

Euler's Constant

Here is the graph $y = \frac{1}{x}$ ADD SHADING



Look at the graph from $x=1$ to $x=4$

The area of the shaded bits is the area of the blocks minus the area under the graph.

The area of the blocks is:

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3}$$

The area under the graph is:

$$\int_1^4 \frac{1}{x} dx = \ln 4$$

So the area of the shaded bits is:

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} - \ln 4$$

Imagine the graph went from $x=1$ to $x=n+1$

The area of the shaded bits is:

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln(n+1)$$

Now let $n \rightarrow \infty$

The total area of all the shaded bits is called γ This is Euler's constant.

$$\gamma = \lim_{n \rightarrow \infty} \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln(n+1) \right)$$

Imagine sliding all the shaded bits horizontally to the left. They will all fit inside the first block with room to spare. So $\gamma < 1$

Incidentally, it is not known if γ is rational or irrational.