Waste}} + \mathrm{w}_{\mathrm{Nam/Waste}} + \mathrm{w}_{\mathrm{P/Waste}} + \mathrm{w}_{\mathrm{K/Waste}} + \mathrm{w}_{\mathrm{Rest/Waste}} = 1 \tag{109}$$

Eq12

$$\begin{aligned}
&\mathrm{fc(C)}_{\mathrm{digestate}} = \mathrm{w'}_{\mathrm{C/Waste}} \cdot \mathrm{w}_{\mathrm{MS/Waste}} \cdot \mathrm{F}_{\mathrm{Waste}} - \\
&\mathrm{fc(CH_4)} \tfrac{\mathrm{MW_C}}{\mathrm{MW_{CH_4}}} - \mathrm{fc(CO_2)} \tfrac{\mathrm{MW_C}}{\mathrm{MW_{CO_2}}}
\end{aligned} \tag{110}$$

Eq13

$$\mathrm{fc(Norg)}_{\mathrm{digestate}} = \mathrm{w'}_{\mathrm{No/Waste}} \cdot \mathrm{w}_{\mathrm{MS/Waste}} \cdot \mathrm{F}_{\mathrm{Waste}} - \mathrm{fc(N_2)} \frac{\mathrm{MW_N}}{\mathrm{MW_{N_2}}} \tag{111}$$

Eq14

$$\mathrm{fc(N)_{digestate}} = \mathrm{w'}_{\mathrm{N/Waste}} \cdot \mathrm{w_{MS/Waste}} \cdot \mathrm{F_{Waste}} - \mathrm{fc(NH_3)} \frac{\mathrm{MW_N}}{\mathrm{MW_{NH_3}}} \tag{112}$$

Eq15

$$\mathrm{fc(P)_{digestate}} = \mathrm{w'}_{\mathrm{P/Waste}} \cdot \mathrm{w_{MS/Waste}} \cdot \mathrm{F_{Waste}} \tag{113}$$

Eq16

$$\mathrm{fc(K)_{digestate}} = \mathrm{w'}_{\mathrm{K/Waste}} \cdot \mathrm{w_{MS/Waste}} \cdot \mathrm{F_{Waste}} \tag{114}$$

Eq17

$$\begin{aligned} &\mathrm{fc(Rest)_{digestate}} = \mathrm{w'}_{\mathrm{rest/Waste}} \cdot \mathrm{w_{MS/Waste}} \cdot \mathrm{F_{Waste}} + \\ &\mathrm{fc(CH_4)_{biogas}} \tfrac{4 \cdot \mathrm{MW_H}}{\mathrm{MW_{CH_4}}} - \mathrm{fc(CO_2)_{biogas}} \tfrac{2 \cdot \mathrm{MW_O}}{\mathrm{MW_{CO_2}}} - \mathrm{fc(NH_3)_{biogas}} \tfrac{3 \cdot \mathrm{MW_H}}{\mathrm{MW_{NH_3}}} - \\ &\mathrm{fc(H_2S)_{biogas}} - \mathrm{fc(O_2)_{biogas}} \end{aligned} \tag{115}$$

Eq18

$$\mathrm{fc(H_2O)_{digestate}} = (1 - \mathrm{w_{MS/Waste}}) \cdot \mathrm{F_{Waste}} - \mathrm{fc(H_2O)_{biogas}} \tag{116}$$

Eq19

$$\begin{aligned} &\mathrm{Q_{digestor}} = \Delta \mathrm{H_{reaction}} - \mathrm{Fcp}(\mathrm{T_{digestor}} - \mathrm{T_{in}}) \\ &\Delta \mathrm{H_{reaction}} = \sum_{\mathrm{prod}} \Delta \mathrm{H_{combus}} - \sum_{\mathrm{reactants}} \Delta \mathrm{H_{combus}} \end{aligned} \tag{117}$$

Eq20

$$T_{out/compresor} = T_{in/compressor} + T_{in/compressor} \left(\left(\frac{P_{out/compressor}}{P_{in/compressor}} \right)^{\frac{z-1}{z}} - 1 \right) \frac{1}{\eta_c} \tag{118}$$

Eq21

$$\mathrm{W}_{(Compressor)} = (F) \cdot \frac{R \cdot z \cdot \left(\mathrm{T}_{in/compressor}\right)}{((\mathrm{MW}) \cdot (z-1))} \frac{1}{\eta_c} \left(\left(\frac{P_{out/compressor}}{P_{in/compressor}} \right)^{\frac{z-1}{z}} - 1 \right) \tag{119}$$

Eq22

$$Q_{(Furnace)} = \sum_h \Delta H_f(h)|_{T(Furnace,GasTurb)} \Big|_{(Furnace,GasTur)} - \sum_h \Delta H_f(h)|_{T(Compres2,Furnace)} \Big|_{(Compres2,Furnace)} - \tag{120}$$

Eq23

$$\eta_s = \frac{\mathrm{H}_{\mathrm{steam-(Turb1,HX5)}} - \mathrm{H}_{\mathrm{steam-(HX4,Turb1)}}}{\mathrm{H}_{steam-(isoentropy)} - \mathrm{H}_{\mathrm{steam-(HX4,Turb1)}}} \tag{121}$$

Eq24

$$H_{steam-(isoentropy)} = \mathrm{f}\left(\mathrm{p}_{\mathrm{(Turb1,HX5)}}, T^{*}{}_{\mathrm{(Turb1,HX5)}}\right) \tag{122}$$

Eq25

$$\mathrm{s}_{\mathrm{steam,(HX4,Turb1)}} = \mathrm{f}\left(\mathrm{p}_{\mathrm{(HX4,Turb1)}}, T_{\mathrm{(HX4,Turb1)}}\right) = \mathrm{f}(\mathrm{p}_{\mathrm{(Turb1,HX5)}}, T^{*}{}_{\mathrm{(Turb1,HX5)}}) \tag{123}$$

Eq26

$$\mathrm{p}_{\mathrm{turb2}} \cdot 760 = e^{\left(A(H_2O) - \frac{B(H_2O)}{(C(H_2O)\mathrm{T}_{\mathrm{turb1min}})}\right)} \tag{124}$$

Eq27

$$\mathrm{T}_{\mathrm{(Turb1,HX5)}} > \mathrm{T}_{\mathrm{turb1min}} \tag{125}$$

Eq28

$$\mathrm{W}_{(\mathrm{Turbine1})} = \mathrm{fc}_{(H2O,\mathrm{HX4},\mathrm{Turb1})} \cdot \left(H_{\mathrm{steam},(\mathrm{HX4},\mathrm{Turb1})} - \mathrm{H}_{\mathrm{steam}(\mathrm{Turb1},\mathrm{HX5})}\right) \tag{126}$$

Eq29

$$\mathrm{Q}_{(\mathrm{HX5})} = \mathrm{fc}_{(H2O,\mathrm{Turb1},\mathrm{HX5})} \cdot \left(H_{steam,(HX5,Turb2)} - \mathrm{H}_{steam,(Turb1,HX5)}\right) \tag{127}$$

Eq30

$$\mathrm{Q}_{(\mathrm{HX5})} = \quad -F(\mathrm{Spl1},\mathrm{HX5}) \cdot \int_{\mathrm{T}_{(Spl1,\mathrm{HX5})}}^{\mathrm{T}_{(\mathrm{HX5},Mix1)}} Cp_{salt} dT \tag{128}$$

Eq31

$$\mathrm{Q}_{(\mathrm{HX5})} = \quad -F(HX4,\mathrm{HX5}) \cdot \int_{\mathrm{T}_{(HX4,\mathrm{HX5})}}^{\mathrm{T}_{(\mathrm{HX5},Mix1)}} Cp_{salt} dT \tag{129}$$

Eq32

$$\mathrm{fc}_{(H2O,\mathrm{HX5},\mathrm{Turb2})} = \mathrm{fc}_{(H2O,\mathrm{Turb2},\mathrm{HX7})} + \mathrm{fc}_{(H2O,\mathrm{Turb2},\mathrm{Turb3})} \tag{130}$$

Eq33

$$\mathrm{Q}_{(\mathrm{HX6})} = \mathrm{fc}_{(H2O,\mathrm{Turb3},\mathrm{HX6})} \cdot \left(\mathrm{H}_{liq,(HX6,HX7)} - \mathrm{H}_{steam(Turb3,HX6)}\right) \tag{131}$$

Eq34

$$\mathrm{T}_{(\mathrm{HX7},\mathrm{HX8})} \leq \mathrm{T}_{\mathrm{turb2min}} \tag{132}$$