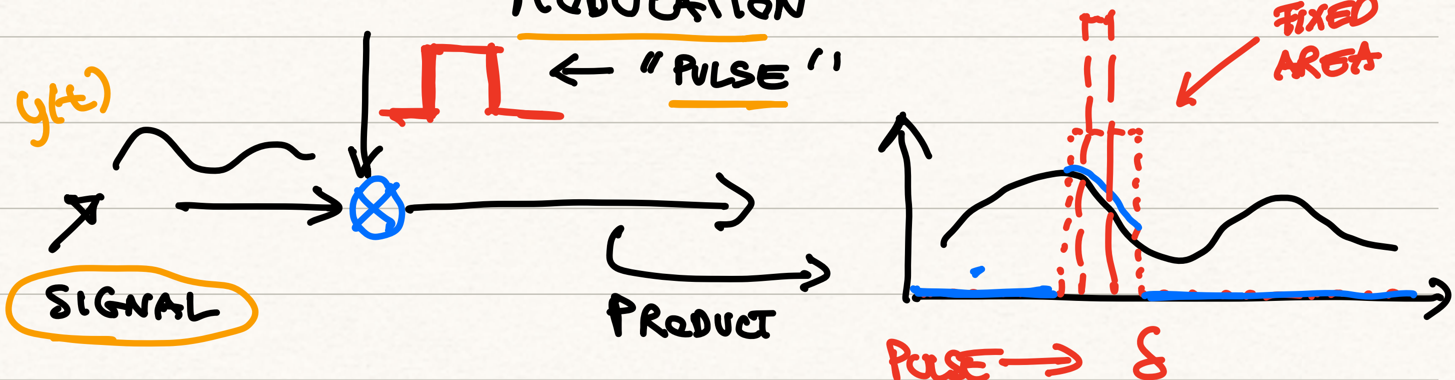
IN THIS VIDEO : WE CONTINUE OUR REVIEW OF SAMPLE-DATA SYSTEMS & CONTROL

PREVIOUSLY :
- → BASIC ASSUMPTION - INTRODUCTION
- → MODELS :
  - Diff. Eqts
  - I/O
  - STATE-SPACE
- → Z - TRANSFORM
- → DISCRETE-TIME MODES
- → STABILITY

NEXT : Brief Review of reps. of
- ↳ SAMPLING (A/D)
- ↳ ZOH (D/A)

FROM A "CONTINUOUS TIME" VIEWPOINT

→ Recall : SAMPLING VIA PULSE AMPLITUDE MODULATION

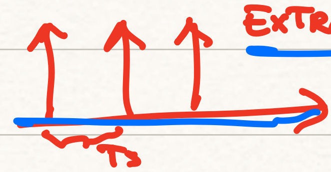


▷ SAMPLING

↳ PRODUCT WITH A TRAIN OF PULSES

$$y(t) \longrightarrow \otimes \xleftarrow{\ \delta_T(t)\ } \longrightarrow \boxed{y_\delta(t) = \sum_k \underbrace{y(t)\,\delta(t - kT_s)}}$$

$$\boxed{\delta_T(t) = \sum_k \delta(t - kT_s)}$$



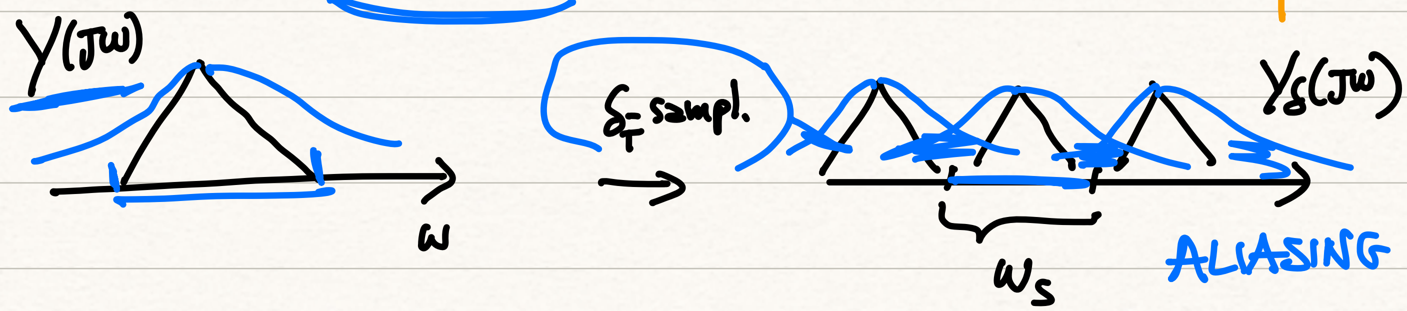
EXTRACTS $y(kT_s)$

▷ USING $\mathcal{L}$ - TRANSF. PROPERTIES , $\omega_s := \frac{2\pi}{T_s}$

$$Y_\delta(s) = \frac{1}{T_s} \sum_k \mathcal{L}\left[e^{jk\omega_s t}\, y(t)\right]$$

$$= \frac{1}{T_s} \sum_k \underbrace{Y(s - jk\omega_s)}$$

TRANSLATED COPIES OF $Y(s)$



• ON THE OTHER HAND :

$$Y_\delta(s) = \underbrace{\int_0^{+\infty} y(\tau)\,\delta(\tau - kT_s)\, e^{-s\tau}\, d\tau}$$

RELATION :

$Y(s) \to Y_\delta(s)$

$\Updownarrow$

$Y(z)$

$$= \sum_k \underbrace{y(kT_s)}\ \underbrace{e^{-sT_s k}}$$

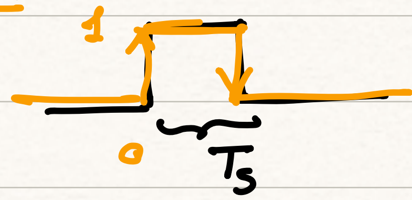
$$= \underbrace{\sum_k y(kT_s)\, (e^{sT_s})^{-k}} = \underbrace{\mathcal{Z}[y(k)]}(e^{sT_s})$$

▷ CAN I MODEL ZOH (D/A) IN CONT. TIME? $z = e^{sT_s}$

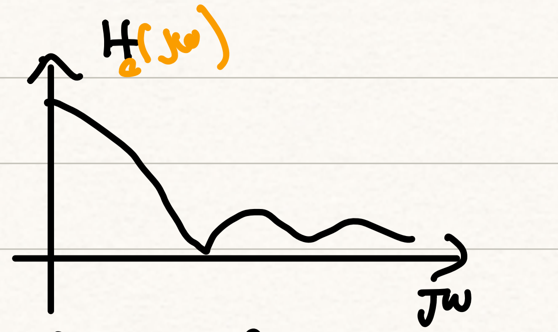
As a filter from $u(k) \approx u_\delta(t) \to \tilde{u}(t)$

▷ CONVOLUTION WITH IMPULSE RESPONSE:

$$h_0(t) = 1\!\!1(t) - 1\!\!1(t - T_s)$$

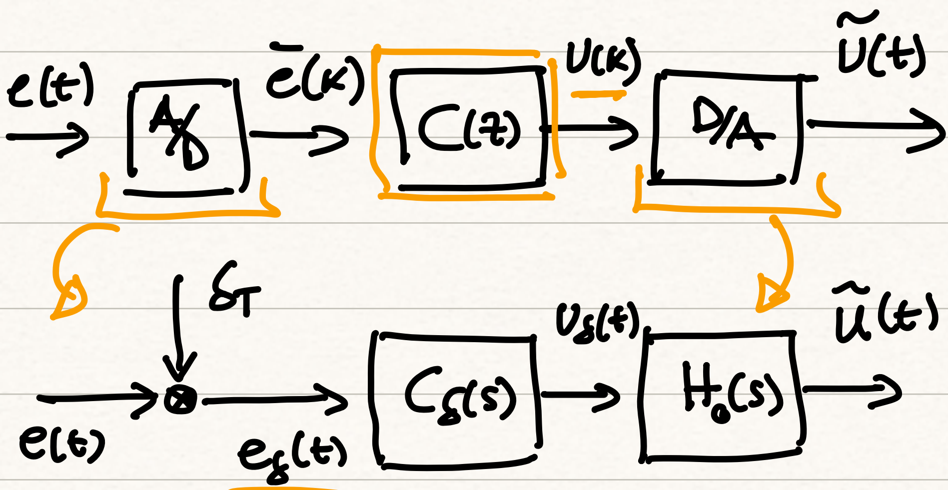


$$\Rightarrow \left\{ H_0(s) = \frac{1}{s} - \frac{e^{sT_s}}{s} = \frac{1 - e^{sT_s}}{s} \right.$$



Low Pass ≈ RECONSTR. FILTER IN SAMPLING

▷ what do we do with this?



THIS IS WHAT THE PLANT "SEES"! $P(s)$

$$\Rightarrow \frac{\tilde{U}(s)}{E(s)} = \frac{1}{T_s} H_0(s)\, C_\delta(s) \leftrightarrow \bar{C}(z)$$

$$= \frac{1 - e^{-sT_s}}{T_s\, s}\, \bar{C}(e^{sT_s})$$

▷ OK, SO WE CAN OBTAIN A CONTINUOUS MODEL FROM A DIGITAL ONE.

▷ WHAT ABOUT THE OPPOSITE?

e.g. $P(s) \longrightarrow P(z)$

$$\underbrace{\longrightarrow \boxed{D/A} \longrightarrow \boxed{P(s)} \longrightarrow \boxed{A/D}}_{P(z)} \longrightarrow$$

why? IT ACCOUNTS FOR THE EFFECTS OF A/D, D/A WHEN DESIGNING $C(s)$, TO BE EMULATED.

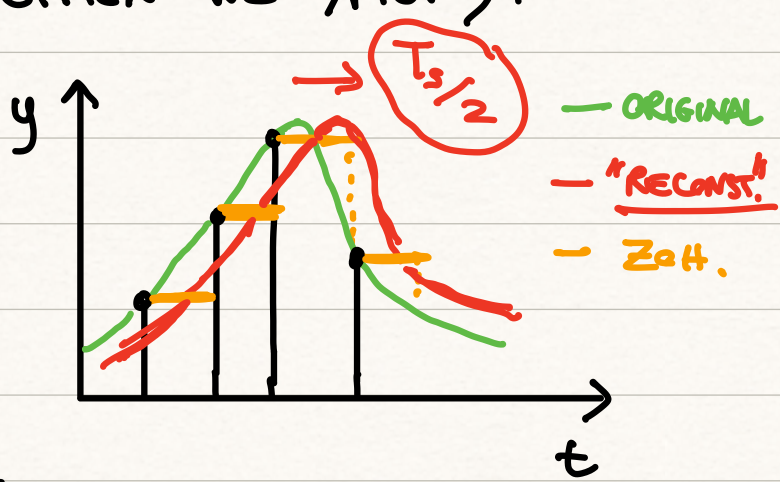
- $\underbrace{\boxed{D/A} \rightarrow \boxed{P(s)}}$ WE ALREADY KNOW:

$$\Rightarrow \tilde{P}(s) = P(s)H_0(s) = P(s)\frac{1-e^{-sT_s}}{s}$$

$\Rightarrow$ FOR DESIGN WITH EMULATION WE TYPICALLY:

$$H_0(s) \simeq e^{-s\frac{T_s}{2}}$$

$\mathcal{L}$- TRANSFORM OF A $\frac{T_s}{2}$ DELAY



$$\Rightarrow \tilde{P}(s) \simeq e^{-s\frac{T_s}{2}}\, P(s)$$

▷ IF WE NEED $P(z)$ WE CAN :

1 - COMPUTE $\mathcal{L}^{-1}[P(s)H_0(s)] = \tilde{p}(t)$

2 - SAMPLE IT $\tilde{p}(t) \to \tilde{p}(kT_s)$

3 - $\mathcal{Z}$-TRANSFORM $\tilde{p}(k) = \tilde{p}(kT_s)$

⇒ OBTAIN EXACT $P(z)$ FOR DIGITAL DESIGN

1 - $\tilde{p}(t) = \mathcal{L}^{-1}\left[\frac{P(s)}{s}\left(1 - e^{sT_s}\right)\right]$

$= \mathcal{L}^{-1}\left[\frac{P(s)}{s}\right](t) - \mathcal{L}^{-1}\left[\frac{P(s)}{s}e^{sT_s}\right](t)$ $(t - T_s)$

2 - $\tilde{p}(kT_s) = \mathcal{L}^{-1}\left[\frac{P(s)}{s}\right](kT_s) - \mathcal{L}^{-1}\left[\frac{P(s)}{s}\right]((k-1)T_s)$

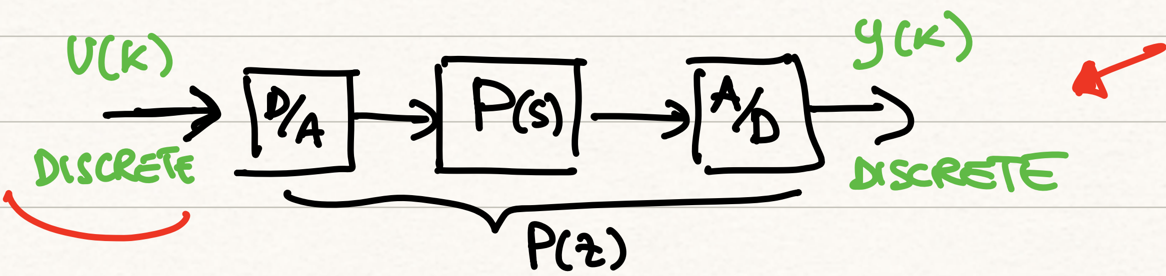
3 - $P(z) = \mathcal{Z}[\tilde{p}(kT_s)](z)$

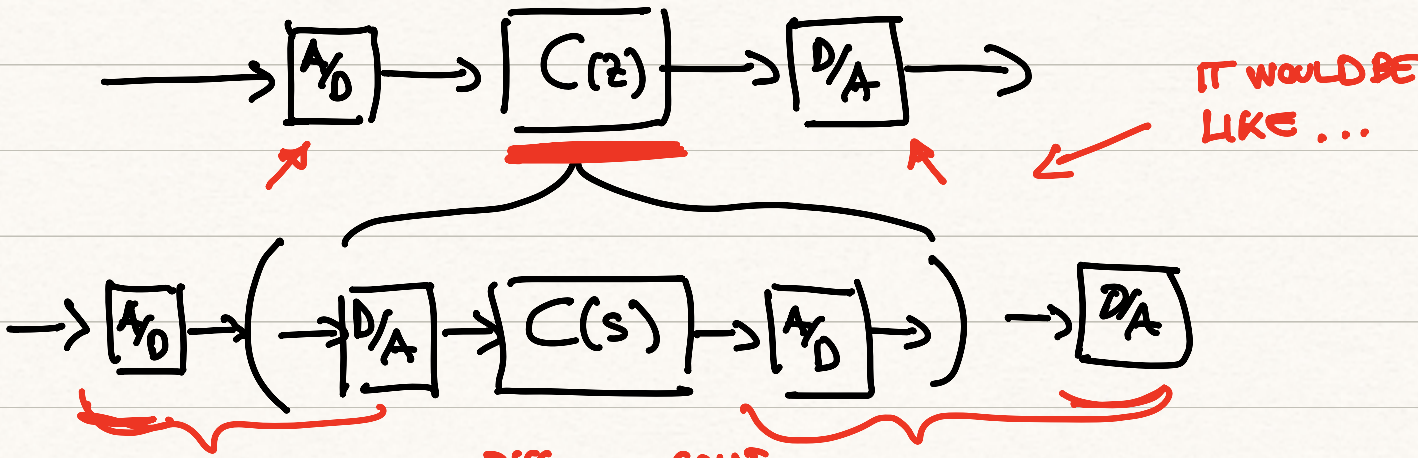
$= (1 - z^{-1})\,\mathcal{Z}\left[\mathcal{L}^{-1}\left[\frac{P(s)}{s}\right](kT_s)\right]$

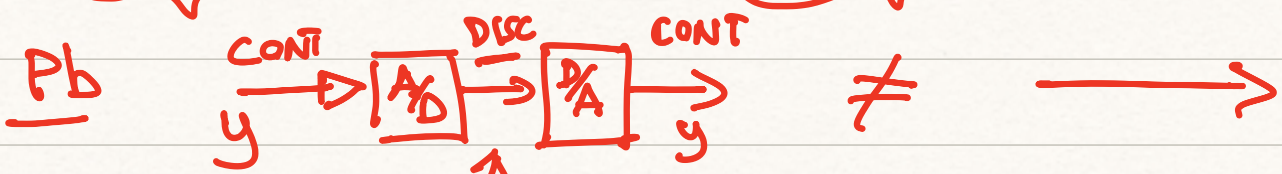
NOTE : This is what MATLAB does with C2D, option 'ZOH'.

- EXACT translation for P
- can I use it for emulation $C(s) \to C(z)$?

NOT EXACT in that case -







I LOSE INTERSAMPLE INFORMATION...

- This loss of information cannot be avoided
- WHAT CAN I USE FOR TRANSLATION ?
  - WE NEED A MAP FROM $S \rightarrow Z$

$\Rightarrow$ APPROXIMATIONS OF

- $z = e^{sT_s}$ Pb NON RATIONAL ! $\Rightarrow$ TUSTIN (APPROX)
- $s \simeq \mathcal{L}[\frac{d}{dt}\cdot] \rightarrow s \simeq$ INCREMENTAL RATIO

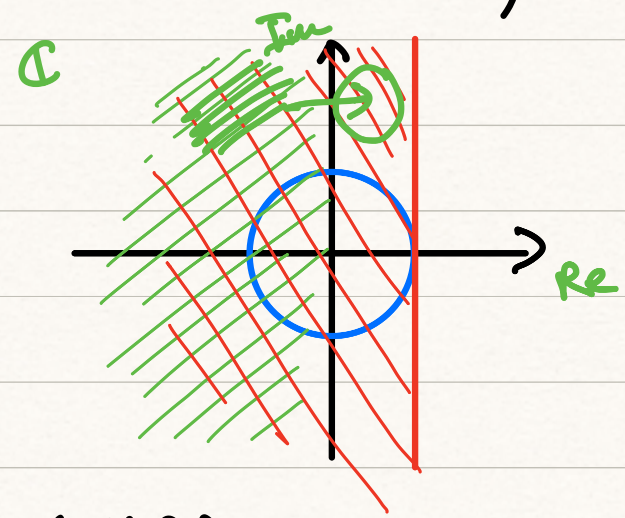
FORWARD EULER

$$\frac{d}{dt}x(t) \simeq \frac{x(k+1) - x(k)}{T_s}$$

Pb - NOT PROPER (NOT CAUSAL)

$$s \simeq \frac{z-1}{T_s}$$

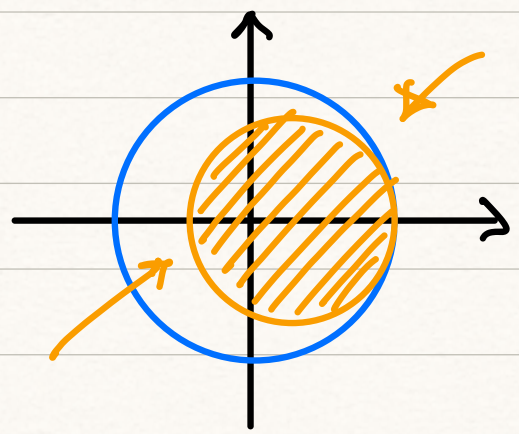
- MAPS STABLE POLES (IN s) TO POTENTIALLY UNSTABLE ONES



//// → STABLE CONT. POLES

//// → "mapped" poles might be unstable

BACKWARD EULER



$$\frac{d}{dt}x(t) \simeq \frac{x(k) - x(k-1)}{T_s}$$

$$s \rightarrow \frac{1 - z^{-1}}{T_s} = \frac{z-1}{T_s z}$$

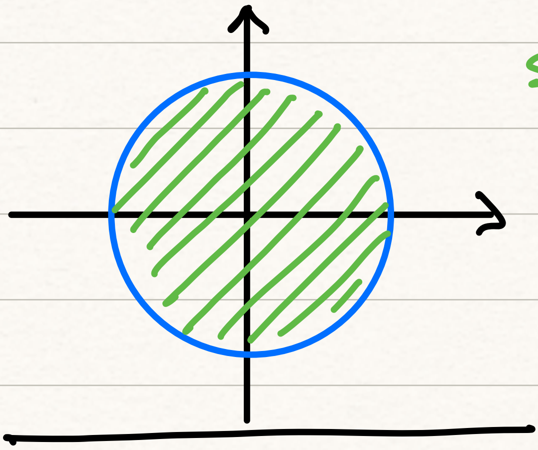
PROPER, MAINTAINS STABILITY

TUSTIN

TRAPEZOIDAL APPROX.

or PADÉ APPROX. OF e^{sT_s} OF ORDER (1,1)

$$s \rightarrow \frac{2}{T}\,\frac{z-1}{z-2}$$



STABLE CONT. POLES MAPPED TO STABLE DISCRETE POLES

$$C(s) \xrightarrow{\text{EMULATE}} \tilde{C}(z) = C(s)\Big|_{s=(\dots)}$$

▷ Same approaches for STATE-SPACE MODELS

↳ FORMULAS IN HANDOUT (EULER, TUSTIN, ...)

↳ MATLAB C2D + OPTIONS

▷ WE CAN ALSO DERIVE MATRICES FOR EXACT TRANSLATION $(A_c, B_c, C_c, D_c) \to (A_D, B_D, C_D, D_D)$

$$x(t) = e^{A(t-t_0)}x_0 + \int_{t_0}^{t} e^{A(t-\tau)} B\, u(\tau)\, d\tau$$

▷ Assume $u(t)$ piecewise constant,

$t_0 = kT_s$, $t = (k+1)T_s$

$$x(k+1) = x((k+1)T_s) = e^{AT_s} x(kT_s) + \int_{kT_s}^{(k+1)T_s} e^{A((k+1)T_s-\tau)} B u(\tau) d\tau$$

$$= \underbrace{e^{AT_s}}_{\Phi} x(k) + \underbrace{\left[\int_{kT_s}^{(k+1)T_s} e^{A(\dots)} B\, d\tau \right]}_{\Gamma} u(k)$$

INDEP. OF k : $\tilde{\tau} := kT_s + \tau$

$$\Rightarrow \quad x(k+1) = \Phi\, x(k) + \Gamma\, u(k)$$

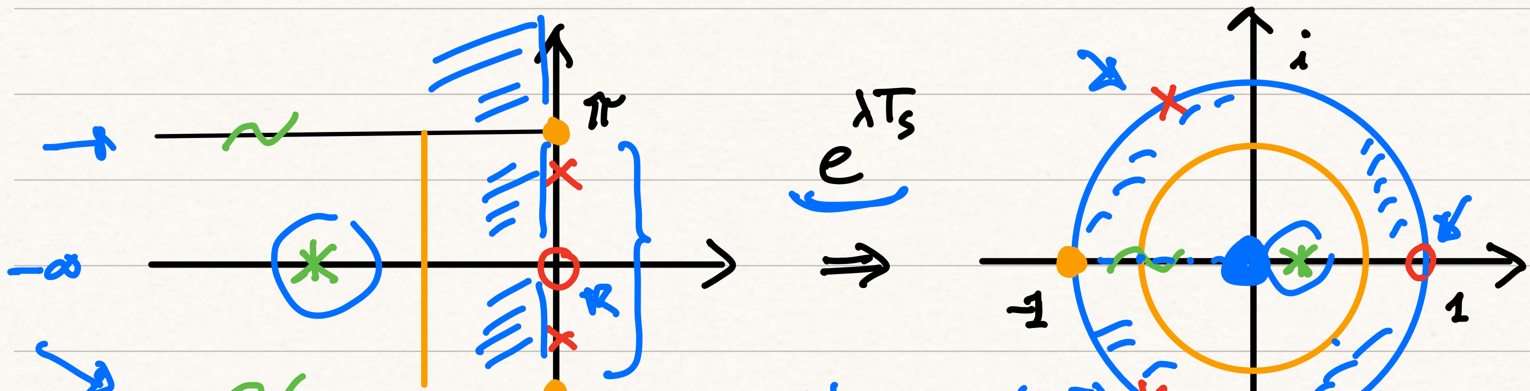
$$y(k) = C\, x(k) + D\, u(k)$$

↑ STATIC, NO NEED FOR CONVERSION.

Note: $A \rightarrow \Phi = e^{AT_s}$

$\lambda(A) \longrightarrow \lambda(\Phi) = e^{\lambda(A)T_s}$

SAME MAP WE ALREADY SAW!



$$e^{a+jb} = e^{a+j(b+2\pi)}$$

▷ The corresponding, exactly translated $P(z)$ is

$$P(z) = C\,(zI - \Phi)^{-1}\,\Gamma + D$$

OK, WE ARE NOW READY TO OUTLINE THE DESIGN APPROACHES ...

APPROACH 1 I/O MODELS, VIA EMULATION

1) Acquire specs., translate them (ω_c, φ; P_1, P_2)

2) Design CONTINUOUS $\boxed{C(s)}$ for $\tilde{P}(s) = P(s)\, e^{-s\frac{T_s}{2}}$

3) EMULATE $C(s) \longrightarrow C(z)$ obtained via TUSTIN, EULER, ..

APPROACH 2 I/O MODELS, VIA DIRECT DIGITAL DESIGN

1) Acquire specs, translate into POLE POSITIONS (in CONT. TIME)

2) TRANSLATE TO DISCRETE POLES:

$$\boxed{p_{DISC} = e^{p_{CONT} T_s}}$$

3) DESIGN $C(z)$: → DIOFANTINE EQTS. ...
→ CONSIDER EXACT. TRANSL $P(z)$

▷ NOT IN THE HANDOUT / ASSIGNMENT :
YOU CAN TRY METHODS YOU HAVE SEEN IN DIGITAL CONTROL.

APPROACH 3 | STATE-SPACE, VIA "EMULATION"

1) Acquire specs, Translate into pole positions (in cont.T)

2) FIND K IN CONT. TIME $\lambda(A-BK) = \dots$

3) DESIGN OBSERVER IN CONT. TIME

4) TRANSLATE OBSERVER USING EMULATION FORMULAS

APPROACH 4 | STATE SPACE, DIRECT DIGITAL DESIGN

1) ACQUIRE SPECS, TRANSLATE IN POLE POSITIONS IN CONT.TIME

2) TRANSLATE TO DISCRETE POLES:

$$p_{DISC} = e^{p_{CONT} T_S}$$

3) FIND EXACT DISCRETE TIME MODEL (Φ, Γ, C, D) (in st. sp.)

4) ALLOCATE POLES IN DISC.TIME

Usual formulas WORK!

if (Φ, Γ) Reachable,

$\exists K$ s.t. $\text{eigs}(\Phi - \Gamma K)$ are the desired ones.

5) DESIGN OBSERVER IN DISCRETE-TIME.

↳ Reduced-ORDER OBSERV.

NOTES : • Feedforward

↳ In emulation, save static gains

↳ In direct digital design:

Eq. EQT.

was $\emptyset$

$$\begin{cases} x_\infty = \Phi x_\infty + \Gamma u_\infty \\ r_\infty = C x_\infty \end{cases} \qquad (D=0)$$

$$\Rightarrow \begin{bmatrix} \Phi - I & \Gamma \\ C & 0 \end{bmatrix} \begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

N_r

• INTEGRAL CONTROL : emulate

$$x_I(k) = x_I(k-1) + \underbrace{T_s\, e(k)}_{\text{INTEGRAL OF CONSTANT FOR } T_s \text{ TIME...}}$$