Pigeon-hole Principle

I have 10 boxes and 14 pigeons. I put each pigeon in a box. Obviously one (or more) box must end up with two (or more) pigeons.

How can something so trivial be of any use? Let's find out.

EXERCISE

1) There are 10 people in a room. Everyone shouts out an integer between 1 and 9

Why must two (or more) people shout out the same integer?

2) There are 40 people at a party. Everyone shakes hands with at least one other person. At the end of the party, everyone is asked: "How many people did you shake hands with?"

Why must two (or more) people give the same reply?

3) Pick 11 different positive integers.

Why must two (or more) of these integers have a difference that is a multiple of ten.

4) Pick 6 different integers from 1, 2, 3, … 10

Why must two (or more) of these integers add up to 11?

SOLUTIONS

1) I have 9 boxes, labelled 1, 2, … 9. I put each person in a box.

If a person shouts out 8 then I put them in the box with 8 on the label. etc

There are 9 boxes and 10 people. One (or more) box must contain two (or more) people.

2) I have 39 boxes, labelled 1, 2, … 39. I put each person in a box.

If a person shakes hands with 17 people, then I put them in the box with 17 on the label. etc

There are 39 boxes and 40 people. One (or more) box must contain two (or more) people.

Note: each person shakes hands with 1 or 2 or 3 … or 39 people.

Note: this argument will apply however many people are at the party.

3) I have 10 boxes, labelled 0, 1, 2, … 9. I put each integer in a box.

If an integer is 3768 then I put it in the box with 8 on the label because 8 is its last digit. etc

There are 10 boxes and 11 integers. One (or more) box must contain two (or more) integers.

Note: integers in the same box have the same last digit so their difference is a multiple of 10

4) I have 5 boxes, labelled A, B, C, D, E. I put each integer in a box.

1 and 10 go in box A.

2 and 9 go in box B.

3 and 8 go in box C.

4 and 7 go in box D.

5 and 6 go in box E.

There are 5 boxes and 6 integers. One (or more) box must contain two (or more) integers.

Note: integers in the same box add up to 11