Pascal's Triangle

The triangle is usually set out like an isosceles triangle but I have set it out slightly differently:

	Col 0	Col 1	Col 2	Col 3	Col 4	Col 5	Col 6	Col 7	Col 8	Col 9
Row 0	1									
Row 1	1	1								
Row 2	1	2	1							
Row 3	1	3	3	1						
Row 4	1	4	6	4	1					
Row 5	1	5	10	10	5	1				
Row 6	1	6	15	20	15	6	1			
Row 7	1	7	21	35	35	21	7	1		
Row 8	1	8	28	56	70	56	28	8	1	
Row 9	1	9	36	84	126	126	84	36	9	1

1) The numbers in the triangle are selection numbers. (see chapter: Arrangements and Selections)

For example, the number in row 9 and column 3 is $(9C3)$

2) We can generate each row of the triangle from the row above. To generate row 10:

$(10C0)=1$

$(10C1)=(9C0)+(9C1)=1+9=10$

$(10C2)=(9C1)+(9C2)=9+36=45$

$(10C3)=(9C2)+(9C3)=36+84=120$ etc

3) Look at the numbers in row 7 of Pascal's triangle: $1,7,21,35,35,21,7,1$

A typical number in this row is $(7C3)=\frac{7\times6\times5\times4\times3\times2\times1}{(3\times2\times1)(4\times3\times2\times1)}$

After lots of cancelling, we are left with a positive integer. The 7 on the top of the fraction can't be cancelled out by numbers on the bottom of the fraction because 7 is prime. So $(7C3)$ must be a multiple of 7

In general:

If p is prime then all the numbers in line p of Pascal's triangle will be a multiple of p (apart from the 1 at each end)

4) Binomial theorem for multiplying out brackets

$(1+x)^1=1+x$

$(1+x)^2=1+2x+x^2$

$(1+x)^3=1+3x+3x^2+x^3$

$(1+x)^4=1+4x+6x^2+4x^3+x^4$

In general: if n is a positive integer

$(1+x)^n=(nC0)+(nC1)x+(nC2)x^2+(nC3)x^3+\ldots+(nCn)x^n$

sub in $x=1$ $(nC0)+(nC1)+(nC2)+\ldots+(nCn)=2^n$

sub in $x=-1$ $(nC0)-(nC1)+(nC2)-(nC3)+\ldots=0$

sub in $x=2$ $(nC0)+2(nC1)+2^2(nC2)+\ldots+2^n(nCn)=3^n$

etc

By adding or subtracting the first two results we get:

$(nC0)+(nC2)+(nC4)+\ldots=2^{n-1}$

$(nC1)+(nC3)+(nC5)+\ldots=2^{n-1}$

EXERCISE

Write down row 10 of Pascal's triangle.

SOLUTION

1, 1+9=10, 9+36=45, 36+84=120, 84+126=210, 126+126=252, 126+84=210, 84+36=120, 36+9=45, 9+1=10, 1