

## Propositions

A proposition is a statement that is either true or false.

Newton was born in England                      this is a true proposition

Fermat was born in Poland                      this is a false proposition

Please shut the door                      this is not a proposition

### Example

$a$  : Today is Monday                       $b$  : I go to work today

We can combine two propositions using AND

$a \wedge b$  : Today is Monday and I go to work today

Today is Monday and I go to work today is true if:

Today is Monday is true

and I go to work today is true

Otherwise it is false.

So  $a \wedge b$  is true if  $a$  is true and  $b$  is true, otherwise it is false.

We can set this out in a truth table where 0 means false and 1 means true:

$a$	$b$	$a \wedge b$
0	0	0
0	1	0
1	0	0
1	1	1

We can combine two propositions using OR

$a \vee b$  : Today is Monday or I go to work today (or both)

Today is Monday or I go to work today is true if:

Today is Monday is true

or I go to work today is true

or both are true

Otherwise it is false.

So  $a \vee b$  is true if  $a$  is true or  $b$  is true (or both), otherwise it is false.

Truth table:

$a$	$b$	$a \vee b$
0	0	0
0	1	1
1	0	1
1	1	1

Notes:

In ordinary English we use OR in two different ways.

I will buy you a beer or a lemonade (but not both)

You will get the job if you can sing or dance (or both)

$a \vee b$  always means  $a$  or  $b$  or both

Negation

The negation of  $a$  is  $a'$

$a$  : Today is Monday

$a'$  : Today is not Monday

Truth table:

$a$	$a'$
0	1
1	0

Examples

Fill in the truth table for  $(a \vee b)'$

$a$	$b$	$a \vee b$	$(a \vee b)'$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Fill in the truth table for  $a \vee (b \wedge c)$

$a$	$b$	$c$	$b \wedge c$	$a \vee (b \wedge c)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

see EXERCISE 1

Look at Exercise 1, questions (3) and (4)

You should have found that the columns for  $(a \wedge b) \vee (a \wedge c)$  and  $a \wedge (b \vee c)$  are the same.

We say  $(a \wedge b) \vee (a \wedge c) = a \wedge (b \vee c)$

See EXERCISE 2

Note:

$a \wedge 1 = 1$  if  $a = 1$  and  $a \wedge 1 = 0$  if  $a = 0$  so  $a \wedge 1 = a$

$a \vee 1 = 1$  if  $a = 1$  and  $a \vee 1 = 1$  if  $a = 0$  so  $a \vee 1 = 1$

Similarly:

$a \wedge 0 = 0$  and  $a \vee 0 = a$

Use truth tables to prove the following rules: (no need to do them all)

$$(a')' = a$$

$$a \wedge a = a$$

$$a \wedge a' = 0$$

$$a \wedge b = b \wedge a$$

$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

$$(a \wedge b)' = a' \vee b'$$

$$a \wedge (a \vee b) = a$$

$$a \vee a = a$$

$$a \vee a' = 1$$

$$a \vee b = b \vee a$$

$$(a \vee b) \vee c = a \vee (b \vee c)$$

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

$$(a \vee b)' = a' \wedge b'$$

$$a \vee (a \wedge b) = a$$

It's like a new type of algebra!

## EXERCISE 1

1) fill in the truth table:

$a$	$b$	$b'$	$a \wedge b'$
0	0		
0	1		
1	0		
1	1		

2) fill in the truth table:

$a$	$b$	$a'$	$a' \vee b$
0	0		
0	1		
1	0		

1	1		
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3) fill in the truth table:

$a$	$b$	$c$	$a \wedge b$	$a \wedge c$	$(a \wedge b) \vee (a \wedge c)$
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

4) fill in the truth table:

$a$	$b$	$c$	$b \vee c$	$a \wedge (b \vee c)$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

## EXERCISE 2

Use truth tables to show that:

1)  $(a \wedge b)' = a' \vee b'$

2)  $(a \vee b)' = a' \wedge b'$

## SOLUTIONS 1

1)

$a$	$b$	$b'$	$a \wedge b'$
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

2)

$a$	$b$	$a'$	$a' \vee b$
0	0	1	1

0	1	1	1
1	0	0	0
1	1	0	1

3)

$a$	$b$	$c$	$a \wedge b$	$a \wedge c$	$(a \wedge b) \vee (a \wedge c)$
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

4)

$a$	$b$	$c$	$b \vee c$	$a \wedge (b \vee c)$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

## SOLUTIONS 2

1)

$a$	$b$	$a \wedge b$	$(a \wedge b)'$	$a'$	$b'$	$a' \vee b'$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

2)

$a$	$b$	$a \vee b$	$(a \vee b)'$	$a'$	$b'$	$a' \wedge b'$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0