

1 Natural Frequency of Rectangular strip with axial load (NAS277)

Description :

Modal analysis of a rectangular strip with axial stress (N_2) on short edge.

Reference :

Arthur W.Leissa ,Vibration of Plates,NASA SP-160, pg:277, Ch:10.2.

Material and Geometric data :

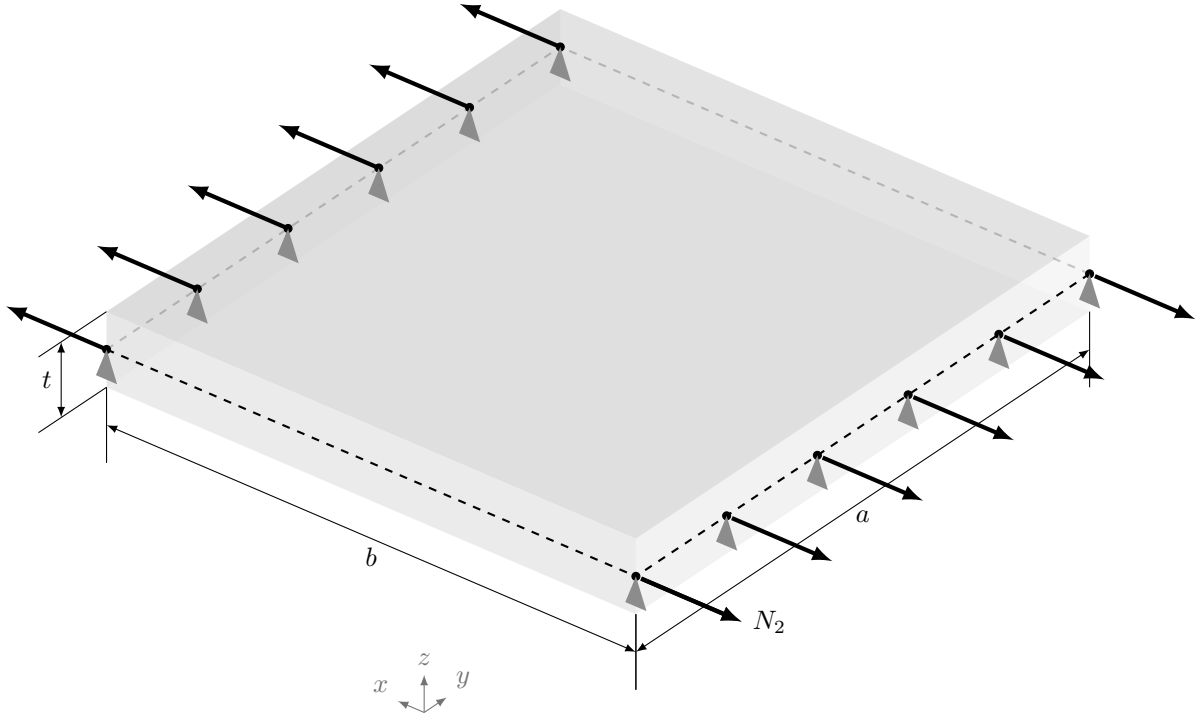


Figure 1: NAS277

Table 1: Input Data

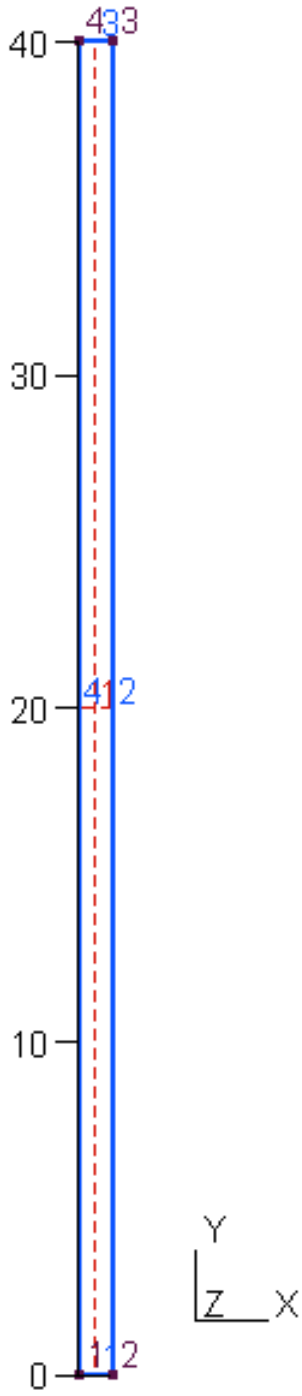
Material Property		Geometric Data		Loading Data
Young's Modulus (E)	1E11 pa	Length (a)	1 m	N_2 3E11 N/m^2
Poission's Ratio (ν)	0.3	Breath (b)	40 m	
Density (ρ)	7810 Kg/m^3	Thickness(t)	1 m	

Mesh and boundary condition :

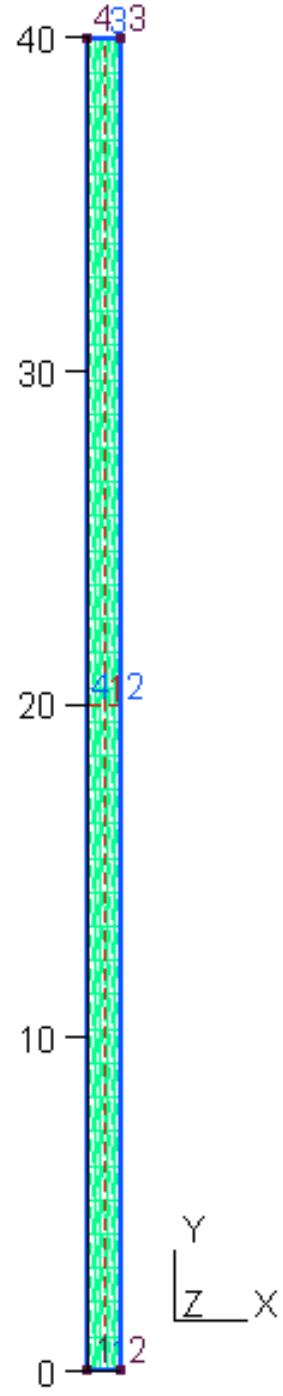
Analytically solution :

The analytical solution of the this problem is given by

$$\omega_{mn} = \sqrt{\frac{1}{\rho} \left(D \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right] + N_1 \left(\frac{m\pi}{a} \right)^2 + N_2 \left(\frac{n\pi}{b} \right)^2 \right)} \quad (1)$$



(a) Geomentry of the problem



(b) Discritization

Table 2: FEM and Boundary condition data

Direchlet Boundary				Loading Conditions	
Geo -Entity	w	θ_x	θ_y	Geo -Entity	N_2
line {1,3}	Fixed	Free	Free	line {1,3}	$3E11 \text{ N/m}^2$

Natural frequencies are
mode 1 : 77.479 Hz

mode 2 : N.A
mode 3 : 155.00 Hz
mode 4 : N.A
mode 5 : 232.61 Hz
mode 6 : N.A

note : modes 2,4 and 6 are twisting modes, which are not given by the formula.

Result and error analysis :

The natural frequencies of the plates are provided below.

mode 1 : 77.458 Hz
mode 2 : 95.610 Hz
mode 3 : 154.98 Hz
mode 4 : 191.46 Hz
mode 5 : 232.63 Hz
mode 6 : 287.38 Hz

So the Error percentage for each mode is :

mode 1 : 0.026 %
mode 3 : 0.012 %
mode 5 : 0.013 %

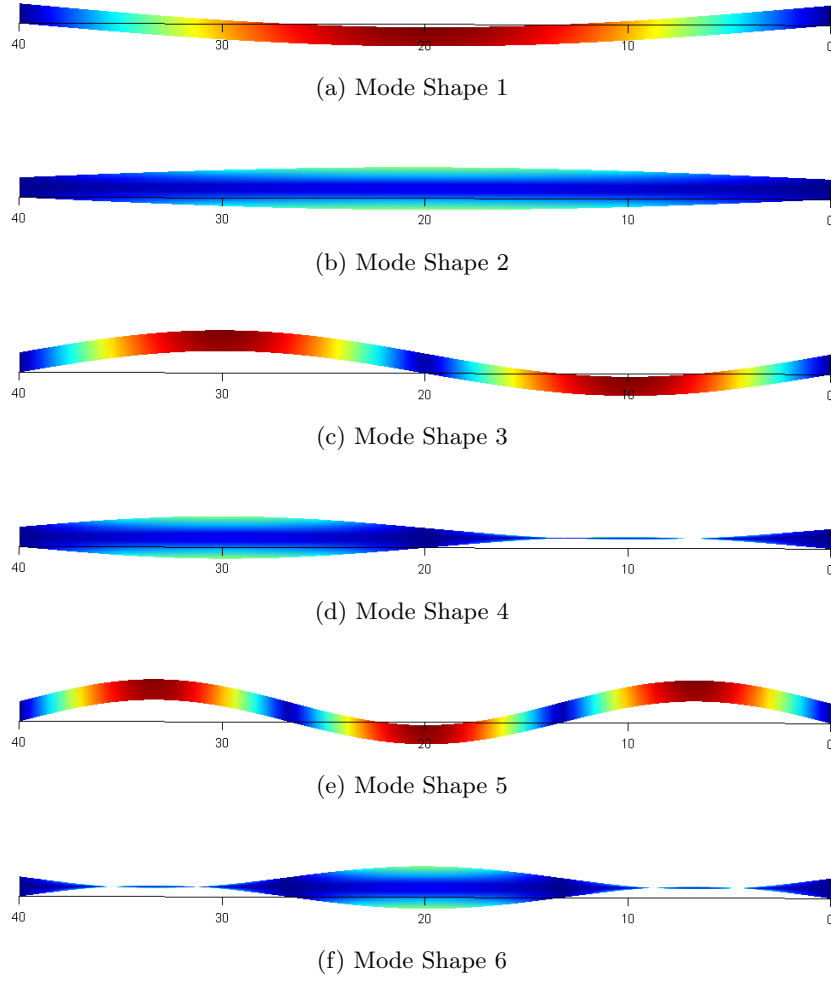


Figure 3: Natural Modes of a rectangular strip

2 Error analysis of strip with various thickness (NAS277)

Description :

It is the continuation of previous problem but the error of the solution is compared for different thicknesses.

Material and Geometric data :

Table 3: Input Data

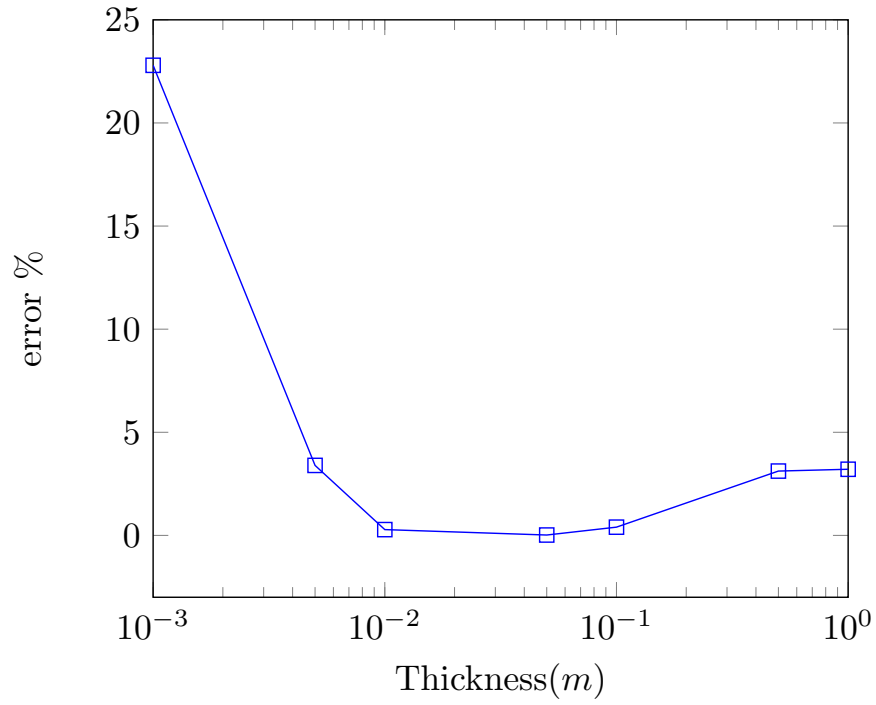
Material Property		Geometric Data		Loading Data	
Young's Modulus (E)	1E11 pa	Length (a)	1 m	N_2	$\frac{T}{t} N/m^2$
Poission's Ratio (ν)	0.3	Breath (b)	40 m	Tension T	3E4 N/m
Density (ρ)	7810 Kg/m^3	Thickness(t)	{0.001,0.005,...,0.5,1} m		

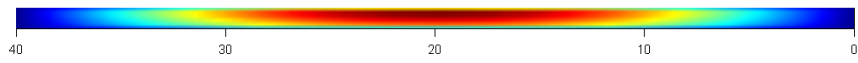
Mesh and boundary condition :

Result and error analysis :

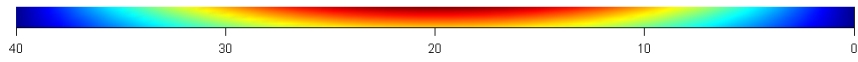
Table 4: FEM and Boundary condition data

Direchlet Boundary				Loading Conditions	
Geo -Entity	w	θ_x	θ_y	Geo -Entity	N_2
line {1,3}	Fixed	Free	Free	line {1,3}	$\frac{T}{t} N/m^2$

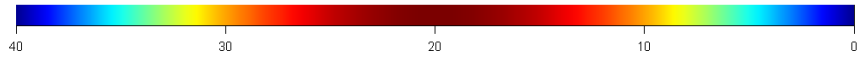




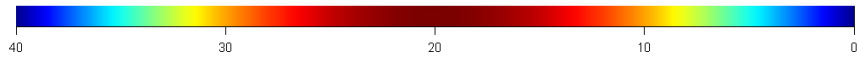
(a) Mode Shape 1 for thickness = 0.001 (m)



(b) Mode Shape 1 for thickness = 0.01 (m)



(c) Mode Shape 1 for thickness = 0.1 (m)



(d) Mode Shape 1 for thickness = 1 (m)

Figure 4: Natural Modes of a rectangular strip