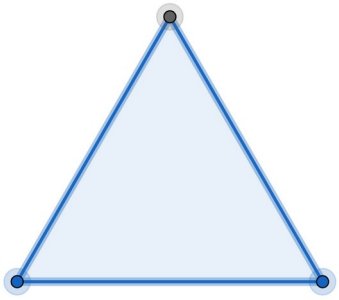
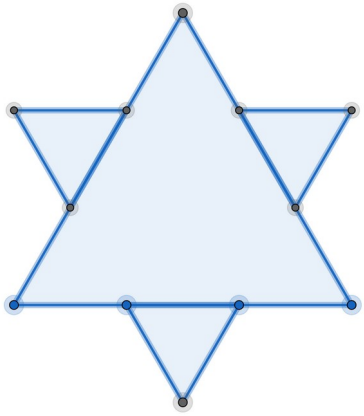
Snow-flake Curve

We start with an equilateral triangle:



Then we add a smaller equilateral triangle to each of the three sides:



Then we add an even smaller equilateral triangle to each of the twelve sides.

We repeat this an infinite number of times to get the snowflake curve. We want to find the perimeter and area of the this curve.

Perimeter

At the start, we have 3 sides of length L so the perimeter is $3L$

The first iteration replaces each side by 4 sides of length $\frac{L}{3}$ so the perimeter is $\frac{4L}{3}$

Each iteration increases the perimeter by a factor of $\frac{4}{3}$

So after an infinite number of iterations, the perimeter is infinite.

Area

At the start, we have a triangle with area 1

The first iteration adds 3 triangles, each of area $\frac{1}{9}$

The second iteration adds 3×4 triangles, each of area $\left(\frac{1}{9}\right)^2$

The third iteration adds $3\times 4\times 4$ triangles, each of area $\left(\frac{1}{9}\right)^3$

etc

So after an infinite number of iterations, the area is:

$$1+\left(3\times\frac{1}{9}\right)+\left(3\times 4\times\frac{1}{9^2}\right)+\left(3\times 4^2\times\frac{1}{9^3}\right)+...$$

Now:

$$\left(3\times\frac{1}{9}\right)+\left(3\times 4\times\frac{1}{9^2}\right)+\left(3\times 4^2\times\frac{1}{9^3}\right)+...=\frac{3/9}{1-4/9}=\frac{3}{5}$$ (see Appendix 2: Geometric Sequence)

So the final area is:

$$1+\frac{3}{5}=\frac{8}{5}$$

So the snow-flake curve has a finite area but an infinite perimeter.