Propositions

A proposition is a statement that is either true or false.

Newton was born in England — this is a true proposition

Fermat was born in Poland — this is a false proposition

Please shut the door — this is not a proposition

Example

a : Today is Monday $\quad$ b : I go to work today

We can combine two propositions using AND

$a \wedge b$: Today is Monday and I go to work today

Today is Monday and I go to work today is true if:

Today is Monday is true

and I go to work today is true

Otherwise it is false.

So $a \wedge b$ is true if a is true and b is true, otherwise it is false.

We can set this out in a truth table where 0 means false and 1 means true:

a	b	$a \wedge b$
0	0	0
0	1	0
1	0	0
1	1	1

We can combine two propositions using OR

$a \vee b$: Today is Monday or I go to work today (or both)

Today is Monday or I go to work today is true if:

Today is Monday is true

or I go to work today is true

or both are true

Otherwise it is false.

So $a \vee b$ is true if a is true or b is true (or both), otherwise it is false.

Truth table:

a	b	$a \vee b$
0	0	0
0	1	1
1	0	1
1	1	1

Notes:

In ordinary English we use OR in two different ways.

I will buy you a beer or a lemonade (but not both)

You will get the job if you can sing or dance (or both)

$a \vee b$ always means a or b or both

Negation

The negation of a is a'

a : Today is Monday a' : Today is not Monday

Truth table:

a	a'
0	1
1	0

Examples

Fill in the truth table for $(a \vee b)'$

a	b	$a \vee b$	$(a \vee b)'$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Fill in the truth table for $a \vee (b \wedge c)$

a	b	c	$b \wedge c$	$a \vee (b \wedge c)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

see EXERCISE 1

Look at Exercise 1, questions (3) and (4)

You should have found that the columns for $(a \wedge b) \vee (a \wedge c)$ and $a \wedge (b \vee c)$ are the same.

We say $(a \wedge b) \vee (a \wedge c) = a \wedge (b \vee c)$

See EXERCISE 2

Note:

$a \wedge 1 = 1$ if $a = 1$ and $a \wedge 1 = 0$ if $a = 0$ so $a \wedge 1 = a$

$a \vee 1 = 1$ if $a = 1$ and $a \vee 1 = 1$ if $a = 0$ so $a \vee 1 = 1$

Similarly:

$a \wedge 0 = 0$ and $a \vee 0 = a$

Use truth tables to prove the following rules: (no need to do them all)

$(a')' = a$

$a \wedge a = a$ $\qquad$ $a \vee a = a$

$a \wedge a' = 0$ $\qquad$ $a \vee a' = 1$

$a \wedge b = b \wedge a$ $\qquad$ $a \vee b = b \vee a$

$(a \wedge b) \wedge c = a \wedge (b \wedge c)$ $\qquad$ $(a \vee b) \vee c = a \vee (b \vee c)$

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ $\qquad$ $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$(a \wedge b)' = a' \vee b'$ $\qquad$ $(a \vee b)' = a' \wedge b'$

$a \wedge (a \vee b) = a$ $\qquad$ $a \vee (a \wedge b) = a$

It’s like a new type of algebra!

EXERCISE 1

1) fill in the truth table:

a	b	b'	$a \wedge b'$
0	0		
0	1		
1	0		
1	1		

2) fill in the truth table:

a	b	a'	$a' \vee b$
0	0		
0	1		
1	0		

1	1		

3) fill in the truth table:

a	b	c	$a \wedge b$	$a \wedge c$	$(a \wedge b) \vee (a \wedge c)$
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

4) fill in the truth table:

a	b	c	$b \vee c$	$a \wedge (b \vee c)$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

EXERCISE 2

Use truth tables to show that:

1) $(a \wedge b)' = a' \vee b'$

2) $(a \vee b)' = a' \wedge b'$

SOLUTIONS 1

1)

a	b	b'	$a \wedge b'$
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

2)

a	b	a'	$a' \vee b$
0	0	1	1

0	1	1	1
1	0	0	0
1	1	0	1

3)

a	b	c	$a \wedge b$	$a \wedge c$	$(a \wedge b) \vee (a \wedge c)$
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

4)

a	b	c	$b \vee c$	$a \wedge (b \vee c)$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

SOLUTIONS 2

1)

a	b	$a \wedge b$	$(a \wedge b)'$	a'	b'	$a' \vee b'$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0-	1	1
1	1	1	0	0	0	0

2)

a	b	$a \vee b$	$(a \vee b)'$	a'	b'	$a' \wedge b'$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0