Coins if you know about the binomial distribution ...

If you spin a coin 20 times, what is the probability you get 10 heads and 10 tails?

Let X be the number of heads in 20 spins

X has a binomial distribution.

So:

$$p(X=10)=(20C10)\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{10}=\frac{(20)!}{(10)!(10)!2^{20}}$$

In general:

If you spin a coin $2n$ times, what is the probability you get n heads and n tails?

Let X be the number of heads in $2n$ spins

X has a binomial distribution.

So:

$$p(X=n)=(2nCn)\left(\frac{1}{2}\right)^{n}\left(\frac{1}{2}\right)^{n}=\frac{(2n)!}{(n)!(n)!2^{2n}}$$

What happens if n is large?

Stirling discovered a remarkable approximation for $m!$ when m is large:

It is:

$m!\approx(m^m)(e^{-m})\sqrt{(2\pi m)}$ what are e and π doing? See Footnote

Using Stirling's approximation show that:

If n is large then:

$$p(X=n)\approx\frac{1}{\sqrt{(n\pi)}}$$

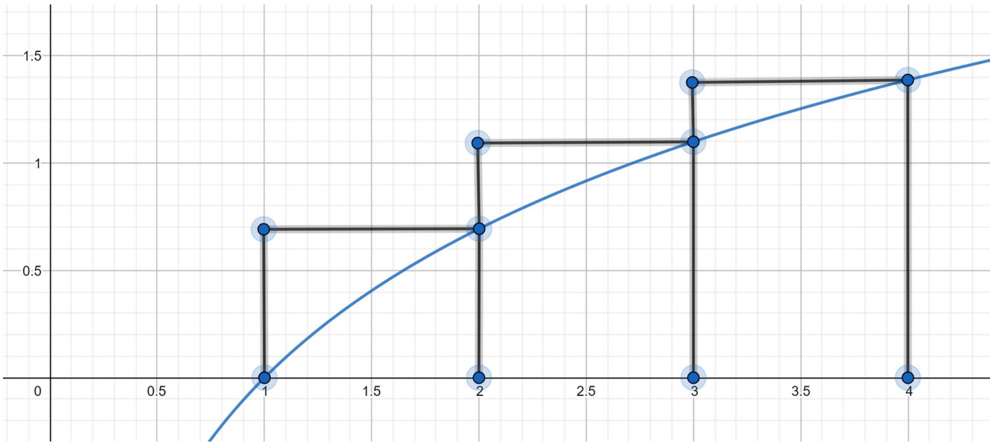
Footnote:

$m!=1\times2\times3\times4\times...\times m$

So:

$\ln(m!)=\ln 1+\ln 2+\ln 3+\ln 4+\ldots+lnm$

Here is the graph $y=lnx$



The diagram shows blocks between $x=1$ and $x=4$

The area of the blocks is: $\ln 2+\ln 3+\ln 4$

The area under the graph is: $\int_1^4 (lnx)\,dx=[xlnx-x]_1^4=4\ln 4-4+1$

So:

$\ln 2+\ln 3+\ln 4\approx 4\ln 4-4+1$

In general:

The diagram shows blocks between $x=1$ and $x=m$

The area of the blocks is: $\ln 2+\ln 3+\ln 4+\ldots+lnm$

The area under the graph is: $\int_1^m (lnx)\,dx=[xlnx-x]_1^m=mlnm-m+1$

So:

$\ln 2+\ln 3+\ln 4+\ldots+lnm\approx mlnm-m+1$

So:

$\ln(m!)\approx mlnm-m+1$

Show that:

$m!\approx(m^m)(e^{-m})e$

This is not as good as Stirling’s approximation but it’s a start.