Appendix 2

1) Arithmetic Sequences

Here is an arithmetic sequence: $7, 10, 13, 16, 19, 22, 25, \ldots$

The first term is 7 and the difference between consecutive terms is 3

$u_1 = 7 \quad u_2 = 7 + (3) = 10 \quad u_3 = 7 + (2 \times 3) = 13 \quad u_4 = 7 + (3 \times 3) = 16 \quad \ldots \quad u_n = 7 + (n-1)3$

In general:

Arithmetic sequence: $a + (a+d) + (a+2d) + (a+3d) + (a+4d) + \ldots$

The nth term is $u_n = a + (n-1)d$

2) Geometric Sequences

Here is a geometric sequence:

$2, 6, 18, 54, 162, 486, 1458, \ldots$

The first term is 2 and the ratio of consecutive terms is 3

$u_1 = 2 \quad u_2 = 2 \times (3) = 6 \quad u_3 = 2 \times (3^2) = 18 \quad u_4 = 2 \times (3^3) = 54 \quad \ldots \quad u_n = 2(3^{n-1})$

In general:

Geometric sequence:

$a, ar, ar^2, ar^3, ar^4, \ldots$

The nth term is:

(ar^{n-1})

Summing an infinite geometric series:

$S = a + ar + ar^2 + ar^3 + ar^4 + \ldots$

So:

$rS = ar + ar^2 + ar^3 + ar^4 + ar^5 + \ldots$

So:

$S - rS = (a + ar + ar^2 + ar^3 + ar^4 + \ldots) - (ar + ar^2 + ar^3 + ar^4 + ar^5 + \ldots) = a$

So:

$S(1-r) = a$

So:

$S = \frac{a}{1-r}$ this result is only valid if $-1 < r < 1$

3) Indices

examples	in general
$3^2 \times 3^4 = (3\times3)\times(3\times3\times3\times3) = 3^6$	$(x^m)(x^n) = x^{m+n}$
$\frac{3^6}{3^2} = \frac{3\times3\times3\times3\times3\times3}{3\times3} = 3^4$	$\frac{x^m}{x^n} = x^{m-n}$
$\frac{3^6}{3^5} = \frac{3\times3\times3\times3\times3\times3}{3\times3\times3\times3\times3} = 3$ but $\frac{3^6}{3^5} = 3^1$	$x^1 = x$
$\frac{3^6}{3^6} = \frac{3\times3\times3\times3\times3\times3}{3\times3\times3\times3\times3\times3} = 1$ but $\frac{3^6}{3^6} = 3^0$	$x^0 = 1$
$(3^4)^2 = (3\times3\times3\times3)\times(3\times3\times3\times3) = 3^8$	$(x^m)^n = x^{mn}$
$3^{-4}\times3^4 = 3^0 = 1$ so $3^{-4} = \frac{1}{3^4}$	$x^{-m} = \frac{1}{x^m}$
$3^{1/2}\times3^{1/2} = 3^1 = 3$ so $3^{1/2} = \sqrt{3}$	$x^{1/2} = \sqrt{x}$
$3^{\frac{5}{2}} = \left(3^{\frac{1}{2}}\right)^5$	$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m$

4) Logarithms

If $125 = 5^3$ then $\log_5 125 = 3$

In general:

If $c = a^b$ then $\log_a c = b$

Now

$8 = 2^3$ so $\log_2 8 = 3$ and $32 = 2^5$ so $\log_2 32 = 5$

examples	in general
$8\times32 = 2^{3+5}$ so $\log_2(8\times32) = \log_2 8 + \log_2 32$	$\log_a(xy) = \log_a(x) + \log_a(y)$

$\frac{32}{8}=2^{5-3}$ so $\log_2\left(\frac{32}{8}\right)=\log_2 32-\log_2 8$ $\log_a\left(\frac{x}{y}\right)=\log_a(x)-\log_a(y)$

$32^4=(2^5)^4=2^{4\times5}$ so $\log_2(32^4)=4\log_2 32$ $\log_a(x^n)=n\log_a(x)$

5) Factor theorem

example

$f(x)=x^2-5x+6$

So:

$f(2)=2^2-(5\times2)+6=0$

The factor theorem tells us that if $f(2)=0$ then $(x-2)$ is a factor of $f(x)$

So:

$f(x)=(x-2)(\ldots)$

In general:

If $f(a)=0$ then $(x-a)$ is a factor of $f(x)$

6) Factorials

$1!=1$

$2!=1\times2$

$3!=1\times2\times3$

$4!=1\times2\times3\times4$ etc

7) Binomial theorem for multiplying out brackets

$(1+x)^1=1+x$

$(1+x)^2=1+2x+x^2$

$(1+x)^3=1+3x+3x^2+x^3$

$(1+x)^4=1+4x+6x^2+4x^3+x^4$

In general: if n is a positive integer

$(1+x)^n=(nC0)+(nC1)x+(nC2)x^2+(nC3)x^3+\ldots+(nCn)x^n$