Maclaurin Series if you know about differentiation and integration ...

Sometimes we can write a function $f(x)$ as a power series:

$f(x)=a+bx+cx^2+dx^3+ex^4+\ldots$

Put $x=0$

$f(0)=a$

Differentiate

$f'(x)=b+2cx+3dx^2+4ex^3+\ldots$

Put $x=0$

$f'(0)=b$

Differentiate

$f''(x)=2c+(3\times 2)dx+(4\times 3)ex^2+\ldots$

Put $x=0$

$f''(0)=2c$

Differentiate

$f'''(x)=(3\times 2)d+(4\times 3\times 2)ex+\ldots$

Put $x=0$

$f'''(0)=(3\times 2)d$

etc

So $f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\frac{f''''(0)}{4!}x^4+\ldots$

Example 1

Lets try this out with $f(x)=sinx$

$f(x)=sinx$	$f(0)=0$
$f'(x)=cosx$	$f'(0)=1$
$f''(x)=-sinx$	$f''(0)=0$
$f'''(x)=-cosx$	$f'''(0)=-1$
$f''''(x)=sinx$	$f''''(0)=0$

etc

So $sinx=x-\frac{1}{3!}x^3+\frac{1}{5!}x^5+\ldots$

Find a graph plotter online and plot the following graphs:

$y=x$

$$y = x - \frac{1}{3!}x^3$$

$$sinx = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$$

etc

And see how these graphs get more and more like $y = sinx$

Example 2

Show that:

$$cosx = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \dots$$

Example 3

Show that:

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots$$

Example 4

There is a sneaky way to find the Maclaurin series for $\ln(1+x)$

We start with:

$1 - x + x^2 - x^3 + ... = \frac{1}{1+x}$ it is a geometric series

So:

$$\int \frac{1}{1+x}dx = \int (1 - x + x^2 - x^3 + ...)dx$$

So:

$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$ this is only valid for $-1 < x \leq 1$

Example 5

There is also a sneaky way to find the Maclaurin series for $\tan^{-1} x$

We start with:

$1 - x^2 + x^4 - x^6 + ... = \frac{1}{1+x^2}$ it is a geometric series

So:

$$\int \frac{1}{1+x^2}dx = \int (1 - x^2 + x^4 - x^6 + ...)dx$$

So:

$$\tan^{-1} x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \ldots$$ this is only valid for $-1 \le x \le 1$

As a bonus:

show that:

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$