Latin Squares

Example 1

Here is a $4 \times 4$ Latin square:

| | | | |
|---|---|---|---|
| A | B | C | D |
| B | A | D | C |
| C | D | A | B |
| D | C | B | A |

There are four symbols A, B, C, D and each symbol appears once (and only once) in each row and once (and only once) in each column.

Latin squares are used in agricultural research. We are growing oats in a field and we want to compare four types of fertilizer, A, B, C, D. We divide the field into 16 plots and apply the fertilizer as in the Latin square. Conditions (drainage etc) might vary across the field, so we want to try each type of fertilizer in each row and each column of the field.

One way to make a Latin square is the diagonal method:

| | | | | | |
|---|---|---|---|---|---|
| A | B | C | D | E | F |
| F | A | B | C | D | E |
| E | F | A | B | C | D |
| D | E | F | A | B | C |
| C | D | E | F | A | B |
| B | C | D | E | F | A |

Look and see how each letter is arranged along diagonals.

See EXERCISE 1

Sudoku Squares

Here is an incomplete sudoku square:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | H | I | E | G | C | D | F | B |
| F | G | D | I | | | | A | E |
| E | | C | A | F | D | H | I | |
| H | I | | | | F | B | D | C |
| C | A | B | H | D | | E | | F |
| | | | | E | | | H | |
| I | | A | | | E | | B | |
| B | E | G | D | I | A | F | C | |
| D | | | | C | | I | E | |

There are nine symbols A, B, C, … I. The challenge is to fill in the square to make a $9\times9$ Latin square. But there is more. If you divide up the board into nine $3\times3$ squares then each $3\times3$ square must contain all nine symbols. So, for example, look at the top-left $3\times3$ square:

| | | |
|---|---|---|
| | H | I |
| F | G | D |
| E | | C |

The two empty cells must contain A and B.

The nine symbols in a sudoku square are usually 1, 2, 3, 4, 5, 6, 7, 8, 9. I've chosen to use letters instead because this puzzle is not really about numbers.

There are just 5,524,751,496,156,892,842,531,225,600 possible $9\times9$ Latin squares but only some of them are Sudoku squares.

See EXERCISE 2

EXERCISE 1

Arrange the letters A, B, C, D, E into a $5\times5$ Latin Square using the diagonal method.

EXERCISE 2

Complete the above sudoku square

SOLUTIONS 1

| | | | | |
|---|---|---|---|---|
| A | B | C | D | E |
| E | A | B | C | D |
| D | E | A | B | C |
| C | D | E | A | B |
| B | C | D | E | A |

SOLUTIONS 2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A | H | I | E | G | C | D | F | B |
| F | G | D | I | B | H | C | A | E |
| E | B | C | A | F | D | H | I | G |
| H | I | E | G | A | F | B | D | C |
| C | A | B | H | D | I | E | G | F |
| G | D | F | C | E | B | A | H | I |
| I | C | A | F | H | E | G | B | D |
| B | E | G | D | I | A | F | C | H |
| D | F | H | B | C | G | I | E | A |