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Image Denoising - Homework 4

9. 1

$$E(P/\tilde{P}) = \frac{\|P-\tilde{P}\|^2}{2\sigma^2} - \log(\mathbb{P}(P)) \rightarrow \text{energy to minimize} \quad (1)$$

$$\mathbb{P}(P/\tilde{P}) = \frac{\mathbb{P}(\tilde{P}/P)\,\mathbb{P}(P)}{\mathbb{P}(\tilde{P})} \quad (2)$$

MAP is found by maximizing (2) respect to P, note that the term $\mathbb{P}(\tilde{P})$ is a constant for this maximization

$$P_{MAP} = \arg\max_P \mathbb{P}(\tilde{P}/P)\,\mathbb{P}(P)$$

$$P_{MAP} = \arg\max_P \mathcal{N}(\tilde{P}/P, \sigma^2)\,\mathbb{P}(P)$$

$$P_{MAP} = \arg\max_P \log(\mathcal{N}(\tilde{P}|P, \sigma^2)) + \log(\mathbb{P}(P))$$

$$\boxed{P_{MAP} = \arg\min_P \frac{\|P-\tilde{P}\|^2}{2\sigma^2} - \log(\mathbb{P}(P))} \quad \square$$

9. 2

If we assume that patches are independent, then the EPLL is the log-likelihood of the image U, because it's the product of all the independent patches probabilities. However, since there is overlap between patches, we know that they can't be independent.
At the end of the day, if patches are small, due to the variations on the image, the independency hypothesis is not completely wrong. Nevertheless, the most "rigorous" interpretation for the EPLL is as the Expected Log Likelihood of a random uniform patch in the image.