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Master's Thesis

Comparative Analysis of Techniques for Spatio-Temporal World Modeling

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I, the undersigned below, declare that this work has not previously been submitted to this or any other university and that it is, unless otherwise stated, entirely my own work.

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Abstract

$ABSTRACT

Acknowledgements

Thanks to $FRIENDS_AND_FAMILY

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1

Introduction

A robots world model is its internal representation of its environment that allows it to reason and make decisions. This ability to make decisions, and thus how well a robot performs in an environment, can be directly correlated to the quality of the model that a robot contains. Historically, these models have been static two or three dimensional representations, but within the past decade or two multiple methods have been developed to introduce an additional dimension to these maps, the dimension of time. The inclusion of times allows for a robot to make decisions about when it may want to accomplish a task, or perhaps know to avoid a certain area at a given time. In the simple case of an office, a robot may learn to avoid areas of high traffic around the cafeteria during lunch time or learn that a shortcut between to builds is open, but only during work hours. These simple examples illustrate the type of knowledge and efficiency that can be gleaned by introducing a temporal component to a robots world model. This new type of world model has come to be known as a spatio-temporal world model as it is a model of the world that contains spatial information, that of the physical environment, as well as temporal information, how the environment changes through time.

Automatically guided vehicles (AVGs), often present in the field of logistics, stand to benefit a great deal from these improvements in world modeling. Moving logistics from point A to point B is an extremely common task in a wide variety of domains spanning industrial, commercial, and even residential applications. AVGs, have most prominently been used in industrial settings for a few decades already, but have been relegated to a discrete and limited set of predictable tasks. This is especially true when logistics must be transported through a particularly dynamic or human environment.

Recent work into introducing a temporal component to this world models has already begun, and is showing great promise. A variety of methods have been introduced to allow for an AVG to observe and make predictions about it's environment through time. However, since this field is relatively knew, and with new advancements and approaches being introduce even within the past view years, it is becoming increasingly harder to evaluate or choose between the different spatio-temporal world modeling options. It is for this reason that a method, or set of criteria, be devised for comparing and contrasting the variety of solutions. The analysis of these methods will not only allow for others to choose the most fitting approach for a given environment, but also expose deficiencies in the current approaches and guide future research efforts. Improvements in this field will ultimately result in more flexible AVGs that can operate in a wider variety of environments and for longer periods of autonomy.

1.1 Challenges and Difficulties

Historically, world modeling techniques could be thought of as simply a mapping and path planning problem in either two dimensional or three dimensional space. These problems have been studied for decades and thus there already exist a handful of well known solutions, each with there own advantages and disadvantages. However, with the fairly recent introduction of the fourth dimension, time, into the equation there has been the introduction of a number of different methods.

The early and simplistic approaches to introducing temporal components into world models started as early as 2002 [1] but within the past decade or so there has been an uptick both in the number of different approaches and the complexity of the methods. [**?**] [TODO ADD OTHER CITATIONS] With this increase in complexity and variety of approaches combined with a lack of historical perspective and analysis, it can be daunting task to select the 'correct' or even a well fitting spatio-temporal world model for a new project. A few papers have introduced some simple means for comparison, but these have been limited to a simple discussion about the space and time complexity of an approach, or an internal reflection on variations of a proposed method.

1.2 Motivation

With so many different methods and no historical knowledge or method of comparison this paper aims to provide a template for comparing existing models that should be extensible to account for the inevitable release of future methods. To that aim the following goals shall be met:

- Summary of the major existing spatio-temporal world modeling techniques.
- Collection of performance measurement or other comparison techniques as defined by the papers themselves.
- Introduction of meta-information in order to better compare the existing world modeling techniques
- A quick and easy to use table for high level overview and comparison of techniques
- Example application of the aforementioned information to select a fitting technique for a real-world application
- Subsequent evaluation and discussion on the appropriateness of technique selected, especially with respect to the introduced meta-information

It is with this collection of existing comparison techniques, new meta-information, and a tangible example that other projects may be able to more easily evaluate and select the best fitting spatio-temporal world modeling technique for the project. Additionally, when new spatio-temporal world model techniques are introduced, it should be with relative easy that their information be integrated into this method for comparative analysis for future use.

1.3 Problem Formulation

In order to best choose between preexisting solutions for spatio-temporal world modeling and guide future development it is vital comparative criteria be established. This comparative analysis will set out to clarify and quantify these approaches. Although the comparative analysis will be general enough to be applicable for any project wanting to incorporate spatio-temporal wold modeling, it will ultimately be viewed through the lens of a specific real-world application with a focus in long-term planning. More details about the specific application will be discussed later however, it is important to note that viewing the various modeling methods through the lens of real-world application, is quite powerful.

Improvements in world modeling, specifically within the domain of spatio-temporal world modeling, have already yielded significant effects on the performance of robotic logistic systems. These improvements directly translate into decreases in travel time as well as increases in reliability that hinges on knowing what areas to avoid a what times. These improvements in turn create a much more powerful and scaleable logistics network with less downtime. This ultimately leads to more goods being delivered which saves both time and money, and in the case of hospitals, possibly lives.

Despite all of these benefits, and the numerous number of different approaches for spatio-temporal world modeling, there currently lacks any method for accurately comparing and contrasting the different approaches. It is with this in mind that this thesis will collect, describe, compare, and contrast these approaches. It will use the preexisting criteria already available when possible as some, but not necessarily all, of the work includes basic performance statistics. Furthermore, in work where these criteria are not mentioned explicitly, or are not otherwise available, an attempt to derive the information either via calculation or collected via simulation. Lastly, new criteria will be devised or otherwise assigned to allow account for information desired and not provided or other meta-information that would aid in comparing these methods.

Finally, an example study will be included which will attempt to select the best-suited approach for a real-world scenario, known as ROPOD. The real-world scenario in question involves moving logistics internally within a hospital. It consists of a central server in charge of planning and routing multiple robots. Additionally, it will be assumed planning will be done with OpenStreetMap and thus will use a graph-based approach. More details and specifics about this project will be discussed in a later section.

TODO: perhaps an introduction or allusion to the experiments to come?

2

State of the Art

Given the range of the different methods for implementing a spatio-temporal world model, the methods have been divided into groups. Most spatio-temporal world models are implemented on top of preexisting world modeling techniques and thus the majority of implementations are tied to a specific spatial representation. There are, however, exceptions to this with some models being built from the ground up effectively intertwining the spatial and temporal components. On the opposite end of this spectrum, there exists currently at least one method that can be used in combination with a multitude of different world models.

2.1 Map Dependent Models

2.1.1 Occupancy Grids

Occupancy grids were introduced in 1985 by Moravec and Elfes. [3] In simple two dimensional terms, they can be thought of as a grid placed over an environment. Each cell then represents the probability or belief that that that cell is either occupied or free. Free in the simplest case meaning that a robot would be able to traverse through the cell. This concept can of course be extended into the third dimension for a more complex world model.

Temporal Occupancy Grids

One of the earliest and most straight forward attempts to introduce a temporal component to a world model were by extending existing world models, occupancy grids in particular. This can be seen in Temporal Occupancy Grids: a Method for Classifying the Spatio-Temporal Properties of the Environment. [1] In this paper Arbuckle et al introduce the concept of temporal occupancy grids (TOGs). The authors noted that the key to these TOGs were that they "can differentiate between different patterns of occupancy, even when the absolute probability of occupancy is the same." That is to say, one could imagine a parking lot where it would be possible with TOGs to distinguish between cells that are parking spaces, cells that are pathways, and cells that are not for driving at all, such as a median. These TOGs additionally made it possible to detect where a door or elevator may be.

Temporal Occupancy Grids were accomplished by generated multiple occupancy grids in the same fashion as was traditionally done but each occupancy grid would represent, and be generated using samples from, multiple different time scales. With multiple occupancy grids spanning multiple time scales, the probability of a cell being occupied could be computed by a simple summation.

Hidden Markov Models

Hidden Markov Models (HHMs), are a type of Markov Chain that can be considered "a doubly embedded stochastic process with an underlying stochastic process that is not observable (it is hidden), but can only be observed through another set of stochastic processes that produce the sequence of observations." [12]. In more general terms, an HMM can be though of as having N number of states S, that are hidden, or otherwise not directly observable. Each state can have M observations made about properties of these states which may reflect indirectly, to varying degrees of certainty, the actual state. Furthermore, each one of these states has a given probability distribution of transitioning from one state to another. It is from this information that a Markov Model or Markov Chain can be constructed.

TODO: Add image?

In the specific case of occupancy grids, each cell can be thought of having two states, free, and occupied. It is not feasible to be able to directly observe every given cell at all times, and specifically at the time of path planning and thus there states can be thought of as hidden. However, through past observation and data collection, there is data know about a cell throughout time. Thus this temporal data can be thought of as the observational data and be used to make predictions about state transitions.

Early combinations of HMMs with occupancy grids differed from previous dynamic world modeling approaches as this approach "does not depend on dynamic object detection and high-level object models; it considers only the occupancy of the space at a lower level of abstraction"[11]. By relying on and collecting lower, more easily observable data, larger amounts of data could be collected and processed over greater periods of time. Since each cell was dependent only on previous observations of that cell throughout time, the increase in data quantity and the discrete nature of the predictions lent themselves would improve state predictions.

Meyer-Delius [11] also introduced the concept of online learning to this approach. Traditionally, offline learning had been used where a robots navigational system would hold copy of a world model produced a some time before operation. It has possible that from the time the map was generated to the time at which the robot was operating that objects in the robots environment may have changed. With the introduction of online learning, the robot would be able to observe these changes and factor them in to its navigational system. This was the first addition to attempt to avoid the static nature of the transition states of the HMM.

Further improvement to occupancy grids with HMMs came with the concept of modeling trajectories of objects in the environment[17]. This is an important improvement because the dynamic motion of objects in an environment, such as humans walking a hallway, could now be better modeled. This process was dubbed Input-Output HMM (IOHMM) due nature of how cells of the grid would communicate with one another. Each cell would not only look through it's own historical data but also be able to communicate with its neighbors. In effect, this could allow a cell in hallway to be able to predict occupancy based off of a nearby cell that is currently occupied.

2.1.2 Spatio-Temporal Hilbert Maps

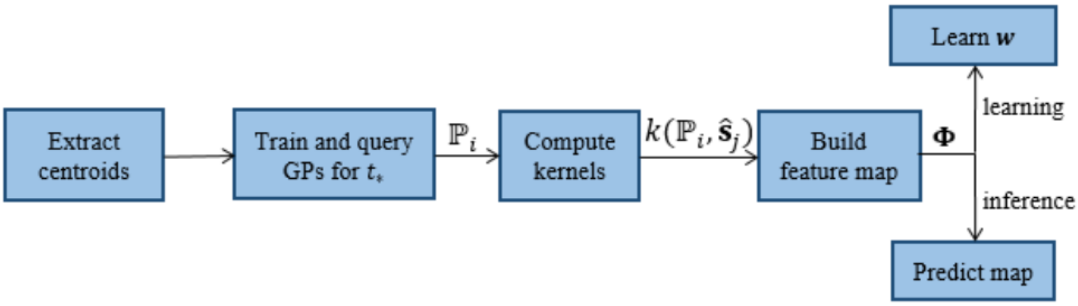

Figure 2.1: Spatio-temporal Hilbert map training process (GP - Gaussian Process) [15]

In contrast to the discrete nature of occupancy grids, Hilbert maps provide a continuous representation of an environment which allows for arbitrary world model resolution. They rely on "fast kernel approximations that project the data in a Hilbert space where a logistic regression classifier is learnt" A stochastic gradient optimization can then applied. This approach is similar to that of a Gaussian processes occupancy map but with a much lower computational cost computational cost computational cost computational cost. Having been introduced as recently as 2016, Hilbert maps, and the addition of a temporal component, are still a fairly new field of research but already some of the authors from the original paper have already begun to introduce a temporal component to this new form of world model. [13, 15]

In static Hilbert maps, the kernel can be thought of as the location of an obstacle or object. When introducing the temporal dimension, the centroid of a moving object is extracted from raw data over time. This data can the be used and trained on to create a model that can predict the direction and speed of an object at a given location at a given time. It is particularly well suited to short-term predictions such as car traffic on a road or at an intersection. [15] [16]

2.2 Map Independent Models

2.2.1 FreMen

Frequency Map Enhancement, or FreMEn as it became known, is a technique for spatio-temporal world modeling that can be used independent of mapping or world modeling technique. It was introduced by Tomáš Krajník, Jaime Pulido Fentanes, Grzegorz Cielniak, Christian Dondrup, an Tom Duckett in 2014. It's original goals focused on improving mapping for long-term scenarios. Additionally, it was notice that a large number of previous approaches had focused on mapping multiple static environments over time which worked well for environments that changed slowly, but were not necessarily well fit for highly dynamic environments. FreMEn was designed to counter these issues. [4] Although initially used with octomaps, three dimensional occupancy grids, it was later decoupled from this mapping technique allowing it to be an extremely diverse and flexible technique.

In it's original and most basic form, FreMEn assumes that an environment can be broken down into multiple independent components. It is then further assumed that these independent components will take one of two binary states. Examples of this include a door being open or shut, a room being occupied, or a cell in an occupancy grid being free or occupied. Each one of these states can not always be directly observed, and the tools e.g. sensors available to the robot my have noise and cannot be taken as one hundred percent ground truth. Thus, each one of these components has a certain probability assigned to it. This probability defines the likelihood of it being in a given state, e.g. a door open or closed. Finally, since these states can be observed multiple times over a given period of time, their probabilities can then be defined as functions dependent on time.

At the heart of FreMEn lies a well known mathematical tool commonly used for signal processing, the Fourier Transform. Since FreMEn focuses on long term observations of dynamic, often human, environments, it is assumed that harmonic patterns will develop over time. The Fourier Transform can then be applied to these long term observations to convert them into the spectral domain for storage. Furthermore, because the Fourier Transform is easily reversible, with the inverse Fourier Transform, one can easily convert between the stored observations in the spectral domain back to the time domain. This allows for predictions at any given time t. Not only is this extremely useful for future predictions, but this method can also be used to analyze the accuracy of the model by comparing previously observed data from the past to the models predictions. Using this historical accuracy one can then tune the order of the spectral model to obtain more accurate historical predictions with hopes of also having more accurate future predictions. More information on this process can be found in the original paper[4] and the follow up FreMEn paper [8].

Improvements and Additions

As ground breaking and as flexible as FreMEn is, due to the many assumptions made, it is not without its flaws. One major assumption made is that areas of observation can be observed not only frequently but periodically in the most strict sense. That is to say, it is not only important that a location be visited and observed, but that the observations follow a pattern of equally spaced and timed observations. This is due to the Fast Fourier Transform (FFT) technique that is used in the original papers. Thus, latter authors devised and implemented other methods of storing data and making predictions. This often involved phase shifting or modifying the amplitude of the observation as well as using a modified equation derived from the Fourier Transform instead of the standard FFT. More information can be found in the paper [14].

Another major limitation of FreMEn is its assumption that all observable behavior can be modeled with binary states. An attempt to solve this issue was to replace the Bernoulli distribution of FreMen with the specific hope of being able to better represent human patterns present in environment such as an office building or hospital. Specifically, they "extend the technique (FreMEn) by employing both Poisson processes as the counting model to replace the binary states of FreMEn and a new way of selecting the most prominent frequency components of the Fourier spectrum."[7] This approach, however, does not come without its own set of assumptions. Since human activity is assumed, the authors also assume behavior would be best grouped by the work week. For example, data sampled for a two months would be broken into 8 week sections. This works extremely well for patterns that are persistent on a weekly basis but perhaps not as well for seasonal changes for example. At the very least, one must apply some critical thinking to how data should be group depending on the desired application.

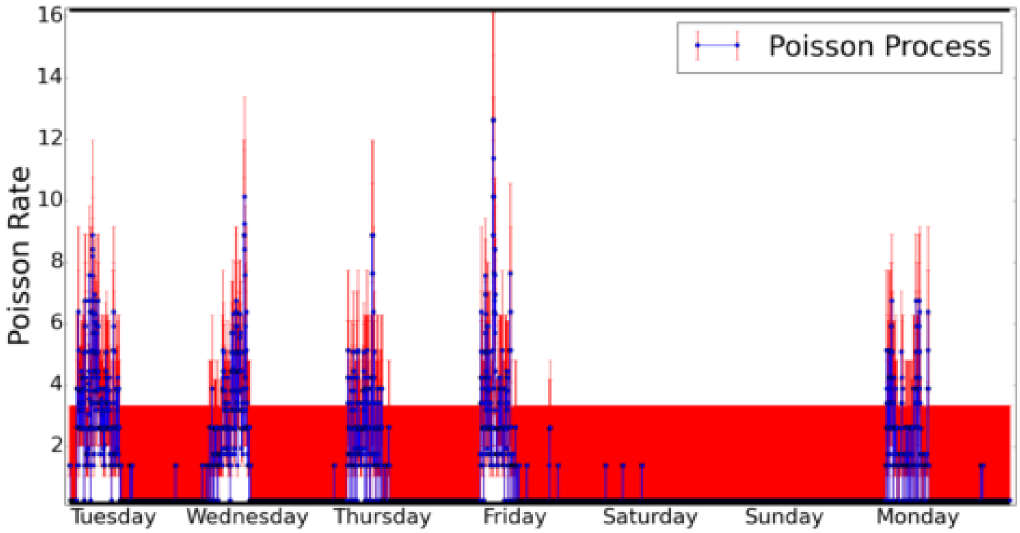

Figure 2.2: Lambda time series of a corridor using Poisson Process [7]

TODO: perhaps a section about the different FreMEn applications available e.g. FreMEn grids, FroctoMaps etc

2.3 Existing Methods for Evaluation or Comparison

2.3.1 Current Methods

2.3.2 Example 1

2.3.3 Example 2

2.3.4 Limitations and Areas of Oversight

3

Criteria for Comparison

How I am planning to compare/evaluate the various methods. TODO this is just some stubbed out notes about what I will cover here
item mostly inward looking. Very little outward looking analysis.

3.1 Criteria

- Space and time complexity (e.g. big O)
- Future Prediction Accuracy
- Historical Recreation Accuracy
- Efficiency of Stroage
- offline vs online learning
- feasibility of use with a multi-robot system
- meta-information
  - availability of work (e.g. libraries or source code)
  - implementation compelxity
  - suitable fields of application

3.2 How to use this criteria

TODO discussion and table goes here

4

ROPOD: A Case Study

4.1 What is ROPOD?

4.2 Evaluation Using Established Criteria

4.2.1 Proposed/Selected Method

4.2.2 Areas of Strength

4.2.3 Areas of Weakness

5

Experimental Setup

Having applied the theoretical knowledge derived in Section 3 to the ROPOD case study in Section 4 we have begun to narrow down the options for spatio-temporal world modeling in this particular case. However, a theoretical comparison will only suffice for so long. Given the focus ROPOD places on real world environments it is critical that some operational tests be preformed before method selection. Not only will these experiments serve as a guide for ROPOD, but they will also act as a template for the comparison of future spatio-temporal world modeling techniques.

5.1 Environmental Representation

Given the complexity and size of the target environment for ROPOD, a large hospital, it is necessary to pair down features of the building until only the core components remain. The three dynamic environmental components being target are doors, elevators, and carts that are often strewn about the hallways and surrounding rooms. Therefore, a model environment has been designed for the simulations to be run on that contains these three key components. The model environment that has been designed takes heavy influence from the actual environment but some notable have been made. The model has a decreased area to allow for faster model training and path planning. Additionally, extraneous rooms and hallways have been removed. A comparison between the actual hospital and the designed model can be seen below.

TODO add picture

5.2 Common Assumptions

In order to insure only the desired component is being tested at any given time a set of assumptions are made.

- All robot components are working correctly (no internal faults)
- Other than the object under test (e.g. doors), all other objects in the environment are static
- All information other than the objects under test are perfectly known
- Observations made/provided by the training data are assumed to be ground-truth

5.3 Commonalities in Approach

Although different components will be under test, each experiment will be run in a similar manor.

The experimental setup is as follows:

- Training data consisting of observations made every 10 minutes over a simulated month will be provided to the models
- 3 training sets will be made:
  - TODO further discuss generation of datasets
  - Highly consistent generation
  - Generation consistent with early periodicity of data obtained
  - Data with higher chance of abnormalities
- Using the same generation procedure, a set of three new months will be generated
- 4 times will be randomly selected each day of the month using a uniform distribution for which the models must generate maps

- An optimal path using the ground truth will be generated for each of these time slots using the selected models
- Additionally, maps will be generated using the ground truth, best case, and worst case, assumptions
- All of these paths will then be compared using the criteria described below.
- TODO what about a test with sparseness of data
- All experiments will be done on the same hardware
  - Desktop PC
  - i7-2600k 3.4GHz
  - 8GB DDR3
  - Ubuntu 14.04 Trusty Tahr

5.4 Comparison Criteria

In order to evaluate the intricacies of the selected models a wide variety of data points have been selected for comparison. A focus has been placed on collecting data relevant to scale-ability of the modeling technique given the eventual scope of the ROPOD project.

- Accuracy to Ground Truth
- Accuracy to Historical Recreations
- Planning Run-time
- Planning Memory Consumption

5.5 Doored Areas

5.5.1 Experimental Motivation

As is the case in many places of employment many areas of a building may not be accessible to the public, and by extension the robots, outside of work hours.

This could come in the form of a given hallway between two areas being locked after 17:00 as the day works go home. In another case, it could be as simple as someone preferring to having a door shut to a hallway during a loud or chaotic time of the day.

Regardless of the reason, it is certain that the states of doors are often both dynamic and periodic. In the ideal case, a robot, much like humans, would learn when certain doors are closed and be able to plan accordingly. Making an accurate prediction can save time, but making an inaccurate predication can also be costly. An in accurate prediction would force a robot to not only backtrack, but also recalculate that path required to get to a target. Additionally, it may not be possible to make deliveries at all times. In the worse case, a robot may even manage to get itself locked in an environment unable to return back to it's base and eventually run out of power requiring human intervention. For these reasons and many more, the door experiment is an excellent example of the benefits of spatio-temporal world modeling.

5.5.2 Experimental Details

In order to keep this test as straight forward as possible, only one door has been included. The door belongs to a room that where a package must be delivered Each model will be directly, or indirectly be tasked with predicting the state of this door. Figure 5.1 displays the starting and goal point of the desired path. A simple 50% confidence value will be used. That is to say, if a model predicts that the door will be 50% or more likely to be open will result in the robot attempting to deliver the package.

TODO should I include something like this here or elsewhere? It is clear that this 50% cutoff does not take into account the penalties of making a wrong prediction, but this experiment was designed to investigate the accuracy of the prediction. How the information of the prediction is handled afterwards is undoubtedly valuable, but is outside the scope of this current research.

TODO include description of how the simulation was generated.

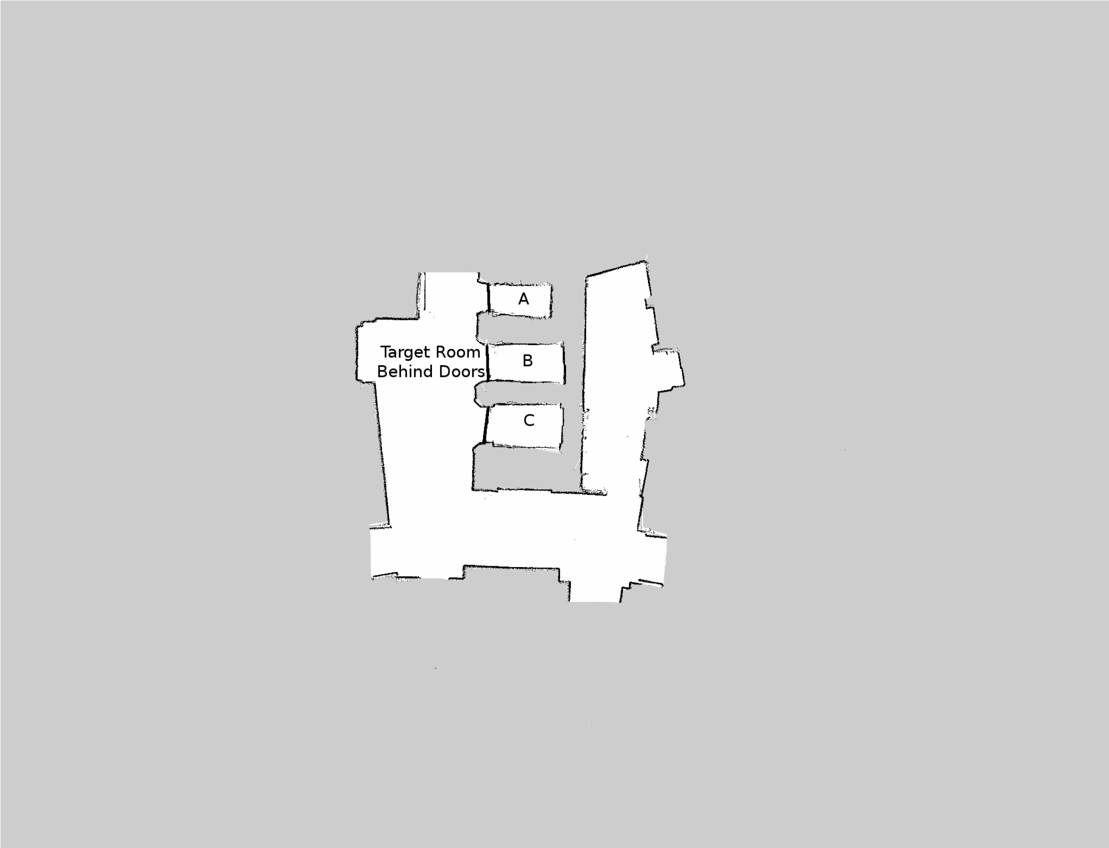

Figure 5.1: Multiple rooms behind doors in ward 24.

5.6 Congested Hallways

5.6.1 Experimental Motivation

5.6.2 Experimental Details

Figure 5.3 displays the starting and goal point of the desired path. In order to keep this test as straight forward as possible, only one door has been included. The door belongs to a hallway that acts like a shortcut between the two other hallways. Each model will be directly, or indirectly tasked with predicting the state of this door. A simple 50% confidence value will be used. That is to say, if a model predicts that the door will be 50% or more likely to be open that path will be taken.

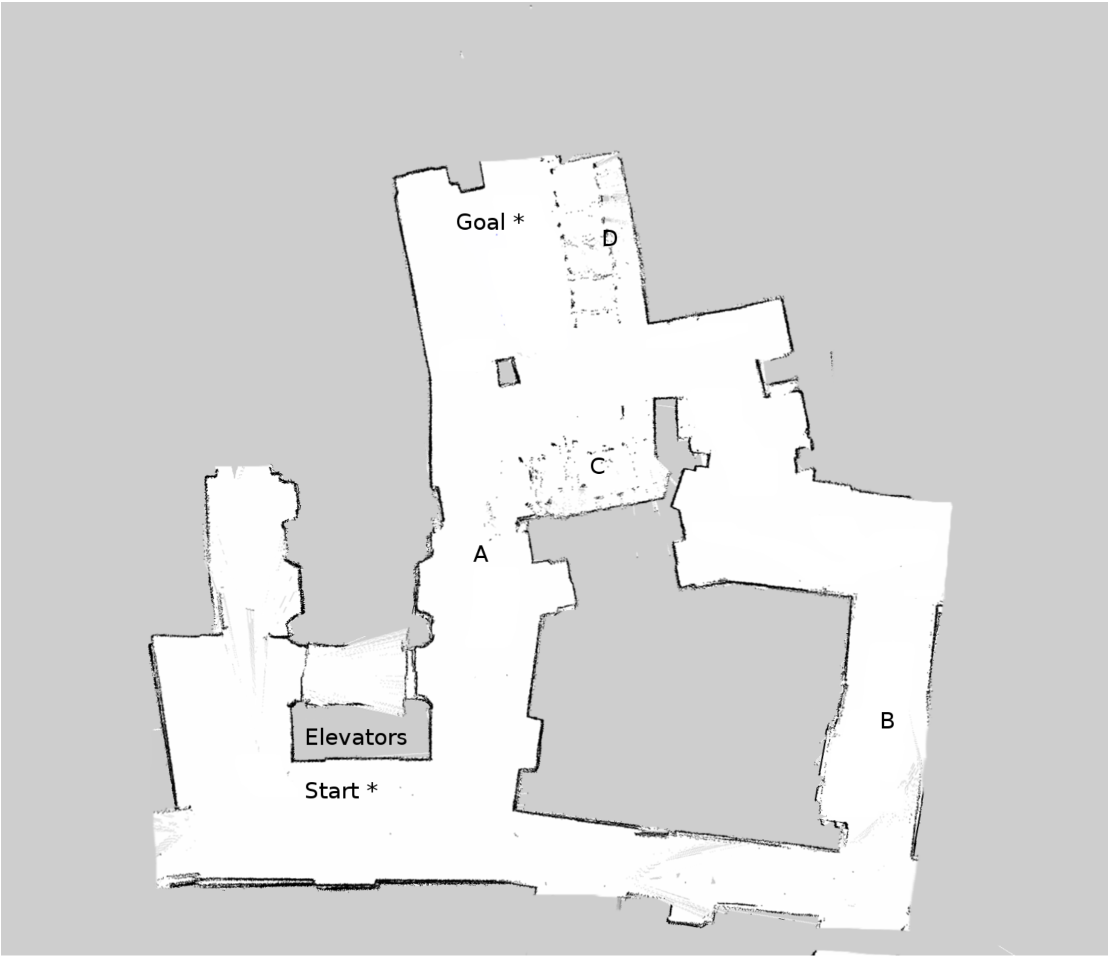

Figure 5.2: The path from the elevator to the storage area is often congested.

TODO include description of how the simulation was generated.

6

Experimental Results

Describe results and analyse them

6.1 Doored Areas

6.1.1 Door A

Pre a sample text blabhyy Pre a sample text blabhyy Pre a sample text blabhyy Pre a sample text blabhyy Pre a sample text blabhyy

Post a sample text blabhyy Post a sample text blabhyy Post a sample text blabhyy Post a sample text blabhyy Post a sample text blabhyy Post a sample text blabhyy

	Duckett	Gaussian	FreMEn	HyperTime
Historical Accuracy	84.11%	77.80%	92.51%	97.66%
Prediction Accuracy	77.15%	79.43%	87.08%	88.87%
Computation Time (Milliseconds)	61	5	7	563
Memory Usage (KB)	31036	34968	34656	37192

Table 6.1: Door A Data Overview

	Duckett	Gaussian	FreMEn	HyperTime
Historical Accuracy	85.71%	59.81%	75.20%	71.55%
Prediction Accuracy	69.24%	62.17%	76.95%	75.78%
Computation Time (Milliseconds)	60	6	8	144
Memory Usage (KB)	31036	34644	34892	37692

Table 6.2: Door B Data Overview

	Duckett	Gaussian	FreMEn	HyperTime
Historical Accuracy	99.56%	62.75%	67.74%	67.74%
Prediction Accuracy	31.82%	14.58%	0.00%	00.00%
Computation Time (Milliseconds)	57	6	7	53
Memory Usage (KB)	31224	35004	34976	37208

Table 6.3: Door C Data Overview

6.1.2 Door B

6.1.3 Door C

6.2 Congested Hallways

6.3 Busy Elevators

7

Conclusions

7.1 Contributions

7.2 Lessons learned

7.3 Future work

A

Design Details

Your first appendix

B

Parameters

Your second chapter appendix

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