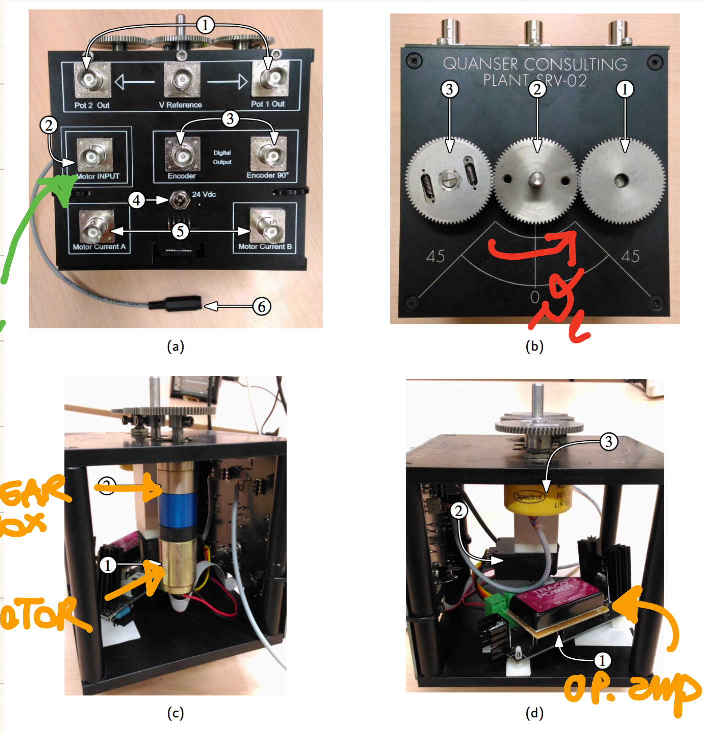
MODELING OF THE DC MOTOR

(INCLUDING ELECTRICAL, GEARBOX, LOAD, ...)



(a) (b) (c) (d)

(MOSTLY) MODELING FROM FIRST PRINCIPLES

QUALITATIVE ANALYSIS:

INPUT   u [VOLT]

from MATLAB → CONN. BOARD

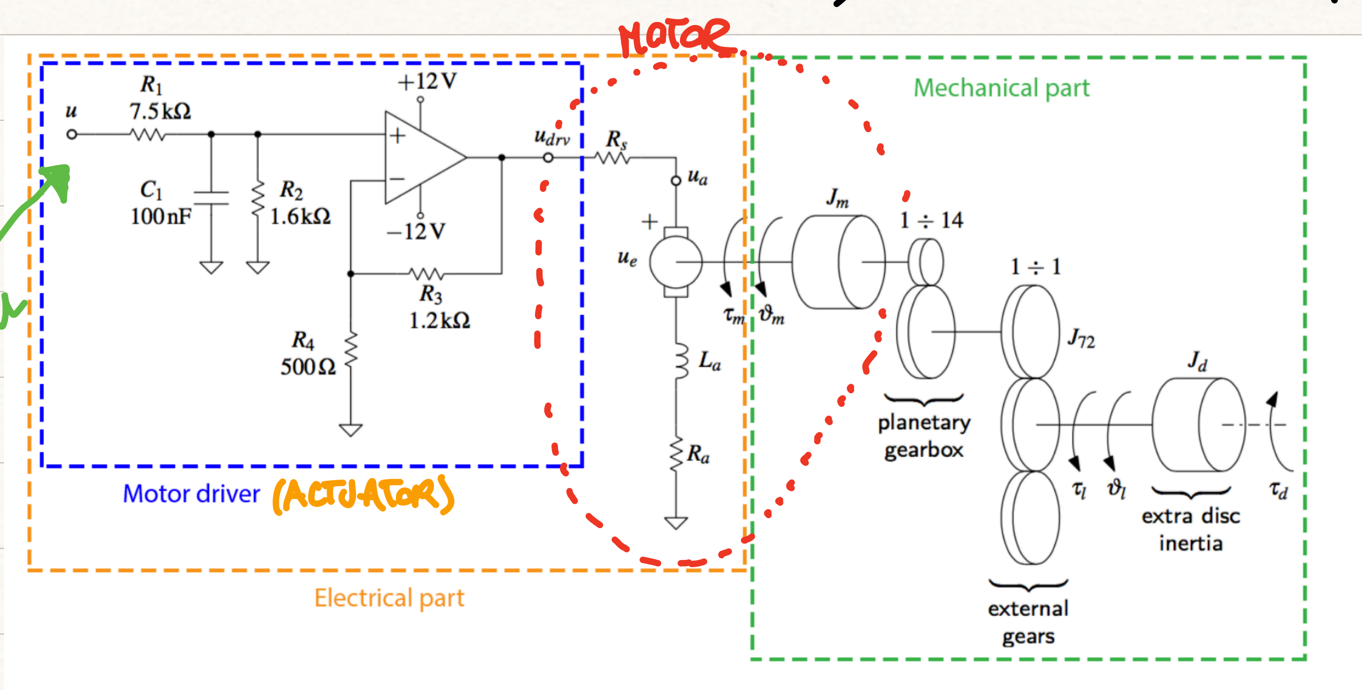
OUTPUT ϑ_L [RAD]

LOAD ANGULAR POSITION

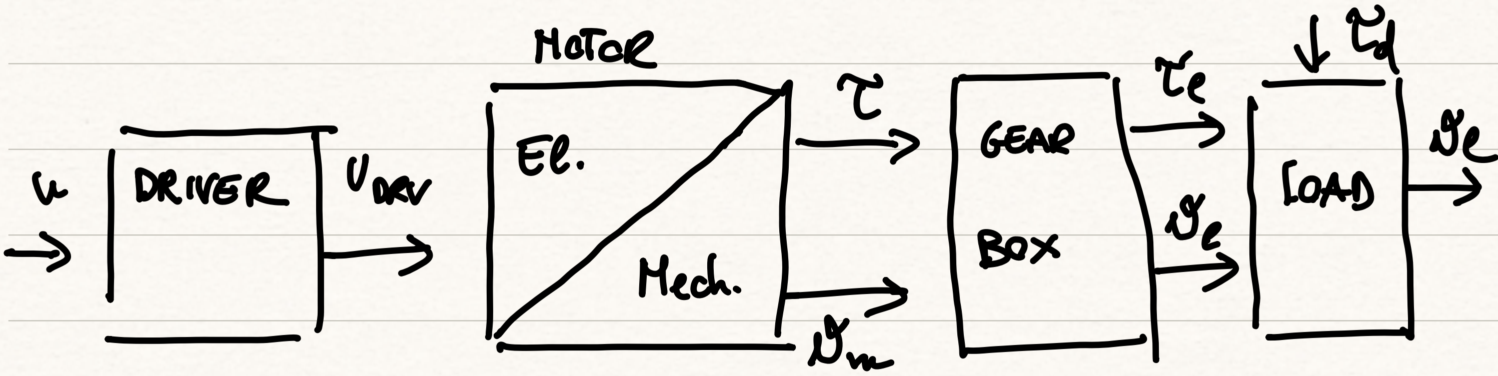
▷ We want :

$$P(s) = \frac{\Omega_L(s)}{U(s)}$$

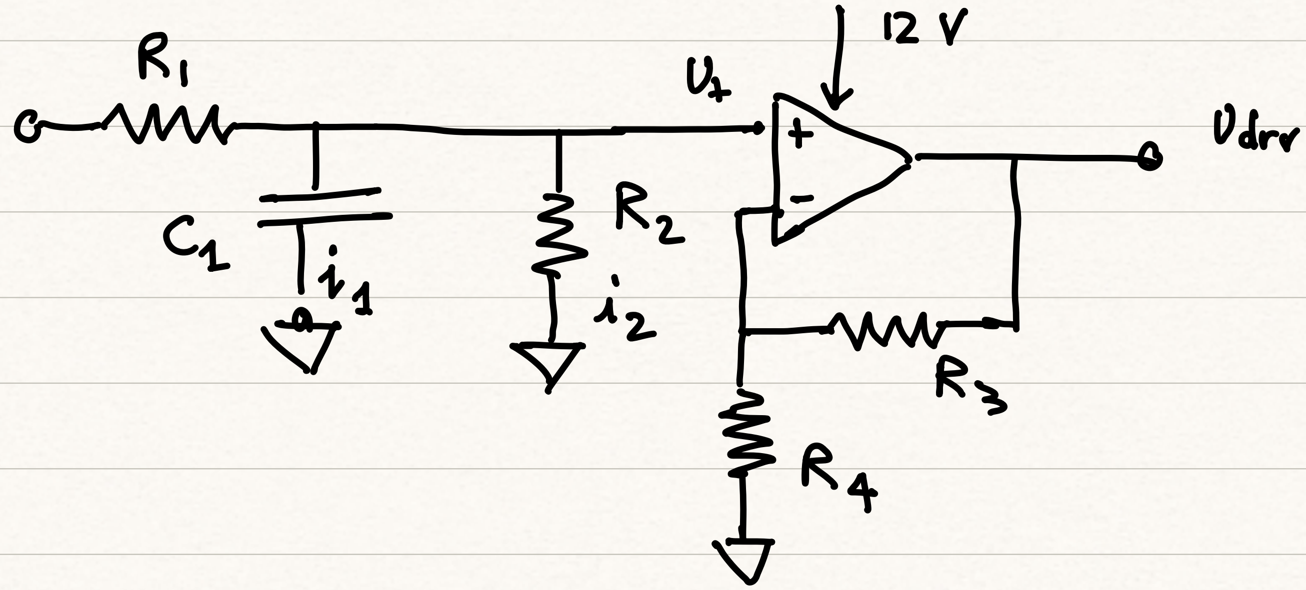
▷ MORE DETAILS : CLAB GUIDE 1 , HANDOUT FOR LAB Ø



WE CAN RE-ORGANIZE INTO BLOCKS (I/O)



PART 1 : DRIVER (INPUT CIRCUIT + POWER AMP)



$$u_+ = u - R_1 (i_1 + i_2) \qquad \begin{cases} i_1 = C_1 \dfrac{du_+}{dt} \\ i_2 = \dfrac{u_+}{R_2} \end{cases}$$

$$\Rightarrow u_+ = u - R_1 \left(C_1 \frac{du_+}{dt} + \frac{u_+}{R_2} \right)$$

$$\Rightarrow \frac{R_1 + R_2}{R_2} u_+ + C_1 R_1 \frac{du_+}{dt} = u$$

NEXT:
OP. AMP.
(Non inv. Config)

$$u_{drv} = \left(1 + \frac{R_3}{R_4} \right) u_+$$

Putting Things Together : $u \rightarrow U_{drv}$

$$\boxed{\underbrace{\frac{R_1 R_2 C_1}{R_1+R_2}}_{=T_{DRV}} \frac{dU_{drv}}{dt} + U_{drv} = \underbrace{\left(1+\frac{R_3}{R_4}\right)\frac{R_2}{R_1+R_2}}_{=K_{DRV}} u}$$

$$\Rightarrow \quad U_{DRV}(s) = \frac{K_{DRV}}{1+T_{DRV}\,s} U(s)$$

$\leftarrow$ IST ORDER

PART 2 : MOTOR

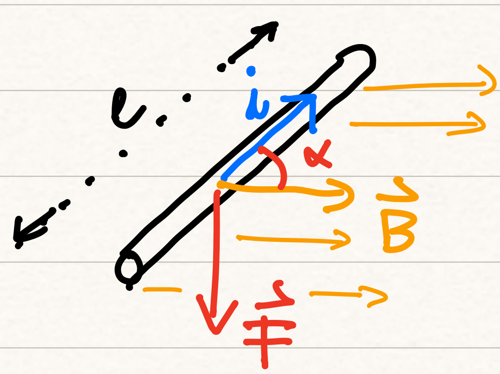
FUNCTIONING PRINCIPLES :

- Electrical current in magnetic field, in a conductor of length ℓ, generates force:

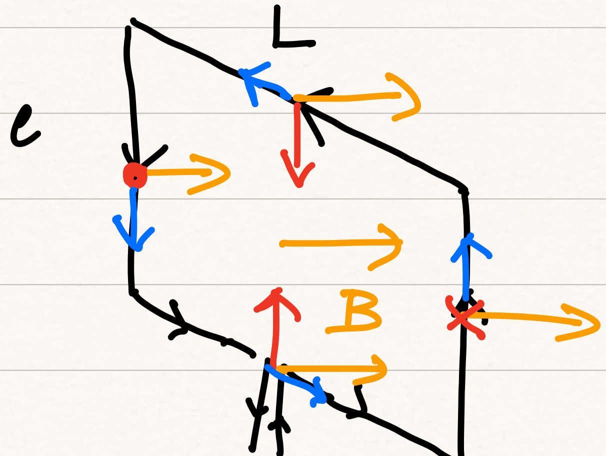
$$\vec{F} = \ell \; \vec{i} \times \vec{B}$$

$$|F| = \ell \, |i|\,|B| \; \sin(\alpha)$$

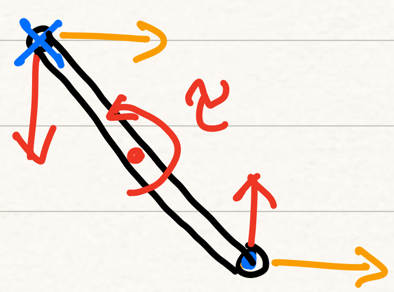
$$\vec{F} \perp \vec{i}, \vec{B}$$



- IF WE MAKE A COIL :



FROM ABOVE

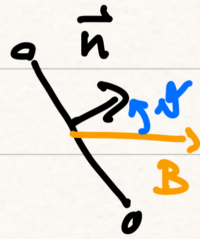


⇒ we get **Torque**: $\tau = 2\frac{L}{2}\, i \ell B \sin\vartheta$

The Mech effect (current → rotation)
creates a "Back action"
BACK - ELECTROMOTIVE FORCE (Lenz Law)

$$V_{em} = F_{em} = -\frac{d\Phi}{dt}$$ ← variation of concatenated FLUX

$$\Phi(t) = \langle \vec{n}, \vec{B} \rangle \cdot \underset{\text{area}}{A} = BL\ell \cos\vartheta$$



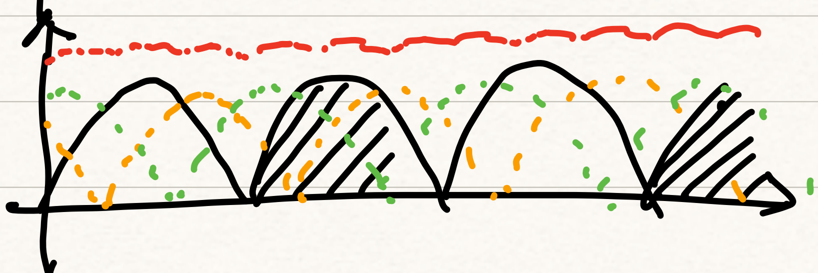
$$V_{em} = -BL\ell \sin(\vartheta)\,\dot{\vartheta} \overset{!}{=} -BL\ell \sin(\vartheta)\,\omega$$

⇒ For each loop (coil): $V_{Loop} = Ri + L\frac{di}{dt} + V_{em}$

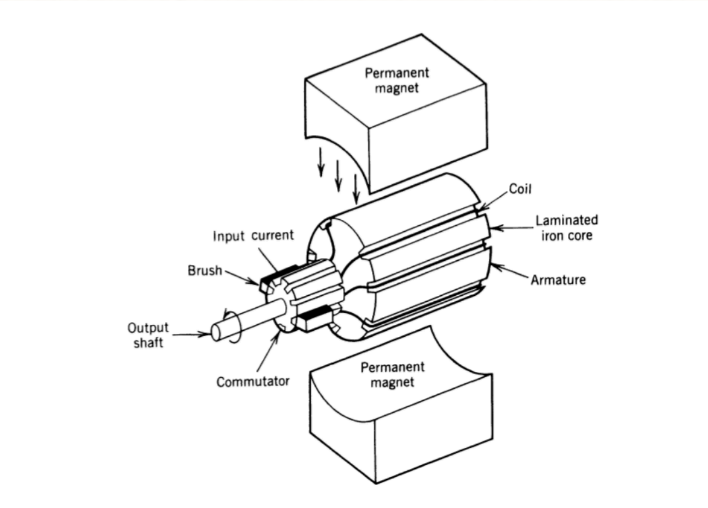
Pb if a single coil rotates:

← Neg. Torque

ROTATING BRUSHES



+MULTIPLE COILS



MOTOR

$$\tau_m \simeq \text{constant} \simeq i\,\underbrace{B A_{eq}}_{K_T}$$

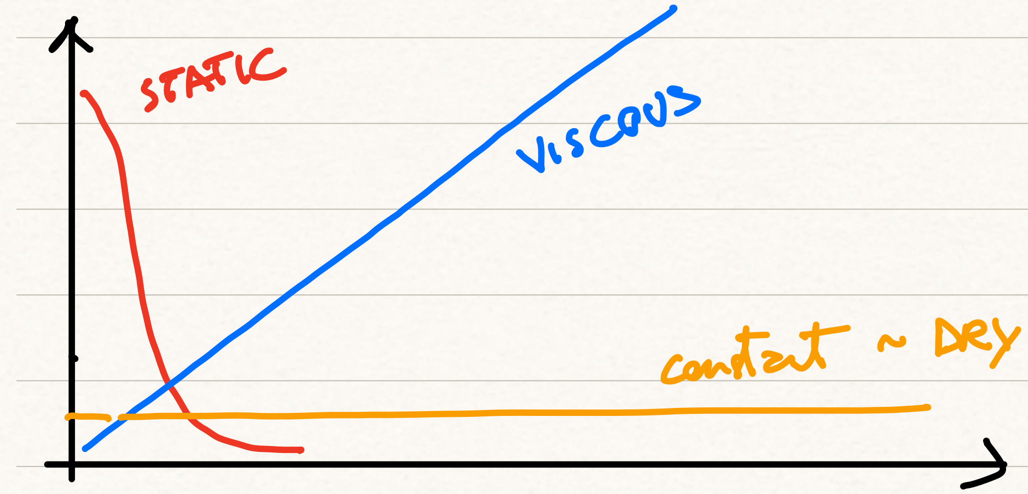
TORQUE CONSTANT

$$\simeq K_T\, i$$

$$V_{em} \simeq B A_{eq}\,\omega_m \simeq K_{el}\,\omega_m$$

K_{el} IS K_T

FRICTION



WE CONSIDER VISCOUS $\tau_v = B_m\,\omega_m$

! FRICTION CONSTANT, NO M.F.

STATIC AS OR

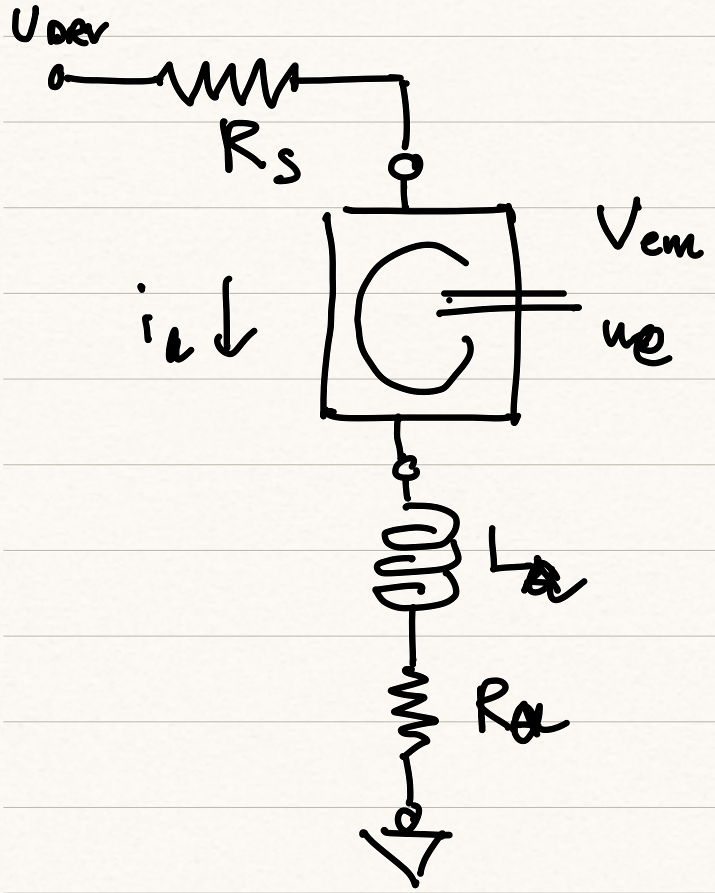
$$\left[\tau_d = \tau_{sF}\,\text{sign}(\omega_m)\right]$$

MOTOR MECH. EQ.

$$J_m \frac{d\omega_m}{dt} = -B_m\,\omega_m + \underset{\uparrow \text{MOTOR}}{\tau_m} - \tau_l'$$

TORQUE OF LOAD, TRANSLATED BEFORE GEARBOX

MOTOR EL. EQ.

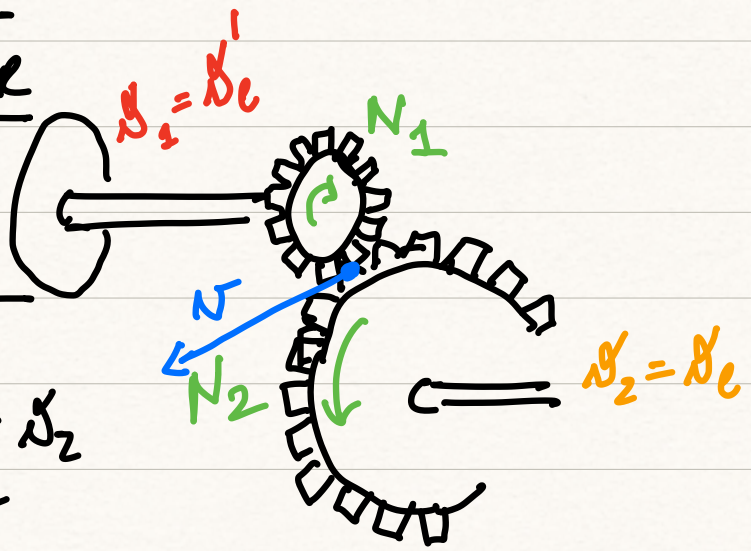


$$L_a \frac{di_a}{dt} + R_a + R_s \, i_a = U_{DRV} + V_e$$

V_{em} induces coupling between mech. & electrical

NEXT : **GEARBOX**
PART 3

MOTOR



$$\vartheta_1 = \frac{N_1}{N_2} \vartheta_2$$

N_i : number of teeth

PROPORTIONAL TO RADIUS

KEY Hyp. :

NO SLIP

same velocity (Linear) at point of contact.

$$|v_1| = K_R N_1 \omega_1 = K_R N_2 \omega_2 = |v_2|$$

$$\Rightarrow \boxed{\omega_1 N_1 = \omega_2 N_2} \quad (\Delta)$$

This can be used to "convert" angular speed

ALSO : ASSUME IDEAL POWER TRANSMISSION

$$P_1 = \omega_1 \tau_1 = \omega_2 \tau_2 = P_2$$

$$\Rightarrow \quad \frac{\tau_2}{\tau_1} = \frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = N = 14$$ (FOR US)

PART 4 : LOAD + WHEELS (mech)

$$\underbrace{J_e}_{J_{DISC} + 3 J_{72}} \frac{d\omega_e}{dt} + B_e \omega_e = \tau_e - \tau_d$$

wheels, 72 Teeth.

Let's put the model together :

- all mech. eqts., at MOTOR SIDE (before gearbox)

$$J_{eq} \frac{d\omega_m}{dt} + B_{eq} \omega_m = \tau_m - \frac{1}{N} \tau_d$$

$$J_{eq} = J_m + \frac{J_e}{N^2}$$

$$B_{eq} = B_m + \frac{B_e}{N^2}$$

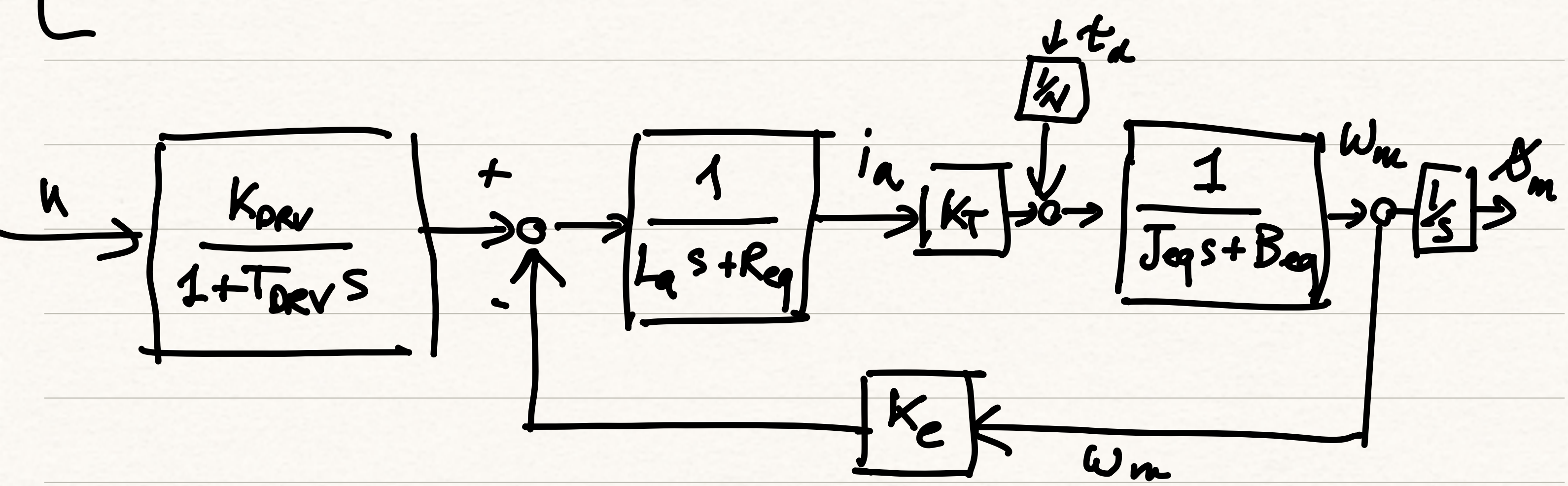
EFFECT OF MOVING LOAD $\longrightarrow$ MOTOR SIDE

MOTOR MODEL

$$\begin{cases} T_{DRV}\dfrac{dU_{drv}}{dt} + U_{drv} = K_{drv}\, u \\[2ex] L_a \dfrac{di_a}{dt} + R_{eq}\, i_a = U_{drv} - \underbrace{K_e\, \omega_m}_{u_e} \sim V_{em} \\[2ex] J_{eq}\dfrac{d\omega_m}{dt} + B_{eq}\,\omega_m = K_T\, i_a - \frac{1}{N}\tau_d \end{cases}$$

$\mathcal{L}$ - TRANSFORM:

$$\begin{cases} (T_{DRV}\, s + 1)\, U_{DRV}(s) = K_{DRV}\, U(s) \\[1ex] (L_a s + R_{eq})\, I_a(s) = U_{DRV}(s) - K_e\, \Omega_m(s) \\[1ex] (J_{eq}\, s + B_{eq})\, \Omega_m(s) = K_T\, I_a(s) - \frac{1}{N} T_d(s) \end{cases}$$



$$P_{u\to\vartheta_m}(s) = \frac{1}{s}\,\frac{K_T}{(L_a s + R_{eq})(J_{eq}\, s + B_{eq}) + K_T K_e}\,\frac{K_{DRV}}{T_{DRV}\, s + 1}$$

REDUCED MODEL (TRY TO DERIVE IT; use $\frac{L_a}{R_{eq}} \ll 1$, $T_{DRV} \ll 1$)

$$\simeq \frac{K_T K_{DRV}}{R_{eq}(J_{eq}s + B_{eq}) + K_T K_e} \frac{1}{s}$$

$$P_{u \to \theta_\ell}(s) \simeq \frac{K_m}{T_m s + 1} \frac{1}{Ns}$$

$$K_m = \frac{K_{DRV} K_T}{R_{eq} B_{eq} + K_T K_e}$$

$$T_m = \frac{R_{eq} J_{eq}}{R_{eq} B_{eq} + K_T K_e}$$