Probability Fallacies

Example 1

I toss a coin 5 times and get 5 tails.

The probability of getting 5 tails with a fair coin is 0.03 So the probability the coin is fair is 0.03

No! To argue like this is incorrect.

$p(5\,tails \mid fair\,coin)=0.03$ and this is not the same as $p(fair\,coin \mid 5\,tails)$

Example 2 Gambler fallacy

A gambler keeps rolling a dice. A six has not occurred for some time. By the law of averages, it should occur soon.

No! To argue like this is to commit the gambler fallacy.

Imagine a conversation between the gambler and the (talking) dice:

Gambler: I notice you haven't come up six for some time.

Dice: Sorry about that. I'll throw in some extra sixes so that things even out.

I don't think so!

Note:

If you are about to roll a dice 20 times then the probability of getting no sixes is $(5/6)^{20}=0.026$

But:

If you have already rolled a dice 19 times and not got any sixes then the probability of not getting a six on the 20th roll is $5/6$

Example 3 Prosecutor fallacy

A murder has been committed by one of the inhabitants of a town. Eric is on trial for the murder, because his DNA matches DNA found at the crime scene. If Eric is not guilty, the probability that his DNA matches DNA found a the crime scene is 0.00002

So the probability that Eric is not guilty is 0.00002

No! To argue like this is to commit the prosecutor fallacy.

$p(match \mid not\,guilty)=0.00002$ and this is not the same as $p(not\,guilty \mid match)$

What the jury wants to know is $p(not\,guilty \mid match)$ This is the probability that Eric is not guilty.

For example, say the town has 250,000 inhabitants. Typically $250{,}000\times 0.00002=5$ people will have DNA that matches the sample found at the crime scene. So, in the absence of any other evidence, the probability that Eric is not guilty is $4/5$

Example 4 False-Positive Fallacy

There is a disease but sufferers show no symptoms. At any given time, 1% of people actually have the disease. A test has been developed that can detect the disease. If a person has the disease, the test result will be positive. If a person does not have the disease, there is a 3% chance that the test result will be positive. Eric has just had the test, and the result is positive. So the probability that Eric does not have the disease is 3%

No! To argue like this is to commit the false-positive fallacy

$p(\textit{positive} \mid \textit{no disease}) = 0.03$ and this is not the same as $p(\textit{no disease} \mid \textit{positive})$

Consider 100 people. On average, 1 person will have the disease and test positive and 99 people will not have the disease and 3 of them will test positive. So of the 4 people who test positive, only one of them has the disease. So the probability that Eric does not have the disease is $3/4$

Example 5 OJ Simpson trial

OJ Simpson is on trial for the murder of his wife. The prosecution has established that he used to beat-up his wife. The defence argue that, if a man beats-up his wife, the probability that he will murder her is extremely small. True but irrelevant. The prosecution should have argued that, if a man beats-up his wife and his wife is murdered, the probability that he is the murderer is very high.

Example 6 Sally Clark trial

Sally Clark is on trial for the murder of her two children. The prosecution argue that, if a woman has 2 children, the probability they both die of natural causes is extremely low. True but irrelevant. The defence should have argued that, if a woman has 2 children and they both die, the probability they both died of natural causes is high.

There were other incorrect uses of probability in this trial. You should read up about it.

Example 7

LAW OF AVERAGES AND LAW OF LARGE NUMBERS