




# Data Structures Review

Quiz 2



Interviewer: Asks me  
a sorting algorithm

Nervous me:



# Quadratic Sorts

# Selection Sort (selecting the smallest element)

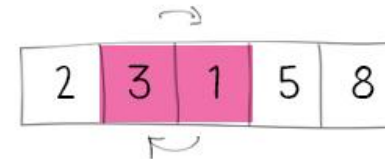
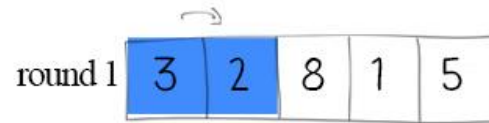
13	46	24	52	20	9	13 > 9 ==> swap
9	46	24	52	20	13	46 > 13 ==> swap
9	13	24	52	20	46	24 > 20 ==> swap
9	13	20	52	24	46	52 > 24 ==> swap
9	13	20	24	52	46	52 > 46 ==> swap
9	13	20	24	46	52	Sorted array



# Insertion Sort (insert next element into sorted list)

8	7	5	9	1	6	2	4	3
7	8	5	9	1	6	2	4	3
5	7	8	9	1	6	2	4	3
5	7	8	9	1	6	2	4	3
1	5	7	8	9	6	2	4	3
1	5	6	7	8	9	2	4	3
1	2	5	6	7	8	9	4	3
1	2	4	5	6	7	8	9	3
1	2	3	4	5	6	7	8	9

# Bubble Sort (compare adjacent elements)





# Practice Problem

**Exercise** Suppose we have the following array contents after the **third pass** of the outer loop of some quadratic sorting algorithms meant to put the array in ascending order:

3, 5, 7, 8, 2, 9, 4, 10, 15, 20

Which sorting algorithm could be operating on this array?

- A) bubble (up) sort
- B) (min) selection sort
- C) insertion sort
- D) none of these



# Linear Data Structures

Stack, Queue, Linked Lists

# Array List

## ARRAY LIST

- Space used is  $O(n)$
- `Size()`, `isEmpty()`, `get()`, and `set()` are  $O(1)$  operations
- `Add()` and `remove()` run in  $O(n)$
- When adding to full array, grow!

## GROW():

- Replace the array  $k = \log_2 n$  times
- What is  $T(n)$ ?
  - $n + 1 + 2 + 4 + 8 + \dots + 2^k = n + 2^{k+1} = 2n - 1$
  - $O(n)$
- The amortized runtime is  $O(1)!!!!$



# Stack

## STACK STUFF

- LIFO (Last in, First out) Data structure
- Push() Inserts element to the top of the stack
- Pop removes(/returns) the last inserted element
- Top() returns last inserted element

## ARRAY BASED IMPLEMENTATION

- We can use an array to implement a stack
- Add elements from left to right
- Keep track of the index of the top element!!
- Popping an element → Decrement the top pointer
- Pushing an element → increment the stack pointer and insert.

# Stack

## ARRAY IMPLEMENTATION

Operation	How?	Time
<code>push</code>	add to end: <code>arr[numElement++]</code>	$O(1)$
<code>pop</code>	delete from end: <code>numElement--</code>	$O(1)$
<code>top</code>	return last: <code>arr[numElement - 1]</code>	$O(1)$
<code>empty</code>	check if <code>numElement == 0</code>	$O(1)$

# Stack

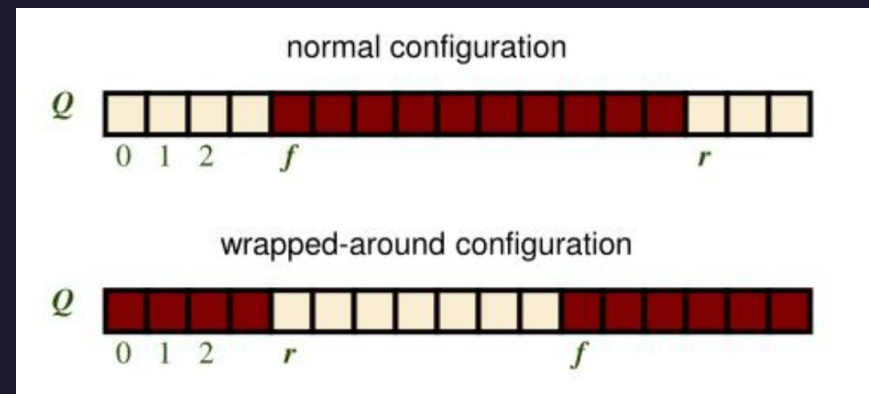
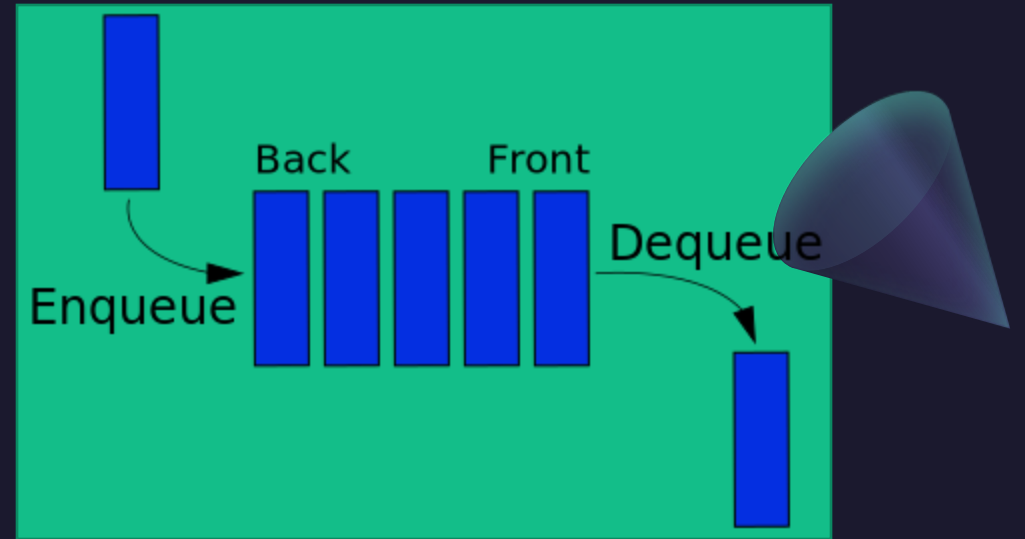
## LINKED LIST IMPLEMENTATION

Operation	How?	Time
<code>push</code>	prepend the list and update the <code>head</code>	$O(1)$
<code>pop</code>	delete from front: <code>head = head.next</code>	$O(1)$
<code>top</code>	return <code>head.data</code>	$O(1)$
<code>empty</code>	check if <code>head</code> is <code>null</code>	$O(1)$

# Queue

## QUEUE STUFF

- Use an array of size  $N$  in a circular fashion
- Two variable to keep track of the front and rear
  - Index of the front element
  - Index immediately past the rear element
- $\text{Size} = (N - f + r) \bmod N$



# Queue

## LINKED LIST IMPLEMENTATION

Operation	How?	Time
<code>enqueue</code>	append the list and update the <code>tail</code>	$O(1)$
<code>dequeue</code>	delete from front: <code>head = head.next</code>	$O(1)$
<code>front</code>	return <code>head.data</code>	$O(1)$
<code>empty</code>	check if <code>head</code> is <code>null</code>	$O(1)$

# Queue

## ARRAY IMPLEMENTATION

Operation	How?	Time
enqueue	<code>data[back] = value</code> and <code>back = ++back % length</code>	$O(1)$
dequeue	<code>front = ++front % length</code>	$O(1)$
front	return <code>arr[front]</code>	$O(1)$
empty	check if <code>numElement == 0</code>	$O(1)$



# Linked Lists

## SENTINEL NODES

- Dummy nodes at either end
- Head & Tail Pointer
- Removes edge cases

## POSITION ADT

- Models the notion of a place within a data structure where a single object is stored.
- Gives a unified view
- One method – `get()`
- Protect the Node Class

# Iterators

## ITERATORS

- An object that enables you to traverse through a collection and to remove elements from the collection selectively if desired.
- Fail Fast & Version Numbers

# Cooler Data Structures

Sets, Trees, Maps

# Sets

## SETS

- **Iterable** collection of unique elements
- `Insert()`, `Remove()` `Has()`
- $O(n)$  – Linked List Implementation
- $O(n)$  – Array (Isn't sorted)
- **DO NOT WORRY ABOUT ORDER WHEN ITERATING!**

## HEURISITICS

- **Transpose Sequential Search**
  - Search an array or list by checking items one at a time. If the value is found, swap it with its predecessor so it is found faster next time.
- **Move to Front**
  - moves the target of a search to the head of a list so it is found faster next time.

# Ordered Set

## SETS

- **Iterable** collection of **ordered** unique elements
  - Generics are bounded by Comparable Interface → Extends Comparable<T>
- $O(n)$  – Linked List Implementation (Find)
- $O(\lg n)$  – Array (Binary Search)
  - Insert and Remove are  $O(n)$  → Elements need to be shifted

# Binary Search Trees

## TREE STUFF

- Leaf – A node at the end of a tree with no children
- Root – node with out a parent
- Depth – number of ancestors deep excluding itself
- Height – Maximum depth of any node

## BST STUFF

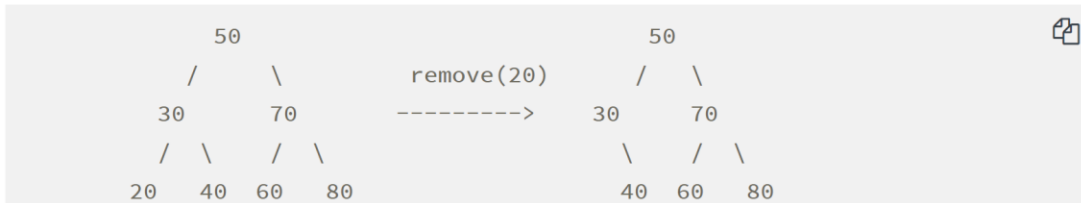
- In a binary tree, each node has at most two children.
- A BST – (Complete Ordering)
  - All elements to the left of a node are smaller.
  - All elements to the right of a node are larger.



# BST Remove

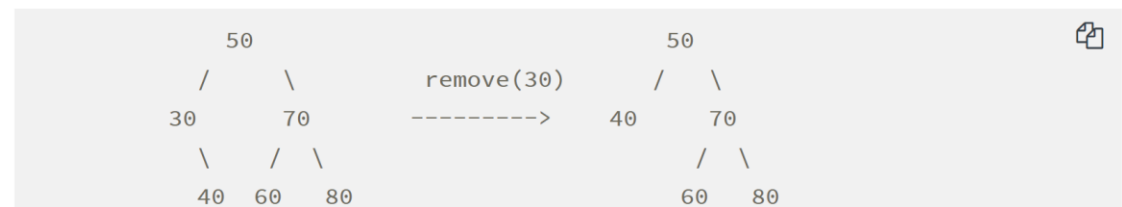
## NODE IS A LEAF

- **Node to be removed is a leaf:** Simply remove from the tree!



## NODE HAS ONE CHILD

- **Node to be removed has only one child:** Copy the child value into the node and delete the child node.

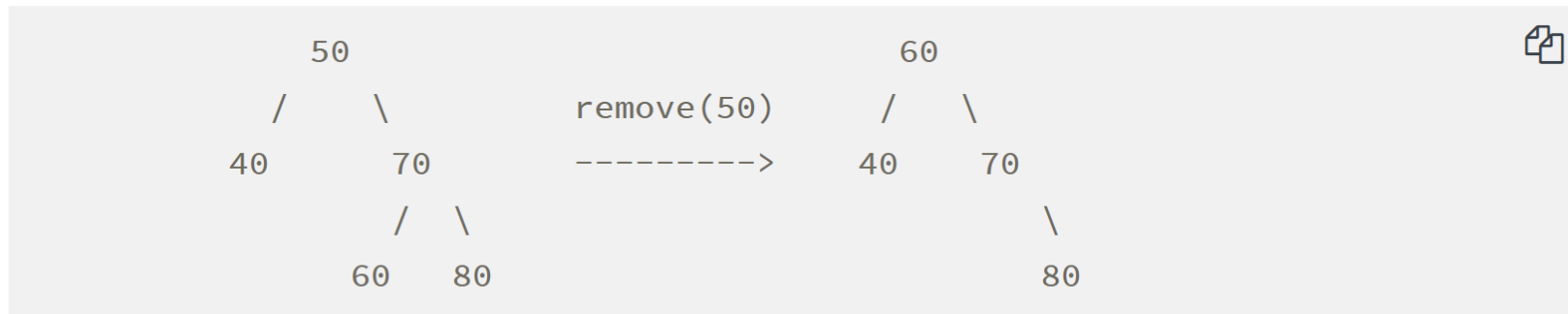


# BST Remove

## NODE HAS TWO CHILDREN

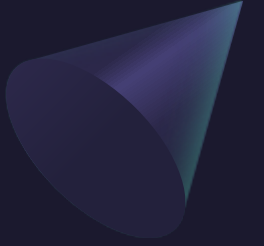
- ***Node to be removed has two children:***

1. Find the smallest value in the node's right subtree (*in-order* successor).
2. Copy the value key of the in-order successor to the target node and delete the in-order successor.



- Note that the largest value in the left subtree (in-order predecessor) can also be used.

# Tree Traversal



## LEVEL-ORDER

- nodes are visited level by level from left to right

## PRE-ORDER

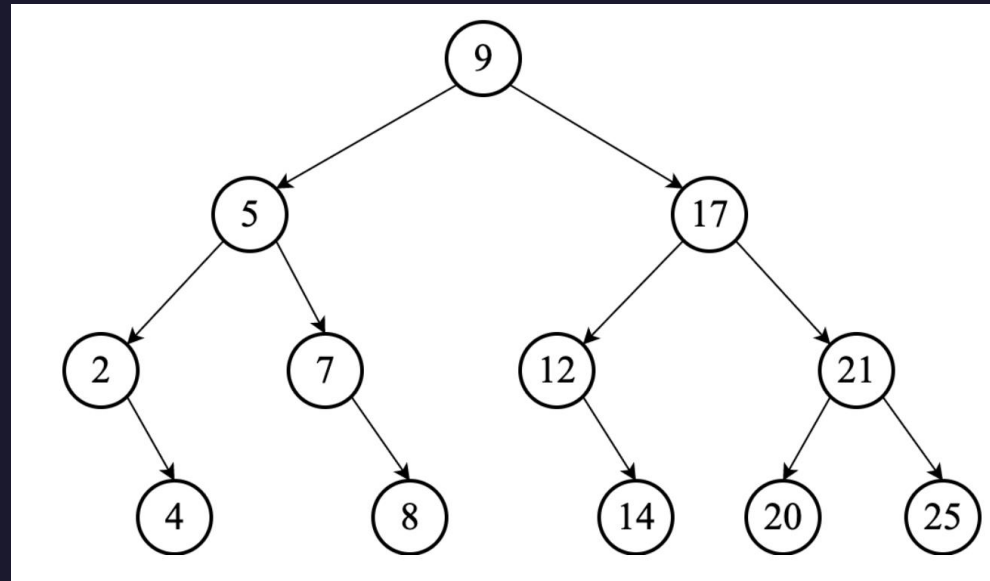
- For every node, visit it, then visit its left subtree, then visit its right subtree.

## POST-ORDER

- For every node, visit its left subtree, then visit its right subtree, then visit the node.

## IN-ORDER

- For every node, visit its left subtree, then visit the node, then visit its right subtree.



N – number of nodes  
E – number of external nodes  
I – Number of internal nodes  
H - height

# Properties

$$E = I + 1$$

$$N = 2e - 1$$

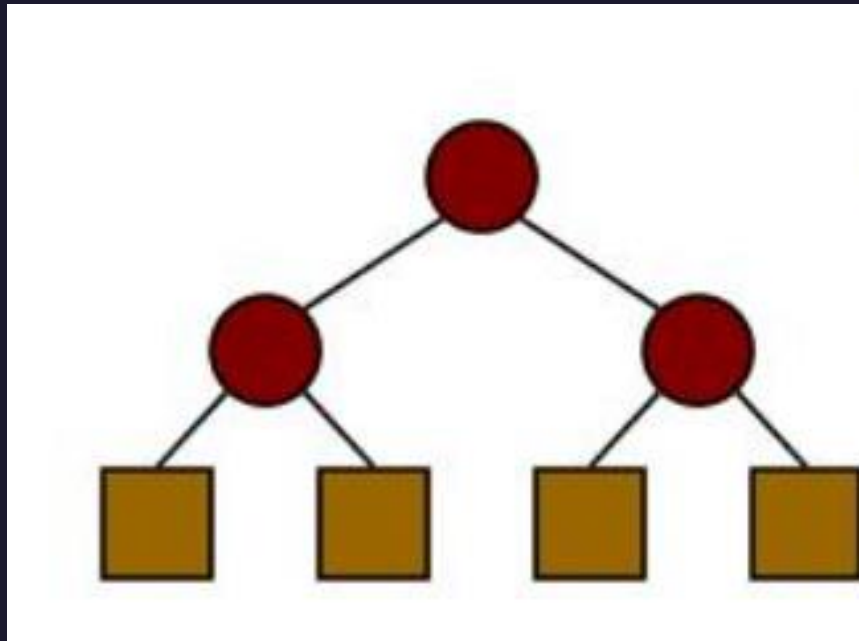
$$H \leq I$$

$$H \leq (n - 1)/2$$

$$E \leq 2^h$$

$$H \geq \log_2 e$$

$$H \geq \log_2(n + 1) - 1$$



Perfect BST has height  $O(\lg n)$

# Balanced Binary Search Trees

## BBST STUFF

- For any node, the heights of the node's left and right subtrees are either equal or differ by at most one.
- Each node's balance factor is the height of the left subtree minus the height of the right subtree.
  - $\text{bf}(\text{node}) = \text{height}(\text{node.left}) - \text{height}(\text{node.right})$
  - Height = for null, 0 for leaf,  $1 + \max(\text{height of children})$

## BST STUFF

- In a binary tree, each node has at most two children.
- A BST – (Complete Ordering)
  - All elements to the left of a node are smaller.
  - All elements to the right of a node are larger.



# General Study/Prep Tips

- Read the notes!
- Ask questions on Courselore
- Complete the practice midterm
- Do the optional exercises (i.e. implement bubble sort recursively)



# Questions?

