Control Theory at Work: Respect the Unstable

A presentation for Control Lab

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Maine source: a Bode Lecture by Gunter Stein

The Role of Automatic Control Systems

- **Closed loop control systems are around us!**
  Where?Almost everywhere electronics are involved...
  ...*home appliances, cars, factories, transportations, defense systems rely on control technology.*

- **The Good:** Basic analysis and design principles are well understood and widely developed...

- **But be careful:**

  - **Society trusts our technology**: We are allowed and expected to design controls for processes humans cannot control, e.g. highly unstable systems...

  - Mathematical advances have to be translated into **practice**...

What is Actually Possible?

- Understanding what you can and **you cannot do** is a key issue in science and engineering;

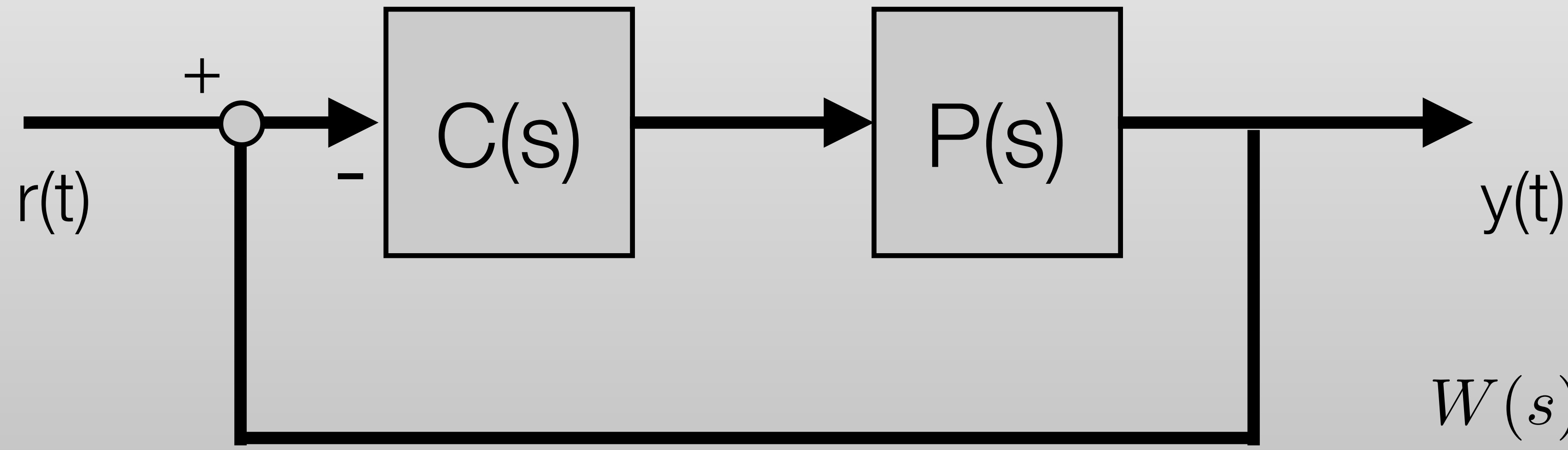
- Think about some of the greatest scientific milestones of the 20th century:

  - Einstein's **Relativity Theories** are "built on" a fundamental limitation on the speed of light;
  - **Quantum Mechanics** poses fundamental limits to deterministic predictions;
  - **Gödel's Theorem** poses ultimate limitations to automatic verification of theorems;

- Also *Information Theory and Communication Technologies* are founded on **Shannon's Coding Theorems** and their bounds on the achievable rates! *[How much can I compress information? How much information can I send over a given channel?]*

Stabilization (I/O) of Unstable Plants

- Define the **Sensitivity Function** $S(s) = \dfrac{\partial W / \partial P}{W/P} = \dfrac{\partial \log W}{\partial \log P} = \dfrac{1}{1 + C(s)P(s)}$



$$W(s) = \frac{C(s)P(s)}{1 + C(s)P(s)}$$

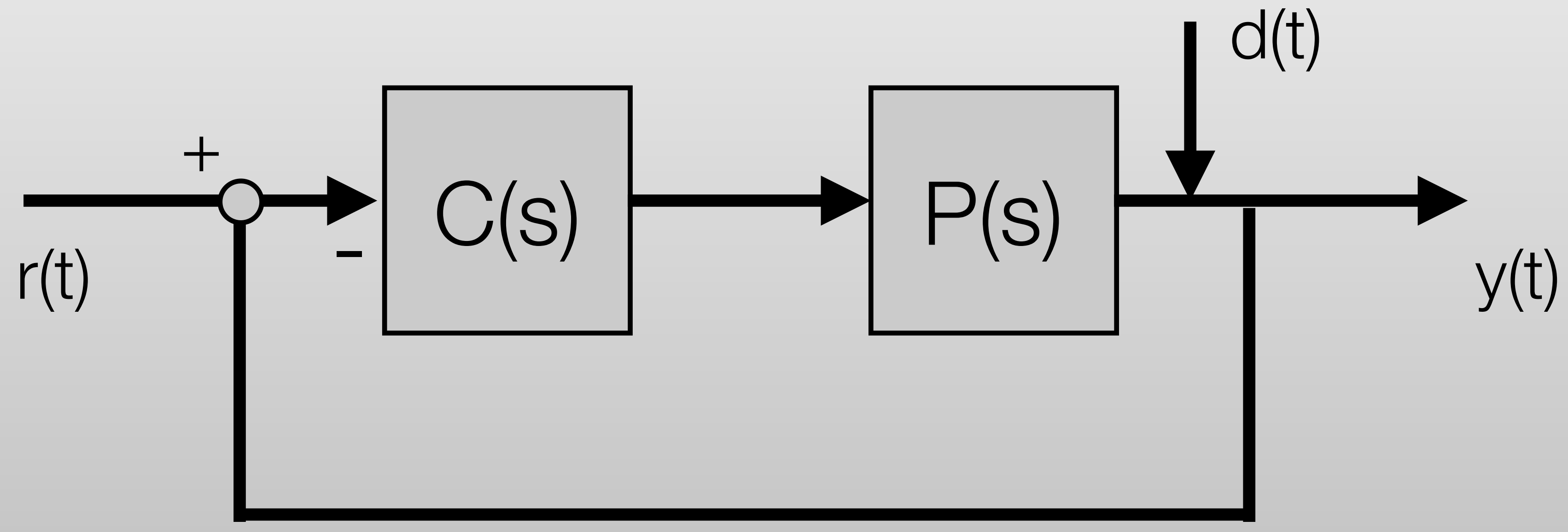
THM. Given a Plant P(s), a compensator C(s), rel.deg>0 is internally BIBO stable (the interconnection is stable) *if and only if:*

1) S(s) has stable poles and is proper;

2) There are no cancellation of unstable poles in C(s)P(s).

Typical Frequency-domain Specifications

- C(s) must be designed so that ***under a given band of interest B*:**



- The **Transfer Function** W(s) = C(s)P(s)/(1+C(s)P(s)) is about 1 (position control/tracking the reference; ~CP>>1);

- The **Sensitivity Function** S(s) = 1/(1+C(s)P(s)) is small;

  Accounts for local insensitivity to modeling errors and d(t) (**Robustness Issues**).

When Does This Result Apply?

- It is "good" theory and... applies also to critically unstable systems!

Basic Facts About Unstable Plants

- Unstable systems are fundamentally, and quantifiably, more difficult to control than stable ones.
- Controllers for unstable systems are operationally critical.
- Closed-loop systems with unstable components are only locally stable.

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Figure 1. *Gripen JAS39 prototype accident on 2 February 1989. The pilot received only minor injuries.*

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Figure 2. *Chernobyl nuclear power plant shortly after the accident on 26 April 1986.*

A familiar name in Control Theory…

- **Hendrik Wade Bode** (24 December 1905 – 21 June 1982)

...was an American engineer, researcher, inventor, author and scientist. As a pioneer of modern control theory and electronic telecommunications he *revolutionized both the content and methodology* of his chosen fields of research.

- Go give *Wikipedia* a look. Influenced Shannon too...

Have you ever heard of...

- **Bode Integrals -** sensitivity of interconnection vs poles of the plant, irrespective of the control:

Stable Plants:

$$\int_0^\infty \ln|s(j\omega)|\, d\omega = 0$$

Unstable Plants:

$$\int_0^\infty \ln|s(j\omega)|\, d\omega = \pi \sum_{p\in P} \mathrm{Re}(p)$$

- *"Quantifies" instability hidden in a* ***closed loop stable system:***
  It grows with the unstable pole real parts of the open-loop plant.
  What is their relevance and use?
  What does it mean for control design?

A Paradigmatic Example (similar to our Segway!)

- **Upside-down Broomstick:**

  Try to stabilize a pen with your finger!

  I might be bad at that, but this guy seems comfortable with a broom...

  Maybe length matters?

- Any kind of inverted pendulum has an unstable pole

$$p_{\text{unstable}} = \sqrt{\frac{g}{L}}$$

It gets worse as L decreases!

So What?

- **In principle the stabilization law should be analogous...**
  [and I probably know more control theory than that guy!]

- But what about the "controller"?

Finite reaction time, neuromuscular lags, limb inertias...
We are not ideal LTI systems! (and most of the world is not so either!)

Summing up: We do ok up to ~ 2 Hz (10-15 rad/s).
Probably I am a little worse than that these days!

- We do not work very well on high frequencies.

  What are the consequences ???

Sensitivity Design

- To be effective, my control law must be “robust” in the band of interest. *Low Sensitivity*, I have to “dig” on the unstable points...and stay low for low frequencies.

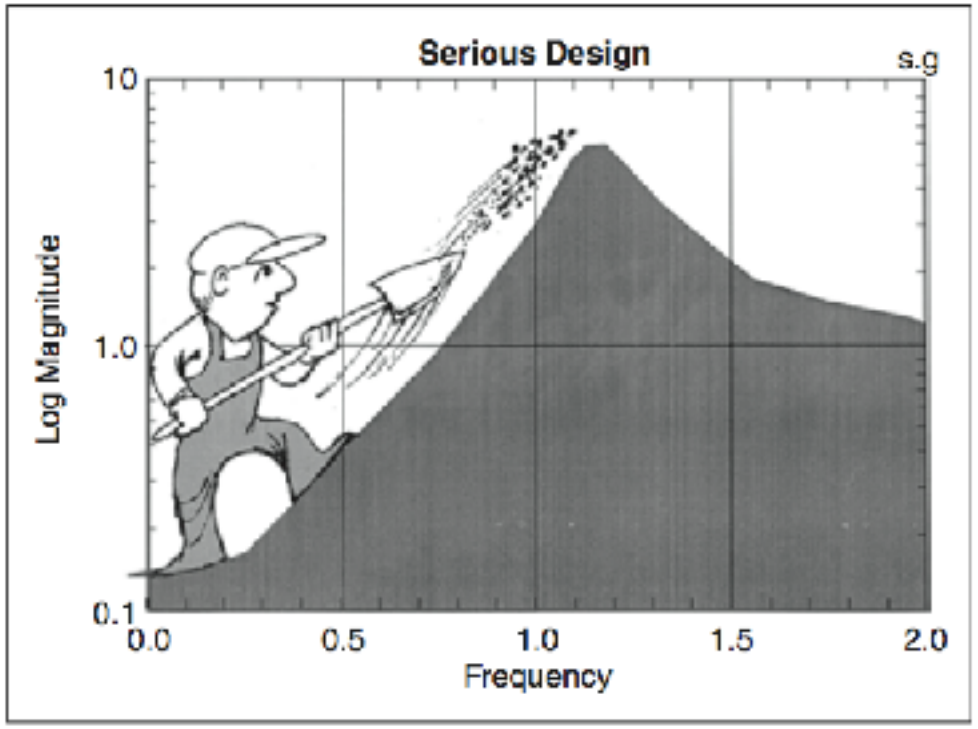


Figure 3. *Sensitivity reduction at low frequency unavoidably leads to sensitivity increase at higher frequencies.*

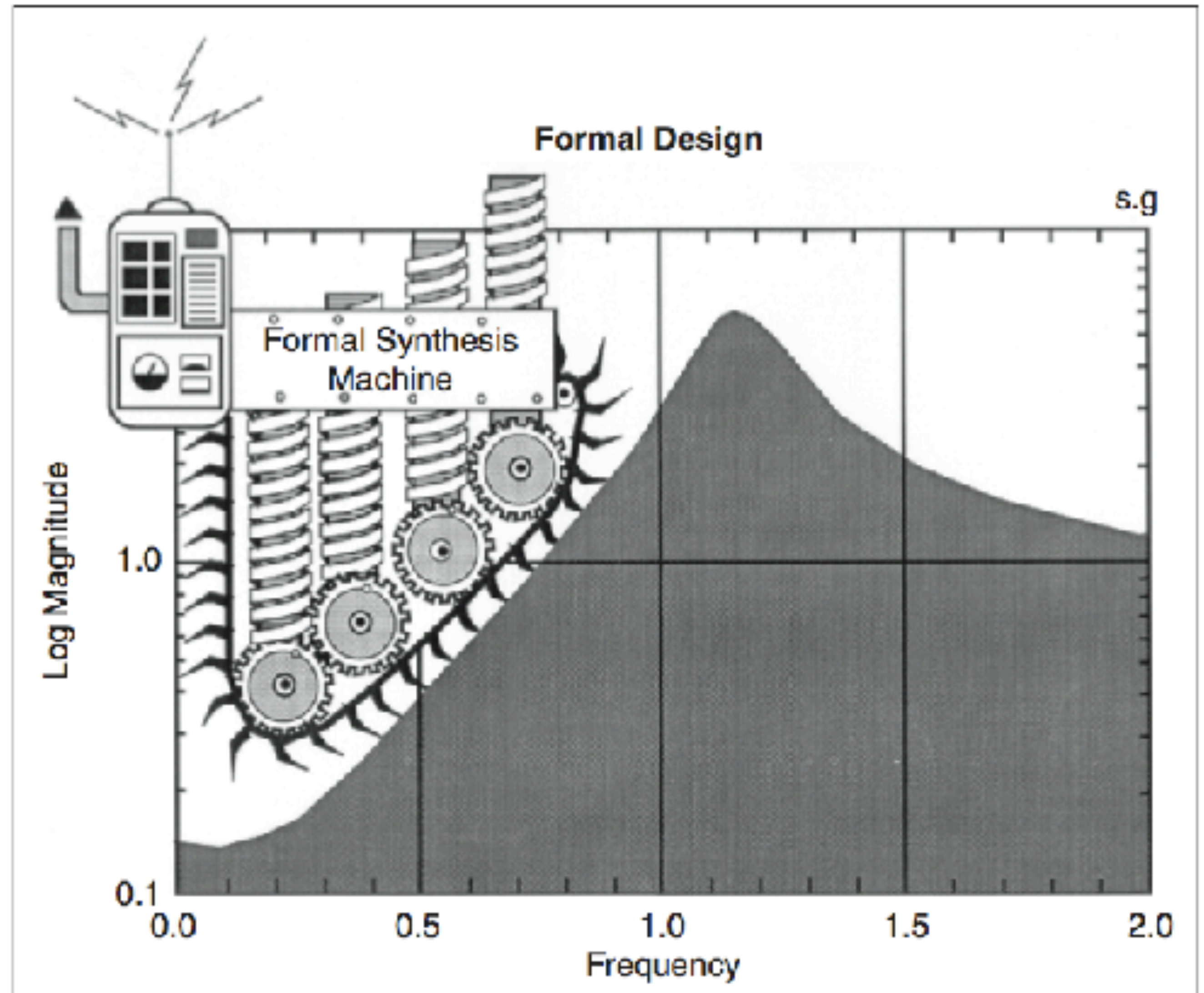


Figure 4. *Sensitivity shaping automated by modern control tools.*

- **The Bode Integral remains constant!!!** I have to move “dirt” around... but the total amount remains!

Can I stabilize the pen, or am I just bad at that?

- What is the minimum sensitivity under B=2Hz that we can achieve for the broomstick?

- **Limits to minimum Sensitivity:** I cannot push “dirt” on higher frequencies, my controller doesn’t work!!!

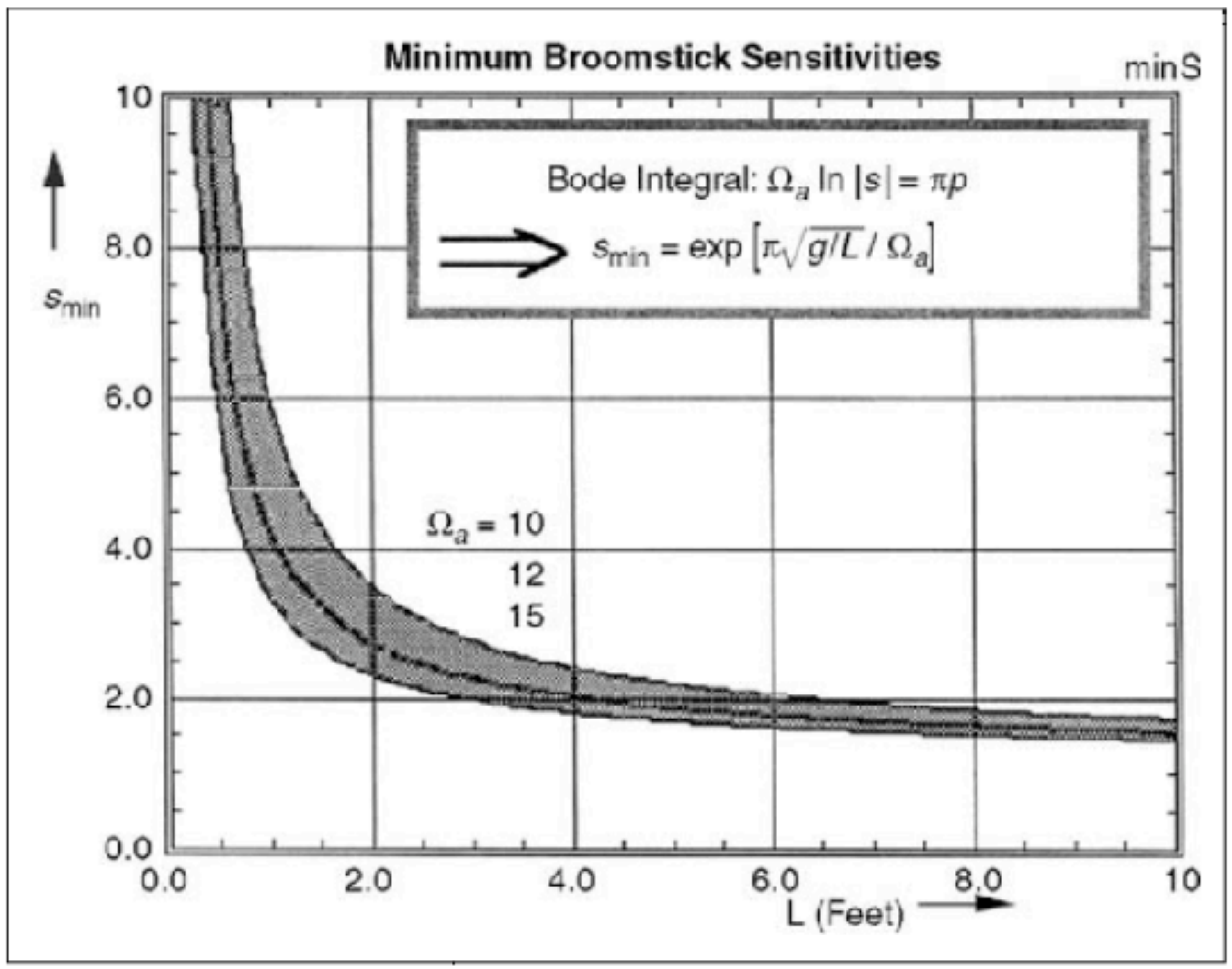


Figure 5. *Sensitivity constraints as a function of broomstick length.*

Are Broomsticks and Planes Related?

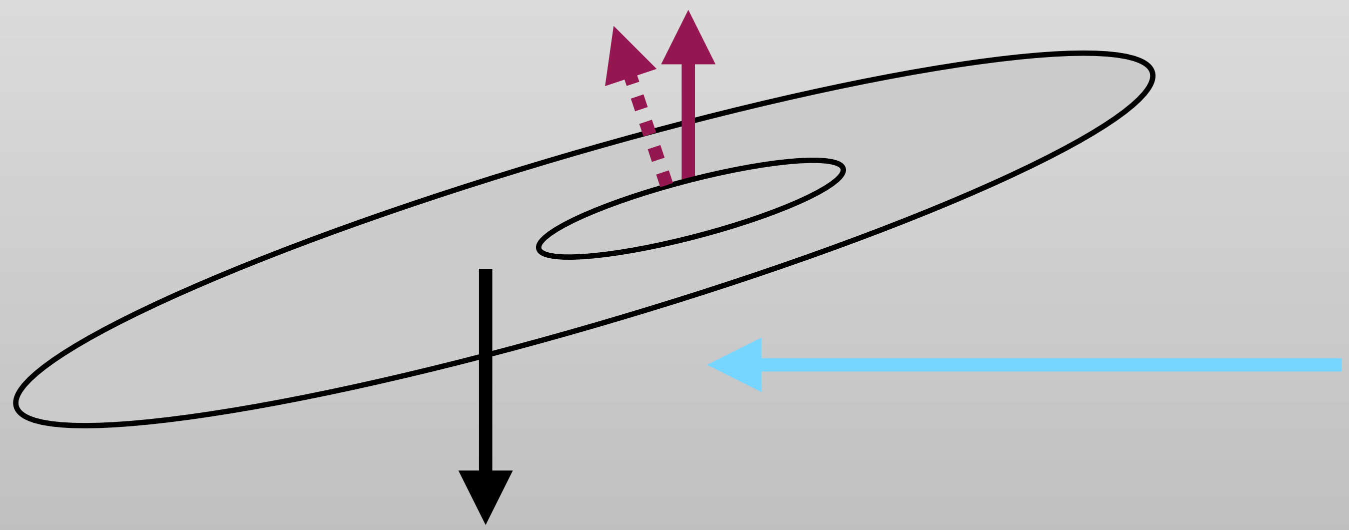
- A more appealing problem to military agencies (I do prefer broomsticks):

  Building a reactive, maneuverable fighter.

Figure 6. *NASA X-29 forward-swept-wing aircraft (photo courtesy of NASA).*

The Flying Broomstick

- After Wright brother’s planes (noteworthy exception!), aircrafts were built “statically stable”, that is:
  Their Center of Gravity is to be ahead of the Lift Center. If **not:**



  Pitch increases lifting force, if **unstable** tends to produce a diverging momentum. Try with paper planes…!

- X-29 was unstable, relying on full-authority automated control.
  Linearized equation *similar* to the inverted pendulum.

  What are the limitation of the controller and effective bandwidth?

Bandwidth Limitations

- Sensors (120 rad/s)
- **Control Processors (30-40 rad/s)**
- Actuators (70-80 rad/s)
- Aerodynamics, how the flow “changes” (100 rad/s)
- Airframe, mechanical structure, rigidity (down to 40 rad/s)

...with this data, the desired control performance (sensitivity) over 40 rad/s could not be guaranteed!!!
Only marginal stability. They had to redesign various parts...

Other notable stability-related accidents...

- **Saab JAS-39 Airplane**

  Unstable oscillations involving actuator saturations;

- **Chernobyl Accident**

  Started from a “human controller mistake”, inducing unstable behavior the human controller couldn’t control...

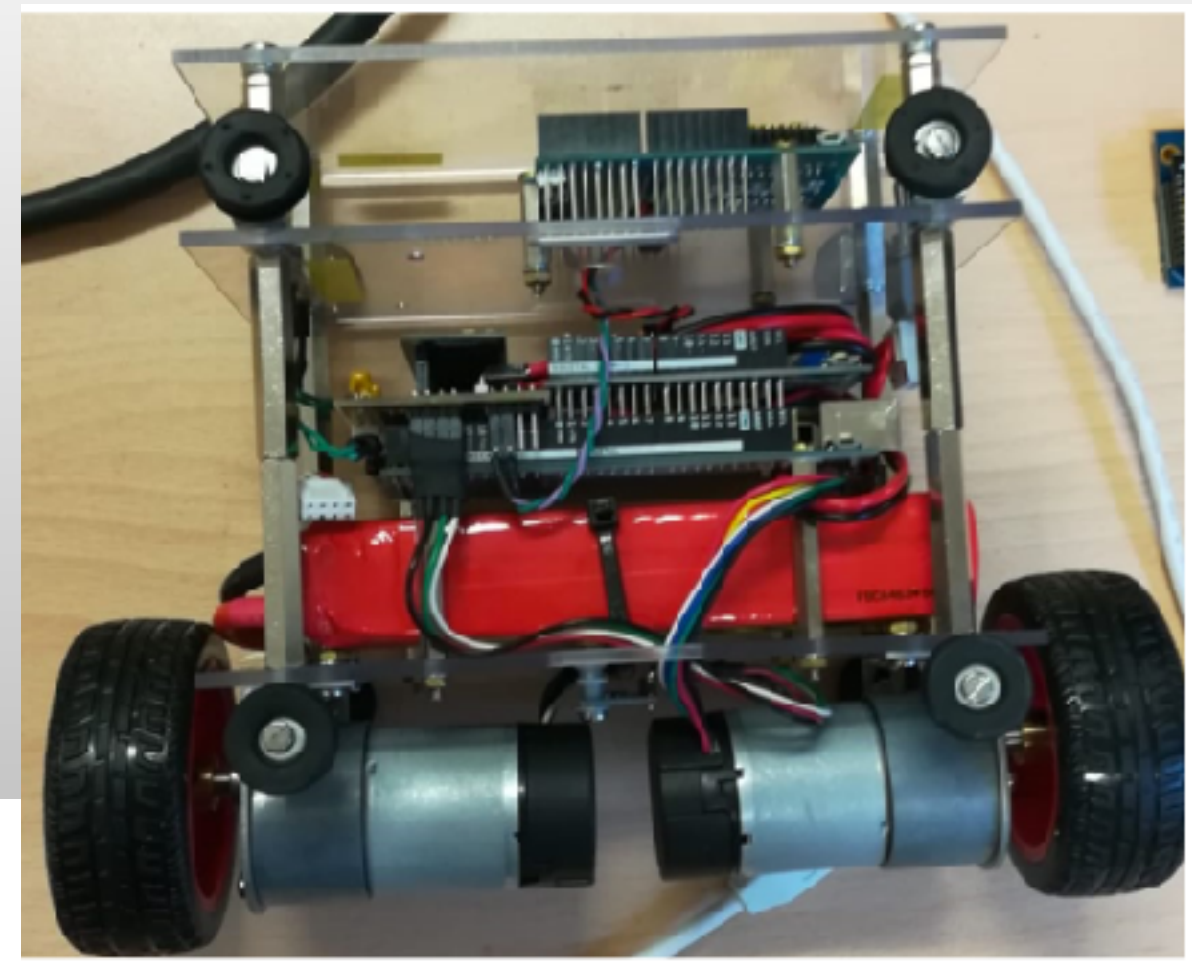
- **Take home message : “Be careful with the unstable!”**

  and also…
  “In theory there is no difference between theory and practice.
  But in practice there is.” Jan L.A. van de Snepscheut

  ...or (for the theoretical crowd) one has to be really careful about the hypothesis and the limitations, carefully tailoring the theory on the application.

Our Unstable System: Balancing Robot

Luckily we are not controlling anything too dangerous!



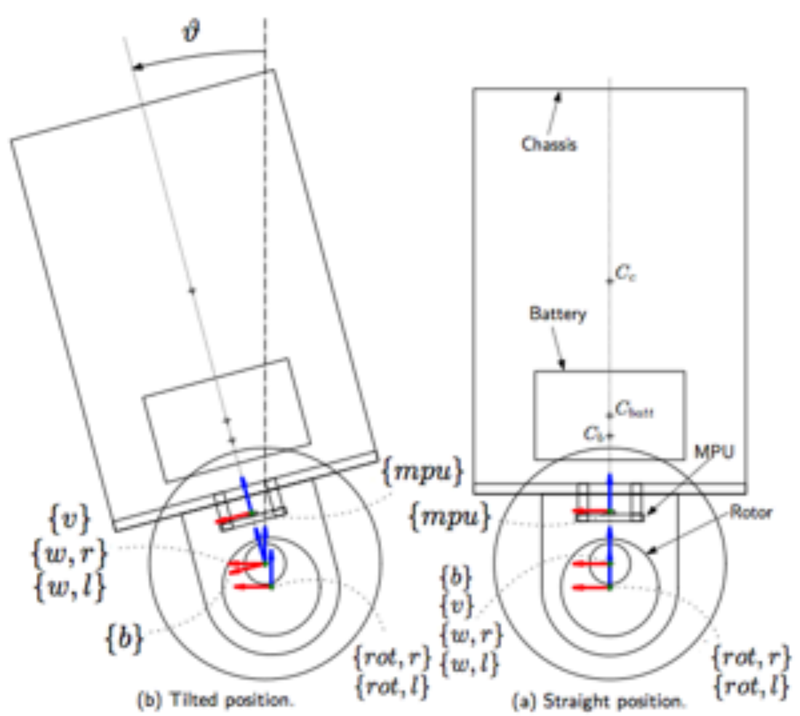


Figure 1: Left side view.

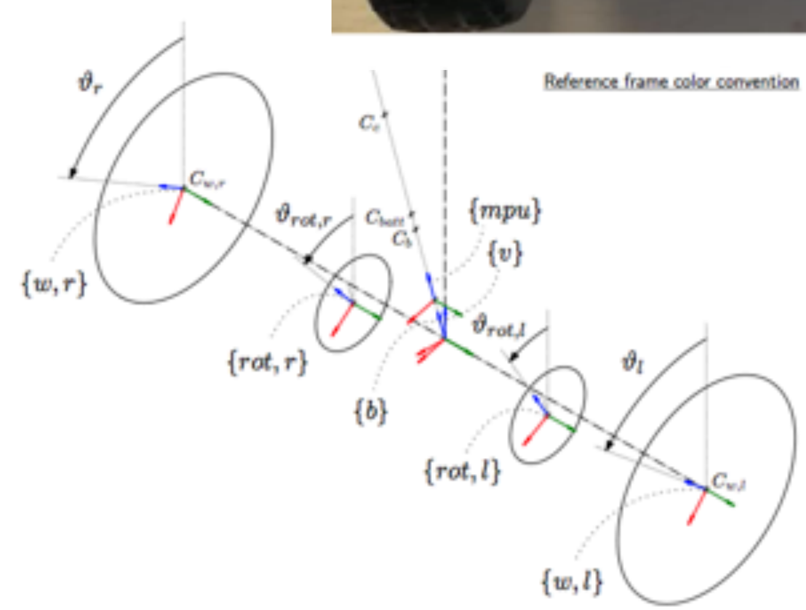


Figure 2: Simplified 3D view.