

A Nice Proof

Theorem

$$1+2^1+2^2+2^3+\dots+2^n=2^{n+1}-1$$

Proof

There are 64 competitors in a knock-out tennis tournament. How many matches will there be during this tournament?

Method 1

First round	32 matches	Second round	16 matches	Third round	8 matches
Fourth round	4 matches	Fifth round	2 matches	Sixth round	1 match

Answer: $1+2+4+8+16+32$

Method 2

Each match knocks-out one competitor. By the end, 63 competitors have been knocked-out

Answer: 63

Comparing our answers we have: $1+2+4+8+16+32=63$

In general:

$$1+2^1+2^2+2^3+\dots+2^n=2^{n+1}-1$$

A useful result and nothing to do with tennis.