

Lanchester model

if you know about differential equations ...

In a tank battle, one army has x tanks and the other army has y tanks. We are going to assume that the rate at which one army's tanks are destroyed is proportional to the number of tanks in the opposing army.

So:

$$\frac{dx}{dt} = -k_1 y \quad \text{and} \quad \frac{dy}{dt} = -k_2 x$$

Let's also assume that each army is equally good at aiming so that:

$$k_1 = k_2$$

Dividing these equations gives:

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\text{So } \int y dy \int x dx \quad \text{So } \frac{1}{2} y^2 = \frac{1}{2} x^2 + c \quad \text{So } y^2 = x^2 + 2c$$

So as the battle proceeds $y^2 - x^2$ will remain constant.

Example

At the start of the battle:

$$x = 24 \quad \text{and} \quad y = 25$$

Throughout the battle:

$$y^2 - x^2 = 49$$

At the end of the battle:

$$x = 0 \quad \text{and} \quad y = 7$$

The strength of a tank army is not proportional to the number of tanks but to the square of the number of tanks. This means weird stuff can happen.

Example

I start with 30 tanks and you start with 42 tanks. In one big battle with all the tanks, I am going to lose. But what if I could arrange some small skirmishes.

If 5 of my tanks engage with 3 of your tanks, then at the end of this skirmish, I'll have 4 tanks and you will have no tanks. If I keep doing this, then I'll soon have more tanks than you!