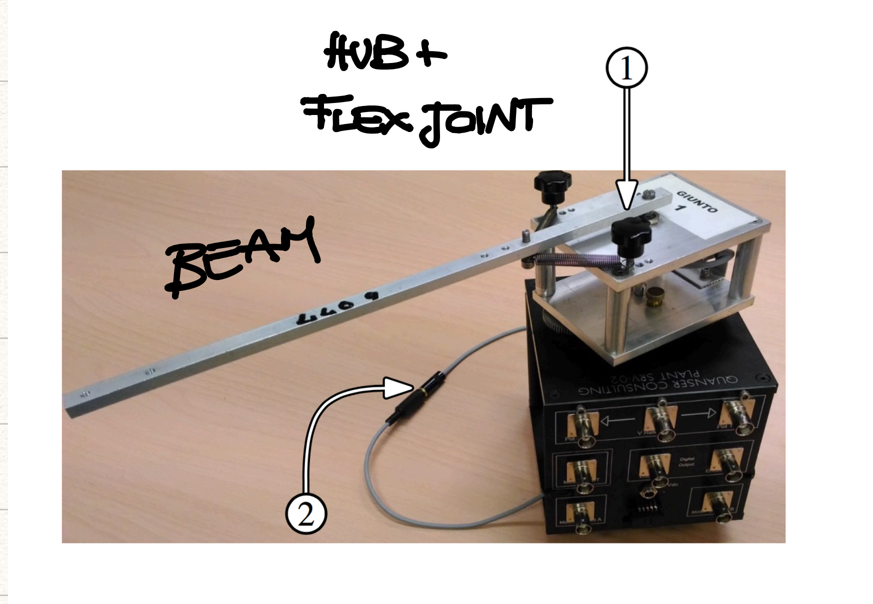
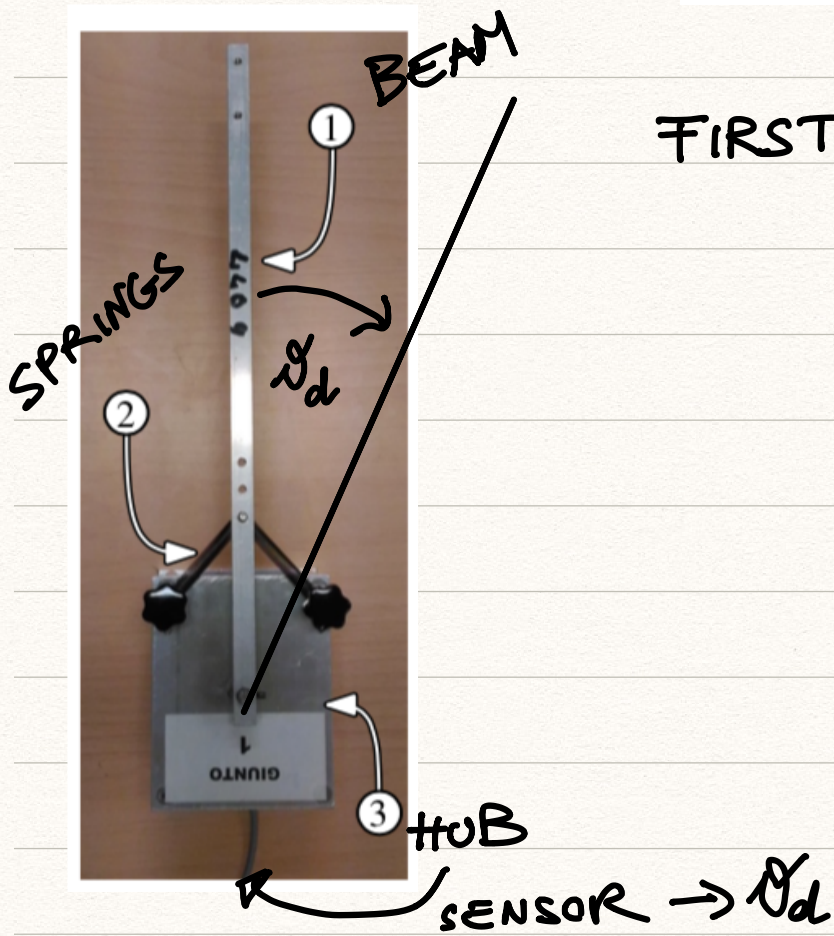
"NEW" SYSTEM

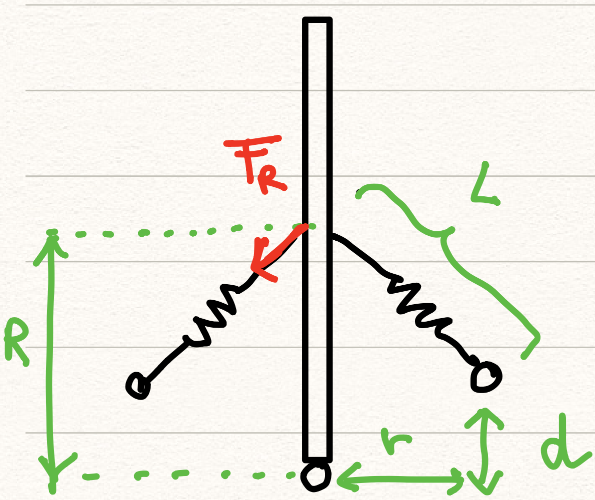
MOTOR + RESONANT LOAD

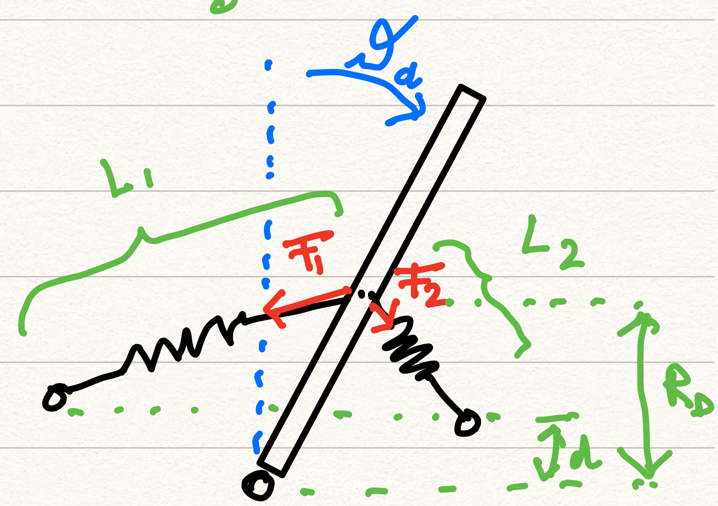




FIRST : FOCUS ON THE PHYSICS OF THE NEW LOAD

$R_D = R\cos\vartheta_d$





A LITTLE GEOMETRY:

$$\begin{cases} L_{1,x} = r + R \sin \vartheta_d \\ L_{2,x} = r - R \sin \vartheta_d \\ L_{i,y} = R \cos \vartheta_d - d \end{cases} \qquad i = 1,2$$

FORCES : ELASTIC CONSTANT × DISPLACEMENT

$$F_i = K_s (L_i - L) + F_R$$

WE NEED : TORQUES (TRANSMITTED TO MOTOR'S SHAFT)

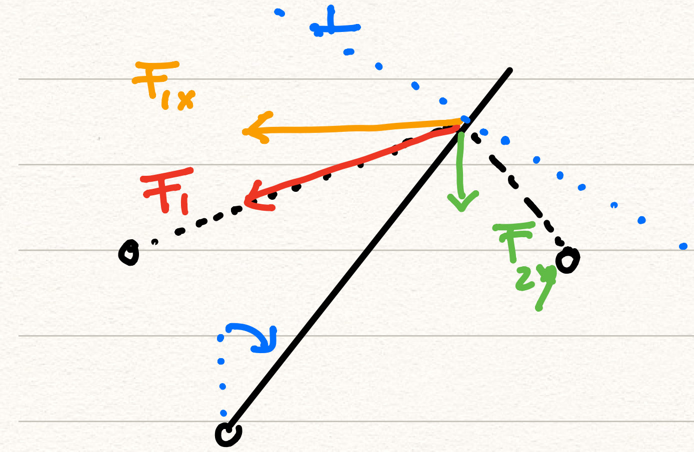
HOW : - WE WRITE $F_{i,x}$, $F_{i,y}$;

- PROJECT each component onto $\perp$ to BEAM;

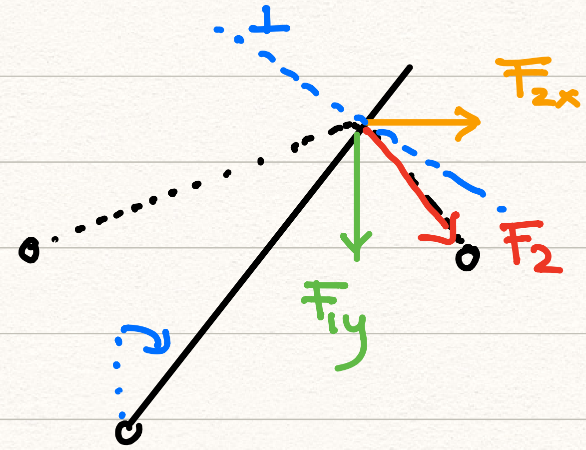
- COMPUTE $\tau = \sum_i F_i^{\perp} \cdot R$

$$F_{i,x} = F_i \frac{L_{ix}}{L_i} \; ; \quad F_{i,y} = F_i \frac{L_{iy}}{L_i}$$

Let's see ...



F_1 DECOMPOSITION & ⊥ COMPONENTS



F_2 DECOMPOSITION & ⊥ COMPONENTS

$$\Rightarrow \tau_e = \left(F_{2,x} - F_{1,x}\right) R \cos(\theta_d) + \left(F_{1,y} + F_{2,y}\right) R \sin(\theta_d)$$

IF θ_d SMALL : LINEARIZE AROUND $\emptyset$ (REST)

$$\tau_e \simeq -K\,\theta_d$$

IF HUB MOVES :

$$\theta_d = \theta_r - \theta_b$$

w.r.t. SAME REFERENCE

$$K = \left| \frac{\partial \tau_e}{\partial \theta_d}(0) \right|$$

$$= \frac{2R}{L^3}\left[\left(L^2 d - R r^2\right) F_R + \left(r^2 R L\right) K_s\right]$$

Pb : F_R , K_s are unknown

↳ WE NEED ESTIMATES !

- ASSUME HUB STILL ("LOCKED" TO TABLE), NO EXTERNAL FORCES

$$\underbrace{J_b \ddot{\vartheta}_d}_{\text{INERTIA Acceleration}} + \underbrace{B_b \dot{\vartheta}_d}_{\text{FRICTION}} + \underbrace{K \vartheta_d}_{\text{ELASTIC}} = 0$$

- REDEFINE $\omega_n = \sqrt{\frac{K}{J_B}}$; $\xi = \frac{B_b}{2\sqrt{J_b K}}$

NOTE: KNOWING ω_n , ξ EQUIVALENT TO K, B_b

$$\Rightarrow \ddot{\vartheta}_d + 2\xi\omega_n \dot{\vartheta}_d + \omega_n^2 \vartheta_d = 0$$

↳ STANDARD FORM FOR II ORDER SYSTEM

↳ IN OUR CASE : $\xi < 1$ UNDERDAMPED

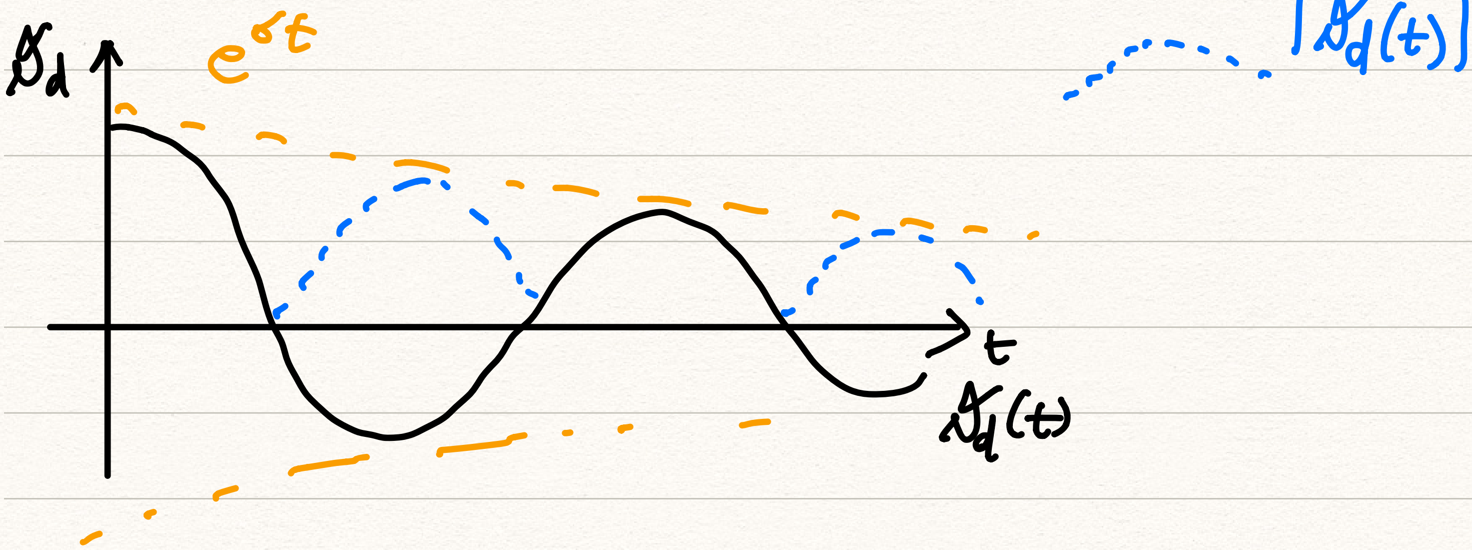
↳ GIVEN INITIAL CONDITIONS $\vartheta_d(0), \dot{\vartheta}_d(0)$

$$\vartheta_d(t) = \alpha e^{\sigma t} \cos(\omega t + \phi)$$

- AS USUAL : $\sigma = -\xi\omega_n$; $\omega = \omega_n\sqrt{1-\xi^2}$
- ALSO

$$\alpha = \sqrt{\vartheta_d^2(0) + \left(\frac{\dot{\vartheta}_d(0) - \sigma\vartheta_d(0)}{\omega}\right)^2} \; ; \; \phi = \operatorname{atan}\left(\frac{\sigma\vartheta_d(0) - \dot{\vartheta}_d(0)}{\omega\vartheta_d(0)}\right)$$

- TYPICAL RESPONSE :



- LET'S FOCUS ON THE MAX OF $|\theta_d(t)|$

$$|\theta_d(t_k)| = \alpha \, e^{\sigma t_k} , \qquad \text{FOR } t_k \text{ S.T. } \dot{\theta}(t_k) = 0$$

$$\Rightarrow \; t_k = \frac{k\pi - \phi}{\omega}$$

- NOTICE :

$$\frac{|\theta_d(t_k)|}{|\theta_d(0)|} = e^{\sigma(t_k - t_0)} = e^{-\frac{\xi}{\sqrt{1-\xi^2}} k\pi}$$

- TAKING LOGARITHMS :

$$\log|\theta_d(t_k)| - \log|\theta_d(t_0)| = -\gamma \, k$$

where $\gamma = \dfrac{\xi \pi}{\sqrt{1-\xi^2}}$

linear ∇_0 IN k

EXPERIMENT :

- Run Test on STILL HUB ;

  Measure $\vartheta_d(t_k)$

- Do a linear Least square fitting

  on the $\log(|\vartheta_d(t_k)|)$

FREE VAR

SETUP : $y \cong -a\,k + b = \varphi_k^T \Theta$

MEAS

$$\varphi_k^T = [-k \ \ 1] \ ; \ \Theta = \begin{bmatrix} a \\ b \end{bmatrix}$$

STACK UP

Define :

$$\Phi = \begin{bmatrix} \varphi_0^T \\ \varphi_1^T \\ \vdots \\ \varphi_M^T \end{bmatrix} \ ; \ Y = \begin{bmatrix} \log|\vartheta_d(t_0)| \\ \vdots \\ \log|\vartheta_d(t_M)| \end{bmatrix}$$

$\Rightarrow$ LEAST SQUARE : $V(\vartheta) = \sum_k \left(\log|\vartheta_d(t_k)| - \varphi_k^T \Theta\right)$

$$= [Y - \Phi\Theta]^T [Y - \Phi\Theta]$$

$$\hat{\Theta}_{LS} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \left(\Phi^T \Phi\right)^{-1} \Phi^T y$$ USUAL FORMULA

- FROM $\hat{a} \equiv \hat{\gamma} \longrightarrow \hat{\xi} = \dfrac{\hat{a}}{\sqrt{\pi^2 + \hat{a}^2}}$

- FOR ω_n ? FIRST ESTIMATE $\left(\omega \simeq \dfrac{\pi}{t_{k+1} - t_k}\right)$

$$\Rightarrow \hat{\omega} = \frac{1}{N} \sum_k \frac{\pi}{t_{k+1} - t_k}$$

then compute :

$$\hat{\omega}_n = \frac{\hat{\omega}}{\sqrt{1 - \xi^2}}$$

- FINALLY, FROM $\hat{\xi}, \hat{\omega}_n \longrightarrow \hat{K}, \hat{B}_{eq}$

#