

1. Find the area under the curve:

$$y = \frac{1}{x^2} \text{ between } x = -1 \text{ and } x = 1$$

DIAGRAM?

The area is above the  $x$  axis, so the integral will be positive.

Show that:

$$\int_{-1}^1 \frac{1}{x^2} dx = \dots = -2$$

2. Now:

$$\int (\sin x \cos x) dx = \frac{1}{2} \sin^2 x \quad \text{check by differentiation}$$

And:

$$\int (\sin x \cos x) dx = -\frac{1}{2} \cos^2 x \quad \text{check by differentiation}$$

So:

$$\frac{1}{2} \sin^2 x = -\frac{1}{2} \cos^2 x$$

So:

$$\sin^2 x + \cos^2 x = 0$$

3. Now:

$$\int \left( \frac{1}{7x} \right) dx = \frac{1}{7} \ln x \quad \text{check by differentiation}$$

And:

$$\int \left( \frac{1}{7x} \right) dx = \frac{1}{7} \ln 7x \quad \text{check by differentiation}$$

So:

$$\ln x = \ln 7x$$

4. Using integration by parts, show that:

$$\int \left( \frac{1}{x} \right) dx = \int 1 \times \left( \frac{1}{x} \right) dx = x \frac{1}{x} + \int \left( \frac{1}{x} \right) dx$$

So:

$$\int \left( \frac{1}{x} \right) dx = 1 + \int \left( \frac{1}{x} \right) dx$$

So:

$$0 = 1$$

5. Now:

$$\int 2 \sin 2x dx = -\cos 2x \quad \text{check by differentiation}$$

And:

$$\int 2 \sin 2x dx = 2 \sin^2 x \quad \text{check by differentiation}$$

So:

$$-\cos 2x = 2 \sin^2 x$$

6. Let:

$$I = \int_0^{\pi} \cos^2 x dx$$

So:

$$I = \int_0^{\pi} \cos x \cos x dx = \int_0^{\pi} \sqrt{1 - \sin^2 x} \cos x dx$$

Use the substitution:

$$\sin x = t$$

Show:

$$\cos x dx = dt$$

Show that:

$$I = \int_0^0 \sqrt{1 - t^2} dt = 0$$

7. Now:

$$\sec^2 x \geq 0$$

So:

$$\int_0^{\pi} \sec^2 x dx > 0$$

But:

$$\int_0^{\pi} \sec^2 x dx = [\tan x]_0^{\pi} = 0$$