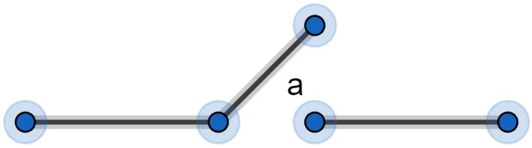
Switching Circuits

Example 1

Here is a switch called a . It can be closed or open.



If a is closed then electric current can flow. We say $a=1$

If a is open then electric current cannot flow. We say $a=0$

Example 2

Here are two switches in series:

$a.b$ denotes switches a and b in series (this is not multiplication!)

If a is closed and b is closed then electric current can flow.

So if $a=1$ and $b=1$ then $a.b=1$ So $1.1=1$

If a is open or b is open (or both) then electric current cannot flow.

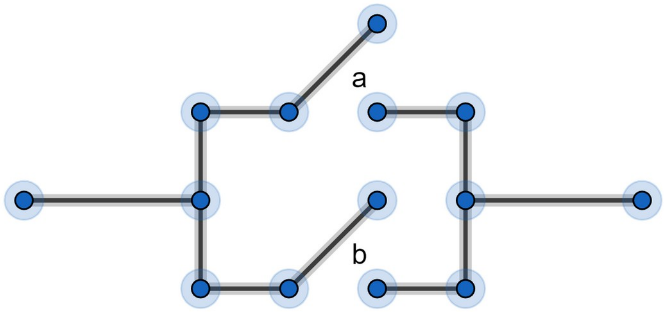
So if $a=0$ or $b=0$ (or both) then $a.b=0$ So $0.1=0$ $1.0=0$ $0.0=0$

We can set this out in a table:

a	b	$a.b$
0	0	0
0	1	0
1	0	0
1	1	1

Example 3

Here are two switches in parallel:



$a+b$ denotes switches a and b in parallel (this is not addition!)

a is closed or b is closed (or both) then electric current can flow.

So if $a=1$ or $b=1$ (or both) then $a+b=1$ So $1+0=1$ $0+1=1$ $1+1=1$

If a is open and b is open then electric current cannot flow.

Soif $a=0$ and $b=0$ then $a+b=0$ So $0+0=0$

We can set this out in a table:

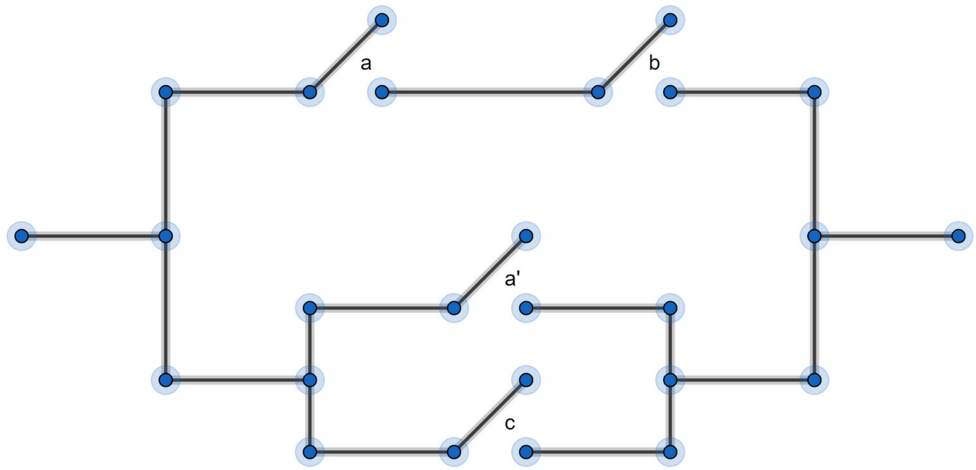
a	b	$a+b$
0	0	0
0	1	1
1	0	1
1	1	1

We can have switches that are linked to each other. If two switches are both called a then they are always in the same state, either both open or both closed. If one switch is called a and another switch is called a' then they are always in opposite states, one open and the other one closed.

If $a=1$ then $a'=0$ If $a=0$ then $a'=1$

Example 4

Here is a circuit:



The mathematical expression for this circuit is: $(a.b)+(a'+c)$ Think about it.

The table for this circuit is:

a	b	c	$a.b$	a'	$a'+c$	$(a.b)+(a'+c)$
0	0	0	0	1	1	1
0	0	1	0	1	1	1
0	1	0	0	1	1	1
0	1	1	0	1	1	1
1	0	0	0	0	0	0
1	0	1	0	0	1	1
1	1	0	1	0	0	1
1	1	1	1	0	1	1

see EXERCISE 1

Look at Exercise 1 questions (4) and (5)

You should have found that the columns for $(a.b)+(a.c)$ and $a.(b+c)$ are the same.

We say $(a.b)+(a.c)=a.(b+c)$

The circuit in (5) does the same thing as the circuit in (4) but uses fewer switches.

$a.1$ denotes switch a in series with a closed switch
electric current can flow if a is closed, so $a.1=1$ if $a=1$
electric current cannot flow if a is open, so $a.1=0$ if $a=0$
So $a.1=a$

$a+1$ denotes switch a in parallel with a closed switch. Electric current can always flow.
So $a+1=1$

$a.0$ denotes switch a in series with an open switch. Electric current can never flow.
So $a.0=0$

$a+0$ denotes switch a in parallel with an open switch
electric current can flow if a is closed, so $a+0=1$ if $a=1$
electric current cannot flow if a is open, so $a+0=0$ if $a=0$
So $a+0=a$

Use tables to prove the following rules: (no need to do them all)

$(a')'=a$

$a.a=a$	$a+a=a$
$a.a'=0$	$a+a'=1$
$a.b=b.a$	$a+b=b+a$
$(a.b).c=a.(b.c)$	$(a+b)+c=a+(b+c)$
$a.(b+c)=(a.b)+(a.c)$	$a+(b.c)=(a+b).(a+c)$
$(a.b)'=a'+b'$	$(a+b)'=a'.b'$
$a.(a+b)=a$	$a+(a.b)=a$

EXERCISE

1) Draw the circuit and fill in the table for $(a+b)+(a.b)$

a	b	$a+b$	$a.b$	$(a+b)+(a.b)$
0	0			
0	1			
1	0			
1	1			

2) Draw the circuit and fill in the table for $a.(a'+b)$

a	b	a'	$a'+b$	$a.(a'+b)$
0	0			
0	1			
1	0			
1	1			

3) Draw the circuit and fill in the table for $(a.b)+c$

a	b	c	$a.b$	$(a.b)+c$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

4) Draw the circuit and fill in the table for $(a.b)+(a.c)$

a	b	c	$a.b$	$a.c$	$(a.b)+(a.c)$
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

5) Draw the circuit and fill in the table for $a.(b+c)$

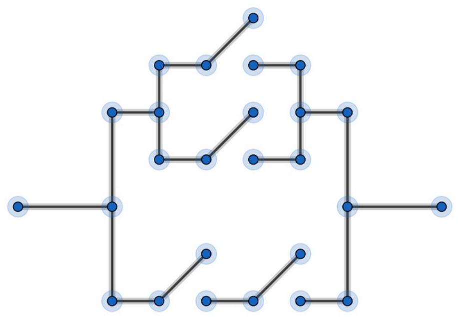
a	b	c	$b+c$	$a.(b+c)$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

SOLUTIONS

1)

a	b	$a+b$	$a.b$	$(a+b)+(a.b)$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1

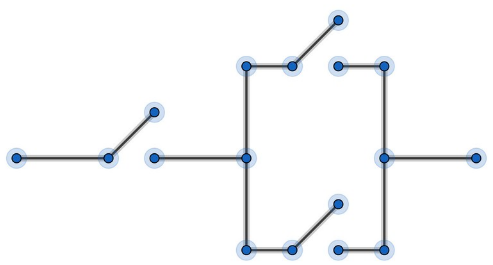
Here is the circuit. Can you label the switches?



2)

a	b	a'	$a'+b$	$a.(a'+b)$
0	0	1	1	0
0	1	1	1	0
1	0	0	0	0
1	1	0	1	1

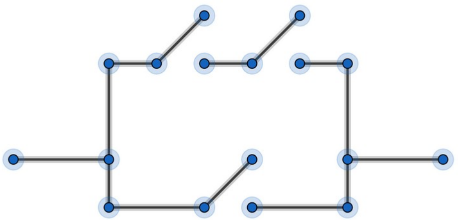
Here is the circuit. Can you label the switches?



3)

a	b	c	$a.b$	$(a.b)+c$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

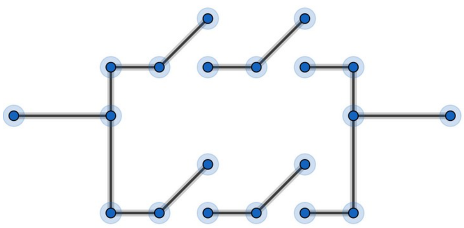
Here is the circuit. Can you label the switches?



4)

a	b	c	$a.b$	$a.c$	$(a.b)+(a.c)$
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

Here is the circuit. Can you label the switches?



5)

a	b	c	$b+c$	$a.(b+c)$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Here is the circuit. Can you label the switches?