Error Detecting Codes

An online book shop sells books. Each book has a four digit code-number. To order a book you have to type this code-number into an online order form. But you might make an error when you do this.

Say the book you want to buy has code-number 4693.

You might type in 4673 instead of 4693. This is a digit error.

You might type in 4963 instead of 4693. This is a swap error.

If you make an error then you get sent the wrong book. It would be great if the shop could detect these errors. There are many ways to do this. We will look at the check-digit method.

Example 1

Sum of digits method:

We add a check-digit at the end of each four digit code-number. So now each book has a five digit code-number. The check-digit is chosen so that the sum of the digits is a multiple of 10.

If a book has code-number 3725, then the check-digit is 3 because $3+7+2+5+3=20=2\times 10$

This book now has code-number 37253

If you make a digit error and type in 37283, then this will be detected because the sum of the digits is no longer a multiple of 10 note: $3+7+2+8+3=23$

The shop will not send you the wrong book because 37283 does not correspond to any book. The shop will know you have made an error and will ask you to re-order.

If you make a swap error and type in 32753, then this will not be detected because the sum of the digits is still a multiple of 10 note: $3+2+7+5+3=20$

The shop will send you the wrong book.

Example 2

Weighted sum of digits method:

We add a check-digit at the end of each four digit code-number. So now each book has a five digit code-number. The check-digit is chosen so that the weighted-sum of the digits is a multiple of 10.

If a book has code-number 3725 and if we use the weights: 1, 3, 1, 3, 1

	3	7	2	5	?
weight	1	3	1	3	1

then the check-digit is 9 because $(3\times 1)+(7\times 3)+(2\times 1)+(5\times 3)+(9\times 1)=50=5\times 10$

The book now has code-number 37259

If you make a digit error and type in 37659

	3	7	6	5	9
weight	1	3	1	3	1

then this will be detected because $(3\times1)+(7\times3)+(6\times1)+(5\times3)+(9\times1)=54$

If you make a swap error and type in 37529

	3	7	5	2	9
weight	1	3	1	3	1

then this will be detected because $(3x1)+(7x3)+(5x1)+(2x3)+(9x1)=44$

Example 3

If a book has code-number 3521 and we are using the weights: 1, 4, 1, 4, 1

then the check-digit is 1.

The book now has code-number 35211.

If you make a digit error and type in 35261

	3	5	2	6	1
weight	1	4	1	4	1

then this will not be detected.

In the code-number, you replaced 1 by 6. The digit has changed by 5.

In the weighted sum, you replaced (1×4) by (6×4) The difference is $5\times4=20$

The weighted sum has changed by a multiple of 10 and so the error will not be detected.

So for digit errors:

If the digit changes by 5 and the weight is a multiple of 2

Or

If the digit changes by a multiple of 2 and the weight is 5

then the error will not be detected.

So we avoid using 2, 4, 5, 6, 8 as weights, and use 1, 3, 7, 9 instead. This means all digit errors will be detected.

We also use different weights on adjacent digits so that most swap errors are detected.

Example 4

If a book has code-number 4273 and we are using the weights: 1, 9, 1, 9, 1

then the check-digit is 4.

The book now has code-number 42734

If you make a swap error and type in 47234

	4	7	2	3	4
weight	1	9	1	9	1

then this will not be detected.

In the code-number you swapped 2 and 7. The difference between these digits is 5.

In the weighted sum, you replaced $(2\times 9)+(7\times 1)$ by $(7\times 9)+(2\times 1)$ The difference is 40.

The weighted sum has changed by a multiple of 10 and so the error will not be detected.

So for swap errors:

If the digits that are swapped differ by 5 and their weights differ by a multiple of 2 (which they will if we only use 1, 3, 7, 9 as weights) then the error will not be detected.

There are lots of other check-digit methods. For example, some books have a 10 digit ISBN number. The book, Symmetry by Hermann Weyl has the number 0691023743

The last digit is the check-digit. This is calculated as follows:

ISBN	0	6	9	1	0	2	3	7	4	3
weight	10	9	8	7	6	5	4	3	2	1

The weighted sum is:

$$(0\times 10)+(6\times 9)+(9\times 8)+(1\times 7)+(0\times 6)+(2\times 5)+(3\times 4)+(7\times 3)+(4\times 2)+(3\times 1)=187$$

The check-digit is chosen so that this weighted sum is divisible by 11.

If the check-digit is 10 then it is denoted by X in the ISBN code.