Dodgy Algebra

Example 1

$3>2$

$3\log(\frac{1}{2})>2\log(\frac{1}{2})$

$\log(\frac{1}{2})^{3}>\log(\frac{1}{2})^{2}$

$\log(\frac{1}{8})>\log(\frac{1}{4})$

$\frac{1}{8}>\frac{1}{4}$

Example 2

$x>3$

$3x>9$

$3x-x^2>9-x^2$

$x(3-x)>(3+x)(3-x)$

$x>3+x$

$0>3$

Example 3

$x=3$

$x^2=3x$

$x^2-9=3x-9$

$(x+3)(x-3)=3(x-3)$

$(x+3)=3$

$x=0$

$3=0$

Example 4

$x+y=2$

$(x+y)(x-y)=2(x-y)$

$x^2-y^2=2x-2y$

$x^2-y^2+(y^2-2x+1)=2x-2y+(y^2-2x+1)$

$x^2-2x+1=y^2-2y+1$

$(x-1)^2=(y-1)^2$

$x-1=y-1$

$x=y$

Example 5

$\sin 70°=\sin 110°$

$70°=110°$

Example 6

$3-\frac{x+4}{x-2}=\frac{2x-10}{x-3}$

$\frac{3(x-2)-(x+4)}{x-2}=\frac{2x-10}{x-3}$

$\frac{2x-10}{x-2}=\frac{2x-10}{x-3}$

$2=3$

EXERCISE

So where did it all go wrong?

SOLUTIONS

1)

We multiplied both sides of an inequality by $\log(1/2)$

But $\log(1/2)$ is negative so we should reverse the inequality sign.

2)

We divided both sides of an inequality by $(3-x)$

But $(3-x)$ is negative so we should reverse the inequality sign.

3)

We divided both sides of an equation by $(x-3)$

But $(x-3)=0$

4)

If $(x-1)^2=(y-1)^2$ then either $(x-1)=(y-1)$ or $(x-1)=-(y-1)$

5)

Look at the graph $y=sinx$

6)

$\frac{2x-10}{x-2}=\frac{2x-10}{x-3}$

So:

$(2x-10)(x-3)=(2x-10)(x-2)$ provided $x\neq 3$ and $x\neq 2$

Either:

$(2x-10)=0$ so $x=5$

Or:

$(x-3)=(x-2)$ which has no solution