

Multiples:

rule of 2: 36478 is a multiple of 2 because 8 (the last digit) is a multiple of 2

$$36478 = 36470 + 8 \text{ and } 36470 \text{ is a multiple of } 2$$

rule of 3: 264 is a multiple of 3 because $2+6+4$ is a multiple of 3

$$264 = 2(100) + 6(10) + 4 = 2(99) + 6(9) + (2+6+4) \text{ and } 2(99) + 6(9) \text{ is a multiple of } 3$$

rule of 4: 94524 is a multiple of 4 because 24 (last two digits) is a multiple of 4

$$94524 = 94500 + 24 \text{ and } 94500 \text{ is a multiple of } 4$$

rule of 5: 743665 is a multiple of 5 because 5 (last digit) is a multiple of 5

$$74365 = 74360 + 5 \text{ and } 74360 \text{ is a multiple of } 5$$

rule of 6: 29622 is a multiple of 6 because 29622 is a multiple of 2 and a multiple of 3

rule of 8: 59136 is a multiple of 8 because 136 (last three digits) is a multiple of 8

$$59136 = 59000 + 136 \text{ and } 59000 \text{ is a multiple of } 8$$

rule of 9: 648 is a multiple of 9 because $6+4+8$ is a multiple of 9

$$648 = 6(100) + 4(10) + 8 = 6(99) + 4(9) + (6+4+8) \text{ and } 6(99) + 4(9) \text{ is a multiple of } 9$$

rule of 10: 89210 is a multiple of 10 because the last digit is 0

rule of 11: 836 is a multiple of 11 because $8-3+6$ is a multiple of 11

$$836 = 8(100) + 3(10) + 6 = 8(99) + 3(11) + (8-3+6) \text{ and } 8(99) + 3(11) \text{ is a multiple of } 11$$

EXERCISE

1)

Is 36470587624275 a multiple of 3?

2)

Is 47385900738828 a multiple of 8?

3)

Is 49775883661205 a multiple of 11?

4)

Show that every palindrome with an even number of digits (like 637736) is a multiple of 11

5)

Show that any 3-digit-repeater (like 726726) is a multiple of 7 and 11 and 13

6)

If n and x are positive integers, prove the following using the factor theorem:

a) $x^n + 1$ is a multiple of $x + 1$ if n is odd

b) $x^n - 1$ is a multiple of $x + 1$ if n is even

c) $x^n - 1$ is a multiple of $x - 1$

SOLUTIONS

1)

Yes. Because $3+6+4+7+0+5+8+7+6+2+4+2+7+5=66$ a multiple of 3

2)

No. Because 828 is not multiple of 8

3)

No. Because $4-9+7-7+5-8+8-3+6-6+1-2+0-5=-9$ not a multiple of 11

4)

$6-3+7-7+3-6=0$ and 0 is a multiple of 11

5)

$7 \times 11 \times 13 = 1001$ and $726726 = 726 \times 1001$

6)

a) if n is odd:

$f(x) = x^n + 1$ so $f(-1) = 0$ so $(x+1)$ is a factor of $f(x)$

b) if n is even:

$f(x) = x^n - 1$ so $f(-1) = 0$ so $(x+1)$ is a factor of $f(x)$

c) $f(x) = x^n - 1$ so $f(1) = 0$ so $(x-1)$ is a factor of $f(x)$