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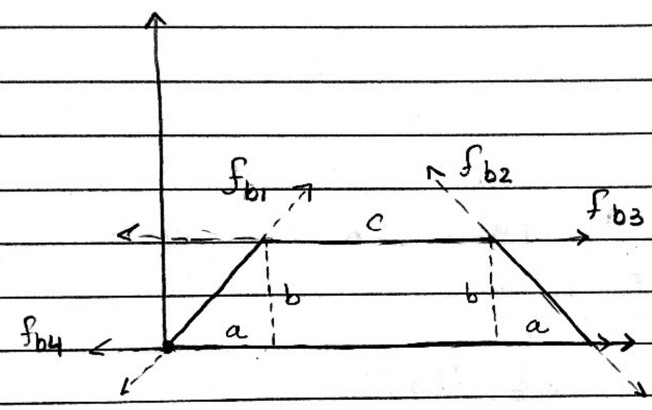
Exercise # 1

a) The hat is made up of several half planes. The hat can be broken up into two sets of primitives: base (b) and point (p)

Base

* Assume symmetry



we will need 4 half planes to define the base

$a, b, c \in \mathbb{R}^+$

$f_{b1})$ $\quad y = \frac{b}{a} x + 0 \longrightarrow f_{b1}(x,y) = ay - bx$

$$\boxed{H_{b1} : f_{b1}(x,y) \leq 0}$$

$f_{b2})$ $\quad y = -\frac{b}{a} x + q$ $\quad$ find q: $\quad$ @ $y = 0, x = 2a + c$

$$-\frac{b}{a} x + q = 0$$

$$-\frac{b}{a}(2a+c) + q = 0$$

$$q = \frac{b}{a}(2a+c)$$

$$y = -\frac{b}{a} x + \frac{b}{a}(2a+c)$$

$$f_{b2}(x,y) = ay + bx - b(2a+c)$$

$$\boxed{H_{b2} : f_{b2}(x,y) \leq 0}$$

$f_{b3})$ $y = b \rightarrow f_{b3}(x,y) = y - b$

$$\boxed{H_{b3} : f_{b3}(x,y) \leq 0}$$

$f_{b4})$ $y = 0 \rightarrow f_{b4}(x,y) = -y$ → we want everything above 0 to be ≤ 0

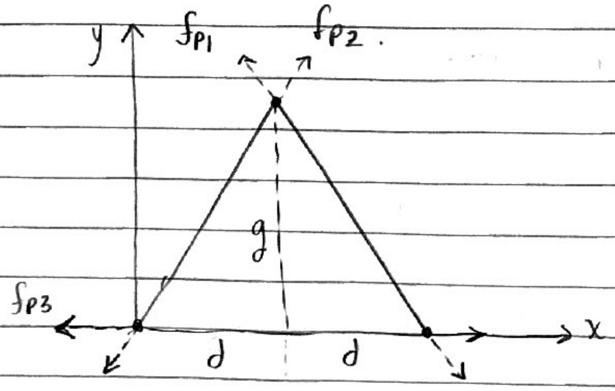
$$\boxed{H_{b4} : f_{b4}(x,y) \leq 0}$$

The base of the "hat" is defined by the primitive H_b

$$\boxed{H_b = \bigcap_{i=1}^{4} H_{bi}}$$

Point

Similar to the base of the hat, the point can be defined by half planes, 3 in this case.



$f_{p1})$ $y = \frac{g}{d}x + 0 \longrightarrow f_{p1}(x,y) = dy - gx$

$$\boxed{H_{p_1} : f_{p1}(x,y) \leq 0}$$

$f_{p2})$ $y = -\frac{g}{d}x + q \rightarrow @\, y=0,\ x = 2d$

$$q = \frac{g\,(2d)}{d} = 2g$$

$$y = -\frac{g}{d}x + 2g$$

$$f_{p2}(x,y) = y + \frac{g}{d}x - 2g$$

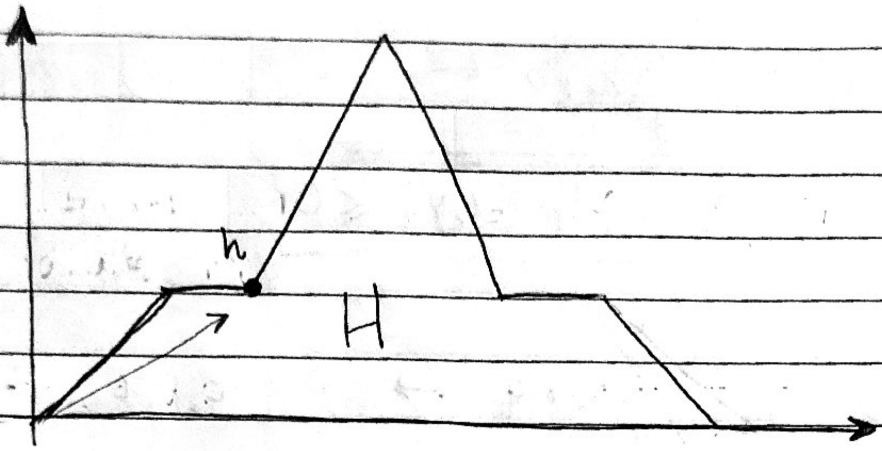
$$\boxed{H_{p2} : f_{p2}(x,y) \leq 0}$$

$f_{p3})$ $f_{p3} = f_{b4}$, $H_{p3} = H_{b4}$

alredy defined for the base

$$H_p = \bigcap_{i=1}^{3} H_{pi}$$

But wait! we have to move the point to the right place!



define Point $h = (x_h, y_h)$ where we have to translate the point to

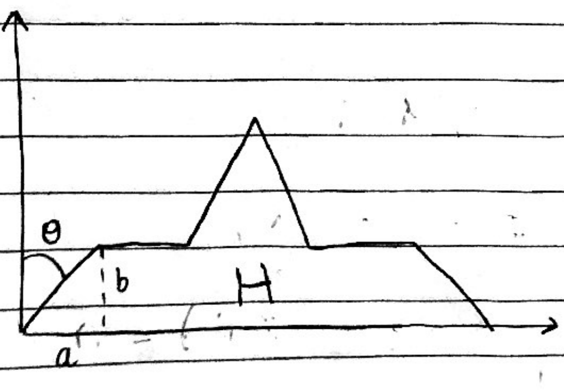
$$f'_{p_1} = f_{p_1}(x - x_p, y - y_p), \quad H'_{p_1} : f'_{p_1} \leq 0$$

$$f'_{p_2} = f_{p_2}(x - x_p, y - y_p), \quad H'_{p_2} : f'_{p_2} \leq 0$$

$$f'_{p_3} = f_{p_3}(x - x_p, y - y_p), \quad H'_{p_3} : f'_{p_3} \leq 0$$

$$H'_p = \bigcap_{i=1}^{3} H'_{pi}$$

Now, we have to rotate by θ

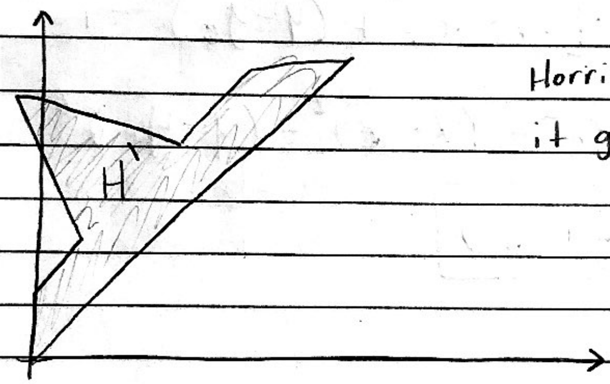


$$\theta = \operatorname{atan}\left(\frac{a}{b}\right)$$

$$H = H_b + H'_p$$

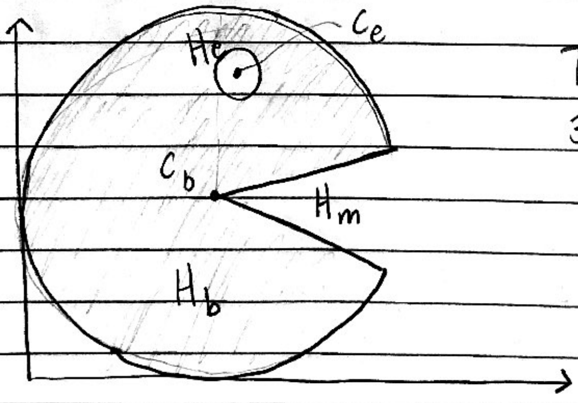
The hat can be represented as H rotated by θ to produce H'

$$\boxed{H_{hat} = R(\theta)H} \qquad \text{where } R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$



Horrible drawing but it gets the job done

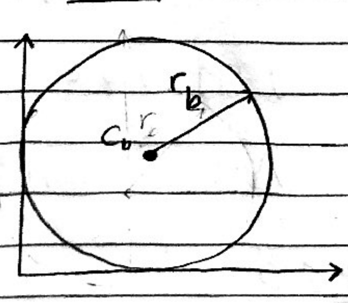
b)



There will be 3 super-primitive:

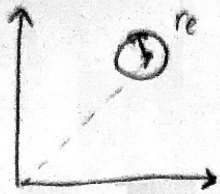
- Body (b)
- Mouth (m)
- Eye (e)

<u>Body</u> $C_b = (x_b, y_b)$



$$f_b = (x - x_b)^2 + (y - y_b)^2 - r_b^2$$

$$\boxed{H_b : f_b(x,y) \leq 0}$$



Eye

Centered at $C_e = (x_e, y_e)$ with radius r_e

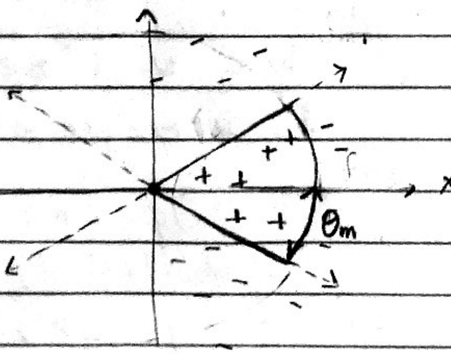
- $f_e(x,y) = (x-x_e)^2 + (y-y_e)^2 - r_e^2$

$$f_e(x,y) = -\left[(x-x_e)^2 + (y-y_e)^2 - r_e^2\right]$$

$$f_e(x,y) = r_e^2 - (x-x_e)^2 - (y-y_e)^2$$

$$\boxed{H_e: \ f_e(x,y) \leq 0}$$

Mouth



$$f_m(x,y) = -\left[(x-x_b)^2 + (y-y_b)^2 - r_b^2\right]$$

$$= r_b^2 - (x-x_b)^2 - (y-y_b)^2$$

$$f_m(x,y) = \{r_b^2 - (x-x_b)^2 - (y-y_b) \mid x \geq 0 \wedge -\theta_m \leq \arctan\left(\tfrac{y}{x}\right) \leq \theta_m\}$$

$$\boxed{H_m: \ f_m(x,y) \leq 0}$$

The entire Pacman is then defined by H_{pm} where

$$\boxed{H_{pm} = H_b \cap H_m \cap H_e}$$

c) Birthday Pacman

To move the hat to $V = (x_v, y_v)$ we have to translate all f_{bi} and f_{pi} for the hat to (x_v, y_v)

$$f_{bi}' = f_{bi}(x - x_v, y - y_v) \; ; \; H_{base}' = \bigcap_{i=1}^{4} f_{bi}'$$

$$f_{pi}'' = f_{pi}(x - x_v, y - y_v) \quad H_p'' = \bigcap_{i=1}^{3} f_{pi}''$$

and now we redefine

$$H_{hat}' = H_{base}' \cap H_p''$$

and finally

$$\boxed{H_{\substack{Birthday\\Pacman}} = H_{pm} \cap H_{hat}'}$$