

Laboratory activity 0 ASSIGNMENTS: Position PID–control of a DC servomotor

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1 Activity goal

The goal of this laboratory activity is to design a position PID controller for the DC servomotor available in laboratory. The design is carried out in frequency domain (i.e. *Bode's method*) with the adoption of a nominal model of the motor, whose parameters are deduced from data–sheets. In the second part of the activity, the values of the motor parameters, including the static and viscous friction, are estimated with simple experimental tests. The parameters will be used in the next laboratory activity to improve the control performances, in particular by implementing a feedforward inertia plus friction compensation scheme.

For remote-only laboratory attendance:

Estimated values of friction and inertia parameter from simulations run on the provided black-box model are to be uploaded in elearning in a (max 2-page) pdf file where the PID parameters values are also included. This will count as your "presence" test for the LAB.

DEADLINE: Monday March 28 2022 (before 23:59:59). A repository will be created on the week of the lab.

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2 Laboratory assignments: numerical simulations

2.1 Simulink model of the DC servomotor with inertial load

1. Implement a Simulink model of the DC gearmotor available in laboratory, according to the block diagram and the the model parameters shown in the Handout. Since the nominal values of the friction parameters are still unknown (they will be estimated in a following laboratory activity), use the following tentative values to test the Simulink model:

$$B_{eq} = 2.0 \times 10^{-6} \text{ Nm/(rad/s)}, \quad \tau_{sf} = 1.0 \times 10^{-2} \text{ Nm} \quad (1)$$

Moreover, remind that $J_l = J_d + 3 J_{72}$. The position and speed variables of the model are measured in [rad] and [rad/s], respectively. However, for analysis purposes, it is preferred to measure the position in [deg] units, and the speed in [rpm] (rotations per minute). Therefore, consider to insert the following conversion gains in the Simulink model

- conversion gain rad2deg from [rad] to [deg] units at the motor position output. The gain is equal to $\text{rad2deg}=180/\pi$.
- conversion gain rads2rpm from [rad/s] to [rpm] units at the motor speed output. The gain is equal to $\text{rads2rpm}=60/2/\pi$.

A possible implementation of the Simulink model is shown in Fig. 1a. The parameters used in the Simulink model are defined in the Listing 1.

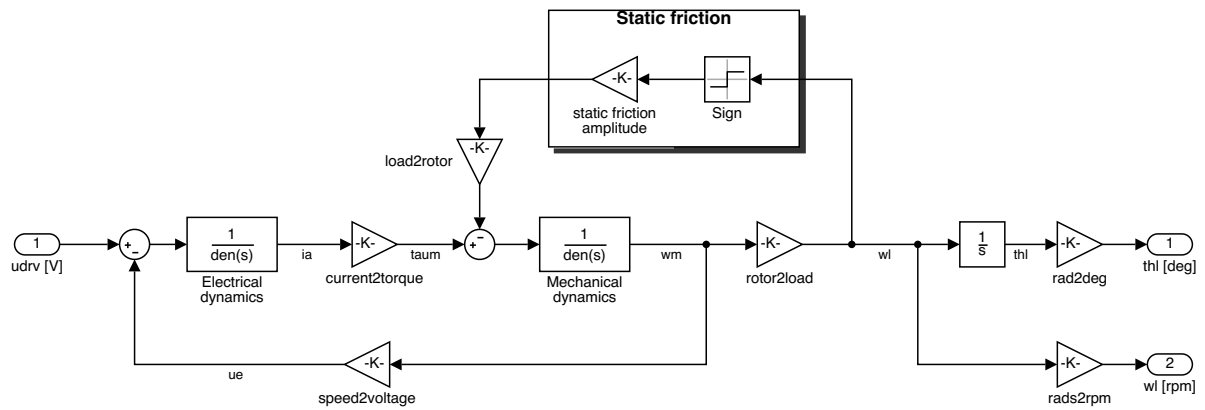
2. Implement the model of the voltage driver as shown in Fig. 1b. The model includes a *Continuous* \rightarrow *Transfer Function* block specifying the transfer function

$$H_{drv}(s) = \frac{k_{drv}}{T_{drv} s + 1} \quad (2)$$

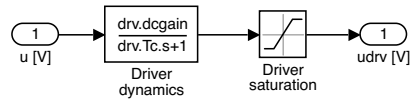
which defines the driver lowpass dynamics, and a *Discontinuities* \rightarrow *Saturation* block that accounts for the voltage limitation of the power amplifier. Since the amplifier output cannot exceed the voltage supply levels of $\pm 12 \text{ V}$, set the saturation levels to those values.

3. Implement the model of the digital-to-analog (DAC) converter, which provides the reference voltage u to the voltage driver, as shown in Fig. 1c. The model includes a *Discrete* \rightarrow *Zero-Order Hold* block to simulate the holding process of the DAC output within each sampling period, and a *Discontinuities* \rightarrow *Quantizer* block to simulate the quantisation of the DAC output due to the finite resolution of the converter. Finally, a *Discontinuities* \rightarrow *Saturation* block accounts for the finite DAC output range. Set the sampling time of the ZOH block to $T_s = 1 \text{ ms}$, which is the sampling time that will be used to run all the experiments. Set the levels of the saturation block to $\pm 10 \text{ V}$, which correspond to the positive/negative full-scale values of the DAC. Since the DAC resolution is equal to 16 bits, set the quantisation interval of the quantiser block equal to $q = 20/(2^{16} - 1) \text{ V}$.
4. Implement the model of the incremental optical encoder (used to measure the load position) as shown in Fig. 1d. The model includes a *Discrete* \rightarrow *Zero-Order Hold* block to simulate

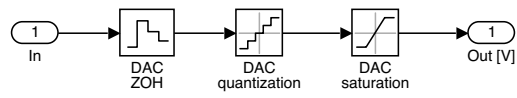
¹Note that there is no specific block for the ideal sampler in Simulink. Although counterintuitive, the conventional way of implementing an ideal sampler in Simulink consists of using a *Discrete* \rightarrow *Zero-Order Hold* block, which is also used (as its name states) to implement the zero-order discrete-to-continuous time interpolator.



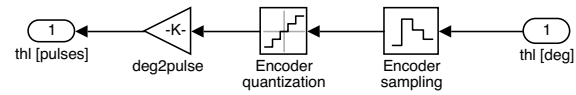
(a) DC motor + gearbox



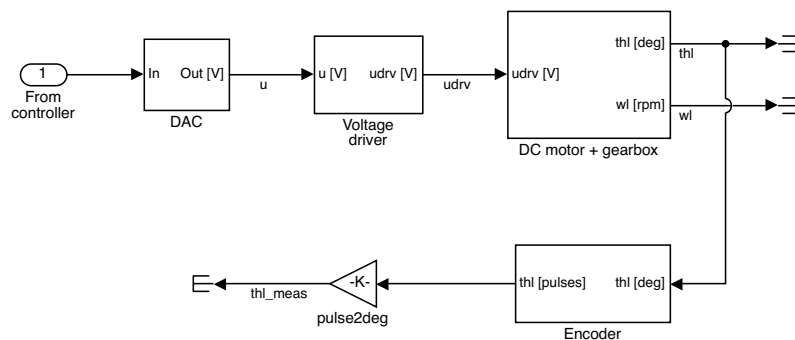
(b) Voltage driver



(c) DAC



(d) Encoder



(e) DC servomotor (top model)

Figure 1: Detailed Simulink model of the DC servomotor.

the ideal sampling of the *load-side* position measurement¹, and a *Discontinuities* \rightarrow *Quantizer* block to account for the finite resolution of the position transducer. Set the sampling time of the ZOH block to $T_s = 1 \text{ ms}$, which is the sampling time that will be used to run all the experiments. Set the quantisation interval q of the quantiser block according to the resolution of the encoder installed on the Quanser SRV-02 unit used for the experiments:

- for all the units except “MOTORE 8” and “MOTORE 10”: $q = 360/(500 \times 4) \text{ deg}$
- for the “MOTORE 8” and “MOTORE 10” units: $q = 360/(1024 \times 4) \text{ deg}$

Since the encoder output is the “pulse count”, rather than the position measurement in [deg] units, insert a *Math* \rightarrow *Gain* block at the quantiser output to perform the conversion from degrees units to pulse count. For such purpose, set the gain equal to $1/q$.

5. Combine all the models prepared in points 1–4, to form a Simulink model of the whole DC servomotor, as shown in Fig. 1e. It is desired to have a position measurement in [deg] units. Therefore, consider to insert a pulse2deg conversion gain at the encoder output, to perform the conversion from “pulse count” to [deg] units. The gain is equal to pulse2deg=360/ppr, where ppr is the number of encoder pulses per rotation (i.e. 500×4 on all the motor units except “MOTORE 8” and “MOTORE 10”; 1024×4 on “MOTORE 8” and “MOTORE 10” units).

Listing 1: load_params_inertial_case.m

```

1 %% General parameters and conversion gains
2
3 % conversion gains
4 rpm2rads = 2*pi/60;           % [rpm] -> [rad/s]
5 rads2rpm = 60/2/pi;          % [rad/s] -> [rpm]
6 rpm2degs = 360/60;           % [rpm] -> [deg/s]
7 degs2rpm = 60/360;           % [deg/s] -> [rpm]
8 deg2rad = pi/180;            % [deg] -> [rad]
9 rad2deg = 180/pi;            % [rad] -> [deg]
10 ozin2Nm = 0.706e-2;          % [oz*inch] -> [N*m]
11
12 %% DC motor nominal parameters
13
14 % brushed DC-motor Faulhaber 2338S006S
15 mot.R = 2.6;                  % armature resistance
16 mot.L = 180e-6;               % armature inductance
17 mot.Kt = 1.088 * ozin2Nm;     % torque constant
18 mot.Ke = 0.804e-3 * rads2rpm; % back-EMF constant
19 mot.J = 5.523e-5 * ozin2Nm;   % rotor inertia
20 mot.B = 0.0;                  % viscous friction coeff (n.a.)
21 mot.eta = 0.69;               % motor efficiency
22 mot.PN = 3.23/mot.eta;        % nominal output power
23 mot.UN = 6;                   % nominal voltage
24 mot.IN = mot.PN/mot.UN;       % nominal current
25 mot.tauN = mot.Kt*mot.IN;     % nominal torque
26 mot.taus = 2.42 * ozin2Nm;    % stall torque
27 mot.w0 = 7200 * rpm2rads;     % no-load speed
28
29 %% Gearbox nominal parameters
30
31 % planetary gearbox Micromotor SA 23/1
32 gbox.N1 = 14;                 % 1st reduction ratio (planetary gearbox)

```

```

33 gbox.eta1 = 0.80; % gearbox efficiency
34
35 % external transmission gears
36 gbox.N2 = 1; % 2nd reduction ratio (external trasmission gears)
37 gbox.J72 = 1.4e-6; % inertia of a single external 72 tooth gear
38 gbox.eta2 = 1; % external trasmission efficiency (n.a.)
39
40 % overall gearbox data
41 gbox.N = gbox.N1*gbox.N2; % total reduction ratio
42 gbox.eta = gbox.eta1*gbox.eta2; % total efficiency
43 gbox.J = 3*gbox.J72; % total inertia (at gearbox output)
44
45 %% Mechanical load nominal parameters
46
47 % inertia disc params
48 mld.JD = 3e-5; % load disc inertia
49 mld.BD = 0.0; % load viscous coeff (n.a.)
50
51 % overall mech load params
52 mld.J = mld.JD + gbox.J; % total inertia
53 mld.B = 2.5e-4; % total viscous fric coeff (estimated)
54 mld.tausf = 1.0e-2; % total static friction (estimated)
55
56 %% Voltage driver nominal parameters
57
58 % op-amp circuit params
59 drv.R1 = 7.5e3; % op-amp input resistor (dac to non-inverting in)
60 drv.R2 = 1.6e3; % op-amp input resistor (non-inverting in to gnd)
61 drv.R3 = 1.2e3; % op-amp feedback resistor (output to inverting in)
62 drv.R4 = 0.5e3; % op-amp feedback resistor (inverting in to gnd)
63 drv.C1 = 100e-9; % op-amp input capacitor
64 drv.outmax = 12; % op-amp max output voltage
65
66 % voltage driver dc-gain
67 drv.dcgain = drv.R2/(drv.R1+drv.R2) * (1 + drv.R3/drv.R4);
68
69 % voltage driver time constant
70 drv.Tc = drv.C1 * drv.R1*drv.R2/(drv.R1+drv.R2);
71
72 %% Sensors data
73
74 % shunt resistor
75 sens.curr.Rs = 0.5;
76
77 % Hewlett-Packard HEDS-5540#A06 optical encoder
78 sens.enc.ppr = 500*4; % pulses per rotation
79 sens.enc.pulse2deg = 360/sens.enc.ppr; % [pulses] → [deg]
80 sens.enc.pulse2rad = 2*pi/sens.enc.ppr; % [pulses] → [rad]
81 sens.enc.deg2pulse = sens.enc.ppr/360; % [deg] → [pulses]
82 sens.enc.rad2pulse = sens.enc.ppr/2/pi; % [rad] → [pulses]
83
84 % potentiometer 1 (Spectrol 138-0-0-103) — installed on motor box
85 sens.pot1.range.R = 10e3; % ohmic value range
86 sens.pot1.range.V = 5; % voltage range
87 sens.pot1.range.th_deg = 345; % angle range [deg]
88 sens.pot1.range.th = sens.pot1.range.th_deg * deg2rad; % angle range [rad]
89 sens.pot1.deg2V = sens.pot1.range.V / sens.pot1.range.th_deg; % sensitivity [V/deg]
90 sens.pot1.rad2V = sens.pot1.range.V / sens.pot1.range.th; % sensitivity [V/rad]
91 sens.pot1.V2deg = 1/sens.pot1.deg2V; % conversion gain [V] → [deg]
92 sens.pot1.V2rad = 1/sens.pot1.rad2V; % conversion gain [V] → [rad]
93
94 %% Data acquisition board (daq) data
95

```

```

96 % NI PCI-6221 DAC data
97 daq.dac.bits = 16;           % resolution (bits)
98 daq.dac.fs = 10;            % full scale
99 daq.dac.q = 2*daq.dac.fs/(2^daq.dac.bits-1); % quantization
100
101 % NI PCI-6221 ADC data
102 daq.adc.bits = 16;          % resolution (bits)
103 daq.adc.fs = 10;            % full scale (as set in SLDRT Analog Input block)
104 daq.adc.q = 2*daq.adc.fs/(2^daq.adc.bits-1); % quantization

```

2.2 Design and numerical validation of the position PID controller

1. Design a position control system for the DC servomotor based on a standard PID controller.

The control design specifications are:

- perfect steady state tracking of step position (load side) references.
- perfect steady state rejection of constant torque disturbances.
- step response (at load side) with settling time $t_{s,5\%} \leq 0.15\text{ s}$ and overshoot $M_p \leq 10\%$

Use the Bode's method for the determination of the three controller gains. For the design, use the simplified plant model. Note that such model is specified with inputs in [V] units, and output in [rad] units. Therefore, the PID controller will be designed to accept a tracking error with [rad] units, and to generate a control signal in [V] units.

Since the total viscous friction is unknown (it will be estimated in the following points), set $B_{eq} = 0$ in the plant transfer function. For the ratio $\alpha = T_I/T_D$, choose $\alpha = 4$ as a first trial.

Consider to implement a Matlab function for the design of the PID controller, which should return the controller gains K_P , K_I and K_D , after selecting, on the basis of the design specifications, the most appropriate controller structure to use (P, PI, PD or PID).

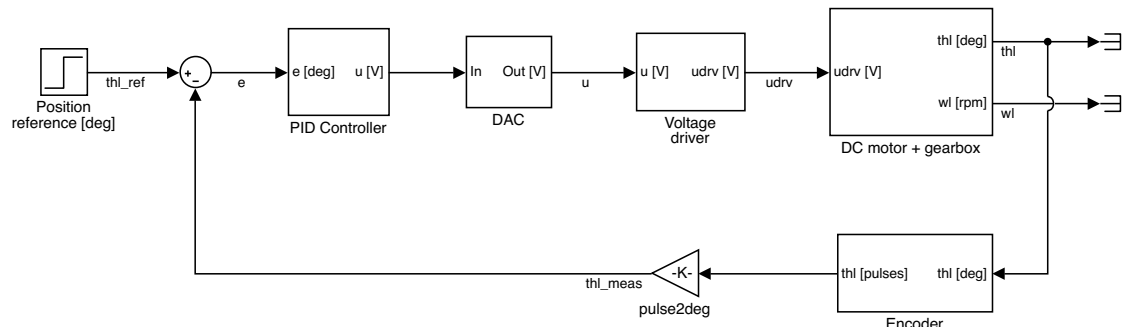
2. Implement a Simulink model of the position PID control loop, that includes the accurate model of the DC servomotor developed in the previous Sec. 2.1. For the PID implementation, use a "real derivative" block based on a first order high-pass filter with T_L selected according to the guideline reported in the Handout.

A possible implementation of the Simulink model is shown in Fig. 2.

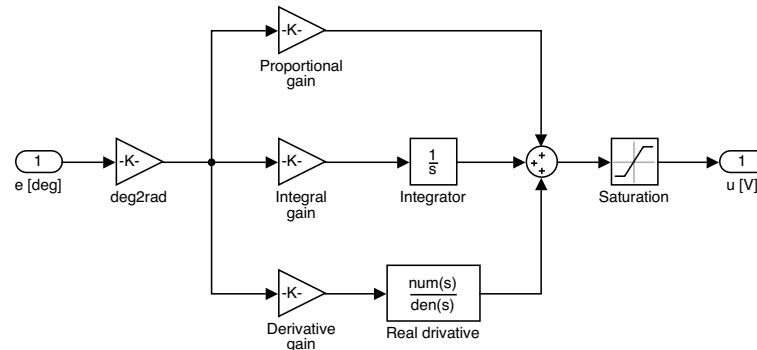
3. Test the performance of the position PID controller in simulation, by using the Simulink model implemented in point 2.

Simulate the response of the position control loop implemented in point 2 to different step reference inputs. Consider the following values for the step amplitude: 10° , 30° , 50° , 90° , 180° , 360° .

For the simulations, use a *Variable-step ode45 (Dormand-Prince)* solver (to be set in *Configuration Parameters* \rightarrow *Solver* \rightarrow *Type* and *Solver*). By choosing a variable-step solver, it could happen that the simulation hangs when the speed becomes very small, because the solver is unable to find a reasonably small step-size where the speed does not change sign (a "too many zero crossings" error will be issued by Simulink). To prevent such circumstance, deselect the option *Enable zero-crossing detection* in the *Block Parameters* window of the *Sign* block used to model the static friction torque.



(a) Position PID control loop (top model)



(b) PID controller

Figure 2: Detailed Simulink model of the position PID control system.

3 Laboratory assignments: experimental tests

3.1 Validation of the position PID controller with the motor black-box model

1. Prepare a Simulink model for testing the position PID controller designed in point 1 of Sec. 2.2 on the real DC servomotor available in laboratory ("real-time simulation"). For such purpose, it is sufficient to proceed as follows:

- make a copy of the Simulink model prepared in point 2 of Sec. 2.2 for the numerical simulations. A possible implementation of such model is shown in Fig. 2 and repeated in Fig. 3a.
- **For in-presence attendance only:** replace the model of the DC servomotor with the blocks of the Simulink Desktop Real-Time (SLDRT) toolbox that allow to communicate with the experimental device, as shown in Fig. 3b.

In addition to send the voltage command to the DAC (using the *Analog Output* block) and read the pulse count from the encoder (using the *Encoder Input* block), consider also to implement a block for sensing the motor current. Follow the instructions provided in the introductory guide to the experimental setup to implement such block (see also the implementation shown in Fig. 3b).

- **For remote attendance only:** replace the model of the DC servomotor with the motor simulink block we provide in the `Quanser_SRV02_block.zip` files - select the build that matches your operating system (Win, OSX or UNIX), use the `.slx` file for the simulink

model, and keep the .slxp file in the same directory. From now on this will be considered your experimental device². This model has the same structure of the “box” representing the DC servomotor in figure Fig. 3b. Thus, it has one input (the voltage input to the driver, u , expected in volts V), and two outputs: thl_meas that corresponds to the output of the encoder and hence it is in pulses, and uRs_meas that corresponds to the voltage at the shunt resistor of the motor armature. The latter allows you to implement a block for sensing the motor current. Follow the instructions provided in the introductory guide to the experimental setup to implement such block (see also the implementation shown in Fig. 3b).

- configure the simulation parameters of the new Simulink model to perform a “real-time simulation”. For the purpose, consult the instructions provided in the introductory guide to the experimental setup.

In particular, choose a *fixed-step ode3* solver, with step size equal to $T_s = 1$ ms (controller sampling time).

2. Repeat the test described in point 3 of Sec. 2.2 on the experimental device, by using the

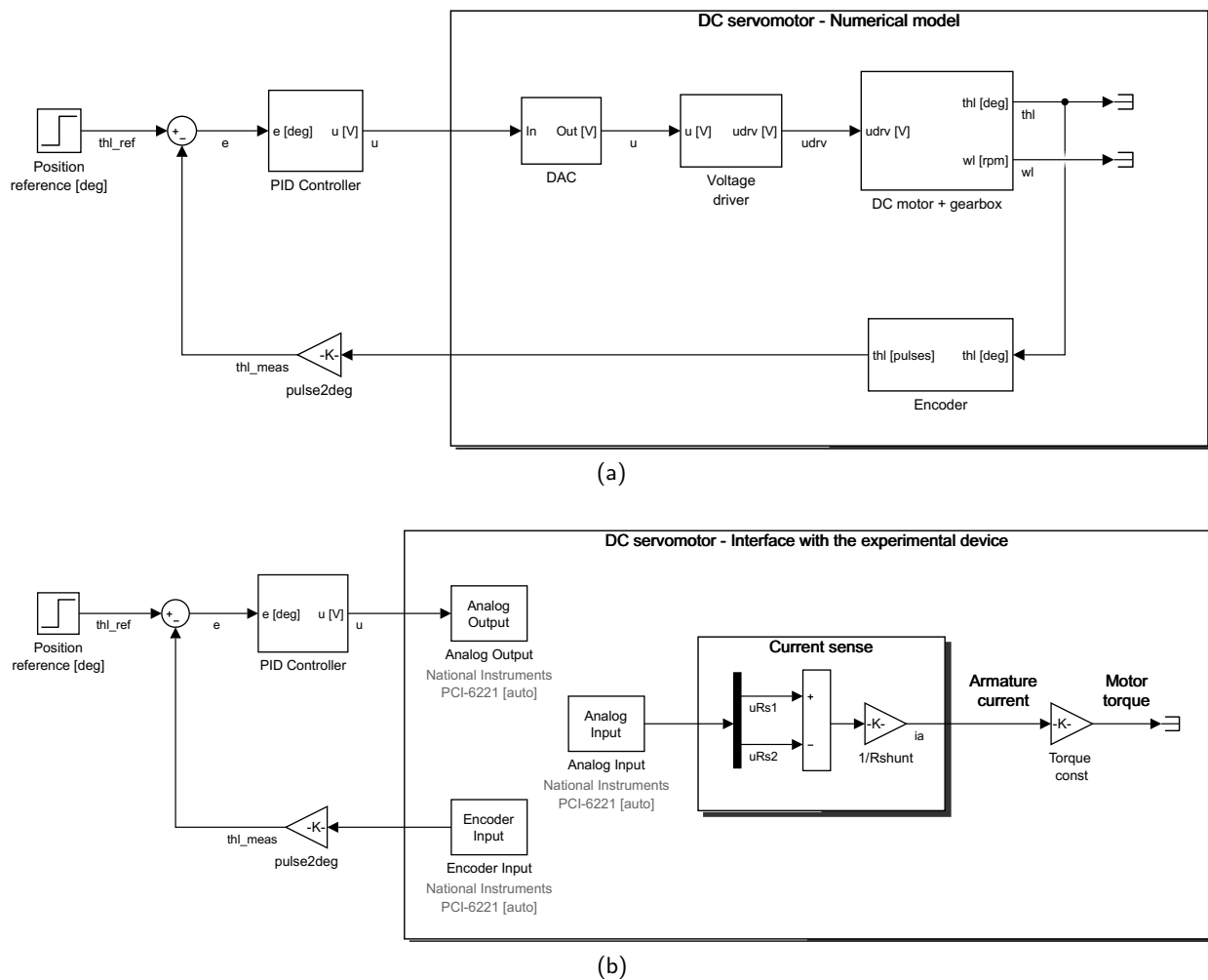


Figure 3: Position PID control loop: (a) Simulink model for numerical simulations; (b) Simulink model for experiments (“real-time simulations”).

²It has been constructed to match the behaviour of one of the motors in the lab so do not expect a perfect match between simulations of the model we derived in class and this one, even when you use the estimated friction parameters.

Simulink model prepared in the previous point.

3.2 Estimation of the friction parameters

1. Apply the procedure outlined in the Handout for estimating the parameters of the static and viscous friction. Since the estimation procedure requires to measure the motor torque (or current, provided that the motor torque constant K_t is known) at *constant* speed, use the position PID controller designed at point 1 to impose constant speed levels to the motor (indeed, verify that the position PID controller can be used for such purpose). Note that if ω_l^* is a desired speed reference to be imposed to the motor, then the corresponding position reference ϑ_l^* to be provided to the position control loop is obtained by integration, i.e.

$$\vartheta_l^*(t) = \int_0^t \omega_l^*(\tau) d\tau \quad (3)$$

For collecting multiple pairs of steady state torque vs speed measurements in a single experiment, consider to use a staircase speed reference of the type:

$$\omega_l^*(t) = k \Delta\omega \quad \text{with} \quad t \in [(k-1)\Delta T, k\Delta T), \quad k = 1, \dots, N^* \quad (4)$$

where $\Delta\omega$ and ΔT are the height and width of each speed step. Consider to perform a 45 s experiment with $\Delta\omega = 50$ rpm and $\Delta T = 5$ s, so that the final load side speed is 450 rpm, just a little below the motor nominal speed, and collect both the motor current and speed³ (see Fig. 4a). After completing the experiment with positive rotations, repeat it by reversing speed direction, i.e. by setting $\Delta\omega = -50$ rpm.

The motor speed can be obtained by differentiating the measured position with a suitable high-pass filter (“real derivative”). The filter cut-off frequency and roll-off must be chosen to avoid excessive amplification of the encoder quantisation noise at high frequency. A possible choice is the following second order Butterworth high-pass filter

$$H_\omega(s) = \frac{\omega_c^2 s}{s^2 + 2\delta\omega_c s + \omega_c^2} \quad \text{with} \quad \omega_c = 2\pi 20, \quad \delta = 1/\sqrt{2} \quad (5)$$

The motor current can be indirectly obtained by measuring the voltage drop across the shunt resistor, as explained in the introductory guide to the experimental setup. Note that the measured current is very noisy, due to the commutations of the brushes contacts occurring while the motor is rotating. To reduce the commutation noise, the current measurement can be filtered with a suitable low-pass filter, before proceeding with the estimation of the friction parameters. A possible choice is the following second order low-pass filter

$$H_i(s) = \frac{\omega_{c,i}^2}{s^2 + 2\delta_i\omega_{c,i}s + \omega_{c,i}^2} \quad \text{with} \quad \omega_{c,i} = 2\pi 20, \quad \delta_i = 1/\sqrt{2} \quad (6)$$

2. For each time interval

$$t \in [(k-1)\Delta T, k\Delta T), \quad k = 1, \dots, N^* \quad (7)$$

³Alternatively, collect the motor position, and then compute the required speed measurement offline, by using the high-pass filter (5).

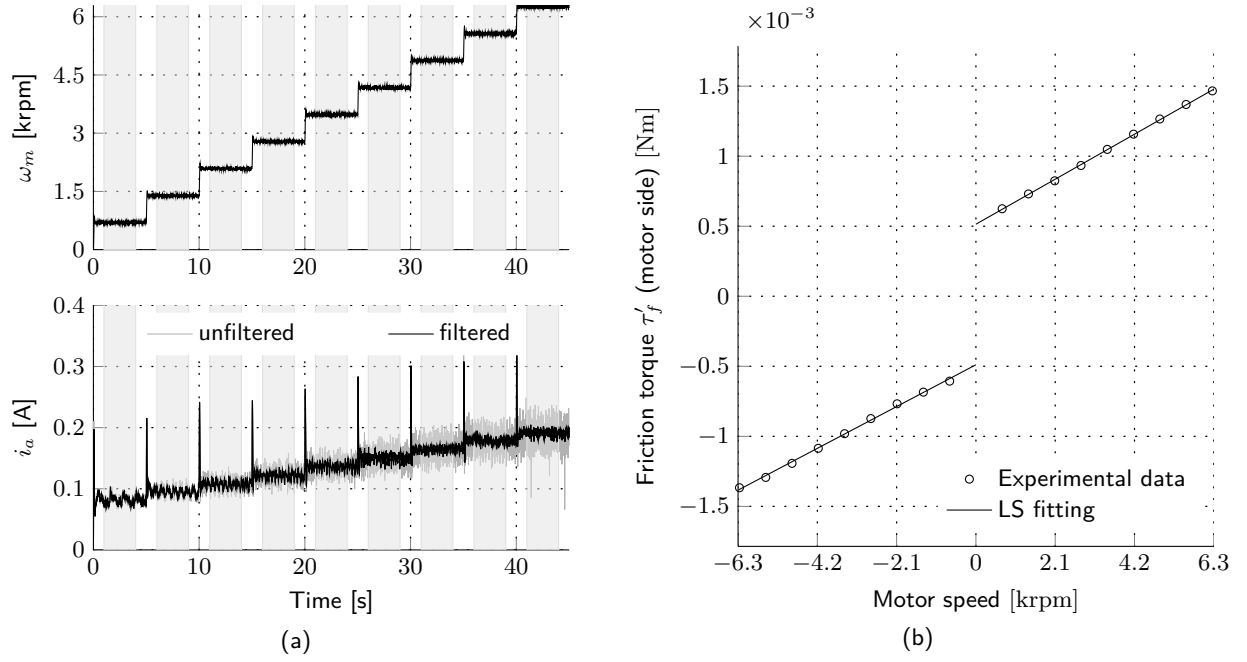


Figure 4: Friction estimation test: (a) experimental data (for positive speed profile); (b) LS estimation of friction parameters. Shaded areas in (a) represent the time interval over which the speed/current averages are computed.

within which the motor speed is constant, compute the average values $\omega_{m,k} = N \omega_{l,k}$ and $\tau_{m,k} = K_t i_{a,k}$ of the motor speed and torque. In each time interval, avoid to consider the initial speed and torque transients in the computation of the average values (i.e. consider only the portion of data where a constant steady state value is reached – see shaded areas in Fig. 4a).

Remind that two tests were performed, one with a positive staircase speed reference, and the other with a negative reference. Consider to compute the speed/torque averages for the data of both tests. Denote with $Z_+^{N*} = \{\tau_{m,k}, \omega_{m,k}\}$ the data set obtained with the positive reference speed, and with $Z_-^{N*} = \{\tau_{m,k}, \omega_{m,k}\}$ the data set obtained with the negative reference speed.

3. With the data set $Z_+^{N*} = \{\tau_{m,k}, \omega_{m,k}\}$ obtained in point 2, obtain the friction parameters \hat{B}_{eq+} and $\hat{\tau}_{sf+}$ using the estimation procedure outlined in the Handout. Please pay attention to use the appropriate measurement units when applying the estimation procedure: in particular, the torque should be expressed in [Nm], and the speed in [rad/s]. In this way, \hat{B}_{eq+} will be expressed in [Nm/(rad/s)], and $\hat{\tau}_{sf+}$ in [Nm].

Repeat the estimation for the data set Z_-^{N*} , obtaining the estimates \hat{B}_{eq-} and $\hat{\tau}_{sf-}$. Then, obtain the overall estimates of the two friction parameters as follows

$$\hat{B}_{eq} = \frac{\hat{B}_{eq+} + \hat{B}_{eq-}}{2}, \quad \hat{\tau}_{sf} = \frac{|\hat{\tau}_{sf+}| + |\hat{\tau}_{sf-}|}{2} \quad (8)$$

The final friction torque characteristic (*motor side*, with speed plotted in [krpm]) should look like that reported in Fig. 4b.

4. Suppose that it is desired to assess the uncertainty that affects the estimates (8). This in turn requires to estimate the covariance matrix of the LS solution. The following result can be used

for such purpose:

Lemma 1 *Suppose that the data used for the estimation of the friction parameters are generated according to the following model:*

$$\tau_{m,k} = \tau'_{f,k} = \varphi_k^T \theta_0 + e_k \quad (9)$$

where φ_k is the vector of regressors, θ_0 is the vector of “true” parameters, and e_k is a stochastic white noise with zero mean and variance λ^2 . Then, the following properties hold:

(a) $\hat{\theta}_{LS}$, as computed in the Handout, is an unbiased estimate of θ_0 .

(b) The covariance matrix of $\hat{\theta}_{LS}$ is given by

$$\text{cov}(\hat{\theta}_{LS}) = \lambda^2 (\Phi^T \Phi)^{-1} \quad (10)$$

where Φ is the regressors matrix.

(c) An unbiased estimate of λ^2 is given by

$$s^2 = \frac{1}{M-p} V(\hat{\theta}_{LS}) \quad (11)$$

where $V(\hat{\theta})$ is the quadratic cost function with M the number of measurements, and $p = 2$ the number of estimated parameters.

Use the results of the previous lemma to evaluate the variances $\text{var}(\hat{B}_{eq})$ and $\text{var}(\hat{\tau}_{sf})$ of the estimates (8) (remind that the two variances are the elements on the leading diagonal of the covariance matrix $\text{cov}(\hat{\theta}_{LS})$).

The confidence intervals for the true parameters B_{eq} and τ_{sf} can then be obtained by taking the least-squares estimates \pm a confidence margin, namely

$$\begin{aligned} B_{eq} &\in \left[\hat{B}_{eq} - c \sqrt{\text{var}(\hat{B}_{eq})}, \quad \hat{B}_{eq} + c \sqrt{\text{var}(\hat{B}_{eq})} \right] \\ \tau_{sf} &\in \left[\hat{\tau}_{sf} - c \sqrt{\text{var}(\hat{\tau}_{sf})}, \quad \hat{\tau}_{sf} + c \sqrt{\text{var}(\hat{\tau}_{sf})} \right] \end{aligned} \quad (12)$$

where c depends on the desired confidence level. For a large number of measurements, the value $c = 1.96$ corresponds to a 95% confidence interval. Instead, when the number of measurements is small, the value of c must be chosen more conservatively, by resorting to the Student distribution with $M - p$ parameters.

3.3 Estimation of the inertia parameters

1. Apply the procedure outlined in the Handout for estimating the total moment of inertia of the DC servomotor (rotor + mechanical load). The estimation procedure requires to measure the motor torque (or current, provided that the motor torque constant K_t is known) during constant acceleration/deceleration phases, which can be imposed again by using the position PID controller designed at point 1. Note that if a_l^* is a desired acceleration reference to be imposed to the motor, then the corresponding position reference ϑ_l^* to be provided to the

position control loop is obtained by double integration, i.e.

$$\vartheta_l^*(t) = \int_0^t \omega_l^*(\tau) d\tau, \quad \omega_l^*(t) = \int_0^t a_l^*(\tau) d\tau \quad (13)$$

For repeating several acceleration/deceleration phases in a single experiment, consider to use a square wave acceleration reference of the type:

$$\omega_l^*(t) = \int_0^t a_l^*(\tau) d\tau \quad (14)$$

with

$$a_l^*(t) = (-1)^k A \quad \text{for} \quad t \in [k \Delta T, (k+1)\Delta T), \quad k = 0, \dots, N^* - 1 \quad (15)$$

where A and ΔT are the amplitude and duration of the acceleration/deceleration phase. Consider to perform a 20 s experiment with $A = 450 \text{ rpm/s}$ and $\Delta T = 1 \text{ s}$, so that the motor speed at the end of each acceleration phase is 450 rpm, just a little below the motor nominal speed, and collect the motor current, speed and acceleration⁴ (see Fig. 5).

The motor speed can be obtained by differentiating the position measurement with the high-pass filter (5). Similarly, the motor acceleration can be obtained by differentiating the speed measurement with the second order Butterworth high-pass filter

$$H_a(s) = \frac{\omega_{c,a}^2 s}{s^2 + 2\delta_a \omega_{c,a} s + \omega_{c,a}^2} \quad \text{with} \quad \omega_{c,a} = 2\pi 20, \quad \delta_a = 1/\sqrt{2} \quad (16)$$

Instead, the motor current is obtained as explained in point 1 of Sec. 3.2, namely by measuring the voltage drop across the shunt resistor, and then using (6) to reduce the effects of the brushes commutation noise.

2. Use the estimated friction parameters (8) to compute the friction torque signal

$$\hat{\tau}_f'(t_k) = \hat{B}_{eq} \omega_m(t_k) + \frac{\hat{\tau}_{sf}}{N} \text{sign}(\omega_m(t_k)) \quad \text{with} \quad t_k = k T_s, \quad k = 0, \dots, M-1 \quad (17)$$

where $\omega_m(t_k)$ is the motor speed measured at the generic sampling time instant t_k , and M is the number of samples collected during the experiment. Then, use (??) to compute the torque signal

$$\hat{\tau}_i(t_k) = \tau_m(t_k) - \hat{\tau}_f'(t_k) \quad (18)$$

which represents the fraction of motor torque used to accelerate the total inertia \hat{J}_{eq} (inertial torque component – see bottom plot in Fig. 5b). In (18), $\tau_m(t_k)$ is the motor torque measured at the generic sampling time instant t_k (obtained by multiplying the measured current $i_a(t_k)$ by the nominal torque constant K_t).

3. For each time interval

$$t \in [k \Delta T, (k+1)\Delta T), \quad k = 0, \dots, N^* - 1 \quad (19)$$

⁴Alternatively, collect the motor position, and then compute the required speed and acceleration measurements offline, by using the high-pass filters (5) and (16).

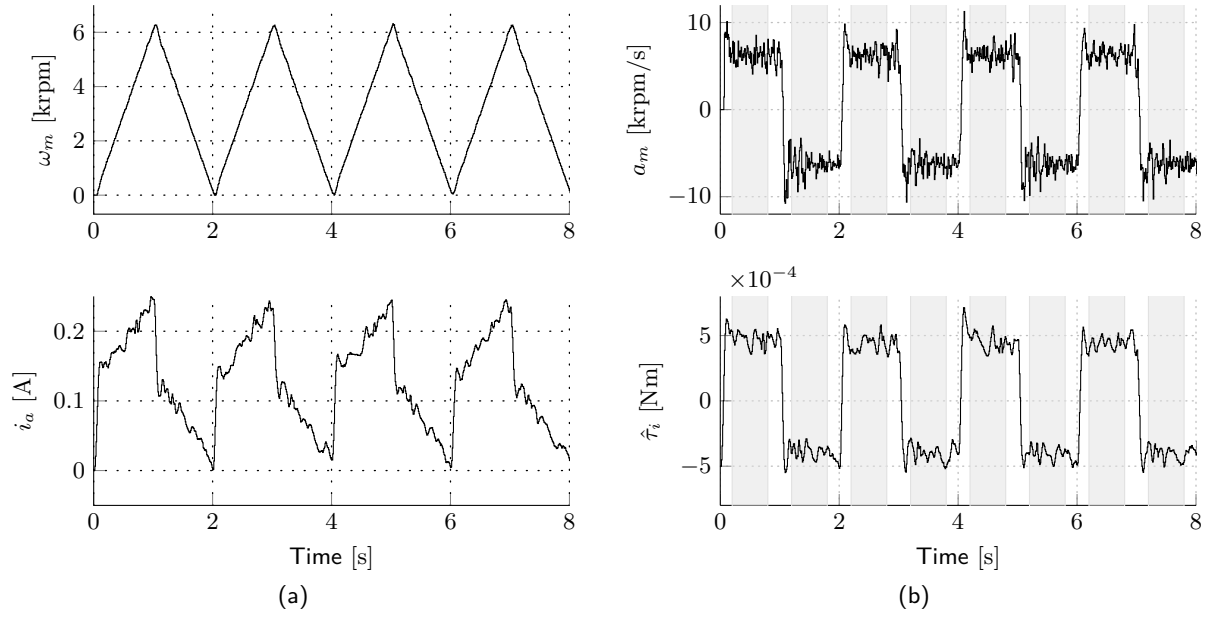


Figure 5: Inertia estimation test: (a) speed and current profiles; (b) acceleration and inertial torque profiles. Shaded areas in (b) represent the time intervals over which the averages of the acceleration and inertial torque measurements are computed.

within which the motor acceleration is constant, compute the average values $a_{m,k} = N a_{l,k}$ and $\hat{\tau}_{i,k}$ of the motor acceleration and inertial torque component. In each time interval, avoid to consider the initial acceleration and torque transients in the computation of the average values (see shaded areas in Fig. 5b).

Note that $a_{m,k}$ is a positive acceleration when k is even, and negative otherwise.

4. Obtain the estimate of the equivalent inertia using the estimation procedure outlined in the Handout yields

$$\hat{J}_{eq} = \frac{1}{P} \sum_{n=0}^{P-1} \frac{\hat{\tau}_{i,2n} - \hat{\tau}_{i,2n+1}}{a_{m,2n} - a_{m,2n+1}} \quad (20)$$

with $P = N^*/2$ (assuming N^* even). In (20) it has been used the fact that given the acceleration reference (15), the average value $a_{m,k}$ of the motor acceleration obtained in point 3 is positive when k is even, and negative otherwise.