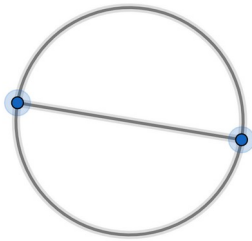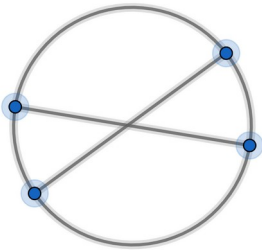Cutting a Pizza

We have a pizza. We run the pizza cutter, in a straight line, across the pizza.
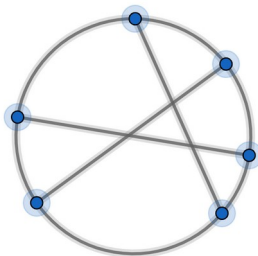
With 1 cut, we get 2 pieces.

With 2 cuts, we get a maximum of 4 pieces.

With 3 cuts, we get a maximum of 7 pieces.

We want the maximum number of pieces, so we want each cut to cross over all the other cuts and we don't want three or more cuts to cross at the same point.

With 4 cuts?

The fourth cut must cross the other 3 cuts, so it must pass through 4 pieces of pizza. Each of these pieces is cut into two pieces, adding 4 more pieces.

Let P_4 be the maximum number of pieces with 4 cuts.

So $P_4 = P_3 + 4$ So $P_4 = P_3 + 4 = 7 + 4 = 11$

In general:

$$P_1=2 \quad \text{and} \quad P_{n+1}=P_n+(n+1)$$

We want a formula for P_n We will use the guess and prove method.

Guess:

$$P_n=\frac{1}{2}n(n+1)+1$$

EXERCISE

Use proof by induction to show that this guess is correct

SOLUTION

Proof part 1:

If $n=1$ then $LHS=P_1=2$ and $RHS=\frac{1}{2}(1)(2)+1=2$ So the formula is true when $n=1$

Proof part 2:

If $P_n=\frac{1}{2}n(n+1)+1$ is true when $n=k$ then:

$$P_k=\frac{1}{2}k(k+1)+1 \quad \text{but} \quad P_{k+1}=P_k+(k+1)$$

$$P_{k+1}=\frac{1}{2}k(k+1)+1+(k+1)=...=\frac{1}{2}(k+1)(k+2)+1$$

So $P_n=\frac{1}{2}n(n+1)+1$ is true when $n=k+1$

So our guess is correct.