

Julia Sets

WE NEED DIAGRAMS

Example 1

Choose a complex number z_0 and look at the sequence:

$$z_0 \quad z_1 \quad z_2 \quad z_3 \quad \dots \text{ where } z_{n+1} = z_n^2$$

If we choose:

$$z_0 = 4e^{i\pi/5}$$

We get:

$$4e^{i\pi/5} \quad 16e^{i2\pi/5} \quad 256e^{i4\pi/5} \quad 65536e^{i8\pi/5} \quad \dots$$

If we choose:

$$z_0 = \frac{1}{3}e^{i\pi/5}$$

We get:

$$\frac{1}{3}e^{i\pi/5} \quad \frac{1}{9}e^{i2\pi/5} \quad \frac{1}{81}e^{i4\pi/5} \quad \frac{1}{6561}e^{i8\pi/5} \quad \dots$$

If $\text{mod}(z_0) > 1$ then $\text{mod}(z_n)$ tends to infinity as n tends to infinity.

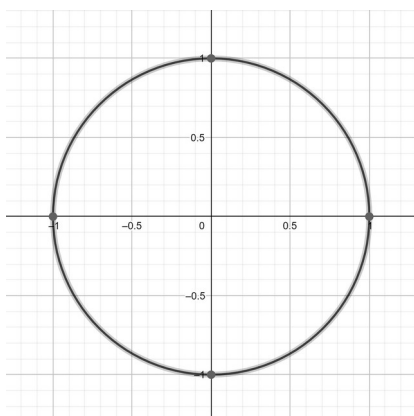
If $\text{mod}(z_0) < 1$ then $\text{mod}(z_n)$ does not tend to infinity as n tends to infinity.

We could colour the number plane.

z_0 is in the red region if $\text{mod}(z_0) > 1$ and z_0 is in the blue region if $\text{mod}(z_0) < 1$

The Julia set is the boundary between the red region and the blue region.

A circle.



So far, so boring ...

Example 2

Choose a complex number z_0 and look at the sequence:

$z_0 \quad z_1 \quad z_2 \quad z_3 \dots$ where $z_{n+1} \rightarrow z_n^2 - 0.5 + 0.3i$

For some values of z_0 we find $\text{mod}(z_n)$ tends to infinity as n tends to infinity.

For other values of z_0 we find $\text{mod}(z_n)$ does not tend to infinity as n tends to infinity.

We could colour the number plane.

z_0 is in the red region if $\text{mod}(z_n)$ tends to infinity as n tends to infinity.

z_0 is in the blue region if $\text{mod}(z_n)$ does not tend to infinity as n tends to infinity.

The Julia set is the boundary between the red region and the blue region.

A slightly deformed circle.

WE NEED A DIAGRAM

So far, so slightly interesting ...

In general:

Choose a complex number z_0 and look at the sequence:

$z_0 \quad z_1 \quad z_2 \quad z_3 \dots$ where $z_{n+1} \rightarrow z_n^2 + c$

Look at the Julia set for different values of c . The results are truly amazing.

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