SO FAR: CONSTRUCTED MODEL FOR MOTOR FROM BASIC LAWS

Mech. Eq. (at MOTOR'S SIDE)

$$\tau_m = J_{eq} \frac{d\omega_m}{dt} + B_{eq}\, \omega_m + \frac{1}{N} \tau_d$$

$$\tau_d = \tau_{SF}\, \mathrm{sign}(\omega_m)$$

? NOT IN DATASHEET ?

→ We need to devise an experiment!

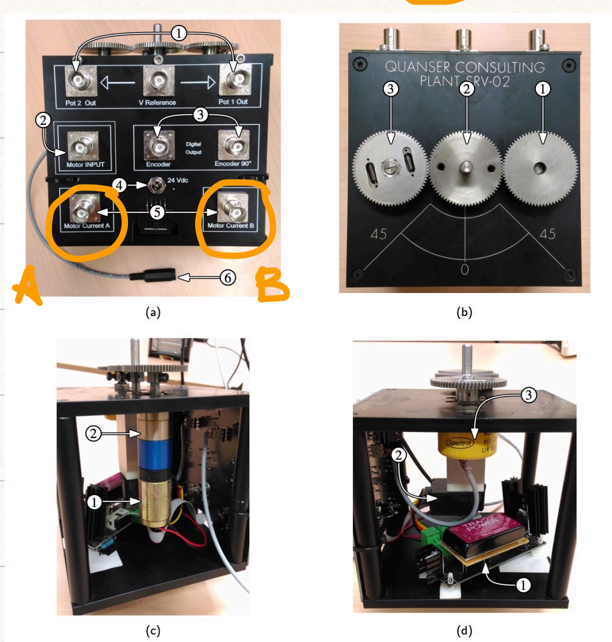
to
- TEST the model
- ESTIMATE MISSING PARAMETERS.

▷ Let's focus on τ_m

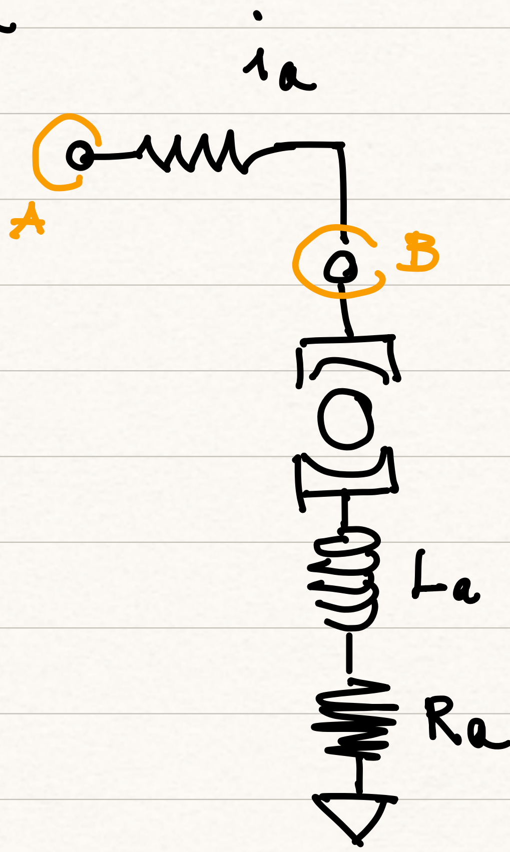
DATASH.

$$\tau_m = K_T\, i_a$$

?



(a) (b) (c) (d)



V_{AB} can be measured; $V_{AB} = R_s \, i_a$ Datash.

$$\Rightarrow \quad i_a = \frac{V_{AB}}{R_s}$$

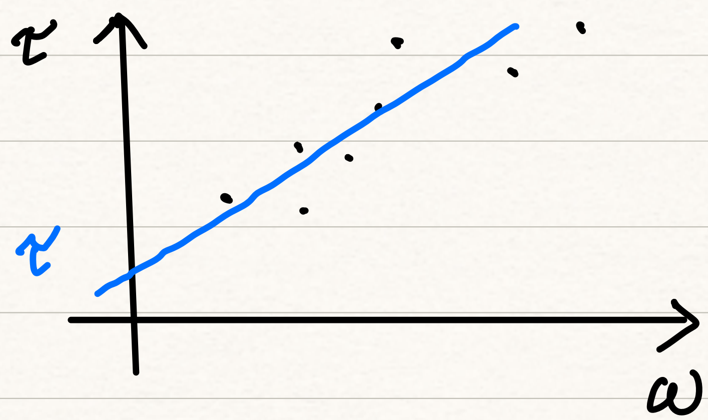
$$\tau_m = \frac{K_T V_{AB}}{R_s} = J_{eq}\,\dot{\omega}_m + B_{eq}\,\omega_m + \tau_d$$

<u>IDEA</u>: Assume we make the MOTOR work at <u>CONSTANT ω_m</u>

$\Rightarrow$ τ_m is an affine function of ω_m

$\Rightarrow$ We can : 1) MEASURE $\tau_{m,k}$ FOR VARIOUS CONSTANT $\omega_{m,k}$

2) PLOT AND OBTAIN LEAST SQUARE INTERPOLATION



LS METHOD :

→ collect $\tilde{\tau}_{m,k}$ (from $V_{AB,k}$)

→ For each k we can define:

"FITTED" TORQUES

$$\tau_{f,k} := \varphi_k^T \vartheta$$

$$\varphi_k^T = \left[\omega_{m,k} \quad \frac{1}{N}\,\mathrm{sign}(\omega_{m,k})\right]$$

$$\vartheta = \begin{bmatrix} B_{eq} \\ \tilde{\tau}_{sf} \end{bmatrix}$$ TO BE ESTIMATED.

→ DEFINE THE COST FUNCTIONAL

$$V(\vartheta) = \sum_{k=1}^{M} \left(\tilde{\tau}_{m,k} - \varphi_k^T \vartheta \right)^2$$

→ MOVE TO MATRIX FORM :

DATA

$$Y = \begin{bmatrix} \tilde{\tau}_{m,1} \\ \vdots \\ \tilde{\tau}_{m,M} \end{bmatrix} = \frac{K_T}{R_S} \begin{bmatrix} V_{AB,1} \\ \\ V_{AB,2} \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \varphi_1^T \\ \vdots \end{bmatrix}$$ ← ANGULAR VELOC.

$$\begin{bmatrix} \vdots \\ \varphi_M^T \end{bmatrix}$$ (MEAS. DATA)

$$\Rightarrow \quad V(\vartheta) = [Y - \Phi\vartheta]^T [Y - \Phi\vartheta]$$

solve L.S. : $\hat{\vartheta}_{LS} = \arg\min_{\vartheta \in \mathbb{R}^2_{(+)}} V(\vartheta)$

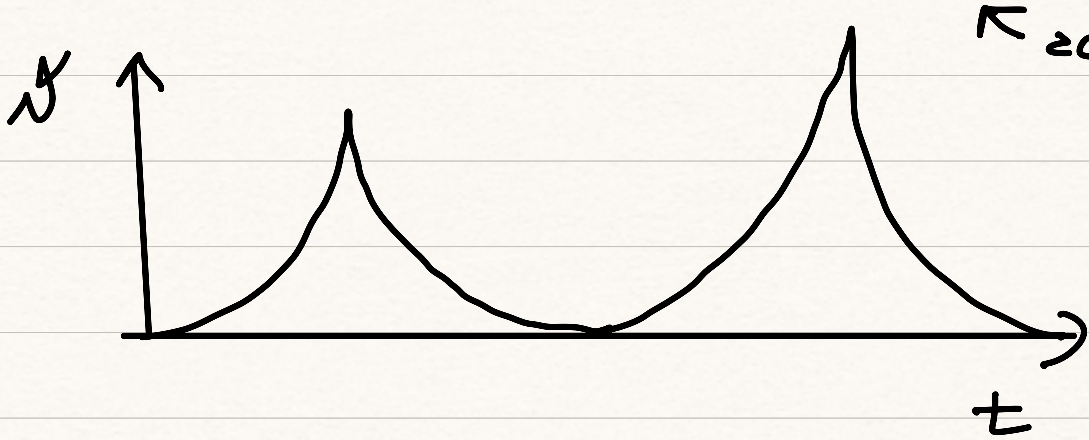
$\Rightarrow$ CLOSED FORM SOLUTION

$$\hat{\vartheta}_{LS} = \begin{bmatrix} \hat{\beta}_{eq} \\ \hat{\tau}_{sf} \end{bmatrix} = \left(\Phi^T \Phi\right)^{-1} \Phi^T Y$$

we can use these to improve the model

ESTIMATION OF J_{eq} : SIMILAR APPROACH

this time we impose $\dot{\omega}_m$ = constant
acceleration



(PARABOLIC RAMPS.)

$$\tau_m = \underbrace{J_{eq}\,\dot{\omega}_m}_{\tilde{\tau}} + \underbrace{B_{eq}\,\omega_m + \tau_{SF}\,\mathrm{sign}(\omega_m)}_{\tau_F}$$

- choose $\dot{\omega}_{m,\ell} = \pm|\dot{\omega}_\ell|$ as WORKING POINTS.
- MEASURE CORRESPONDING $V_{AB}(t_{k,\ell})$ $\omega_m(t_{k,\ell})$ AT DIFFERENT TIMES $t_{k,\ell}$ ← same ℓ same $\dot{\omega}_\ell$
- ESTIMATE (from ℓ- meas) $\hat{\tau} = \tau_m - \tau_F$

$$\hat{\tau}_{k\ell} = \frac{K_T}{R_S} V_{AB}(t_{k\ell}) - \tau_F(t_{k,\ell})$$

FORMULA ABOVE

- Take the average over $\underline{\underline{k}}$: $\hat{\tau}_\ell$ and

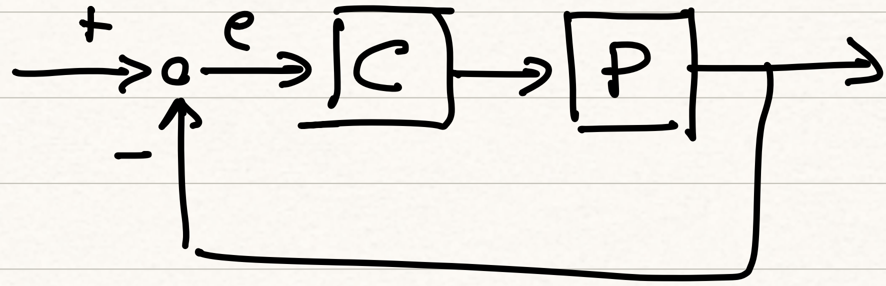
$$\hat{J}_{eq,\ell} := \frac{\tau_{\ell,+} - \tau_{\ell,-}}{\dot{\omega}_{\ell,+} - \dot{\omega}_{\ell,-}}$$

Repeat and average over ℓ

SOME PRACTICAL ISSUES :

(1) How do we impose $\dot{\omega} = 0$, cst ?

TRACKING RAMPS $\longrightarrow$ IMP



$$P(s) = \frac{K_m}{1 + T_m s} \; \frac{1}{N s}$$

$$C(s) = C_{PID}(s) = K_p \frac{s^2 (T_I T_L + T_I T_D) + s (T_I + T_L) + 1}{T_I s \,(1 + T_D s)(1 + T_L s)}$$

Poles of CP in $s = 0$? #2

IF ONLY #1 → FINITE ERROR.

Verify : (e.g. F.V.T.)

$$\lim_{t \to +\infty} e(t) = \lim_{s \to 0} s E(s) \quad \left(\text{WITH ONE POLE}\right)$$

$$(\ldots)$$

$$= \frac{T_I}{T_I + \frac{K_m}{N} K_p}$$

(2) OUR OUTPUT : ϑ_m

$$\omega_m = \frac{d}{dt}\vartheta_m$$

$$\downarrow$$

$\sim s$ NOT PROPER, SENSITIVE TO NOISE

(3) $\dot{i}_{AB} = \frac{V_{AB}}{s}$: NOISY MEASUREMENTS.

Same Solution : USE LOW-PASS FILTERS (II° ORDER)

$$H_{LP}(s) = \frac{\omega_c^2}{s^2 + 2\xi\omega_c s + \omega_c^2} \qquad \xi = 1/\sqrt{2}$$

$\Rightarrow s\,H_{LP}(s)$

Filtered derivative