Proof using Selections

Example 1

We have 10 books and we want to select 4 of them.

Method 1:

We select 4 books and keep them.

The number of ways this can be done is: $(10C4)$

Method 2:

We select 6 books and throw them away (keeping the remaining 4 books)

The number of ways this can be done is: $(10C6)$

So $(10C4)=(10C6)$

In general:

$(mCk)=(mC(m-k))$

Example 2

We have 12 books and we want to select 5 of them.

Method 1:

The number of ways this can be done is: $(12C5)$

Method 2:

One of the books is Alice's Adventures In Wonderland.

There are $(11C4)$ selections which include Alice's Adventures In Wonderland.

There are $(11C5)$ selections which exclude Alice's Adventures In Wonderland.

So there are $(11C4)+(11C5)$ selections in total.

So $(11C4)+(11C5)=(12C5)$

In general:

$(mCk)+(mC(k+1))=((m+1)C(k+1))$

Example 3

We have 4 books and we want to select some (or none) of them.

Method 1:

There are $(4C0)$ selections of 0 books.

There are $(4C1)$ selections of 1 book.

There are $(4C2)$ selections of 2 books.

There are $(4C3)$ selections of 3 books.

There are $(4C4)$ selections of 4 books.

So there are $(4C0)+(4C1)+(4C2)+(4C3)+(4C4)$ selections in total.

Method 2:

For each book, there are 2 choices. Either the book is selected or the book is not selected.

So there are 2^4 selections in total.

So $(4C0)+(4C1)+(4C2)+(4C3)+(4C4)=2^4$

In general:

$$(nC0)+(nC1)+(nC2)+...+(nCn)=2^n$$