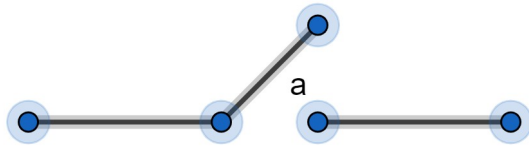


## Switching Circuits

### Example 1

Here is a switch called  $a$ . It can be closed or open.



If  $a$  is closed then electric current can flow. We say  $a=1$

If  $a$  is open then electric current cannot flow. We say  $a=0$

### Example 2

Here are two switches in series:



$a.b$  denotes switches  $a$  and  $b$  in series (this is not multiplication!)

If  $a$  is closed and  $b$  is closed then electric current can flow.

So if  $a=1$  and  $b=1$  then  $a.b=1$  So  $1.1=1$

If  $a$  is open or  $b$  is open (or both) then electric current cannot flow.

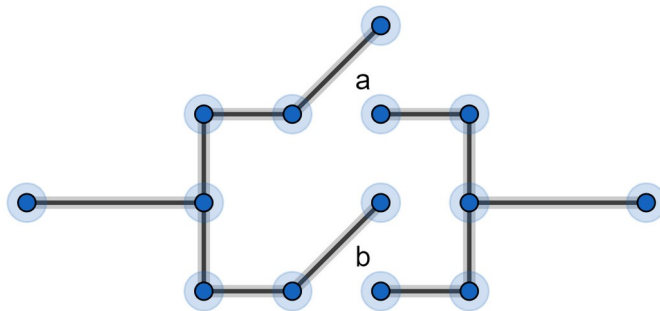
So if  $a=0$  or  $b=0$  (or both) then  $a.b=0$  So  $0.1=0$   $1.0=0$   $0.0=0$

We can set this out in a table:

$a$	$b$	$a.b$
0	0	0
0	1	0
1	0	0
1	1	1

### Example 3

Here are two switches in parallel:



$a+b$  denotes switches  $a$  and  $b$  in parallel (this is not addition!)

$a$  is closed or  $b$  is closed (or both) then electric current can flow.

So if  $a=1$  or  $b=1$  (or both) then  $a+b=1$  So  $1+0=1$   $0+1=1$   $1+1=1$

If  $a$  is open and  $b$  is open then electric current cannot flow.

So if  $a=0$  and  $b=0$  then  $a+b=0$  So  $0+0=0$

We can set this out in a table:

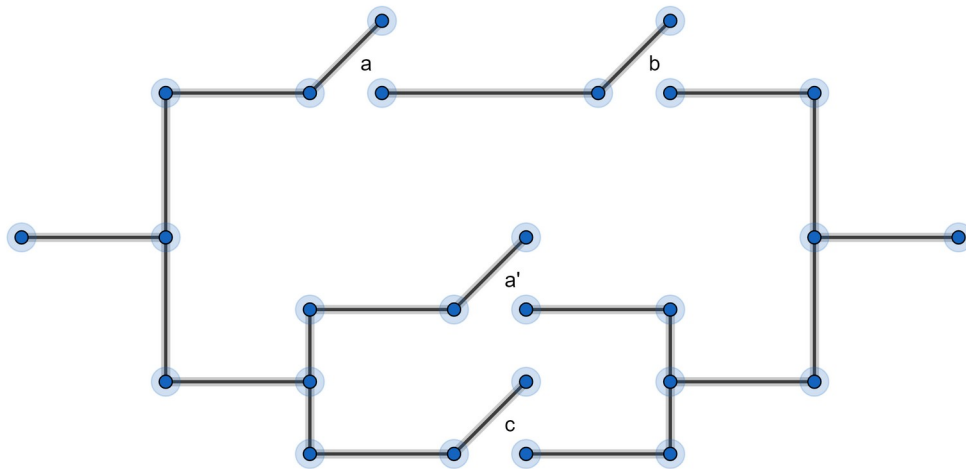
$a$	$b$	$a+b$
0	0	0
0	1	1
1	0	1
1	1	1

We can have switches that are linked to each other. If two switches are both called  $a$  then they are always in the same state, either both open or both closed. If one switch is called  $a$  and another switch is called  $a'$  then they are always in opposite states, one open and the other one closed.

If  $a=1$  then  $a'=0$  If  $a=0$  then  $a'=1$

#### Example 4

Here is a circuit:



The mathematical expression for this circuit is:  $(a.b) + (a'.c)$  Think about it.

The table for this circuit is:

$a$	$b$	$c$	$a.b$	$a'$	$a'+c$	$(a.b) + (a'+c)$
0	0	0	0	1	1	1
0	0	1	0	1	1	1
0	1	0	0	1	1	1
0	1	1	0	1	1	1
1	0	0	0	0	0	0
1	0	1	0	0	1	1
1	1	0	1	0	0	1
1	1	1	1	0	1	1

see EXERCISE 1

Look at Exercise 1 questions (4) and (5)

You should have found that the columns for  $(a.b) + (a.c)$  and  $a.(b+c)$  are the same.

We say  $(a.b) + (a.c) = a.(b+c)$

The circuit in (5) does the same thing as the circuit in (4) but uses fewer switches.

$a.1$  denotes switch  $a$  in series with a closed switch

electric current can flow if  $a$  is closed, so  $a.1=1$  if  $a=1$

electric current cannot flow if  $a$  is open, so  $a.1=0$  if  $a=0$

So  $a.1=a$

$a+1$  denotes switch  $a$  in parallel with a closed switch. Electric current can always flow.

So  $a+1=1$

$a.0$  denotes switch  $a$  in series with an open switch. Electric current can never flow.

So  $a.0=0$

$a+0$  denotes switch  $a$  in parallel with an open switch

electric current can flow if  $a$  is closed, so  $a+0=1$  if  $a=1$

electric current cannot flow if  $a$  is open, so  $a+0=0$  if  $a=0$

So  $a+0=a$

Use tables to prove the following rules:

(no need to do them all)

$$(a')' = a$$

$$a.a = a$$

$$a.a' = 0$$

$$a.b = b.a$$

$$(a.b).c = a.(b.c)$$

$$a.(b+c) = (a.b) + (a.c)$$

$$(a.b)' = a' + b'$$

$$a.(a+b) = a$$

$$a+a = a$$

$$a+a' = 1$$

$$a+b = b+a$$

$$(a+b)+c = a+(b+c)$$

$$a+(b.c) = (a+b).(a+c)$$

$$(a+b)' = a'.b'$$

$$a+(a.b) = a$$

## EXERCISE

1) Draw the circuit and fill in the table for  $(a+b) + (a.b)$

a	b	$a+b$	$a.b$	$(a+b) + (a.b)$
0	0			
0	1			
1	0			
1	1			

2) Draw the circuit and fill in the table for  $a.(a'+b)$

a	b	$a'$	$a'+b$	$a.(a'+b)$
0	0			
0	1			
1	0			
1	1			

3) Draw the circuit and fill in the table for  $(a.b)+c$

a	b	c	$a.b$	$(a.b)+c$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

4) Draw the circuit and fill in the table for  $(a.b)+(a.c)$

a	b	c	$a.b$	$a.c$	$(a.b)+(a.c)$
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

5) Draw the circuit and fill in the table for  $a.(b+c)$

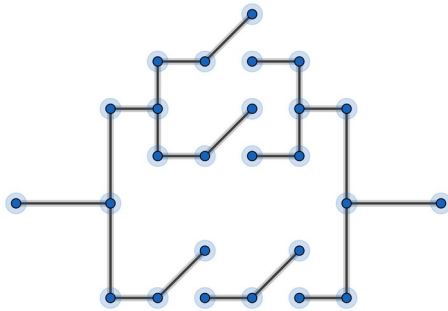
a	b	c	$b+c$	$a.(b+c)$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

## SOLUTIONS

1)

a	b	$a+b$	$a.b$	$(a+b)+(a.b)$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1

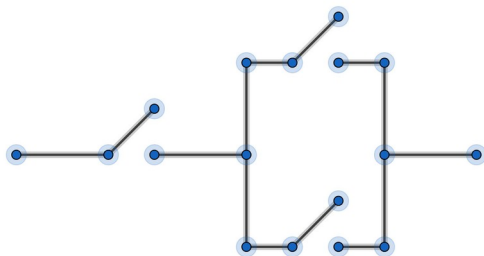
Here is the circuit. Can you label the switches?



2)

a	b	$a'$	$a'+b$	$a.(a'+b)$
0	0	1	1	0
0	1	1	1	0
1	0	0	0	0
1	1	0	1	1

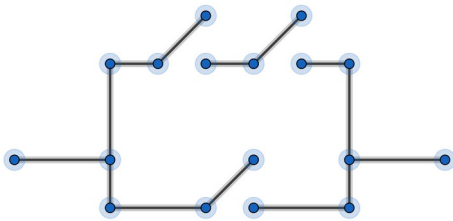
Here is the circuit. Can you label the switches?



3)

a	b	c	$a \cdot b$	$(a \cdot b) + c$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

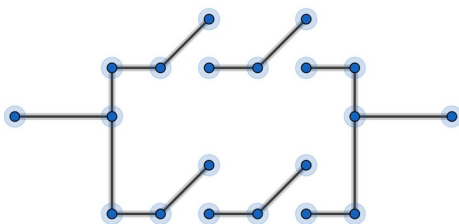
Here is the circuit. Can you label the switches?



4)

a	b	c	$a \cdot b$	$a \cdot c$	$(a \cdot b) + (a \cdot c)$
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

Here is the circuit. Can you label the switches?



5)

a	b	c	$b+c$	$a \cdot (b+c)$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Here is the circuit. Can you label the switches?

