ASEN 5519 - Algorithmic Motion Planning
Fall 2021

Homework 8

Assigned October 15; Due October 23

Exercise 1. Extend your implementation of the GoalBiasRRT planner in **Homework 7** to plan for m disk robots translating and rotating in a workspace $W \subset \mathbb{R}^2$ using a coupled (centralized) approach. Every robot A^i for all $i \in \{1, \dots, m\}$ is a disk with radius R. The initial position of robot i (center of disk) is at x^i_{start}, and its goal is (center of disk) x^i_{goal}. Further assume that all m robots move at the same speed. That is, 1 step for robot A^i is equal to 1 step for robot A^j for all $i, j \in \{1, \dots, m\}$.

Your planner should take a step size r, a goal bias probability p_{goal}, maximum number of iterations n, workspace obstacles, workspace boundaries, m robots and their corresponding initial position x^i_{start} and goal position x^i_{goal}, and radius ϵ (centered at x^i_{goal}) for the termination condition at goal as input and return a valid path for every robot from x^i_{start} to x^i_{goal}, the size of the RRT tree, and the computation time.

- Consider a 2D rectangular workspace $W = [0, 16] \times [0, 16]$, with obstacle space $WO \subset W$. Let $WO = \bigcup_{i=1}^{6} \overline{WO}_i$, where each $\overline{WO}_i$ is a rectangle in W with the following vertices:

$\overline{WO}_1$: $v_1^1 = (4, 6)$, $v_1^2 = (6, 6)$, $v_1^3 = (6, 7)$, $v_1^4 = (4, 7)$
$\overline{WO}_2$: $v_2^1 = (4, 6)$, $v_2^2 = (5, 6)$, $v_2^3 = (5, 10)$, $v_2^4 = (4, 10)$
$\overline{WO}_3$: $v_3^1 = (4, 9)$, $v_3^2 = (6, 9)$, $v_3^3 = (6, 10)$, $v_3^4 = (4, 10)$
$\overline{WO}_4$: $v_4^1 = (10, 6)$, $v_4^2 = (12, 6)$, $v_4^3 = (12, 7)$, $v_4^4 = (10, 7)$
$\overline{WO}_5$: $v_5^1 = (11, 6)$, $v_5^2 = (12, 6)$, $v_5^3 = (12, 10)$, $v_5^4 = (11, 10)$
$\overline{WO}_6$: $v_6^1 = (10, 9)$, $v_6^2 = (12, 9)$, $v_6^3 = (12, 10)$, $v_6^4 = (10, 10)$

- Consider a set of $m = 6$ disk robots, where $R = 0.5$ with the following start and goal configurations:

$x^1_{\text{start}} = (2, 2)$, $x^1_{\text{goal}} = 14, 14)$
$x^2_{\text{start}} = (2, 14)$, $x^2_{\text{goal}} = (14, 2)$
$x^3_{\text{start}} = (8, 14)$, $x^3_{\text{goal}} = (8, 2)$
$x^4_{\text{start}} = (2, 8)$, $x^4_{\text{goal}} = (14, 8)$
$x^5_{\text{start}} = (11, 2)$, $x^5_{\text{goal}} = (5, 14)$
$x^6_{\text{start}} = (11, 14)$, $x^6_{\text{goal}} = (5, 2)$

(a) Answer the following questions:

i. What is the C-space of each individual disk robot?
ii. What is the C-space of the meta-agent (composed space)?
iii. Describe how the C-space grows with respect to the number of agents.

iv. Is the growth rate a concern for scaling to larger number of agents? Justify your ansewr.

(b) Solve planning problem for $m = 2$ using the first two robots ($i = \{1, 2\}$). Let $n = 7,500$, $r = 0.5$, $p_{\text{goal}} = 0.05$, and $\epsilon = 0.25$. Plot the solution in the workspace showing *both* agents reaching their respective goals.

(c) Use 100 runs to benchmark your implementation in two categories: computation time and size of the tree. Show your results using boxplots. Additionally save the data in whatever format you choose – it will be needed later.

(d) Repeat parts (b) and (c) for all $m = 3, 4, 5$, and 6 by incrementally adding agents from the list in logical order (i.e. $m = 3$ uses agents 1, 2, and 3 etc.). Show your results for every benchmark using boxplots. Be sure to save the data separately for your own use later on.

(e) Compute the average computation time and average size of tree over the 100 runs for each benchmark in (c) and (d). Use the 6 average computation times (1 for every value of m) and 6 average tree sizes to produce two plots:

i. an average computation time (y-axis) vs. number of agents m (x-axis), and

ii. an average size of tree (y-axis) vs. number of agent m (x-axis).

Comment on what these plots tell us about increasing the number of agents.

Exercise 2. Implement a priority based planning framework (decoupled/decentralized approach) using your GoalBiasRRT planner. Your program should take an ordered list of m start/goal configuration pairs, a fully defined GoalBiasRRT planner and a timing function as input and return a set of paths for m robots and computation time.

(a) Consider the multi-robot motion planning problem described in Exercise 1.

i. What is the C-space used for planning in the priority based planning framework?

ii. Describe how the C-space grows with respect to the number of agents.

iii. How does this differ from Exercise 1 part (a)?

(b) Solve the planning problem for $m = 2$ using the first two agents in the list. Let the underlying GoalBiasRRT planner be defined by $n = 7,500$, $r = 0.5$, $p_{\text{goal}} = 0.05$, and $\epsilon = 0.25$. Describe the timing function used and plot the solution in the workspace showing *both* agents reaching their respective goals.

(c) Use 100 runs to benchmark your implementation on computation time. Show your results using a boxplot. Additionally save the data in whatever format you choose – it will be needed later.

(d) Repeat parts (b) and (c) for all $m = 3, 4, 5$, and 6 by incrementally adding agents from the list in logical order (i.e. $m = 3$ uses agents 1, 2, and 3 etc.). Show your results for every benchmark using boxplots. Be sure to save the data separately for your own use later on.

(e) Compute the average computation time over the 100 runs for each benchmark. Use the 6 average computation times (1 for every value of m) to create a computation time vs. m plot. Comment on the differences between this scatter plot and the analogous one from Exercise 1 part (e).