

Dodgy Algebra

Example 1

$$3 > 2$$

$$3 \log\left(\frac{1}{2}\right) > 2 \log\left(\frac{1}{2}\right)$$

$$\log\left(\frac{1}{2}\right)^3 > \log\left(\frac{1}{2}\right)^2$$

$$\log\left(\frac{1}{8}\right) > \log\left(\frac{1}{4}\right)$$

$$\frac{1}{8} > \frac{1}{4}$$

Example 2

$$x > 3$$

$$3x > 9$$

$$3x - x^2 > 9 - x^2$$

$$x(3-x) > (3+x)(3-x)$$

$$x > 3+x$$

$$0 > 3$$

Example 3

$$x = 3$$

$$x^2 = 3x$$

$$x^2 - 9 = 3x - 9$$

$$(x+3)(x-3) = 3(x-3)$$

$$(x+3) = 3$$

$$x = 0$$

$$3 = 0$$

Example 4

$$x + y = 2$$

$$(x+y)(x-y) = 2(x-y)$$

$$x^2 - y^2 = 2x - 2y$$

$$x^2 - y^2 + (y^2 - 2x + 1) = 2x - 2y + (y^2 - 2x + 1)$$

$$x^2 - 2x + 1 = y^2 - 2y + 1$$

$$(x-1)^2 = (y-1)^2$$

$$x-1 = y-1$$

$$x = y$$

Example 5

$$\sin 70^\circ = \sin 110^\circ$$

$$70^\circ = 110^\circ$$

Example 6

$$3 - \frac{x+4}{x-2} = \frac{2x-10}{x-3}$$

$$\frac{3(x-2) - (x+4)}{x-2} = \frac{2x-10}{x-3}$$

$$\frac{2x-10}{x-2} = \frac{2x-10}{x-3}$$

$$2 = 3$$

EXERCISE

So where did it all go wrong?

SOLUTIONS

1)

We multiplied both sides of an inequality by $\log(1/2)$

But $\log(1/2)$ is negative so we should reverse the inequality sign.

2)

We divided both sides of an inequality by $(3-x)$

But $(3-x)$ is negative so we should reverse the inequality sign.

3)

We divided both sides of an equation by $(x-3)$

But $(x-3) = 0$

4)

If $(x-1)^2 = (y-1)^2$ then either $(x-1) = (y-1)$ or $(x-1) = -(y-1)$

5)

Look at the graph $y = \sin x$

6)

$$\frac{2x-10}{x-2} = \frac{2x-10}{x-3}$$

So:

$$(2x-10)(x-3)=(2x-10)(x-2) \text{ provided } x \neq 3 \text{ and } x \neq 2$$

Either:

$$(2x-10)=0 \text{ so } x=5$$

Or:

$$(x-3)=(x-2) \text{ which has no solution}$$