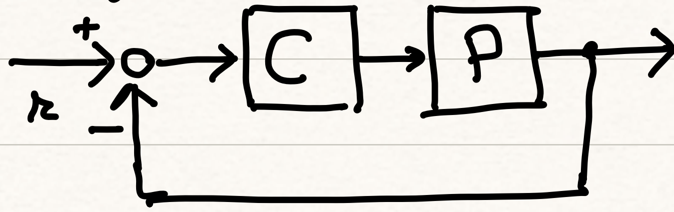
# PERFORMANCE SPECIFICATIONS

FOR THE CONTROLLED SYSTEM (I/O approach)

- Assume we have a model for the plant $P(s)$
- Consider typical control architecture




- TO START OUR CONTROL DESIGN:

we need controlled system's desired behavior

(0) STABILITY : (minimal) BIBO

INTERNAL STABILITY OF INTERCONNECTION

either : - Design $C(s)$ and then verify

↳ $D = D_P D_C + N_P N_C$ HURWITZ

↳ $\deg D = \deg D_P D_C$

- Use DIOFANTINE EQ. FOR $D$

(I) ASYMPTOTIC REGIME PERFORMANCE

what do we want the system to do at $t \to +\infty$?

TYPICAL : ABILITY OF $W = \frac{CP}{1+CP}$ TO <u>TRACK</u>

STANDARD CLASSES OF INPUTS.

↙ asympt. exact tracking

$$r(t) - y(t) = e(t) \xrightarrow[t \to +\infty]{} 0$$

↘ BOUNDED ERROR

$$|e(t)| < \varepsilon \quad \text{when } t \to +\infty$$

RECALL: For STANDARD REFERENCES

$\delta_{-1}(t) = 1\!\!1(t)$, $\delta_{-2}(t) = t1\!\!1(t)$, ...

it depends on the "TYPE" OF LOOP

TYPE $\equiv$ # OF POLES OF $C(s)P(s)$ in $s=0$

FACT: if $C(s)P(s) = \frac{1}{s^\ell} C^\circ(s)P^\circ(s)$

$\Rightarrow$ W tracks exactly up to $\delta_{-\ell}(t)$

- This can be proved easily as a particular case of the

INTERNAL MODEL PRINCIPLE (I/O VERSION)

Task: ASYMPT. TRACKING of $\{r(t)\}$, SOLUTIONS OF ODE

$$r^{(q)}(t) + \alpha_{q-1} r^{(q-1)}(t) + \ldots + \alpha_0 r(t) = 0 \quad (r)$$

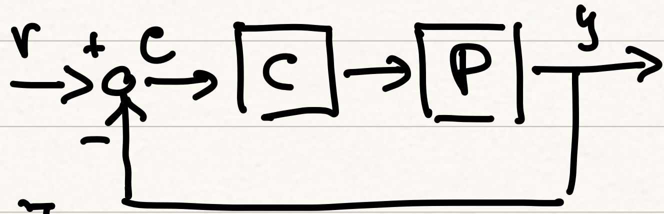
Use $\mathcal{L}$-TRANSF.: call $\mathcal{L}[r] = R$, $A(s) = \sum_i \alpha_i s^i$

$$(r) \Rightarrow \quad A(s)R(s) + C(s) = 0$$

$\uparrow$ all the init. conditions

$$\Rightarrow \quad R(s) = \frac{C(s)}{A(s)} = \frac{N_R(s)}{D_R(s)}$$

▷ Let's see what happens with W :




$C = \frac{N_C}{D_C} \quad P = \frac{N_P}{D_P}$

$$\underset{\displaystyle E}{\overset{\mathcal{L}[e]}{\|}} = R - Y = R - \frac{CP}{1+CP} R$$

$$= \frac{1}{1+CP} R = \frac{D_C D_P}{\underbrace{D_C D_P + N_P N_C}_{D\text{, HURWITZ FOR (0)}}} \frac{N_R}{D_R} = \frac{N_E}{D_E}$$

NOTICE :

$\underset{t \to +\infty}{e(t) \to 0} \iff D_E$ HURWITZ (after simplifications) $\iff$ All modes are convergent.

$$\text{if } D_R \begin{cases} \text{HURWITZ} \Rightarrow \text{ok} \\ D_R = D_R^0 \, U_R \end{cases}$$

$D_R^0$: STABLE POLES; $U_R$: NON-CONVERGENT POLES

- ASYMPT. TRACKING POSSIBLE IFF $U_R$ SIMPLIFIES WITH $D_C D_P$.

Assume $U_R(s) = \prod_i (s - p_i)$ ; FOR EACH $p_i$

↳ either $D_P(p_i) = 0$ ($\nabla$)

↳ OR WE MUST INCLUDE $(s - p_i)$ in $D_C$ ($\nabla$)

WARNING ($\nabla$) : - IF $D_P$ IS NOT EXACTLY KNOWN, CANCELLATION WILL NOT WORK; CONST.

SUMMING UP : <u>INTERNAL MODEL PRINCIPLE</u> (I/O)

$R(s) = \frac{N_R(s)}{D_R(s)}$ will be tracked asympt.

For <u>any</u> $N_R(s)$ if the UNSTABLE ROOTS OF $D_R$ ARE ALSO ROOTS OF EITHER $D_P$ OR $D_C$.

TO DESIGN $C(s) = C_o(s) \frac{1}{U_R(s)}$ :

1) Define $P_v(s) = \frac{1}{U_R(s)} P(s)$

2) DESIGN $C_o(s)$ FOR $P_v(s)$ [ACCORDING TO OTHER SPECS.]

3) Use $C(s) = C_o(s) \frac{1}{U_R(s)}$

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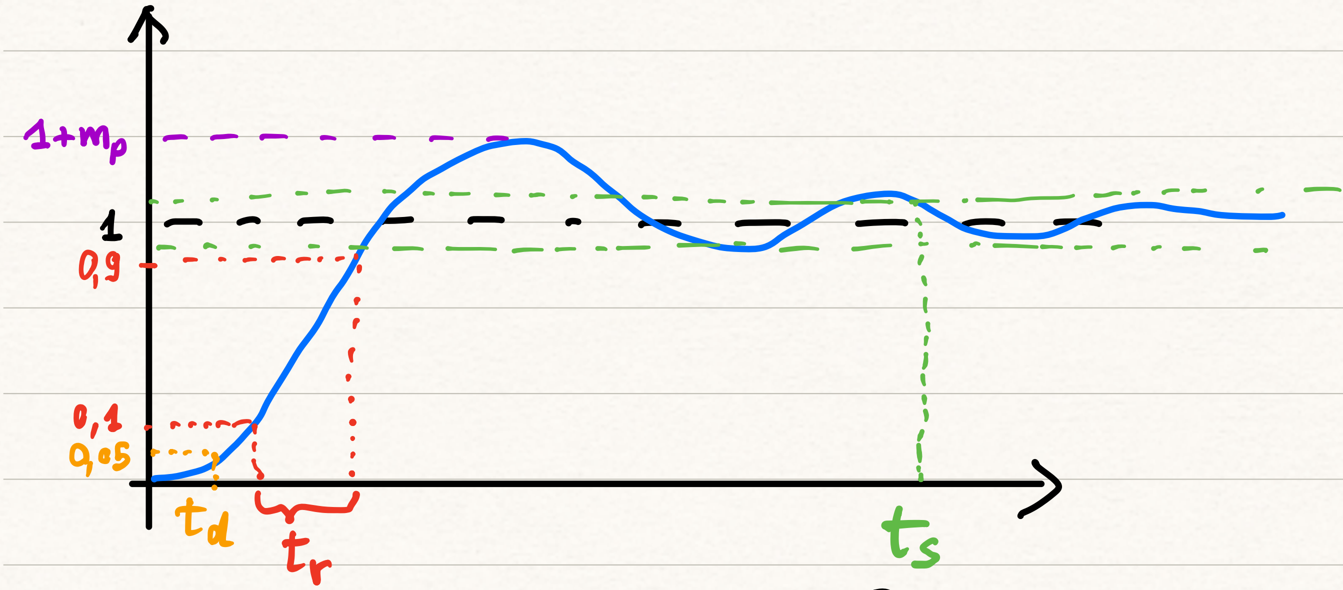
# (II) CONTROL PERFORMANCE IN THE <u>TRANSIENT</u>

TYPICAL APPROACH :

- APPROX. $W = \frac{CP}{1+CP} \approx$ II ORDER (DOMINANT POLES)
- CONSIDER (UNIT) STEP RESPONSE

# DESCRIPTIVE PARAMETERS




$m_p$ : MAX PEAK

$t_r$ : RISE TIME

$t_s$ : SETTLING TIME

$t_d$ : DELAY TIME

HOW TO TRANSLATE IN SOMETHING MORE DIRECTLY USEFUL ?

## $1^{st}$ APPROACH : ADMISSIBLE REGION FOR POLES

▷ $W(s) \approx$ II° ORDER APPROX. , DOM. POLES

$$p_{1,2} = \sigma \pm j\omega = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

STEP RESPONSE:

$$\mathcal{L}^{-1}\Big[\underbrace{W(s)\frac{1}{s}}_{W_{-1}(t)}\Big] = 1\!\!1(t) - e^{\sigma t}\left[\sin(\omega_d t + \varphi)\right]$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$m_p$ : compute $1^{st}$ zero of derivative

$$\Rightarrow \quad t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \qquad m_p = e^{\frac{-\xi}{\sqrt{1-\xi^2}}\pi}$$

$t_s$ : can be estimated via $e^{\sigma t}$

$$e^{\sigma t_s} \simeq 0{,}05 \Rightarrow t_{s,5\%} = \frac{\log(20)}{|\sigma|} \simeq \frac{3}{|\sigma|}$$

$$\simeq 0{,}01 \Rightarrow t_{s,1\%} \simeq \frac{4.6}{|\sigma|}$$

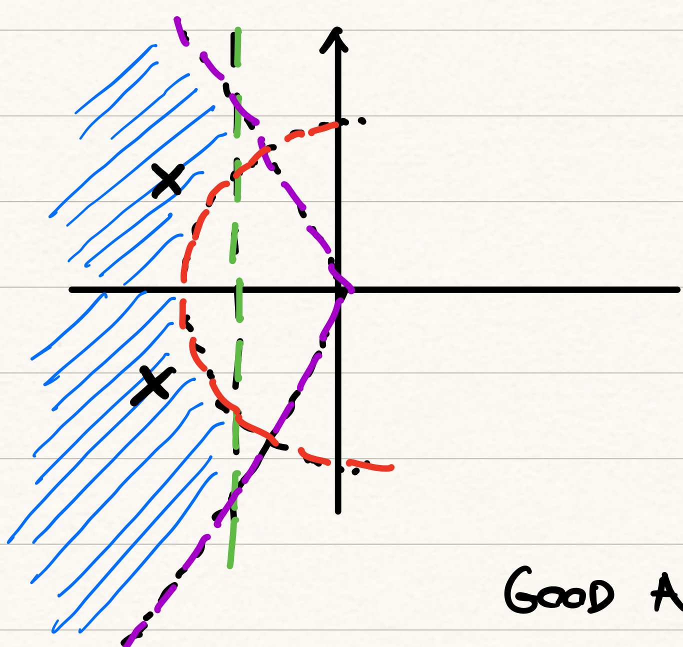
$t_r$ : RISE TIME (a bit harder)

$$\text{if } \xi \in [0{,}5\,,\,0{,}7] \Rightarrow t_r \simeq \frac{1{,}8}{\omega_n}$$

▷ We can use these formulas to translate SPECS

Assume we want $m_p \leq m_p^\circ$ , $t_s \leq t_s^\circ$ , $t_r \leq t_r^\circ$

We can traslate into :



$$\omega_n \geq \frac{1.8}{t_r^\circ}$$

$$\xi \geq \left( \frac{\ln(m_p^\circ)^2}{\ln(m_p^\circ)^2 + \pi^2} \right)^{1/2}$$

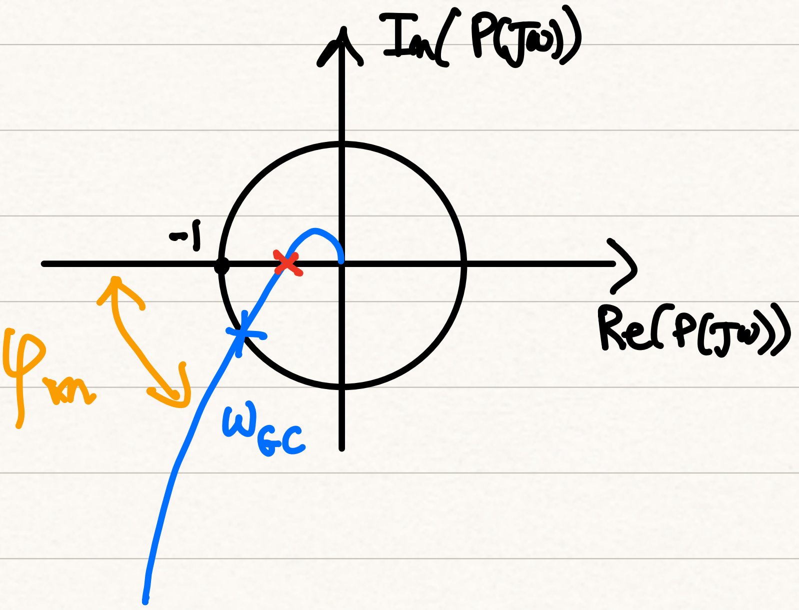
$$|\sigma| \geq \frac{3}{t_a^\circ}$$

GOOD APPROACH FOR POLE ALLOCATION DESIGN METHODS

## $2^{nd}$ APPROACH — Properties of $C(s)P(s)$ (BODE METHOD)

① Assume $P(s) = \dfrac{K}{s(s-p_1)(s-p_2)\dots}$

② GRAPH OF $P(j\omega)$ IN $\mathbb{C}$ PLANE (Nyquist plot)




$\omega_{GC} : |P(j\omega_{GC})| = 1$

$\omega_M : \arg P(j\omega_M) = \pi$

SYNTHESIS APPROACH WITH BODE METHOD

a) From SPECS, $t_r$, $m_p$, ESTIMATE $\omega_{GC}^*$, $\varphi_m^*$

$$\omega_{GC}^* \simeq 2/t_r \quad ; \quad \varphi_m^* \simeq 1{,}04 - 0{,}8\, m_p$$

[ b) Define $P'(s) = \dfrac{1}{U_R(s)} P(s)$ ← asymp. regime ]

c) FIND $P(j\omega_{gc}^*)$, $\arg P(j\omega_{gc}^*)$

d) IMPOSE $|C(j\omega_{gc}^*)| = \dfrac{1}{|P(j\omega_{gc}^*)|}$; $\arg C(j\omega_{gc}) = -\pi + \varphi_m - \arg P(j\omega_{gc})$

We can use it for <u>P.I.D design</u>

$$C(s) = K_P + \frac{K_I}{s} + K_D s$$

$$= K_P \left(1 + \frac{1}{T_I s} + T_D s\right)$$

$T_I = \frac{K_P}{K_I}$

$T_D = \frac{K_D}{K_P}$

- We need to IMPOSE :

$$C(j\omega_{gc}^*)\, P(j\omega_{gc}^*) = e^{j(-\pi + \varphi_m^*)}$$

- DEFINE : $\Delta K = |P(j\omega_{gc}^*)|^{-1}$

$$\Delta\varphi = -\pi + \varphi_m^* - \arg(P(j\omega_{gc}^*))$$

$$\Rightarrow \quad C(j\omega_{gc}^*) = \Delta K \cdot e^{j\Delta\varphi} \qquad (1)$$

- Use PID TF in (1) :

↳ TAKE REAL PART ON BOTH SIDES

$$K_P = \Delta K \cos(\Delta\varphi) \qquad (2)$$

↳ IMAGINARY PART :

$$K_P\left(-\frac{1}{T_I \omega_{gc}^*} + T_D\, \omega_{gc}^*\right) = \Delta K \sin(\Delta\varphi) \qquad (3)$$

Use (2) in (3), devide by $K_P$

$$\left( -\frac{1}{T_I \omega_{gc}^*} + T \omega_{gc}^* \right) = \frac{\Delta K \sin(\Delta\varphi)}{\Delta K \cos(\Delta\varphi)} = \tan(\Delta\varphi)$$

- FROM EXPERIENCE / GOOD PRACTICE CHOOSE $T_I = \alpha T_D$

  $\alpha \gtrsim 4$

$\Rightarrow$ P.I.D Parameters can be computed:

$$\begin{cases} K_p = \Delta K \cos(\Delta\varphi) \\ T_D = \dfrac{\tan(\Delta\varphi) + \sqrt{\tan(\Delta\varphi)^2 + 4/\alpha}}{2\omega_{gc}^*} \\ T_I = \alpha T_D \end{cases}$$

SIMILAR APPROACH FOR PD:

$$K_p = \Delta K \cos(\Delta\varphi)$$

$$K_D = \frac{1}{\omega_{gc}^*} \Delta K \sin(\Delta\varphi)$$

SIMILAR APPROACH FOR PI:

$$K_p = \Delta K \cos(\Delta\varphi)$$

$K_I = -\omega_{gc}\,\Delta K\,\sin(\Delta\varphi)$

WARNING : $T_D s$ NOT PROPER !

$\Rightarrow$ we use $\dfrac{T_D s}{T_L s + 1}$ , $T_L \geq \beta\,\omega_{gc}^*$

$\beta \in [2, 5]$