Topology

In geometry we study rigid objects and we are concerned with lengths, areas, volumes, angles, etc

This is not the case in topology.

Example 1

When you are planning how to travel between two stations on the London underground, you need to know the order of the stations along the lines and where the lines connect. You don't need to know anything about lengths or angles. The London underground map is topological.

Example 2

In the chapter: Polyhedrons, we proved there are only 5 regular polyhedrons.

The proof relied on all the faces being regular polygons. This means that every edge has the same length and every interior angle is the same. We were doing geometry.

Here is another proof:

Consider a polyhedron where each face has n sides and r faces meet at each vertex.

Note:

If r faces meet at each vertex then r edges meet at each vertex.

Note:

$n \geq 3$ and $r \geq 3$

There are F faces and each face has n edges.

So: $E = Fn$ No!

Each edge is shared by 2 faces.

So $E = \frac{Fn}{2}$

There are V vertices and each vertex is joined to r edges.

So $E = Vr$ No!

Each edge is shared by 2 vertices.

So $E = \frac{Vr}{2}$

Now $E = \frac{Fn}{2}$ and $E = \frac{Vr}{2}$ so $\frac{Fn}{2} = \frac{Vr}{2}$ so $V = \frac{Fn}{r}$

Now $F + V = E + 2$

So $F + \frac{Fn}{r} = \frac{Fn}{2} + 2$

So $2rF + 2Fn = Fnr + 4r$

So $F(2n - nr + 2r) = 4r$

So $2n - nr + 2r > 0$

So $2n+2r>nr$

Together with $n\geq 3$ and $r\geq 3$ we get just five possible n and r values:

n	r	$2n+2r$	nr
3	3	12	9
3	4	14	12
3	5	16	15
4	3	14	12
5	3	16	15

In this proof we have not talked about regular polygons. The proof works even if the edges have different lengths and the interior angles are not the same. The edges don't even have to be straight. We have only assumed that all the faces have the same number of edges and all the vertices are connected to the same number of edges. So our result is more general. It is really a topological not a geometrical result.

Example 3

Can we find any polyhedrons whose faces are pentagons and hexagons with three faces meeting at each vertex?

If there are P pentagons and H hexagons then:

$F=P+H$

Each pentagon has 5 edges and each hexagon has 6 edges.

So $E=5P+6H$ No!

Each edge is shared with 2 faces.

So $E=\frac{1}{2}(5P+6H)$

Three faces meet at each vertex so 3 edges meet at each vertex.

So $E=3V$ No!

Each edge is shared with 2 vertices.

So $E=\frac{3V}{2}$

So $V=\frac{1}{3}(5P+6H)$

Now $F+V=E+2$

So $P=12$