Finite Element Simulation of 2D Metal Strip Vibration in Hot-Dip Galvanization Process

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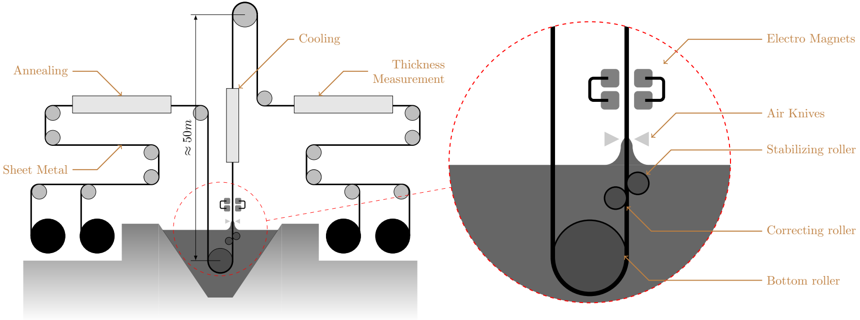
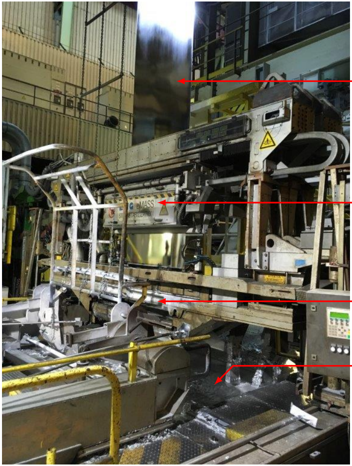
Objective of the internship

- To understand the Hot-Dip Galvanization process to have an idea about the loads and boundary condition.
- To find existing research about finite element of strip used in this process.
- To formulate plate equation of motion
- To code accurate and efficient finite element program from scratch
- To test the finite element code with analytical and existing numerical methods
- To find a way to integrate with existing control algorithms.

Hot-Dip Galvanization Process



(a) Schematics of the Hot-Dip Galvanization Line



(b) Hot-Dip Galvanization Line

- A thin Layer of Zinc is coated to Increase the corrosion resistance of steel
- Air knives control the thickness of the Zinc layer
- Excessive Vibration results in uneven coating.
- Electromagnets are used to control the vibration of the strip.

Plan

Axially Moving Plate

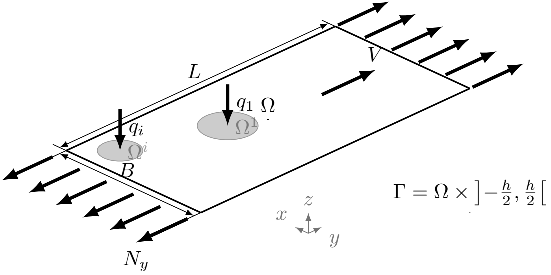


Figure: Description of domain

- Ω is the two dimensional domain strictly in xy plane
- h is the thickness of the plate
- $\Omega_1 ... \Omega_i$ are the sub-domains where pressure forces $q_1 .. q_i$ are applied
- V is the Line speed and Nx is the tension on the line
- L is the length and B is the width

The modified form of the Hamilton principle is taken as.

$$\delta H = \int_{t_0}^{t_1} (\delta U - \delta K + \delta W + \delta M)\, dt = 0 \tag{1}$$

$$\delta \mathbf{u}\Big|_{t_0}^{t_1} = 0 \tag{2}$$

δ is the variation, t_0 and t_1 are any arbitrary temporal points, U is the total potential energy , K is the kinematic energy and W is the work performed by external forces on the system. $\mathbf{u}$ is the total displacement of the plate. M is the momentum transports at boundaries y=0 and y = L.

$$\delta M = \int_0^W \int_{-h/2}^{h/2} L\rho \mathbf{v} \delta \mathbf{u}\Big|_{y=0}^{y=L} dzdx = 0 \tag{3}$$

Here, $\mathbf{v}$ is the total velocity vector of the plate. M becomes zero because the line speed is equal at the boundaries. So, there is no overall change in the mass of the plate.

Final Weak form

Substituting everything in the Hamilton principle and taking integration by parts gives the final weak form of the axially moving and axially tensed plate.

$$\int \int_{\Omega} \rho \ddot{\tilde{\boldsymbol{u}}}^T \boldsymbol{Z} \delta \tilde{\boldsymbol{u}} + \rho V_1 \delta \tilde{\boldsymbol{u}} \boldsymbol{Z} \dot{\tilde{\boldsymbol{u}}}_{,2} + \rho V_1 \tilde{\boldsymbol{u}} \boldsymbol{Z} \delta \dot{\tilde{\boldsymbol{u}}}_{,2} - \rho V_1^2 \tilde{\boldsymbol{u}}_{,2} \boldsymbol{Z} \delta \tilde{\boldsymbol{u}}_{,2}$$

$$+ \boldsymbol{\kappa}^T \tilde{\boldsymbol{D}} \delta \boldsymbol{\kappa} + \left(\boldsymbol{\epsilon}^S \right)^T \tilde{\boldsymbol{D}}_{\boldsymbol{c}} \delta \boldsymbol{\epsilon}^S + w_{,2} \tilde{\boldsymbol{\sigma}}^A \delta w_{,2} d\Omega = \sum_j^{nb} \int_{\Omega^j} q_j \delta \tilde{\boldsymbol{u}} d\Omega^j$$

First term in the equation corresponds to the local acceleration. second and third terms are the Coriolis acceleration and the fourth term is the centripetal acceleration. Because of these terms, this equation is gyroscopic.

Reissner Mindlin Plate element(QUAD4)

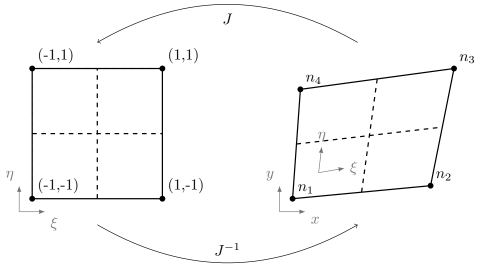


Figure: Jacobian transformation

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$
$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$
$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

$$\begin{Bmatrix} \frac{\partial N}{\partial \xi} \\ \frac{\partial N}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \frac{\partial N}{\partial x} \\ \frac{\partial N}{\partial y} \end{Bmatrix} \quad J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \tag{4}$$

The triangle element with three nodes, is given here.

$$N = \begin{bmatrix} P(1) - P(4) + P(6) + 2*(P(7) - P(9)) \\ -b(2)*(P(9) - P(6)) - b(3)*P(7) \\ -c(2)*(P(9) - P(6)) - c(3)*P(7) \\ P(2) - P(5) + P(4) + 2*(P(8) - P(7)) \\ -b(3)*(P(7) - P(4)) - b(1)*P(8) \\ -c(3)*(P(7) - P(4)) - c(1)*P(8) \\ P(3) - P(6) + P(5) + 2*(P(9) - P(8)) \\ -b(1)*(P(8) - P(5)) - b(2)*P(9) \\ -c(1)*(P(8) - P(5)) - c(2)*P(9) \end{bmatrix}$$

The element is based on a polynomial expression of nine terms.

$$\begin{aligned} \mathbf{P} = [&L_1 \quad L_2 \quad L_3 \quad L_1L_2 \quad L_2L_3 \quad L_3L_1 \\ &L_1^2L_2 + \tfrac{1}{2}L_1L_2L_3(3(1-\mu_3)L_1 - (1+3\mu_3)L_2 + (1+3\mu_3)L_3)) \\ &L_2^2L_3 + \tfrac{1}{2}L_1L_2L_3(3(1-\mu_1)L_2 - (1+3\mu_1)L_3 + (1+3\mu_1)L_1)) \\ &L_3^2L_1 + \tfrac{1}{2}L_1L_2L_3(3(1-\mu_2)L_3 - (1+3\mu_2)L_1 + (1+3\mu_2)L_2))] \end{aligned}$$

Weak Form to FE format

The Finite Element Matrix equation is given as

$$\begin{aligned}\int\int_{\Omega} &\left(\rho\,[\mathbf{N}]\,[\mathbf{Z}]\,[\mathbf{N}]\,\{\ddot{\tilde{\mathbf{u}}}^e\}\right)\delta\tilde{\mathbf{u}}^e + \left(2\rho V_1\,[\mathbf{N}]\,[\mathbf{Z}]\,[\mathbf{H_v}]\,\{\dot{\tilde{\mathbf{u}}}^e\}\right)\delta\tilde{\mathbf{u}}^e\\ &- \left(\rho V_1^2\,[\mathbf{H_v}]\,[\mathbf{Z}]\,[\mathbf{H_v}]\,\{\tilde{\mathbf{u}}^e\}\right)\delta\tilde{\mathbf{u}}^e + \left([\mathbf{B}]\left[\tilde{\mathbf{D}}\right][\mathbf{B}]\,\{\tilde{\mathbf{u}}^e\}\right)\delta\tilde{\mathbf{u}}^e\\ &+ \left([\mathbf{B_S}]\left[\tilde{\mathbf{D}}_\mathbf{S}\right][\mathbf{B_S}]\,\{\tilde{\mathbf{u}}^e\}\right)\delta\tilde{\mathbf{u}}^e + \left([\mathbf{H_A}]\left[\tilde{\mathbf{N}}_\mathbf{A}\right][\mathbf{H_A}]\,\{\tilde{\mathbf{u}}^e\}\right)\delta\tilde{\mathbf{u}}^e\,d\Omega\\ &= \sum_i^{nb}\int\int_{\Omega_i}\left(q_i\left[\tilde{\mathbf{N}}_\mathbf{f}\right]\right)\delta\tilde{\mathbf{u}}^e\,d\Omega_i\end{aligned}$$

After rearranging them to their respective groups we get.

$$[\mathbf{M}^e]\,\{\ddot{\mathbf{u}}^e\} + [\mathbf{C}^e]\,\{\dot{\mathbf{u}}^e\} + [\mathbf{K}^e]\,\{\mathbf{u}^e\} = \{\mathbf{F}^e\}$$

All the Element mass Matrices $[\mathbf{M^e}]$ are assembled in the final Mass Matrix $[\mathbf{M}]$.Doing same for other matrices gives us the ODE in terms of FE matrices.

$$[\mathbf{M}]\,\{\ddot{\mathbf{u}}\} + [\mathbf{C}]\,\{\dot{\mathbf{u}}\} + [\mathbf{K}]\,\{\mathbf{u}\} = \{\mathbf{F}\}$$

Solution Procedure

Dynamic Analysis:
To solve the dynamic system

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\} + [\mathbf{C}]\{\dot{\mathbf{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{F}\}$$

Newmark time integration scheme is employed.

Newmark algorithm

$$R = F_t + \mathbf{M}\left(a_0 u_t + a_2 \dot{u}_t + a_3 \ddot{u}_t\right) + \mathbf{C}\left(a_1 u_t + a_4 \dot{u}_t + a_5 \ddot{u}_t\right)$$

$$\begin{aligned} u_{t+1} &= \left[a_0\mathbf{M} + a_1\mathbf{C} + \mathbf{K}\right]^{-1} R \\ \dot{u}_{t+1} &= a_1\left(u_{t+1} - u_t\right) - a_4\dot{u}_t - a_5\ddot{u}_t \\ \ddot{u}_{t+1} &= a_0\left(u_{t+1} - u_t\right) - a_2\dot{u}_t - a_3\ddot{u}_t \end{aligned}$$

a_0 .. a_5 are the integration variables which depends on the Integration parameters θ, α and time step size h. u_t,$\dot{u}_t$ and $\ddot{u}_t$ are the displacement , velocity and acceleration of current time step. u_{t+1},$\dot{u}_{t+1}$ and $\ddot{u}_{t+1}$ are the displacement , velocity and acceleration of next time step.

Unconditionally Stable for

$$\theta \geq \frac{1}{2}$$

$$\alpha \geq \frac{1}{4}\left(\frac{1}{2} + \theta\right)^2$$

Code Features

- MATLAB is used as the language to program the FEM from scratch.
- GMSH open source tool is used for pre processing and post processing.
- Object oriented programming style is adopted.

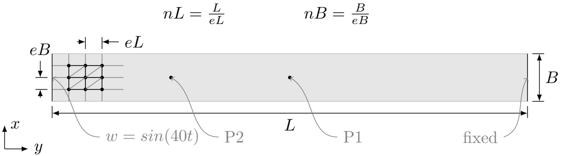
Listing 1: Example input script

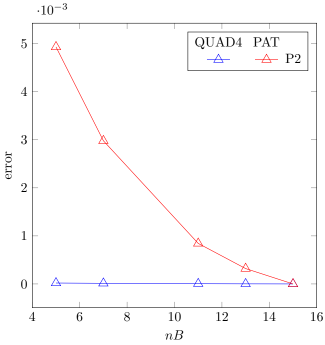
```
FEM=DYNAMIC('Dynamic_WC1');
FEM.ReadMesh('strip.msh');
FEM.Mat(1)=MATDat("Mat.dat.txt");

t=linspace(0,10,2001);
x1=sin(25*t);

FEM.Pp(1)=ProbeOnPhyEn([21 22 23 24],[1 0 0]);
FEM.T=FEMTime(0.01,3);
FEM.TS(1)=TimeSeries(t,x1);


FEM.Up(1)=DirichletOnPhyEn(FEM, [12], [1 0 0], 0 );
FEM.Up(2)=DirichletOnPhyEn(FEM, [11], [1 0 0], 0.01 );
FEM.Fp(1)=NeumannOnPhyEn(FEM, [22 23 24 21], [1 0 0], 20000 );
```

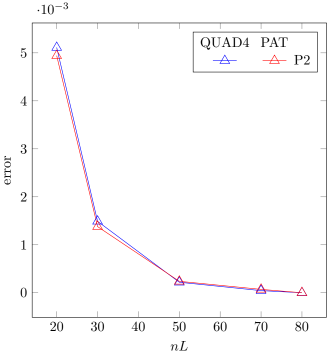
```
FEM.ImposeU( FEM. Up(2)  *  FEM.TS(1)  );
FEM.ImposeU( FEM. Up(1)   );

K1=10;
K2=2;
K3=10;
P1 = K1 * FEM.Pp(1).forT(0);
P2 = K2 *(FEM.Pp(1).forT(0)-FEM.Pp(1).forT(-1))/(FEM.T.forT(0)-
    FEM.T.forT(-1));
P3 = K3 * Int(FEM.Pp(1));
 FEM.ApplyF( -1 *  FEM.Fp(1)  * (P1 + P2 + P3) );


 FEM.SetDomain([1 21 22 23 24],[1],'MITC4');
%
 FEM.InitialX('zero');
 FEM.InitialV('zero');
%
  tic;
 FEM.SolveLU();
  toc;
%
 FEM.WritePos();
 FEM.WriteProbe();
 FEM.PlotProbe();
 FEM.PlotF();
 FEM.WriteF();
```

Plan

Study on Directional mesh density







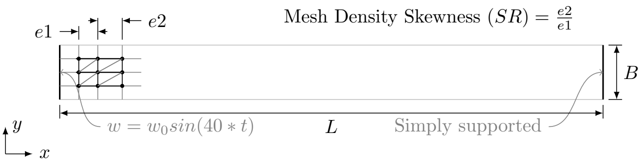


Figure: Mesh density skewness

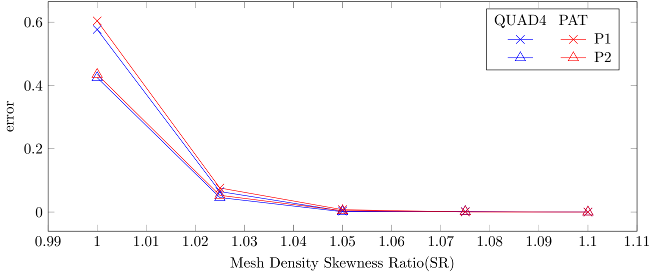
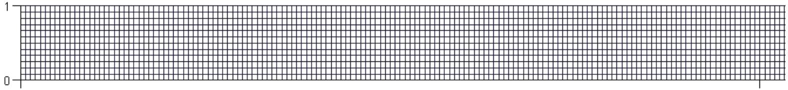
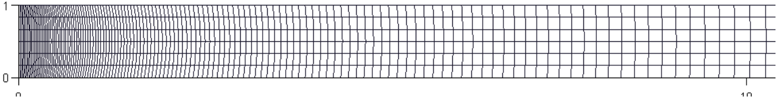


Figure: Comarison of PAT and MITC4 elements for different mesh density skewness

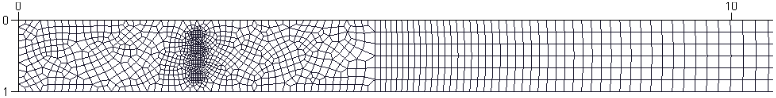
Optimized Finite Element Mesh



(a) Mesh : 4 , Nn = 7813, Solution Time = 929s

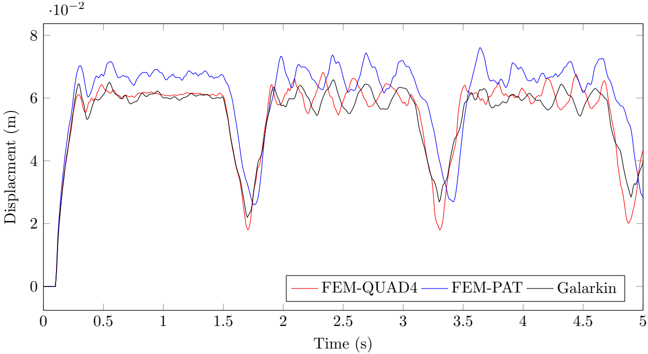


(b) Mesh : 2_3 , Nn = 1407, Solution Time = 13s

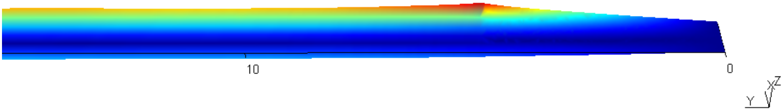


(c) Mesh : strip , Nn = 1886, Solution Time = 21s

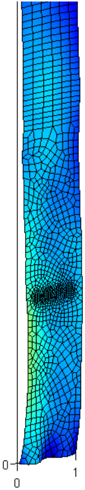
Comparison of FEM solution with Galarkin method



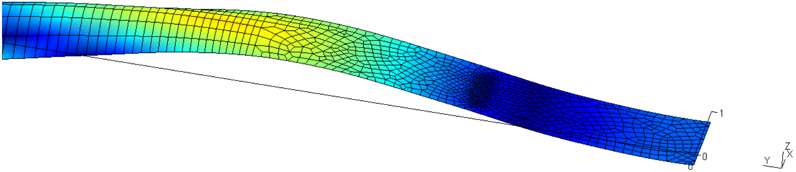
(d) Mesh : strip , Nn = 1886, Solution Time = 21s

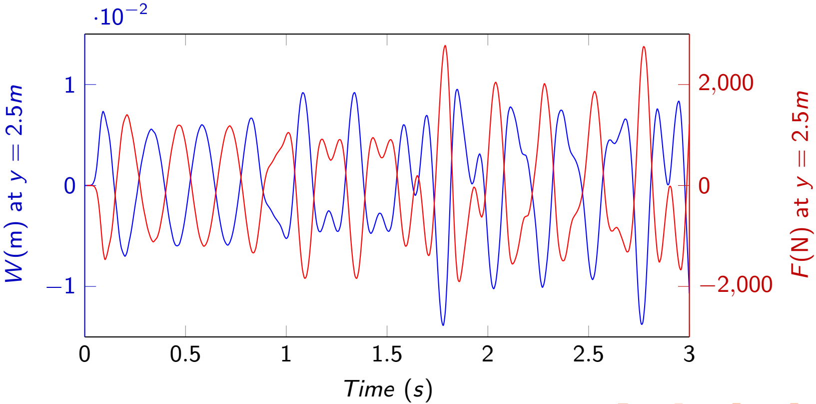


Strip with imposed displacement load using real data



Basic Control Demonstration





Plan

Conclusion

- In most of the cases the QUAD4 element performs better. Particular when there is point load and simply supported boundary condition.
- PAT element shows good convergence property for distributed load.
- Inclusion of axial velocity makes Coriolis and centripetal acceleration components appear in the equation which makes it gyroscopic.
- PAT element is unaffected by the low directional mesh density in direction perpendicular to the axial velocity.
- The element size has to be very small only on the boundary where the Dirichlet load is applied and also at the direction of the line speed.
- FEM program converges to the solution produced by the Galarkin method.
- Techniques like pre-factorization and Modal Superposition techniques drastically decreased the solution time.

Suggestions for future improvements

- Better Shape function(MITC family)
- Including non-linearity and Multi physics
- Including Contact between plate and rollers
- Better Linear algebra solver packages (LAPACK, cuBLAS ..)
- Modal order reduction techniques
- Creating a user friendly GUI

Thank you for your attention!!!