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Image Denoising - Part 2 HW 3

Ex 2.1

$$U^{t+1} = U^{t} - \eta \nabla E_\theta^{FoE}(U_t, V)$$

$$E_\theta^{FoE}(U,V) = -\log p(V|U)\, p_\theta(U)$$

$$= \frac{1}{2\sigma^2}\|U-V\|^2 + \sum_{x\in\Omega}\sum_{i=1}^{N} \phi_i(k_i * U(x)) + \text{constants}$$

We're minimizing the energy on U, so we take the gradient respect to U

$$(\nabla E_\theta^{FoE})_U = \frac{1}{\sigma^2}(U^t - V) + \frac{d}{dU}\left[\sum_i \sum_x \phi_i(k_i * U(x)^t)\right]$$

$$= \frac{1}{\sigma^2}(U^t - V) + \sum_i \frac{d}{dU}\left[\sum_x \phi_i(k_i * U(x)^t)\right]$$

$$= \frac{1}{\sigma^2}(U^t - V) + \sum_i \sum_x \phi_i'(k_i * U(x)^t)\, \underbrace{\frac{d(k_i * U(x)^t)}{dU}}_{\text{flips the convolution}}$$

$$= \frac{1}{\sigma^2}(U^t - V) + \sum_i \sum_x \phi_i'(k_i * U^t(x))\, \bar{k}_i$$

$$= \frac{1}{\sigma^2}(U^t - V) + \sum_i \phi_i'(k_i * U^t) * \bar{k}_i$$

$$\Rightarrow U^{t+1} = U^t - \eta\left(\frac{U^t - V}{\sigma^2} + \sum_i \bar{k}_i * \phi'(k_i * U^t)\right)$$

Ex 2.2

$$L_\infty(\theta) = E\{\log p_\theta(u)\} = \int \log p_\theta(u)\, p(u)\, du = \lim_{m \to \infty} \frac{1}{m} \sum_{i=1}^{m} \log p_\theta(u_i)$$

Show that maximizing L_∞, minimizes:

$$KL(p(u) \,\|\, p_\theta(u)) = \int \log \frac{p(u)}{p_\theta(u)}\, p(u)\, du$$

$$= \int p(u) \log p(u) - \underbrace{\int \log p_\theta(u)\, p(u)\, du}_{L_\infty(\theta)}$$

Notice the - sign, if we maximize L_∞ then we minimize KL.