Proof by Contradiction

To prove a theorem is true, we assume it is false and then show that this cannot be the case as it leads to a contradiction.

Theorem 1

$4x-2y=1$ has no solution where x and y are integers.

Proof

Assume we have found a solution where x and y are integers:

LHS is even. RHS is odd.

Contradiction.

Theorem 2

No prime (except 3) is one less than a square.

Proof

Assume the prime p is one less than the square n^2

So:

$p=n^2-1=(n-1)(n+1)$

So:

the prime p can be written as the product of two integers.

Contradiction. Unless $(n-1)=1$ So $n=2$ so $p=3$

Theorem 3

$\log 5$ is irrational

Proof

Assume $\log 5$ is rational

So:

$\log 5=\frac{p}{q}$ where p and q are positive integers.

So:

$5=10^{p/q}$ So $5^q=10^p$

Now:

LHS is odd. RHS is even.

Contradiction

Theorem 4

$\sqrt{2}$ is irrational

Proof

Assume $\sqrt{2}$ is rational

So:

$\sqrt{2}=\frac{p}{q}$ where p and q are integers

We can say p and q are not both multiples of 2 because if they had both been multiples of 2 then we would have cancelled them down before we started.

Now:

$2q^2=p^2$

So:

p^2 is a multiple of 2. So p is a multiple of 2. Let $p=2r$

So:

$2q^2=4r^2$ So $q^2=2r^2$ So q^2 is a multiple of 2. So q is a multiple of 2.

So:

p and q are not both multiples of 2 but p is a multiple of 2 and q is a multiple of 2.

Contradiction.