Voting Systems

Example 1

There are 3 candidates A, B and C. We need to elect one of them. There are 62 voters and each voter has put the candidates in order of preference:

Order of preference	Number of voters
ABC	18
ACB	15
BAC	5
BCA	14
CAB	8
CBA	2

This means:

18 voters put A as their first choice, B as their second choice and C as their third choice, etc.

Let's look at three different voting systems:

a) First-Past-The-Post

If a voter puts a candidate first choice, then that candidate gets 1 point.

A gets $18+15=33$ points, B gets $5+14=19$ points and C gets $8+2=10$ points.

The winner is the candidate with the most points.

So A is the winner.

b) Alternative-Vote

If a voter puts a candidate first choice, then that candidate gets 1 point.

A gets $18+15=33$ points, B gets $5+14=19$ points and C gets $8+2=10$ points.

The candidate with the fewest points is eliminated. So C is eliminated.

The 8 voters whose order of preference was CAB now put A as their first choice and B as their second choice.

The 2 voters whose order of preference was CBA now put B as their first choice and A as their second choice.

A now gets $18+15+8=41$ points and B now gets $5+14+2=21$ points.

So A is the winner.

c) Most-Popular

In an election between just A and B:

$18+15+8=41$ voters prefer A to B and $5+14+2=21$ voters prefer B to A

So A is more popular than B.

In an election between just A and C:

$18+15+5=38$ voters prefer A to C and $8+2+14=24$ voters prefer C to A

So A is more popular than C.

In an election between just B and C:

$5+14+18=37$ voters prefer B to C and $8+2+15=25$ voters prefer C to B

So B is more popular than C.

A is more popular than B and A is more popular than C.

So A is the winner.

See EXERCISE

This all seems straight forward, but ...

Example 2

Order of preference	Number of voters
ABC	40
ACB	10
BAC	5
BCA	30
CAB	8
CBA	40

B is more popular than A and C is more popular than A.

But with First-Past-The-Post, A is the winner.

Example 3

Order of preference	Number of voters
ABC	6
ACB	15
BAC	5
BCA	15
CAB	8
CBA	10

C is more popular than A and C is more popular than B.

But with Alternative-Vote, C is eliminated.

Example 4

Order of preference	Number of voters
ABC	14
ACB	7
BAC	8
BCA	12
CAB	12
CBA	6

A is more popular than B and B is more popular than C and C is more popular than A.

So with Most-Popular, there is no winner.

Example 5

Order of preference	Number of voters
ABC	20
ACB	20
BAC	5
BCA	24
CAB	17
CBA	14

With Alternative-Vote:

A gets 40 points, B gets 29 points and C gets 31 points. So B is eliminated.

A now gets 45 points and C now gets 55 points. So C is the winner.

If 3 of the voters whose order of preference was ABC, had voted tactically and voted BAC then:

Order of preference	Number of voters
ABC	17
ACB	20
BAC	8
BCA	24
CAB	17
CBA	14

A gets 37 points, B gets 32 points and C gets 31 points. So C is eliminated.

A now gets 54 points and B now gets 46 points. So A is the winner.

So tactical voting paid off. But you need to be careful ...

If 10 of the voters whose order of preference was ABC, had voted tactically and voted BAC then:

Order of preference	Number of voters
ABC	10
ACB	20
BAC	15
BCA	24
CAB	17
CBA	14

A gets 30 points, B gets 39 points and C gets 31 points. A is eliminated. Whoops!

Example 6

Order of preference	Number of governors
ABC	5
ACB	4
BAC	5
BCA	3
CAB	1
CBA	3

There are 3 candidates for a job at a school. Each of the governors has put the candidates in order of preference.

With First-Past-The-Post:

A gets 9 points, B gets 8 points, C gets 4 points. So the governors decide to appoint A.

However, just before the Principal announces the result, C gets a call on her phone, offering her a job at a different school, which she accepts. “Never mind” says the Principal “we were not going to give her the job anyway”. Not so fast! If C is no longer available:

A gets 10 points, B gets 11 points.

There are many other voting systems for electing one candidate. However ...

Arrow’s Theorem:

There is no perfect voting system. Arrow wrote a list of the features you would certainly want in any voting system. Arrow’s theorem proves that no voting system can have all these features.

Gibbard–Satterthwaite theorem:

We would like a voting system where there is no benefit in tactical voting. The Gibbard–Satterthwaite theorem proves that this is not possible.

EXERCISE

1)

Order of preference	Number of voters
ABC	5
ACB	7
BAC	1
BCA	9
CAB	2
CBA	7

a) Who wins with First-Past-The-Post?

b) Who wins with Alternative-Vote?

c) Who wins with Most-Popular?

2)

Another voting system is Borda score.

If a voter puts a candidate first choice, then the candidate gets 3 points.

If a voter puts a candidate second choice, then the candidate gets 2 points.

If a voter puts a candidate third choice, then the candidate gets 1 point.

The winner is the candidate with the most points.

Look at Example 1 at the start of this chapter. Who is the winner with Borda score?

SOLUTIONS

1)

a) A gets $5+7=12$ B gets $1+9=10$ C gets $2+7=9$

A is the winner.

b) A gets $5+7=12$ B gets $1+9=10$ C gets $2+7=9$ C is eliminated.

A now gets $5+7+2=14$ B now gets $1+9+7=17$

B is the winner.

c) A against B A gets $5+7+2=14$ B gets $1+9+7=17$

A against C A gets $5+7+1=13$ C gets $2+7+9=18$

B against C B gets $1+9+5=15$ C gets $2+7+7=16$

C is the winner.

2)

A gets $(18+15)\times3+(5+8)\times2+(14+2)\times1=141$

B gets $(5+14)\times3+(18+2)\times2+(15+8)\times1=120$

C gets $(8+2)\times3+(15+14)\times2+(18+5)\times1=111$

A is the winner.