

A Nice Sum

E	D	C	B	A
D	E	D	C	B
C	D	E	D	C
B	C	D	E	D
A	B	C	D	E

In the above table we have:

1 letter A, two letter B, three letter C, four letter D, five letter E, four letter D, three letter C, two letter B and one letter A.

How many letters have we got?

We have got $1+2+3+4+5+4+3+2+1$ letters. But we have got 5^2 letters.

So $1+2+3+4+5+4+3+2+1=5^2$

In general:

$$1+2+3+\dots+n+\dots+3+2+1=n^2$$

Now let's add up all the numbers in this table:

1×1	1×2	1×3	1×4
2×1	2×2	2×3	2×4
3×1	3×2	3×3	3×4
4×1	4×2	4×3	4×4

First method:

The numbers in the table add up to $(1+2+3+4)^2$ (check by multiplying out the brackets)

Second method:

W	X	Y	Z
X	X	Y	Z
Y	Y	Y	Z
Z	Z	Z	Z

The number in the W cell:

$$1 \times 1 = 1$$

The numbers in the X cells:

$$(2 \times 1) + (2 \times 2) + (1 \times 2) = 2(1 + 2 + 1) = 2(2^2) = 2^3$$

The numbers in the Y cells:

$$(3 \times 1) + (3 \times 2) + (3 \times 3) + (2 \times 3) + (1 \times 3) = 3(1 + 2 + 3 + 2 + 1) = 3(3^2) = 3^3$$

The numbers in the Z cells:

$$(4 \times 1) + (4 \times 2) + (4 \times 3) + (4 \times 4) + (3 \times 4) + (2 \times 4) + (1 \times 4) = 4(1 + 2 + 3 + 4 + 3 + 2 + 1) = 4(4^2) = 4^3$$

So the numbers in the table add up to $1^3 + 2^3 + 3^3 + 4^3$

So comparing our results using the first method and the second method:

$$(1 + 2 + 3 + 4)^2 = 1^3 + 2^3 + 3^3 + 4^3$$

In general:

$$(1 + 2 + 3 + \dots + n)^2 = 1^3 + 2^3 + 3^3 + \dots + n^3$$