Paradoxes

Example 1

Achilles and the tortoise decide to run a race. Because Achilles can run 10 times as fast as the tortoise, the tortoise is given a head start of 100m. The tortoise starts at point A.

By the time Achilles reaches point A, the tortoise has moved on by 10m, to point B.

By the time Achilles reaches point B, the tortoise has moved on by 1m, to point C.

By the time Achilles reaches point C, the tortoise has moved on by 0.1m, to point D.

Each time Achilles reaches the point where the tortoise was, the tortoise has moved on.

So Achilles can never catch up with the tortoise.

Example 2

In the first half of the cricket season:

Fred faced 475 deliveries and scored 170 runs. John faced 717 deliveries and scored 250 runs.

So Fred had the better batting average.

In the second half of the cricket season:

Fred faced 725 deliveries and scored 185 runs. John faced 483 deliveries and scored 112 runs.

So Fred had the better batting average.

Over the whole season:

Fred faced 1200 deliveries and scored 355 runs. John faced 1200 deliveries and scored 362 runs.

So John had the better batting average.

Example 3

I claim that all ravens are black. You wish to investigate my claim, so you look at lots of ravens.

Every time you see a raven and it turns out to be black, your confidence in my claim increases.

All ravens are black, means the same as, all non-black things are non-ravens.

So every time you see a non-black thing and it turns out to be a non-raven, your confidence in my claim increases. Suppose you see a yellow thing and it turns out to be a banana. This should increase your confidence in my claim.

Example 4

There are two envelopes on the table. One envelope contains twice as much money as the other envelope. You can keep one of these envelopes and the money inside. But you are only allowed to look inside one envelope before making your decision. What should you do?

You look inside one envelope. It contains £100. So the other envelope must contain £50 or £200. If you choose to keep the other envelope, you could lose £50 but you are just as likely to gain £100.

So the best plan is to keep the other envelope. You come to this conclusion however much money is in the first envelope. So to save time, choose an envelope, don't bother to look inside it, and keep the other one.

Example 5

Teacher Alice sets her pupils a test and their mean score is 60%. Teacher Bill sets his pupils the same test and their mean score is 50%. Susan is in Alice's class. Susan scored 54%. If Susan is moved from Alice's class to Bill's class then the mean score for both classes would increase.

Example 6

A naughty girl did not complete her maths homework, so she is to be punished. She is allowed to make one statement. If the statement is true, she will have to clean the board. If the statement is false, she will have to pick-up litter. The girl makes the statement: "I shall have to pick-up litter". So what happens?

Example 7

I teach my maths class every Monday, Tuesday, Wednesday, Thursday and Friday. I tell them that they are going to have a test next week. But to add to their misery, they will not know, at the start of each day, if they are getting the test that day. Then one of my brighter students says:
"We can't have the test on Friday, because if we haven't had the test by then, we will know at the start of Friday that we are getting the test that day"
So the test has to take place on Monday, Tuesday, Wednesday or Thursday.
The student then argues that the test can't take place on Thursday or Wednesday or Tuesday or Monday.
The student therefore concludes that I can't give them an unexpected test.
Imagine their surprise when they get the test on Tuesday!