Recurrence Relations

Example 1

We have a sequence of numbers $u_1, u_2, u_3, \ldots$

$u_1 = 3$ and $u_{n+1} = 4u_n + 1$ This is a recurrence relation. If you know a number in this sequence then the recurrence relation will tell you how to calculate the next number in this sequence.

Now $u_1 = 3$

put $n = 1$ into $u_{n+1} = 4u_n + 1$ and we get $u_2 = 4u_1 + 1 = (4 \times 3) + 1 = 13$

put $n = 2$ into $u_{n+1} = 4u_n + 1$ and we get $u_3 = 4u_2 + 1 = (4 \times 13) + 1 = 53$

put $n = 3$ into $u_{n+1} = 4u_n + 1$ and we get $u_4 = 4u_3 + 1 = (4 \times 53) + 1 = 213$

etc

Example 2

We can define factorials using a recurrence relation:

$1! = 1$ $\qquad (n+1)! = (n+1)n!$

Now $1! = 1$

put $n = 1$ into $(n+1)! = (n+1)n!$ and we get $2! = (2)1! = 2 \times 1 = 2$

put $n = 2$ into $(n+1)! = (n+1)n!$ and we get $3! = (3)2! = 3 \times 2 = 6$

put $n = 3$ into $(n+1)! = (n+1)n!$ and we get $4! = (4)3! = 4 \times 6 = 24$

etc

Many problems give rise to recurrence relations as will will see in the next few sections.

Exercise

Write down the first 5 terms of the sequence

$u_1 = 2$ and $u_2 = 3$ $\qquad u_{n+2} = u_n \times u_{n+1}$

Solution

$u_1 = 2$

$u_2 = 3$

$u_3 = u_1 \times u_2 = 2 \times 3 = 6$

$u_4 = u_2 \times u_3 = 3 \times 6 = 18$

$u_5 = u_3 \times u_4 = 6 \times 18 = 108$