Supplementary Information:
Phosphorus in Ontario's economic sectors: mapping flows and assessing recovery and recycling potential

Université Laval team[a,*], McGill University team[b], University of Waterloo team[c]

[a] *Université Laval*
[b] *McGill University*
[c] *University of Waterloo*

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1. Distribution of CAFOs size in regions of the the Great Lakes area

The size distribution of Ontario's CAFOs is not reported by public databases, but the number of animals is aggreggated at Census Division level REF, REF. As an approximation, the size distribution of CAFOs of other regions in the vicinity of the Great Lakes area reporting the size of CAFOs is calculated and extrapolated to the province of Ontario. The size distribution of CAFOs is determined for the US states of Ohio REF, Pennsylvania REF, Indiana REF, Michigan REF, and Wisconsin REF. The distribution of CAFOs size has been fit to a truncated normal distribution, since the possible size of livestock facilities is bounded between 300 animal units for being considered as an intensive livestock production facility REF, and 10,000 animal units in order to remove extra-large CAFOs that are outliers in the size distribution, avoiding excessive long tails distorting the distributions. The probability density distribution of a truncated normal distribution is shown in Eq. 1, where μ, σ^2, a, and b denote the mean, variance, and lower and upper bounds respectively, while Θ denotes the probability density distribution of the standard normal distribution, as shown in Eq. 2, and Φ denotes the cumulative distribution function of the standard normal distribution, as

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*Corresponding author
Email address:* `edgar.martin-hernandez.1@ulaval.ca` (Université Laval team)

shown in Eq. 3. Figure 1 represent the distribution of CAFOs using the kernel density estimation (KDE) REF, and Table 1 collect the truncated normal distribution fitting parameters for each evaluated region.

$$f\left(x\right) = \frac{1}{\sigma} \frac{\Theta\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)} \tag{1}$$

$$\Theta\left(\xi\right) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\xi^2} \tag{2}$$

$$\Phi\left(\xi\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-\xi^2}{2}} d\xi \tag{3}$$

Table 1: Truncated normal distribution fitting parameters for the distribution of cAFOs sizes in regions of the Great Lakes area.

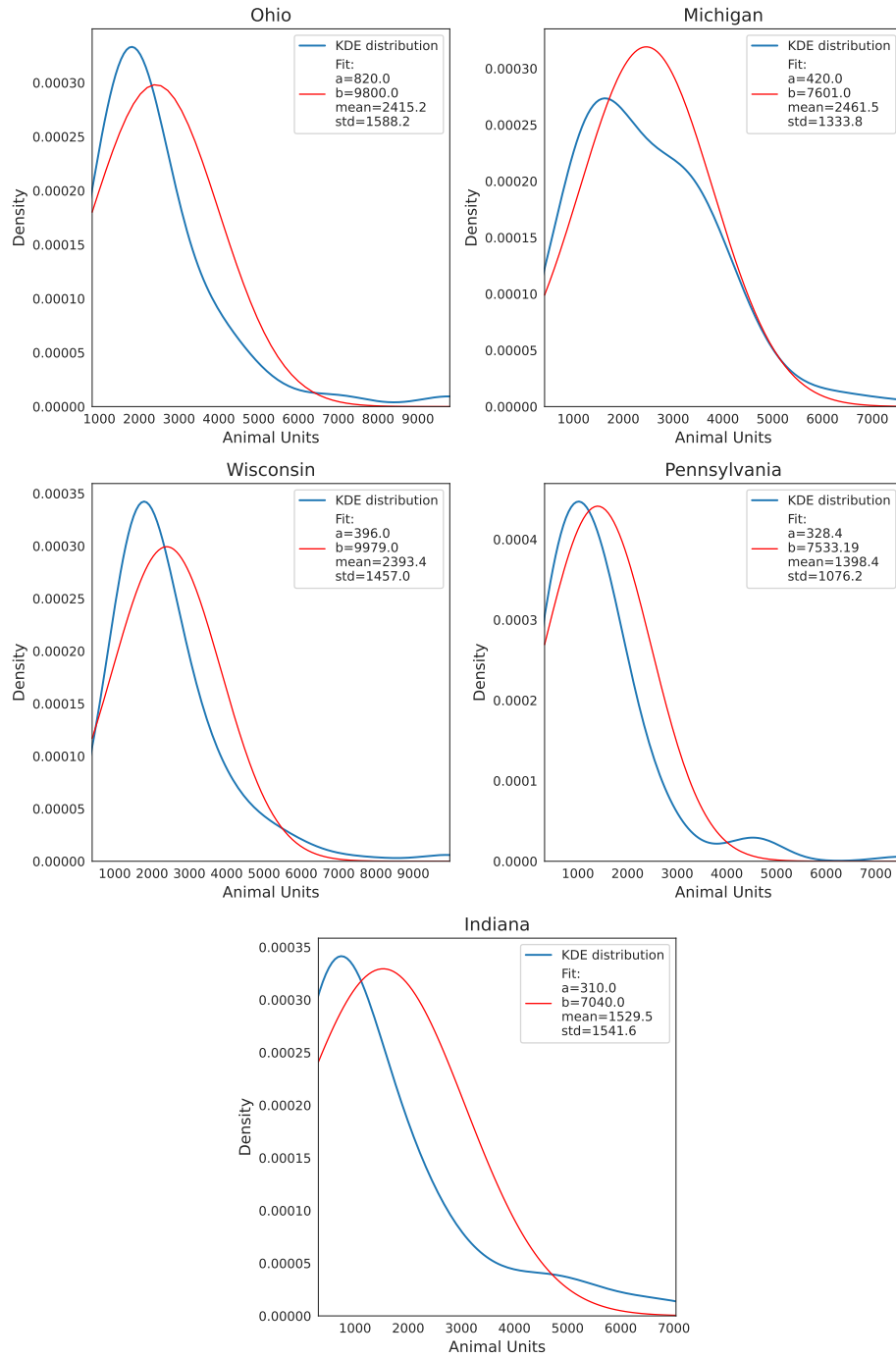| Parameters	Ohio	Michigan	Wisconsin	Pennsylvania	Indiana
mean	2415.245	2461.528	2393.431	1398.358	1529.522
std	1588.247	1333.813	1457.033	1076.217	1541.599
a	820.000	420.000	396.000	328.000	310.000
b	9800.000	7601.000	9979.000	7533.000	7040.000

Figure 1: Distribution of CAFOs size in regions of the the Great Lakes area.

27 References