Arguments with If … Then

Example 1

      If today is Monday then I go to work today

      Today is Monday

      So I go to work today

This argument consists of three propositions:

Premise: If today is Monday then I go to work today

Premise: Today is Monday

Conclusion: I go to work today

We are not interested in whether these propositions are true or false. We are interested in whether this argument is valid or invalid. An argument is only valid if the conclusion must be true whenever both the premises are true.

p : Today is Monday q : I go to work today

p' : Today is not Monday q' : I do not go to work today

The above argument is valid and has the form:

      $p \Rightarrow q$

      p

    So q

Example 2

      If today is Monday then I go to work today

      I do not go to work today

      So today is not Monday

This argument is valid and has the form:

      $p \Rightarrow q$

      q'

    So p'

Example 3

      If today is Monday then I go to work today

      Today is not Monday

      So I do not go to work today

This argument is invalid and has the form:

      $p \Rightarrow q$

p'

So q'

Example 4

  If today is Monday then I go to work today

  I go to work today

  So today is Monday

This argument is invalid and has the form:

$p \Rightarrow q$

q

So p

See Exercise 1

I have stated which of the above arguments are valid and which are invalid. It is just obvious.

Well … we can do better than this. We can test arguments using truth tables.

Example

Test the argument:

$p \Rightarrow q$

q'

So p'

p	q	$p \Rightarrow q$	q'	p'
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	1	0	0

Remember, an argument is only valid if the conclusion must be true whenever both the premises are true.

Both the premises are true in line 1 and the conclusion is true.

So this argument is valid.

Example

$$p \Rightarrow q$$

$$p'$$

So q'

p	q	$p \Rightarrow q$	p'	q'
0	0	1	1	1
0	1	1	1	0
1	0	0	0	1
1	1	1	0	0

Both the premises are true in line 2 but the conclusion is false.
So this argument is invalid.

See Exercise 2

If an argument is invalid then we might be able to show this by finding a counter-example.

Consider the argument

$$p \Rightarrow q$$

$$p'$$

So q'

If we put:

  p dogs are birds q dogs are animals

then we get:

      If dogs are birds then dogs are animals

      Dogs are not birds

      So dogs are not animals

An argument is only valid if the conclusion must be true whenever both the premises are true. But here, both premises are true and the conclusion is false. So the argument must be invalid.

See Exercise 3

EXERCISE 1

  p : It is raining q : I take my umbrella

  p' : It is not raining q' : I do not take my umbrella

Which of the following arguments are valid?

1) If it is raining then I take my umbrella

   I do not take my umbrella

   So it is not raining

2) If it is raining then I take my umbrella

   I take my umbrella

   So it is raining

3) If it is raining then I take my umbrella

   It is not raining

   So I do not take my umbrella

4) If it is raining then I take my umbrella

   It is raining

   So I take my umbrella

EXERCISE 2

Test the following arguments using truth tables:

1.

$p \Rightarrow q$

p

So q

2.

$p \Rightarrow q$

q

So p

3.

$(p \wedge q)'$

p'

So q

4.

$p \vee q$

p'

So q

5.

$p \vee q$

$p \Rightarrow r$

$q \Rightarrow r$

So r

6.

$p \Rightarrow q$

$q \Rightarrow r$

So $p \Rightarrow r$

EXERCISE 3

Show that the negation of $p \Rightarrow q$ is $p \wedge q'$

SOLUTIONS 1

1) This argument has the form:

$p \Rightarrow q$

q'

So p'

Like example 2, this is valid.

2) This argument has the form:

$p \Rightarrow q$

q

So p

Like example 4, this is invalid.

3) This argument has the form:

$p \Rightarrow q$

p'

So q'

Like example 3, this is invalid.

4) This argument has the form:

$p \Rightarrow q$

p

So   *q*

Like example 1, this is valid.

SOLUTIONS 2

1.

p	*q*	*p*⇒*q*	*p*	*q*
0	0	1	0	0
0	1	1	0	1
1	0	0	1	0
1	1	1	1	1

Look at line 4. This argument is valid.

2.

p	*q*	*p*⇒*q*	*q*	*p*
0	0	1	0	0
0	1	1	1	0
1	0	0	0	1
1	1	1	1	1

Look at line 2. This argument is invalid.

3.

p	*q*	*p*∧*q*	(*p*∧*q*)′	*p*′	*q*
0	0	0	1	1	0
0	1	0	1	1	1
1	0	0	1	0	0
1	1	1	0	0	1

Look at line 1. This argument is invalid.

4.

p	*q*	*p*∨*q*	*p*′	*q*
0	0	0	1	0
0	1	1	1	1
1	0	1	0	0
1	1	1	0	1

Look at line 2. This argument is valid.

5.

p	q	r	$p \vee q$	$p \Rightarrow r$	$q \Rightarrow r$	r
0	0	0	0	1	1	0
0	0	1	0	1	1	1
0	1	0	1	1	0	0
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	0	1	1	1	1	1
1	1	0	1	0	0	0
1	1	1	1	1	1	1

Look at lines 4, 6 and 8. This argument is valid.

6.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1

Look at lines 1, 2, 4, 8. This argument is valid

SOLUTIONS 3

p	q	$p \Rightarrow q$	$(p \Rightarrow q)'$	q'	$p \wedge q'$
0	0	1	0	1	0
0	1	1	0	0	0
1	0	0	1	1	1
1	1	1	0	0	0

The columns for $(p \Rightarrow q)'$ and $p \wedge q'$ are the same.