A Nice Proof

Theorem

$$1+2^1+2^2+2^3+...+2^n=2^{n+1}-1$$

Proof

There are 64 competitors in a knock-out tennis tournament. How many matches will there be during this tournament?

Method 1

| First round | 32 matches | Second round | 16 matches | Third round | 8 matches |
| Fourth round | 4 matches | Fifth round | 2 matches | Sixth round | 1 match |

Answer: $1+2+4+8+16+32$

Method 2

Each match knocks-out one competitor. By the end, 63 competitors have been knocked-out

Answer: 63

Comparing our answers we have: $1+2+4+8+16+32=63$

In general:

$$1+2^1+2^2+2^3+...+2^n=2^{n+1}-1$$

A useful result and nothing to do with tennis.