Finite Element simulation of 2D metal strip in Hot-Dip galvanization Process

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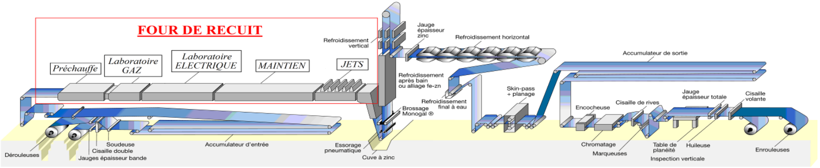
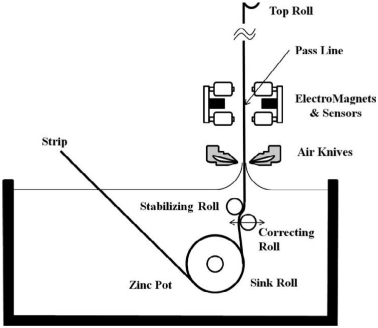
Hot-Dip Galvanization Process

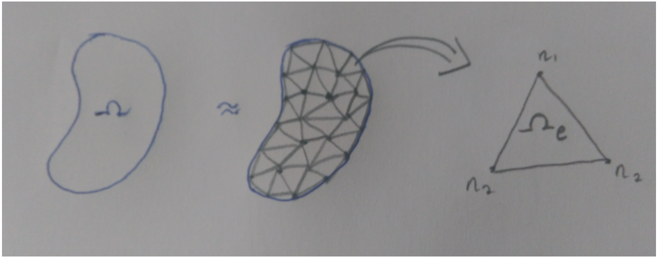




- A thin Layer of Zinc is coated to Increase the corrosion resistance of steel
- Air knives control the thickness of the Zinc layer
- Excessive Vibration results in uneven coating.
- Electromagnets are used to control the vibration of the strip.

Finite Element Modelling

In Finite element a continuous domain is discretized into elements. Each element is connected by nodes.



$$\Omega \approx \sum_{i}^{nE} \Omega_e^i$$

- Complex behaviour of the metal strip.
- Two dimensional domain and Three Dimensional Displacement field.
- Complex and multiple boundary condition.
- Free control over discretization of the domain.
- Intuitive Solution Procedure.

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Hamilton principle

Hamilton principle is used to derive the equation of motion and the Hamilton is given as

$$H = \int_{t_0}^{t_1} (T - V + W)\, dt$$

The Hamilton Principle states that variation of Hamilton is zero

$$\delta H = \int_{t_0}^{t_1} (\delta T - \delta V + \delta W)\, dt = 0$$

The variation of displacement δu is zero at the beginning and end time.

$$\delta u \Big|_{t_0}^{t_1} = 0$$

T is the kinetic energy, V in the potential energy and W is the work done to the system.

Final Equation of Motion

Final weak form

$$\int \int_{\Omega} \rho \ddot{\tilde{u}}_i Z_{ij} \delta \tilde{u}_i + 2\rho V_1 \delta \tilde{u}_i Z_{ij} \dot{\tilde{u}}_{j,1} - \rho V_1^2 \tilde{u}_{j,1} Z_{ij} \delta \tilde{u}_{j,1}$$

$$+\kappa^T \tilde{D} \delta \kappa + \left(\epsilon^S\right)^T \tilde{D}_c \delta \epsilon^S + w_{,\alpha} \tilde{\sigma}^A \delta w_{,\alpha} d\Omega = \sum_i^{nb} \int \int_{\Omega_i} q_i \delta \mathbf{u_i} d\Omega_i dt$$

Final strong Form

$$\rho h \left(\frac{\partial^2 w}{\partial t^2} + 2V_1 \frac{\partial^2 w}{\partial x \partial t} + V_1^2 \frac{\partial^2 w}{\partial x^2} \right) + D \nabla^4 w - N_x h \frac{\partial^2 w}{\partial x^2} = F$$

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial Y^4} \qquad D = \frac{Eh^3}{12\left(1 - \nu^2\right)}$$

Where, ρ = Density, N_x = Axial Stress, h = thickness, V_1 = Line speed, F = Distributed Force

Plan

Shape function of a rectangular element

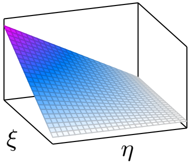
The displacement field over the element is given as the sum of product of shape function and nodal displacements. Here n is the total number of nodes in an element

Displacement field of an element

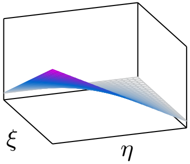
$$\tilde{\mathbf{u}} \approx \sum_{i=1}^{n} \left(N_i w_i + \overline{N}_i \theta_{x_i} + \overline{\overline{N}}_i \theta_{y_i} \right)$$

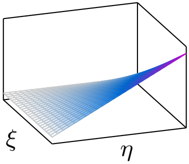
$$\tilde{u} = [w, \theta_x, \theta_y]^T \; \theta_x = \frac{\partial w}{\partial x} \theta_y = \frac{\partial w}{\partial y}$$

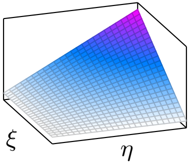
$$N_1 = \tfrac{1}{4}(1-\xi)(1-\eta)$$



$$N_2 = \tfrac{1}{4}(1+\xi)(1-\eta) \qquad N_3 = \tfrac{1}{4}(1+\xi)(1+\eta) \qquad N_4 = \tfrac{1}{4}(1-\xi)(1+\eta)$$







The FE approximation

$$\tilde{\mathbf{u}} \approx \sum_{i=1}^{n} \left(N_i w_i + \overline{N}_i \theta_{x_i} + \overline{\overline{N}}_i \theta_{y_i} \right)$$

is written in matrix format as

$$\tilde{\mathbf{u}} \approx \begin{bmatrix} N_1 & 0 & 0 & \cdots & N_{nN} & 0 & 0 \\ 0 & \overline{N}_1 & 0 & \cdots & 0 & \overline{N}_{nN} & 0 \\ 0 & 0 & \overline{\overline{N}}_1 & \cdots & 0 & 0 & \overline{\overline{N}}_{nN} \end{bmatrix} \left\{ \begin{array}{c} w_1 \\ \theta_{x_1} \\ \theta_{y_1} \\ \vdots \\ w_{nN} \\ \theta_{x_{nN}} \\ \theta_{y_{nN}} \end{array} \right\} = \mathbf{N}\tilde{\mathbf{u}}^e$$

Similarly other terms of the Finite Element Matrices are

$$\dot{\tilde{\mathbf{u}}} \approx \mathbf{N}\dot{\tilde{\mathbf{u}}}^e \qquad \ddot{\tilde{\mathbf{u}}} \approx \mathbf{N}\ddot{\tilde{\mathbf{u}}}^e \qquad \kappa \approx \mathbf{B}\tilde{\mathbf{u}}^e \qquad \tilde{\epsilon}^S \approx \mathbf{B_S}\tilde{\mathbf{u}}^e$$

$$\tilde{u}_{1,\alpha} \approx \mathbf{H_A}\tilde{\mathbf{u}}^e \qquad \tilde{u}_{\alpha,1} \approx \mathbf{H_v}\tilde{\mathbf{u}}^e \qquad \tilde{w} \approx \mathbf{N_f}\tilde{\mathbf{u}}^e$$

Weak Form to FE format

The Finite Element Matrix equation is given as

$$\int\int_{\Omega}\left(\rho\left[\mathbf{N}\right]\left[\mathbf{Z}\right]\left[\mathbf{N}\right]\left\{\ddot{\tilde{\mathbf{u}}}^e\right\}\right)\delta\tilde{\mathbf{u}}^e+\left(2\rho V_1\left[\mathbf{N}\right]\left[\mathbf{Z}\right]\left[\mathbf{H_v}\right]\left\{\dot{\tilde{\mathbf{u}}}^e\right\}\right)\delta\tilde{\mathbf{u}}^e$$
$$-\left(\rho V_1^2\left[\mathbf{H_v}\right]\left[\mathbf{Z}\right]\left[\mathbf{H_v}\right]\left\{\tilde{\mathbf{u}}^e\right\}\right)\delta\tilde{\mathbf{u}}^e+\left(\left[\mathbf{B}\right]\left[\tilde{\mathbf{D}}\right]\left[\mathbf{B}\right]\left\{\tilde{\mathbf{u}}^e\right\}\right)\delta\tilde{\mathbf{u}}^e$$
$$+\left(\left[\mathbf{B_S}\right]\left[\tilde{\mathbf{D}}_\mathbf{S}\right]\left[\mathbf{B_S}\right]\left\{\tilde{\mathbf{u}}^e\right\}\right)\delta\tilde{\mathbf{u}}^e+\left(\left[\mathbf{H_A}\right]\left[\tilde{\mathbf{N}}_\mathbf{A}\right]\left[\mathbf{H_A}\right]\left\{\tilde{\mathbf{u}}^e\right\}\right)\delta\tilde{\mathbf{u}}^e d\Omega$$
$$=\sum_i^{nb}\int\int_{\Omega_i}\left(q_i\left[\tilde{\mathbf{N}}_\mathbf{f}\right]\right)\delta\tilde{\mathbf{u}}^e d\Omega_i$$

After rearranging them to their respective groups we get.

$$\left[\mathbf{M}^e\right]\left\{\ddot{\mathbf{u}}^e\right\}+\left[\mathbf{C}^e\right]\left\{\dot{\mathbf{u}}^e\right\}+\left[\mathbf{K}^e\right]\left\{\mathbf{u}^e\right\}=\left\{\mathbf{F}^e\right\}$$

All the Element mass Matrices $[\mathbf{M^e}]$ are assembled in the final Mass Matrix $[\mathbf{M}]$.Doing same for other matrices gives us the ODE in terms of FE matrices.

$$\left[\mathbf{M}\right]\left\{\ddot{\mathbf{u}}\right\}+\left[\mathbf{C}\right]\left\{\dot{\mathbf{u}}\right\}+\left[\mathbf{K}\right]\left\{\mathbf{u}\right\}=\left\{\mathbf{F}\right\}$$

Static Analysis:
To solve a static system

$$[\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{F}\}$$

$u = K \backslash F$ command is used since, $u = K^{-1}F$ is a very expensive command. $u = K \backslash F$ command first factorizes the K matrix into upper and lower triangles then it is solved, which is a much more efficient process.
Modal Analysis:

$$\mathbf{M\ddot{u}} + \mathbf{Ku} = 0$$

is converted in to a eigenvalue problem

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\overline{\mathbf{u}} = 0 \qquad \mathbf{u} = \overline{\mathbf{u}}e^{i\omega t}$$

ω is the natural frequency and $\overline{\mathbf{u}}$ is the natural mode. In MATLAB, [V,D]=eig(K,M) function is used to do the modal analysis.

Dynamic Analysis:
To solve the dynamic system

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\} + [\mathbf{C}]\{\dot{\mathbf{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{F}\}$$

Newmark time integration scheme is employed.

Newmark algorithm

$$R = F_t + \mathbf{M}\left(a_0 u_t + a_2 \dot{u}_t + a_3 \ddot{u}_t\right) + \mathbf{C}\left(a_1 u_t + a_4 \dot{u}_t + a_5 \ddot{u}_t\right)$$

$$\begin{aligned} u_{t+1} &= \left[a_0\mathbf{M} + a_1\mathbf{C} + \mathbf{K}\right]^{-1} R \\ \dot{u}_{t+1} &= a_1\left(u_{t+1} - u_t\right) - a_4\dot{u}_t - a_5\ddot{u}_t \\ \ddot{u}_{t+1} &= a_0\left(u_{t+1} - u_t\right) - a_2\dot{u}_t - a_3\ddot{u}_t \end{aligned}$$

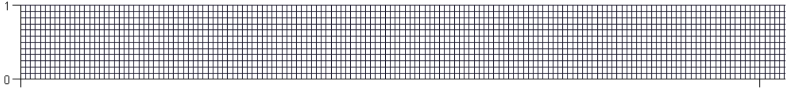
a_0 .. a_5 are the integration variables which depends on the Integration parameters θ, α and time step size h. u_t,$\dot{u}_t$ and $\ddot{u}_t$ are the displacement , velocity and acceleration of current time step. u_{t+1},$\dot{u}_{t+1}$ and $\ddot{u}_{t+1}$ are the displacement , velocity and acceleration of next time step.

Unconditionally Stable for

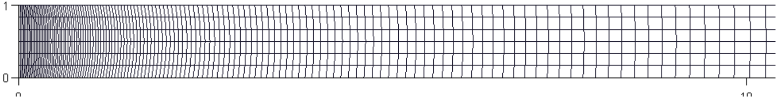
$$\theta \geq \frac{1}{2}$$

$$\alpha \geq \frac{1}{4}\left(\frac{1}{2} + \theta\right)^2$$

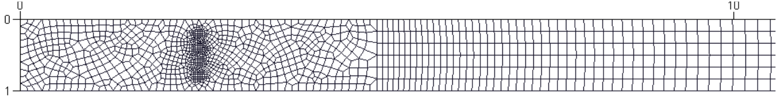
Finite Element Discretization



(a) Mesh : 4 , Nn = 7813, Solution Time = 929s

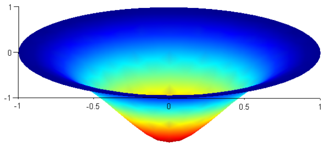


(b) Mesh : 2_3 , Nn = 1407, Solution Time = 13s



(c) Mesh : strip , Nn = 1886, Solution Time = 21s

Plan



The target analytically solution given as

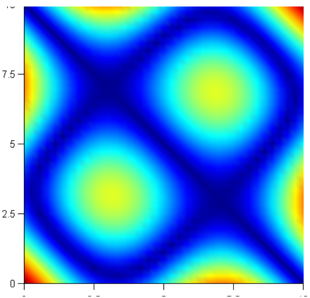
$$w = \frac{F_z}{16\pi D}\left[r^2 - a^2\right] + \frac{F_z r^2}{8\pi D}\left[log\,\frac{a}{r}\right] \quad (1)$$

The analytical solution is -0.000434 *in*.
Numerical solution is $-0.000429 in$.
So the Error percentage is 1.26%.

Material Property	
Young's Modulus (E)	5E11 Pa
Poission's Ratio (ν)	0.3
Geometric Data	
Radius (r)	1 m
Thickness(t)	0.01 m
Loading Data	
Point Load (F_z)	-1000 N

Reference

S.Timoshenko , S . Woinowsky , Theory of Plates and Shells , pg:69, Article : 19 .



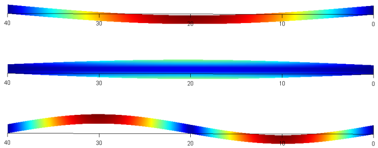
The analytical frequency is 1.632 Hz.
The numerical frequency is 1.626 Hz.
So the Error percentage is 0.32 %

Material Property	
Young's Modulus (E)	25E11 Pa
Poission's Ratio (ν)	0.3
Density(ρ)	8000
Geometric Data	
length (l)	10 m
breath (b)	10 m
Thickness(t)	0.01 m

Reference

NAFEMS Manual. Solution Retrieved from Ansys verification problem (VMP09-T12).

NAS227 Modal analysis of thin plate with axial load



Analytical Solution = 77.47 Hz
Numerical Solution = 77.45 Hz
Error % = 0.01 %.

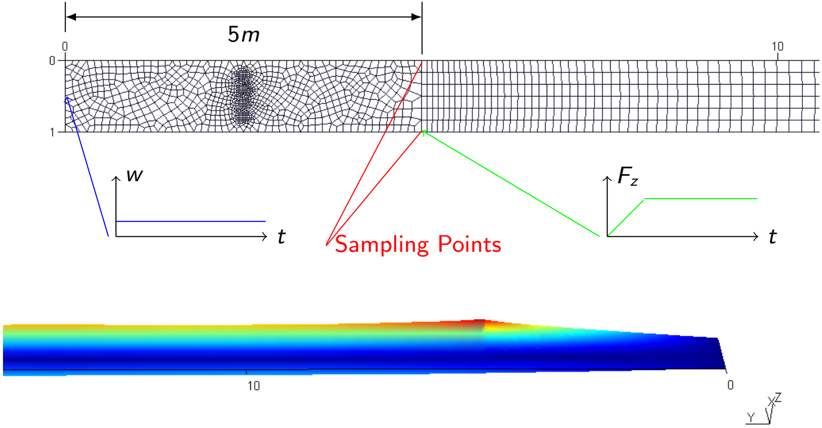
$$\rho^2\omega_{mn}^2 = D\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$
$$+N_1\left(\frac{m\pi}{a}\right)^2 + N_2\left(\frac{n\pi}{b}\right)^2$$

Material Property	
Young's Modulus (E)	1E11 Pa
Poission's Ratio (ν)	0.3
Density(ρ)	7810
Geometric Data	
length (l)	1 m
breath (b)	40 m
Thickness(t)	0.5 mm
Loading Data	
Axial load (N_x)	6E7 N/m^2

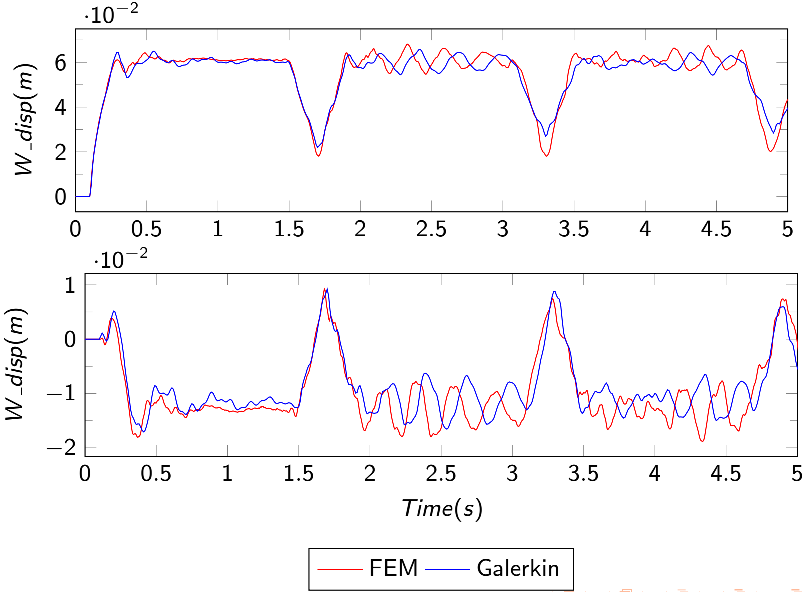
Reference

Arthur W.Leissa ,Vibration of Plates,NASA SP-160, pg:277, Ch:10.2.

Comparison of FEM solution with Galerkin method

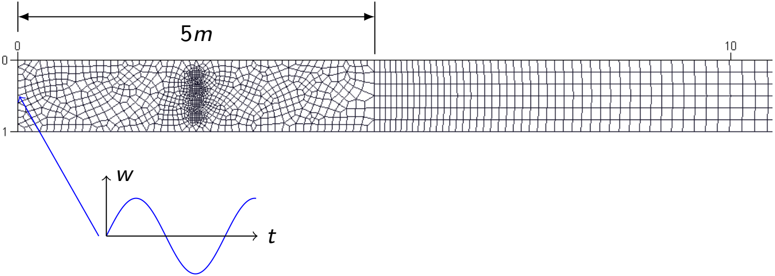


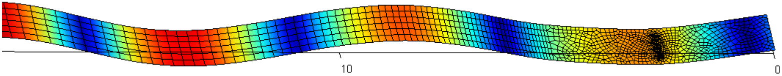
Comparison of FEM solution with Galerkin method



Plan

Same Geometry from previous problem with harmonic load at one end

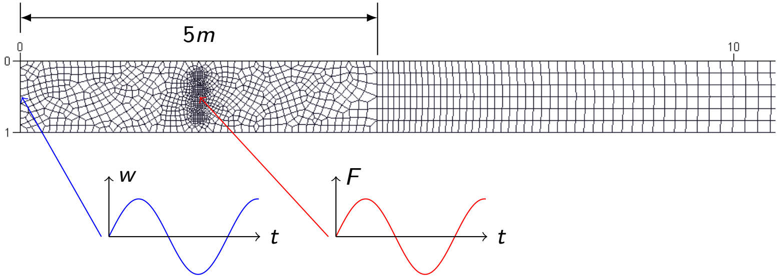


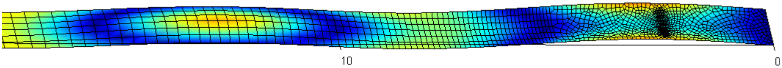


Analysis Statistics

nT=1000 , nN = 1886 , nE = 1836, nDOF = 5640, Solution Time = 145 s

Along with previous loading, forces are applied at designated spots.

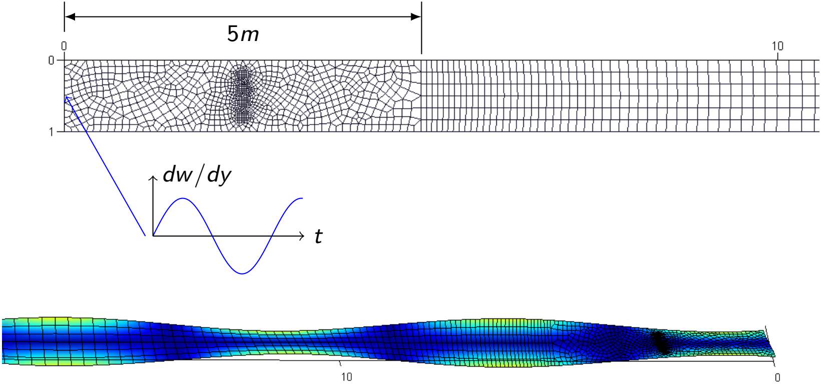




Analysis Statistics

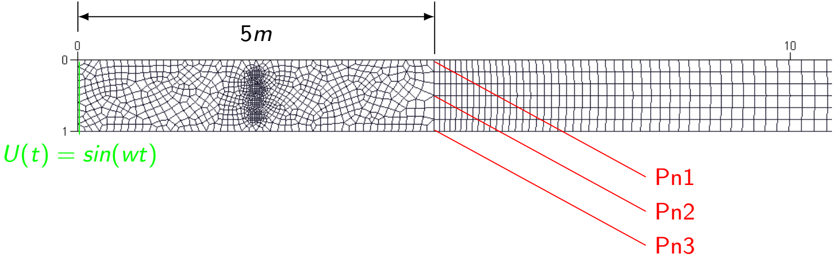
nT=1000 , nN = 1886 , nE = 1836, nDOF = 5640, Solution Time =182 s

Twisting deformation is applied at one end.

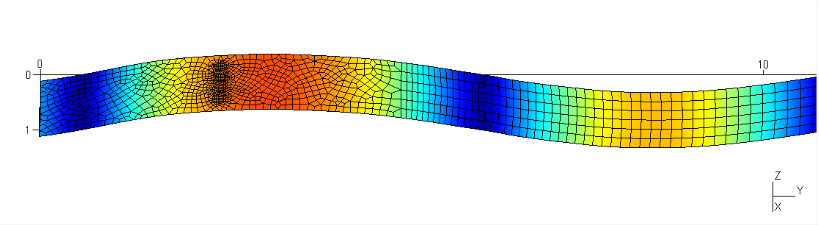


Analysis Statistics

nT=200 , nN = 1886 , nE = 1836, nDOF = 5640, Solution Time = 39s



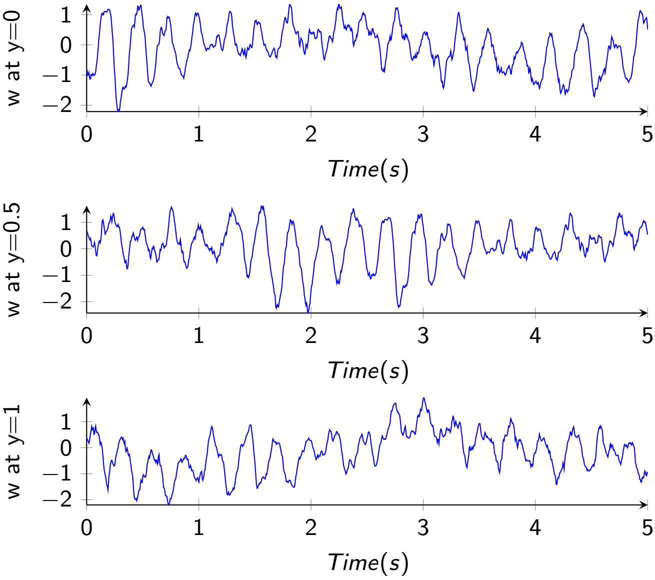
5m
0
10
0
1
$U(t) = sin(wt)$
Pn1
Pn2
Pn3

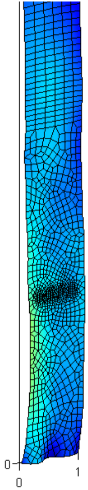


0
10
0
1
Z
Y
X

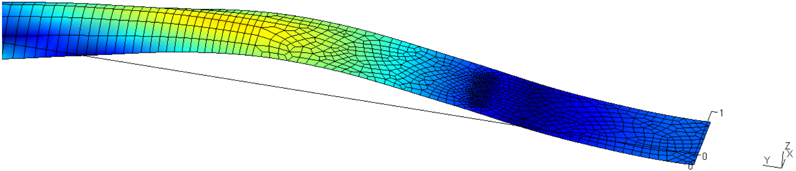
Response of the plate for different Axial velocities(V) m/s

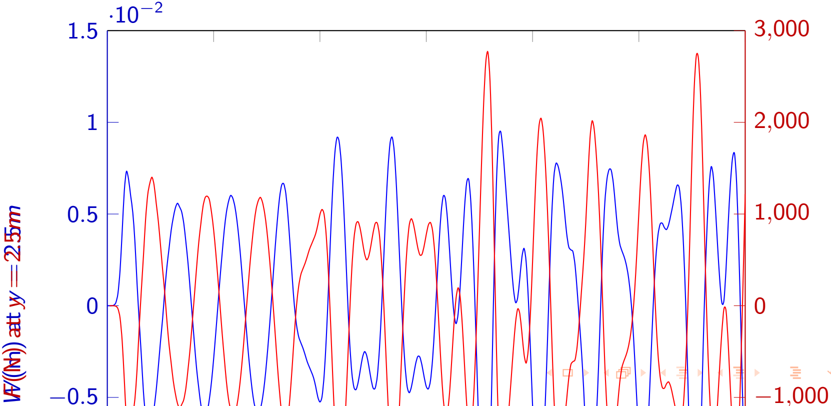
Strip with displacement from real world data





Basic Control Demonstration





Plan

Work to be Done

- Interfacing with control law
- Optimizing the code to increase efficiency
- To clear bugs and bottle necks
- Creating well documentation for the FEM program.

Suggested Future Work

- Better Shape function
- Including non-linearity and Multi physics
- Including Contact between plate and rollers
- Better Linear algebra solver packages (LAPACK, cuBLAS ..)
- Modal superposition and Modal order reduction techniques
- Creating a user friendly GUI

Conclusion

Advantages of FEM

- Better control over accuracy.
- Once coded successfully, It is very easy to implement even for complex geometry and mesh.
- Higher dimensions can be easily modelled.

Disadvantages of FEM

- Computationally expensive.
- Complexity in coding may be overwhelming .
- Suffers from "The curse of dimensionality!".

Thank you for your attention!!!