

Casting-out Nines

Now:

$$8263 = (8 \times 1000) + (2 \times 100) + (6 \times 10) + (3)$$

So:

$$8263 = (8 \times 999) + (2 \times 99) + (6 \times 9) + (8 + 2 + 6 + 3)$$

So:

$$8263 = (9 \times \dots) + (8 + 2 + 6 + 3)$$

So:

$$8263 = 8 + 2 + 6 + 3, \text{mod } 9$$

So to find $N, \text{mod } 9$ we just add up the digits of N

If you do an addition, subtraction or multiplication then the answer must be correct in mod 9.

Example 1

Eric says $123 + 35 = 157$

mod 9:

$$LHS = 123 + 35 = (1 + 2 + 3) + (3 + 5) = 6 + 8 = 14 = 5$$

$$RHS = 157 = 1 + 5 + 7 = 13 = 4$$

end of mod 9

So Eric's answer must be incorrect.

Example 2

Eric says $3647 \times 7298 = 26615797$

mod 9:

$$LHS = 3647 \times 7298 = (3 + 6 + 4 + 7) \times (7 + 2 + 9 + 8) = 20 \times 26 = 2 \times 8 = 16 = 7$$

$$RHS = 26615797 = 2 + 6 + 6 + 1 + 5 + 7 + 9 + 7 = 43 = 7$$

end of mod 9

Be careful. We have not shown that Eric's answer must be correct.

We have shown that Eric's answer is either correct or out by a multiple of 9