A Nice Sum

E	D	C	B	A
D	E	D	C	B
C	D	E	D	C
B	C	D	E	D
A	B	C	D	E

In the above table we have:

1 letter A, two letter B, three letter C, four letter D, five letter E, four letter D, three letter C, two letter B and one letter A.

How many letters have we got?

We have got $1+2+3+4+5+4+3+2+1$ letters. But we have got 5^2 letters.

So $1+2+3+4+5+4+3+2+1=5^2$

In general:

$1+2+3+...+n+...+3+2+1=n^2$

Now let's add up all the numbers in this table:

1×1	1×2	1×3	1×4
2×1	2×2	2×3	2×4
3×1	3×2	3×3	3×4
4×1	4×2	4×3	4×4

First method:

The numbers in the table add up to $(1+2+3+4)^2$ (check by multiplying out the brackets)

Second method:

W	X	Y	Z
X	X	Y	Z
Y	Y	Y	Z
Z	Z	Z	Z

The number in the W cell:

$1\times1=1$

The numbers in the X cells:

$(2\times1)+(2\times2)+(1\times2)=2(1+2+1)=2(2^2)=2^3$

The numbers in the Y cells:

$(3\times1)+(3\times2)+(3\times3)+(2\times3)+(1\times3)=3(1+2+3+2+1)=3(3^2)=3^3$

The numbers in the Z cells:

$(4\times1)+(4\times2)+(4\times3)+(4\times4)+(3\times4)+(2\times4)+(1\times4)=4(1+2+3+4+3+2+1)=4(4^2)=4^3$

So the numbers in the table add up to $1^3+2^3+3^3+4^3$

So comparing our results using the first method and the second method:

$(1+2+3+4)^2=1^3+2^3+3^3+4^3$

In general:

$(1+2+3+...+n)^2=1^3+2^3+3^3+...+n^3$