ASEN 5519 - Algorithmic Motion Planning
Fall 2021

Homework 3

Assigned September 10; Due September 17

Exercise 1. Consider the following Euler representation of rotation:

$$R(\alpha, \beta, \gamma) = R_z(\gamma) R_y(\beta) R_z(\alpha).$$

(a) Determine matrix $R(\alpha, \beta, \gamma)$.

(b) Show that $R(\alpha, \beta, \gamma) = R(\alpha - \pi, -\beta, \gamma - \pi)$.

(c) Given a rotation matrix R', determine α, β, and γ in terms of elements of R'. (Hint: denote the element of R' in the i^{th} row and j^{th} column by R'_{ij}, and write your solutions in terms of these elements.)

Exercise 2. Consider the 3-link manipulator in Figure 1. The links $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ are identical.

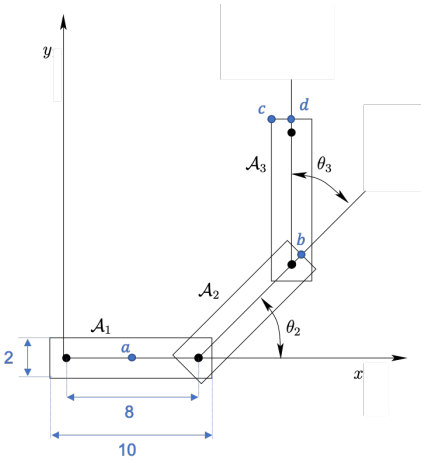

Figure 1: 3-link manipulator.

(a) For the configuration $(\theta_1, \theta_2, \theta_3) = (\pi/4, \pi/2, -\pi/6)$, determine the locations of points a, b, and c.

(b) Find the configuration(s) of the robot when point d is at $(0, 4)$.

Exercise 3. Express the configuration spaces of the following systems in terms of a Cartesian product of simpler spaces (such as $\mathbb{R}^n$, $\mathbb{S}^n$, etc.) and determine their dimensions. Justify your answer.

(a) Two trains on two train tracks.

(b) A spacecraft that can translate and rotate in 2D.

(c) Two mobile robots rotating and translating in the plane.

(d) Two translating and rotating planar mobile robots connected rigidly by a bar.

(e) A cylindrical rod that can translate and rotate in 3D. (Hint: if the rod is rotated about its central axis, it is assumed that the rod's position and orientation are not changed in any detectable way.)

(f) A spacecraft that can translate and rotate in 3D and is equipped with a 3-link robot arm (revolute joints only).

(g) The manipulator in Figure 2.

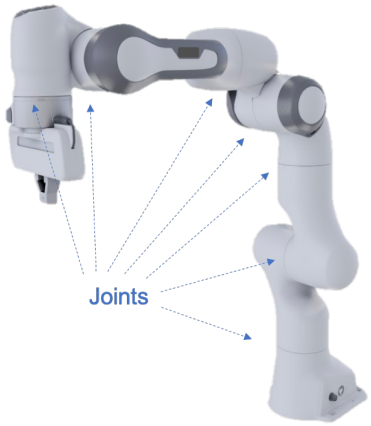

Figure 2: Robotic manipulator with revolute joints

Exercise 4. Consider workspace $W \subseteq \mathbb{R}^n$ with convex obstacles. Show that the C-space obstacles are also convex for a convex robot with transitional motion in W.