

## Julia Sets

## WE NEED DIAGRAMS

### Example 1

Choose a complex number  $z_0$  and look at the sequence:

$$z_0 \quad z_1 \quad z_2 \quad z_3 \quad \dots \text{ where } z_{n+1} = z_n^2$$

If we choose:

$$z_0 = 4e^{i\pi/5}$$

We get:

$$4e^{i\pi/5} \quad 16e^{i2\pi/5} \quad 256e^{i4\pi/5} \quad 65536e^{i8\pi/5} \quad \dots$$

If we choose:

$$z_0 = \frac{1}{3}e^{i\pi/5}$$

We get:

$$\frac{1}{9}e^{i\pi/5} \quad \frac{1}{81}e^{i2\pi/5} \quad \frac{1}{6561}e^{i4\pi/5} \quad \frac{1}{6561}e^{i8\pi/5} \quad \dots$$

If  $\text{mod}(z_0) > 1$  then  $\text{mod}(z_n)$  tends to infinity as  $n$  tends to infinity.

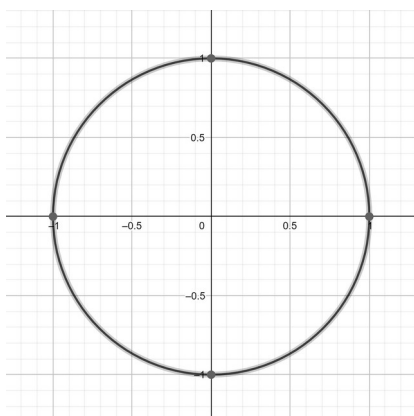
If  $\text{mod}(z_0) < 1$  then  $\text{mod}(z_n)$  does not tend to infinity as  $n$  tends to infinity.

We could colour the number plane.

$z_0$  is in the red region if  $\text{mod}(z_0) > 1$  and  $z_0$  is in the blue region if  $\text{mod}(z_0) < 1$

The Julia set is the boundary between the red region and the blue region.

A circle.



So far, so boring ...

### Example 2

Choose a complex number  $z_0$  and look at the sequence:

$$z_0 \quad z_1 \quad z_2 \quad z_3 \quad \dots \text{ where } z_{n+1} \rightarrow z_n^2 - 0.5 + 0.3i$$

For some values of  $z_0$  we find  $\text{mod}(z_n)$  tends to infinity as  $n$  tends to infinity.

For other values of  $z_0$  we find  $\text{mod}(z_n)$  does not tend to infinity as  $n$  tends to infinity.

We could colour the number plane.

$z_0$  is in the red region if  $\text{mod}(z_n)$  tends to infinity as  $n$  tends to infinity.

$z_0$  is in the blue region if  $\text{mod}(z_n)$  does not tend to infinity as  $n$  tends to infinity.

The Julia set is the boundary between the red region and the blue region.

A slightly deformed circle.

WE NEED A DIAGRAM

So far, so slightly interesting ...

In general:

Choose a complex number  $z_0$  and look at the sequence:

$$z_0 \quad z_1 \quad z_2 \quad z_3 \quad \dots \text{ where } z_{n+1} \rightarrow z_n^2 + c$$

Look at the Julia set for different values of  $c$ . The results are truly amazing.

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