Euler's Constant

Here is the graph $y=\frac{1}{x}$ ADD SHADING

Look at the graph from $x=1$ to $x=4$

The area of the shaded bits is the area of the blocks minus the area under the graph.

The area of the blocks is:

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}$$

The area under the graph is:

$$\int_1^4 \frac{1}{x}\,dx=\ln 4$$

So the area of the shaded bits is:

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}-\ln 4$$

Imagine the graph went from $x=1$ to $x=n+1$

The area of the shaded bits is:

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}-\ln(n+1)$$

Now let $n\to\infty$

The total area of all the shaded bits is called γ This is Euler's constant.

$$\gamma=limit(n\to\infty)\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}-\ln(n+1)$$

Imagine sliding all the shaded bits horizontally to the left. They will all fit inside the first block with room to spare. So $\gamma < 1$

Incidently, it is not known if γ is rational or irrational.