Verification Problems for FEM matlab program

Emayavaramban ELANGO

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Contents

Formulae used:

$$error\% = \left| \frac{w_{analytical} - w_{FEM}}{w_{analytical}} \right| \times 100 \tag{1}$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

1 Triangular Plate With Point Load (VM34)

Description :

A triangular plate with point load (P) on one corner is tested and its opposite edge is build-in.

Reference :

C.O.Harris , Introduction to Stress Analysis, The Macmillan Co., pg:114, Pr:61.
Solution Retrieved from Ansys verification problem (VM34).

Material and Geometric data :

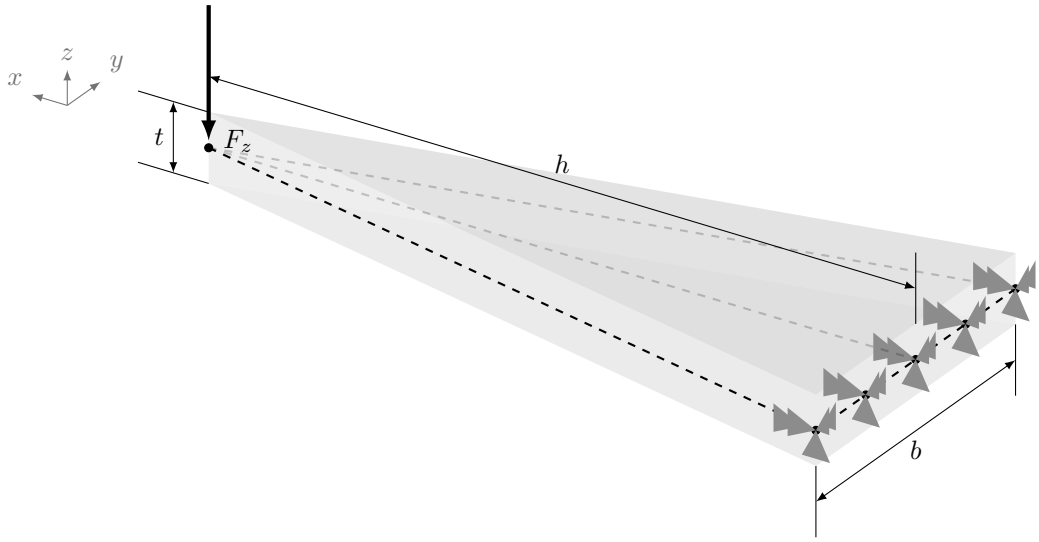


Figure 1: VM34

Table 1: Input Data

Material Property		Geometric Data		Loading Data	
Young's Modulus (E)	3E7 psi	Height (h)	20 in	Point Load (F_z)	10 lbs
Poission's Ratio (ν)	0.3	Breath (b)	3 in		
		Thickness(t)	0.5 in		

Mesh and boundary condition :

Table 2: FEM and Boundary condition data

Direchlet Boundary				Neumann Boundary			
Geo - Entity	w	θ_x	θ_y	Geo - Entity	F_z	M_x	M_y
line {3}	Fixed	Fixed	Fixed	Point {2}	-10 lbs		

Analytically solution :

The target analytically solution is given as -0.042677 in at the corner where the loading is applied.

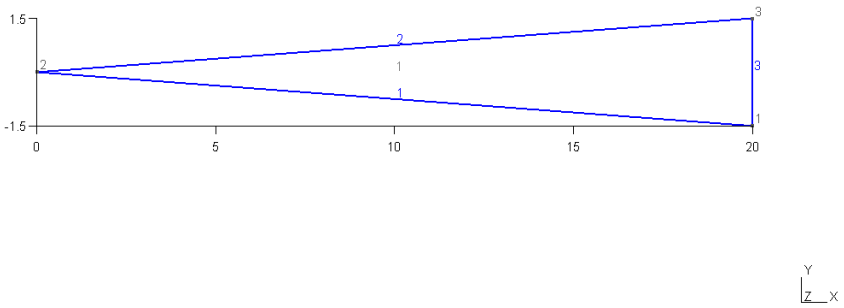


Figure 2: Geomentry of the problem

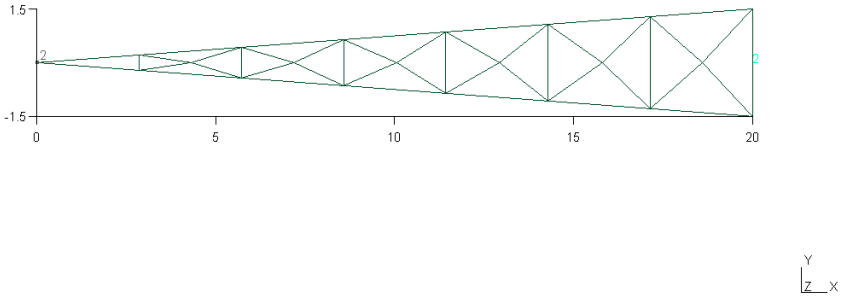


Figure 3: Discritization

Result and error analysis :

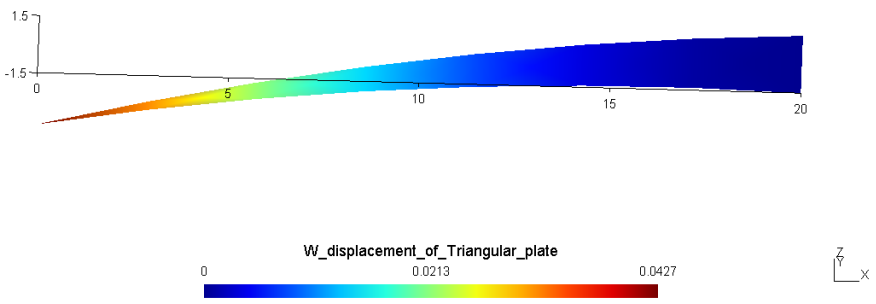


Figure 4: FEM solution plot

The maximum displacement of the domain is our solution . w displacement at point 2 is $-0.0426677in.$

So the Error percentage is 0.00234%.

2 Simply Supported Circular plate with point load (TIM68)

Description :

Simply Supported Circular plate with point load at the center.

Reference :

S.Timoshenko , S . Woinowsky , Theory of Plates and Shells , pg:68, Article : 19 .

Material and Geometric data :

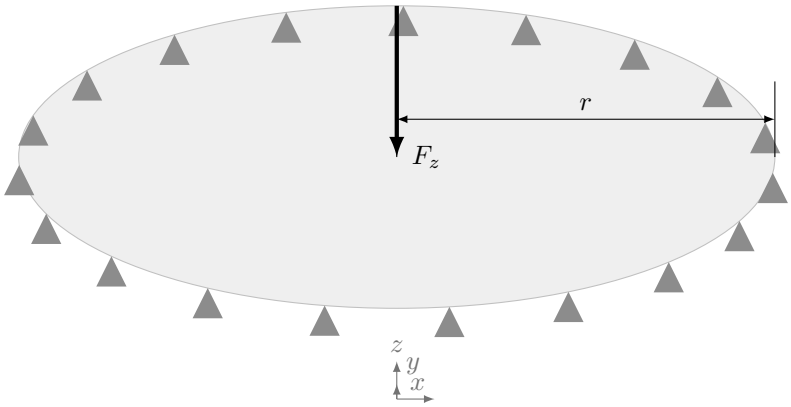


Figure 5: TIM68

Table 3: Input Data

Material Property		Geometric Data		Loading Data	
Young's Modulus (E)	5E11 Pa	Radius (r)	1 m	Point Load (F_z)	1000 N
Poission's Ratio (ν)	0.3	Thickness(t)	0.01 m		

Mesh and boundary condition :

Table 4: FEM and Boundary condition data

Direchlet Boundary				Neumann Boundary			
Geo -Entity	w	θ_x	θ_y	Geo -Entity	F_z	M_x	M_y
line {1,2,3,4}	Fixed	Free	Free	Point {1}	-1000 N		

Analytically solution :

The target analytically solution given as

$$w_{at\_center} = \frac{F_z}{16\pi D}\left[\frac{(3+\nu)r^2}{1+\nu}\right] \tag{3}$$

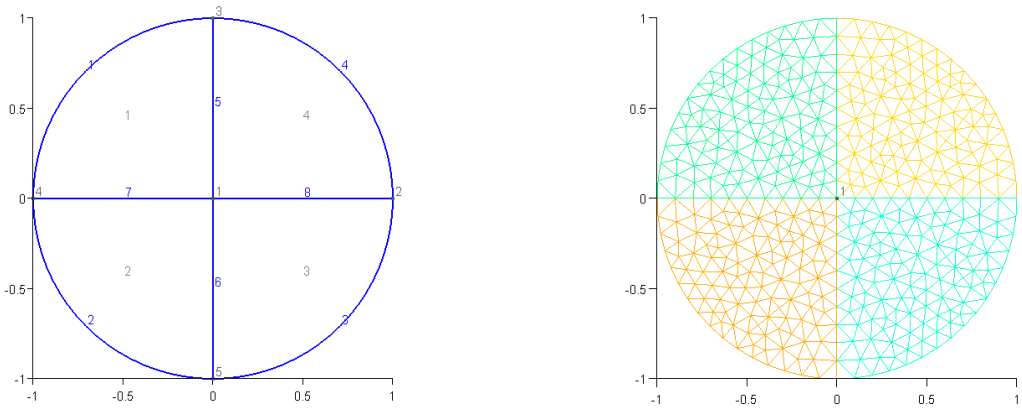


Figure 6: Geomentry and Mesh of TIM68

Result and error analysis :

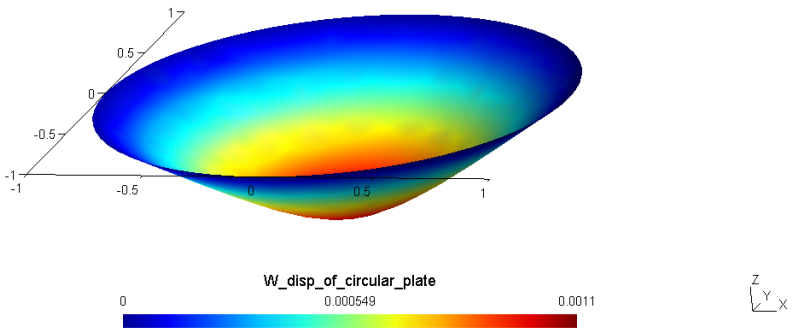


Figure 7: FEM solution plot

The maximum displacement of the domain is our solution . w displacement at center is $-0.0010983in$.

So the Error percentage is 0.42103%.

3 Build-in Circular plate with point load (TIM69)

Description :

Simply Supported Circular plate with point load at the center.

Reference :

S.Timoshenko , S . Woinowsky , Theory of Plates and Shells , pg:69, Article : 19 .

Material and Geometric data :

Table 5: Input Data

Material Property		Geometric Data		Loading Data	
Young's Modulus (E)	5E11 Pa	Radius (r)	1 m	Point Load (F_z)	-1000 N
Poission's Ratio (ν)	0.3	Thickness(t)	0.01 m		

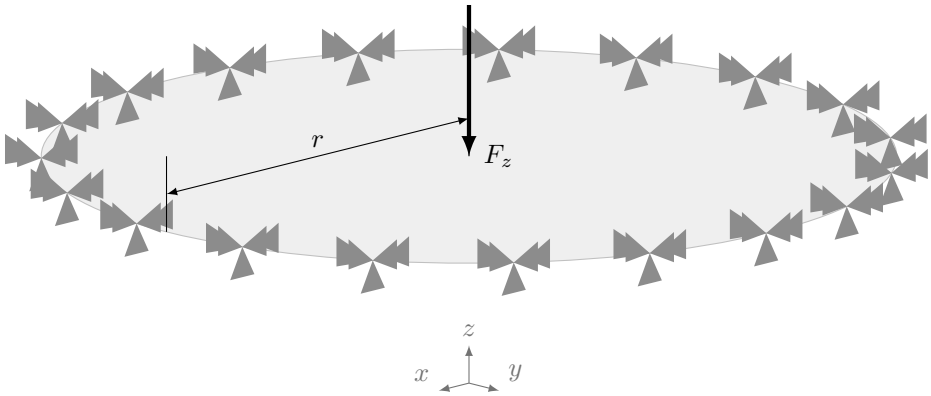


Figure 8: TIM69

Mesh and boundary condition :

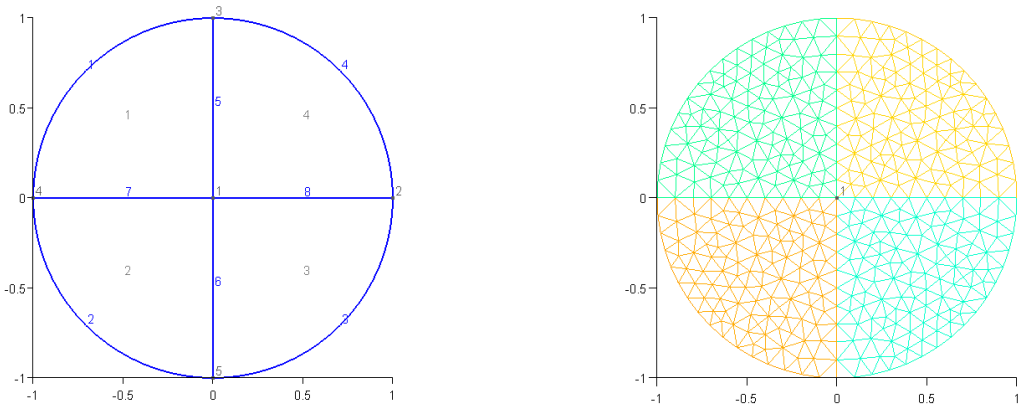


Figure 9: Geomentry and Mesh of TIM68

Table 6: FEM and Boundary condition data

Direchlet Boundary				Neumann Boundary			
Geo -Entity	w	θ_x	θ_y	Geo -Entity	F_z	M_x	M_y
line {1,2,3,4}	Fixed	Fixed	Fixed	Point {1}	-1000 N		

Analytically solution :

The target analytically solution given as

$$w = \frac{F_z}{16\pi D}\left[r^2 - a^2\right] + \frac{F_z r^2}{8\pi D}\left[log\frac{a}{r}\right] \tag{4}$$

The displacement at center is obtained by substituting $a \approx 0$. The analytical solution is -0.000434 m.

Result and error analysis :

The maximum displacement of the domain is our solution . w displacement at center is $-0.000429in$. So the Error percentage is 1.26%.

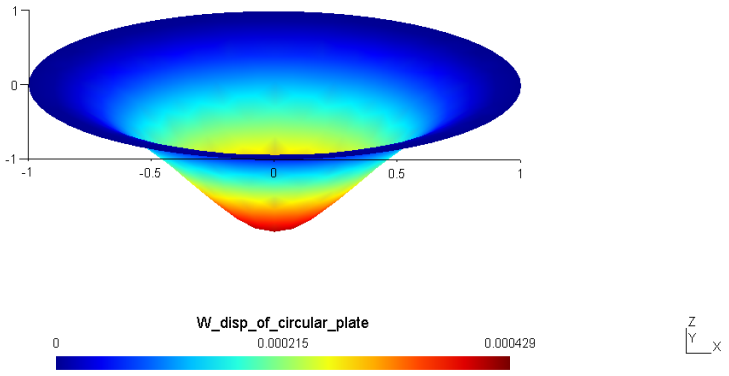


Figure 10: FEM solution plot

4 Natural Frequency of Square plate (VMP09)

Description :

Natural frequency of a square plate is analyzed and compared.

Reference :

NAFEMS Manual. Solution Retrieved from Ansys verification problem (VMP09-T12).

Material and Geometric data :

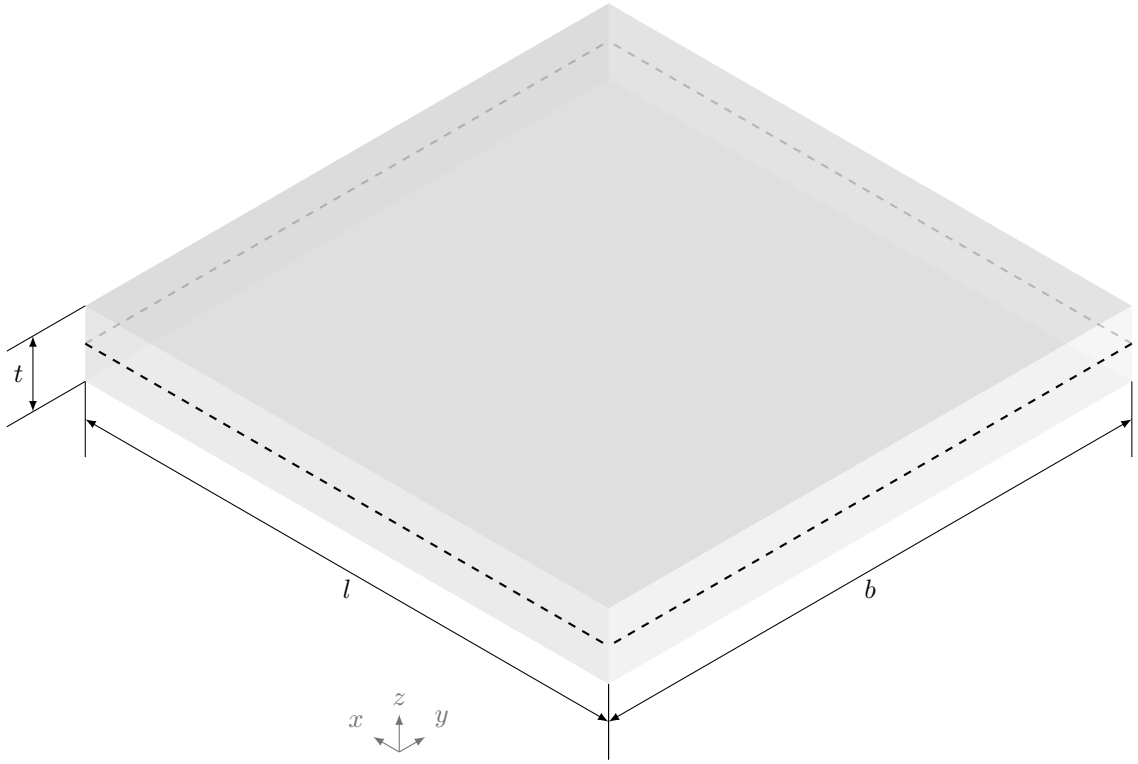


Figure 11: VMP09

Table 7: Input Data

Material Property		Geometric Data		Loading Data
Young's Modulus (E)	2E11 pa	Length (l)	10 m	
Poission's Ratio (ν)	0.3	Breath (b)	10 m	Nil
Density (ρ)	8000 Kg/m^3	Thickness(t)	0.05 m	

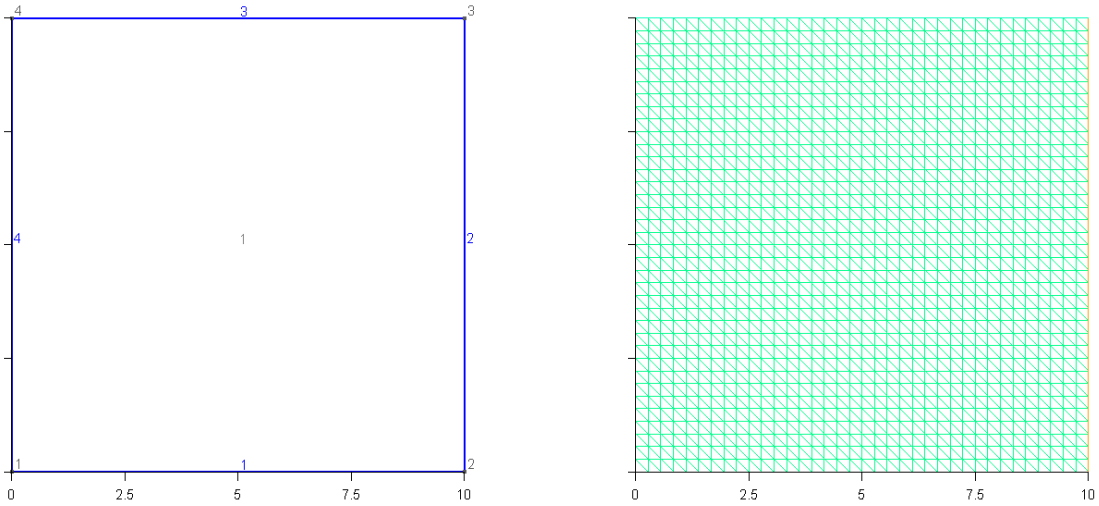


Figure 12: Geomentry and Mesh of TIM68

Table 8: FEM and Boundary condition data

Direchlet Boundary				Neumann Boundary			
Geo -Entity	w	θ_x	θ_y	Geo -Entity	F_z	M_x	M_y
line {1,2,3,4}	Free	Free	Free	Nil			

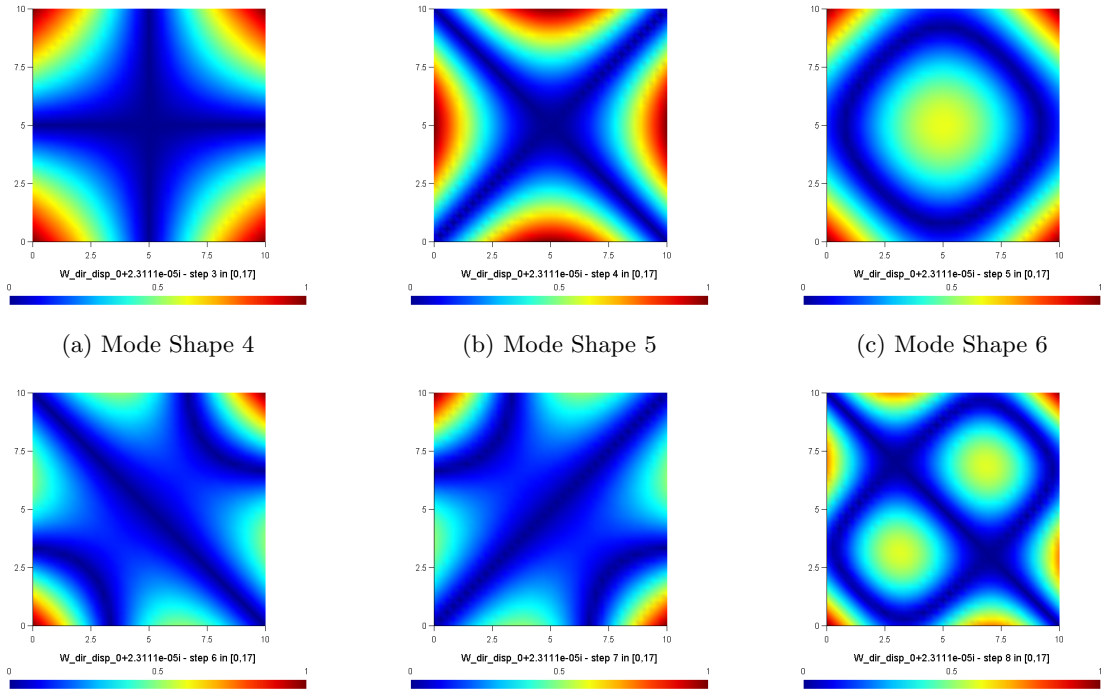
Mesh and boundary condition :

Analytically solution :

Retrieved Natural frequencies from reference manuals are
Mode 4 = 1.622 Hz
Mode 5 = 2.360 Hz
Mode 6 = 2.922 Hz
Mode 7 = 4.233 Hz
Mode 8 = 4.233 Hz
Mode 9 = 7.416 Hz

Result and error analysis :

The Natural modes obtained are plotted in the below figures.
For each natural frequency the error percentage is
Mode 4 = 0.0018 %
Mode 5 = 0.0042 %
Mode 6 = 0.061 %
Mode 7 = 0.911 %
Mode 8 = 0.902 %



(a) Mode Shape 4 (b) Mode Shape 5 (c) Mode Shape 6

(d) Mode Shape 7 (e) Mode Shape 8 (f) Mode Shape 9

Figure 13: Natural Modes of a Square Plate

Mode 9 = 0.645 %

5 Square plate with distributed load(TIM116)

Description :

A simply Supported square plate with distributed load (q).

Reference :

S.Timoshenko , S . Woinowsky , Theory of Plates and Shells , pg:116, Article : 30 .

Material and Geometric data :

Table 9: Input Data

Material Property		Geometric Data		Loading Data	
Young's Modulus (E)	2E11 pa	Length (a)	10 m	Distributed Load (q)	1000 N/m^2
Poission's Ratio (ν)	0.3	Breath (b)	10 m		
		Thickness(t)	0.1 m		

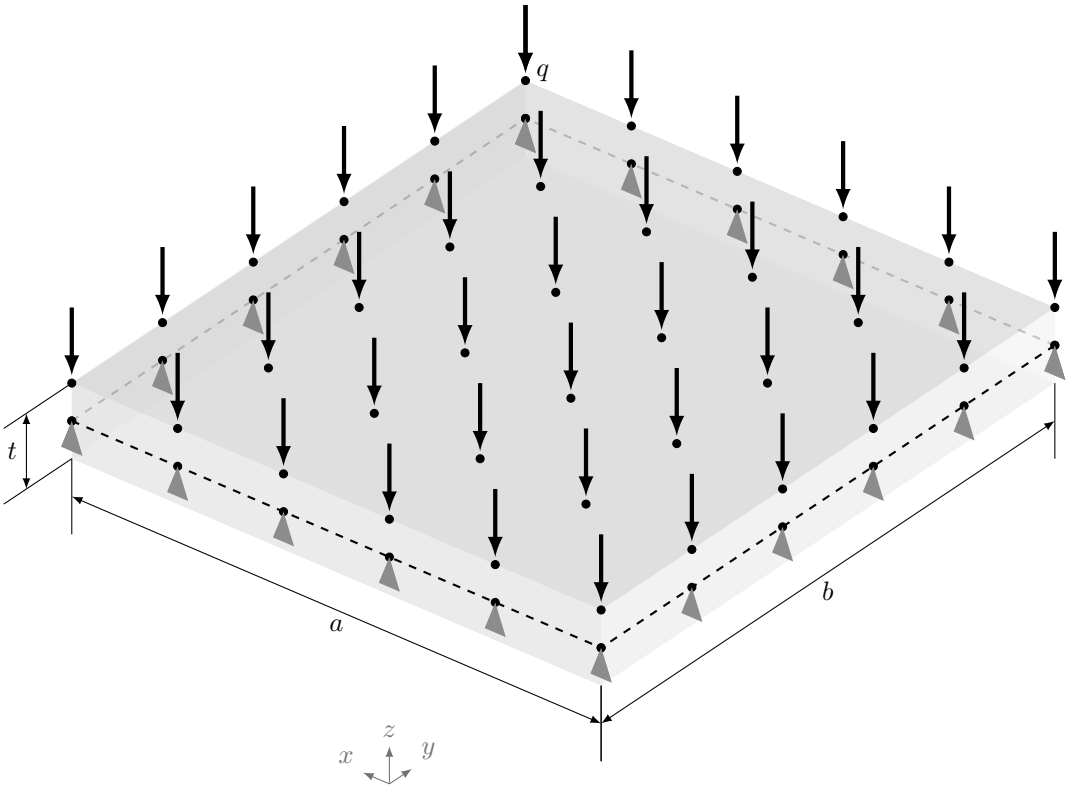


Figure 14: TIM116

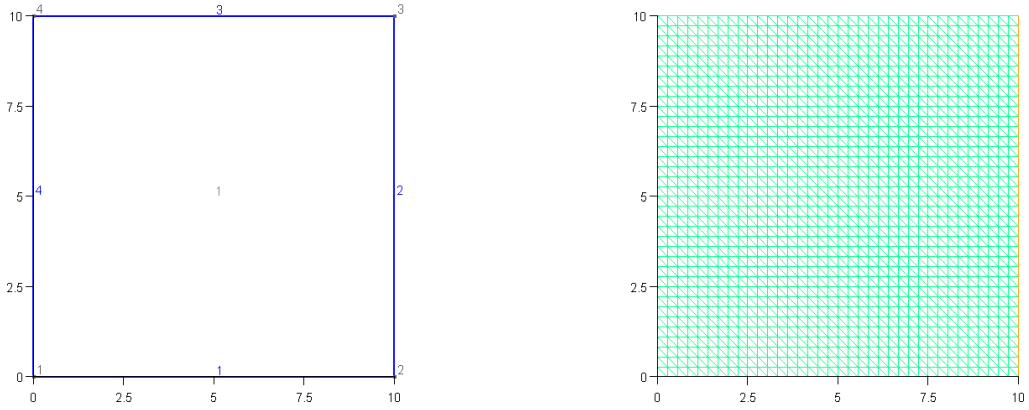


Figure 15: Geomentry and Mesh of TIM116

Table 10: FEM and Boundary condition data

Direchlet Boundary				Neumann Boundary			
Geo -Entity	w	θ_x	θ_y	Geo -Entity	F_z	M_x	M_y
line {1,2,3,4}	Fixed	Free	Free	Area {1}	1000 N/m^2		

Mesh and boundary condition :

Analytically solution :

The w_{max}which is the w displacement at the middle of the plate is given by

$$w_{max} = 0.00406\frac{qb^4}{D} \tag{5}$$

The analytically solution of the problem is calculated as $w_{max} = -0.0022167m$

Result and error analysis :

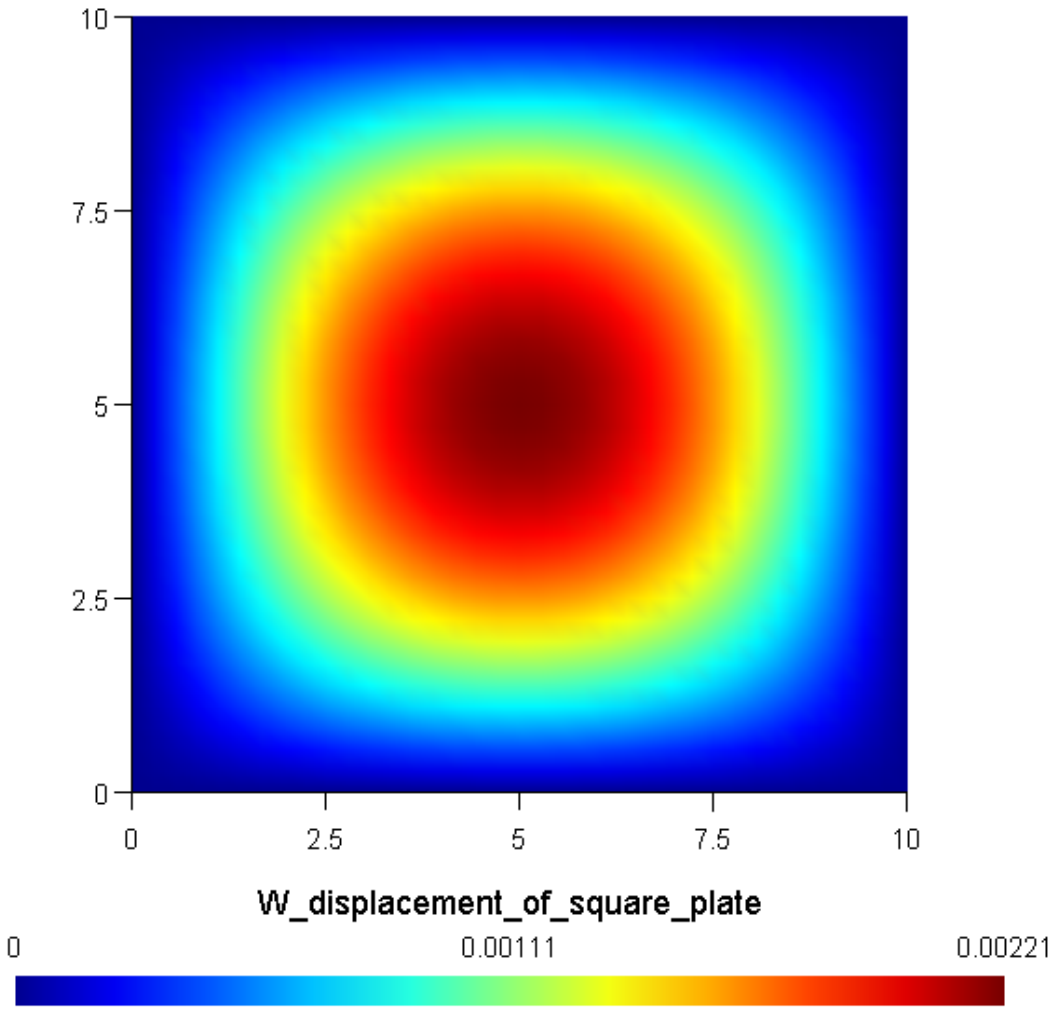


Figure 16: FEM solution plot

The maximum displacement of the domain is our solution . w displacement at middle is $-0.0022144m$.

$$error\% = \left| \frac{w_{analytical} - w_{FEM}}{w_{analytical}} \right| \times 100 \tag{6}$$

So the Error percentage is 0.1%.

6 Square plate with combined load(TIM381)

Description :

A simply Supported square plate with distributed load (q).

Reference :

S.Timoshenko , S . Woinowsky , Theory of Plates and Shells , pg:381, Article : 30 .

Material and Geometric data :

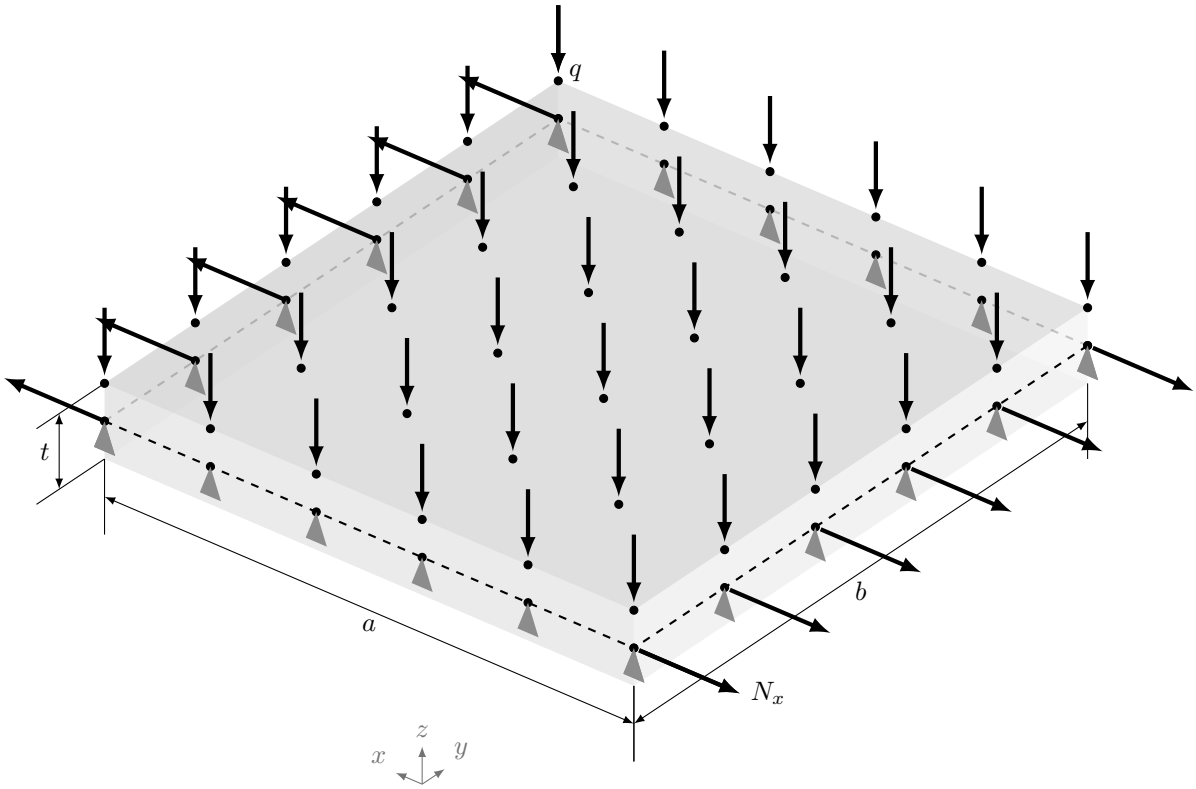


Figure 17: TIM381

Table 11: Input Data

Material Property		Geometric Data		Loading Data	
Young's Modulus (E)	2E11 pa	Length (a)	10 m	Axial Tension(N_x)	2E7 N/m
Poission's Ratio (ν)	0.3	Breath (b)	10 m	Distributed Load (q)	1000 N/m^2
		Thickness(t)	0.1 m		

Mesh and boundary condition :

Table 12: FEM and Boundary condition data

Direchlet Boundary				Neumann Boundary			
Geo -Entity	w	θ_x	θ_y	Geo -Entity	F_z	M_x	M_y
line {1,2,3,4}	Fixed	Free	Free	Area {1}	1000 N/m^2		
				Geo -Entity	Fx		
				line{1,3}	2E7 N/m		

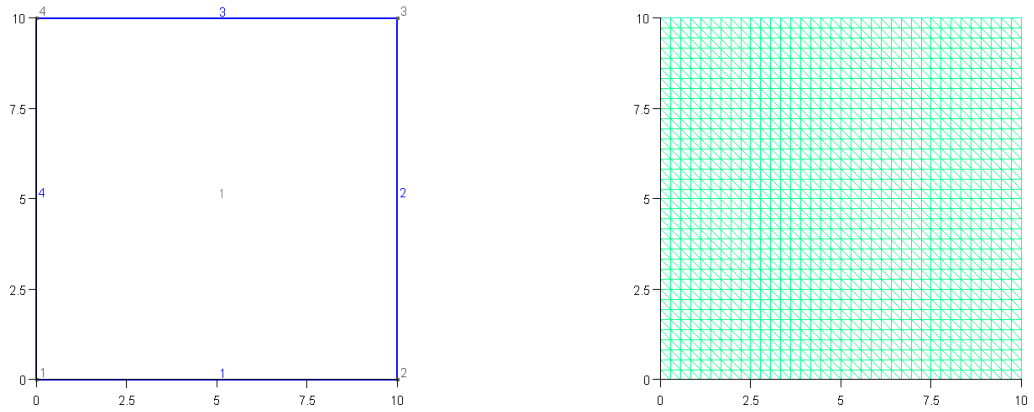


Figure 18: Geomentry and Mesh of TIM381

Analytically solution :

The w_{max}which is the w displacement at the middle of the plate is given by

$$w_{max} = \alpha \frac{qb^4}{Et^3} \tag{7}$$

Where α can be taken from the graph given in the figure : **??**. For this problem alpha is taken as $\alpha \approx 0.0113$.

The parameter γ is given by

$$\gamma = \frac{N_x b^2}{4\pi^2 D} \tag{8}$$

The analytically solution of the problem is calculated as $w_{max} = 0.000565m$

Result and error analysis :

The maximum displacement of the domain is our solution . w displacement at middle is $0.000566m$.

So the Error percentage is 0.17%.

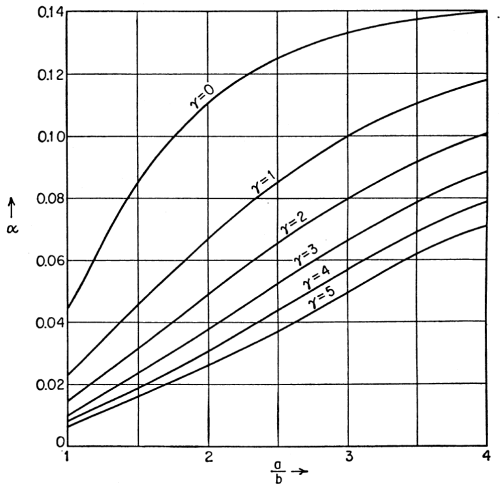


Figure 19: Graph to find α .

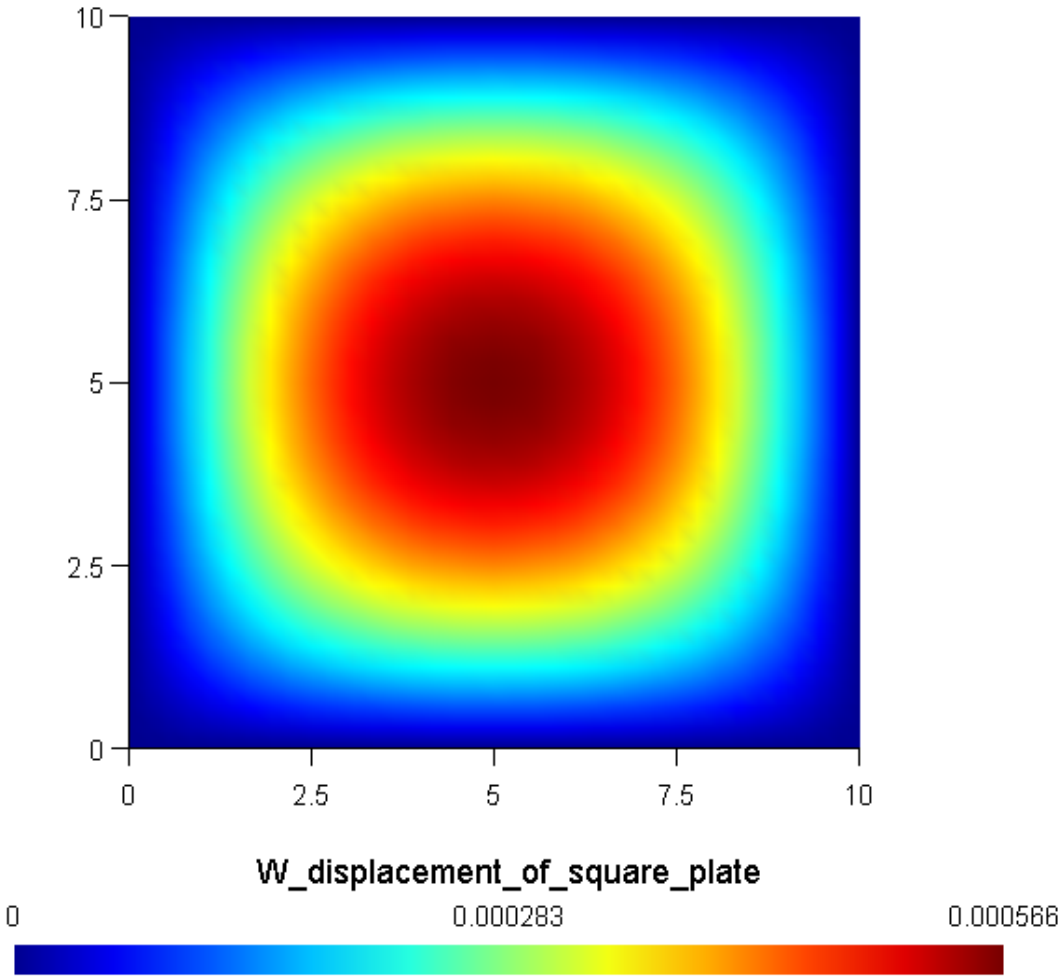


Figure 20: FEM solution plot

7 Natural Frequency of Rectangular strip with axial load (NAS277)

Description :

Modal analysis of a rectangular strip with axial stress (N_2) on short edge.

Reference :

Arthur W.Leissa ,Vibration of Plates,NASA SP-160, pg:277, Ch:10.2.

Material and Geometric data :

Mesh and boundary condition :

Analytically solution :

The analytical solution of the this problem is given by

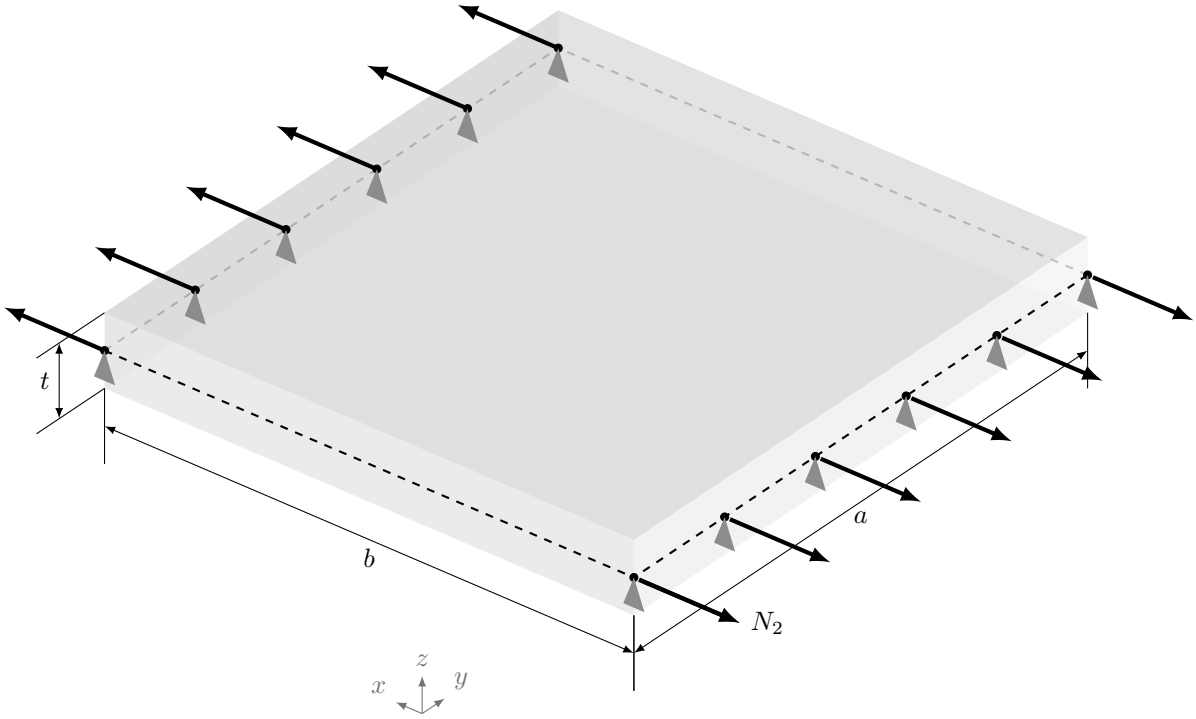


Figure 21: NAS277

Table 13: Input Data

Material Property		Geometric Data		Loading Data	
Young's Modulus (E)	1E11 pa	Length (a)	1 m	N_2	3E11 N/m^2
Poission's Ratio (ν)	0.3	Breath (b)	40 m		
Density (ρ)	7810 Kg/m^3	Thickness(t)	1 m		

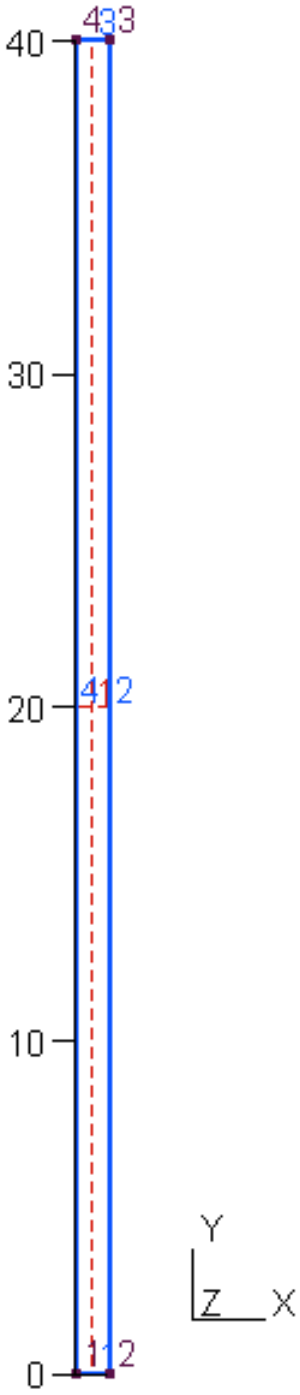
Table 14: FEM and Boundary condition data

Direchlet Boundary				Loading Conditions	
Geo -Entity	w	θ_x	θ_y	Geo -Entity	N_2
line {1,3}	Fixed	Free	Free	line {1,3}	3E11 N/m^2

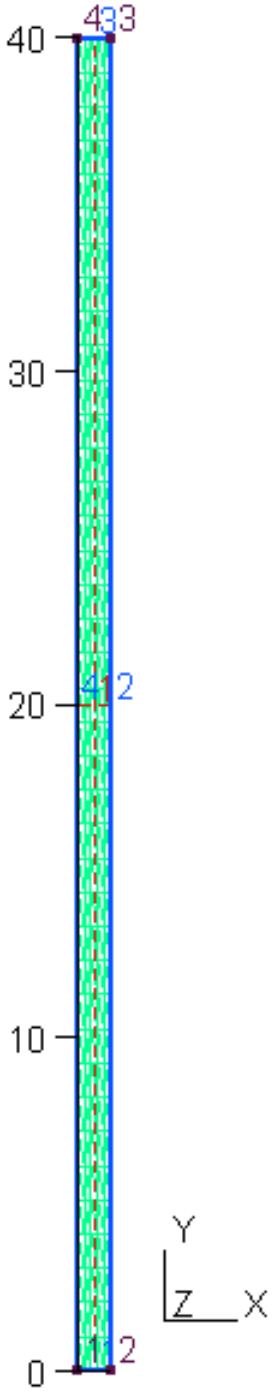
$$\omega_{mn} = \sqrt{\frac{1}{\rho}\left(D\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] + N_1\left(\frac{m\pi}{a}\right)^2 + N_2\left(\frac{n\pi}{b}\right)^2\right)} \tag{9}$$

Natural frequencies are
mode 1 : 77.479 Hz
mode 2 : N.A
mode 3 : 155.00 Hz
mode 4 : N.A
mode 5 : 232.61 Hz
mode 6 : N.A

note : modes 2,4 and 6 are twisting modes, which are not given by the formula.



(a) Geomentry of the problem

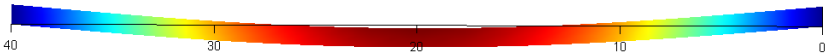


(b) Discritization

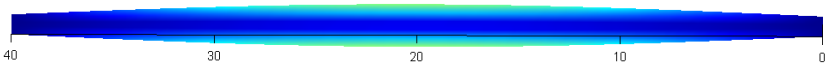
Result and error analysis :

The natural frequencies of the plates are provided below.
mode 1 : 77.458 Hz
mode 2 : 95.610 Hz
mode 3 : 154.98 Hz
mode 4 : 191.46 Hz
mode 5 : 232.63 Hz
mode 6 : 287.38 Hz

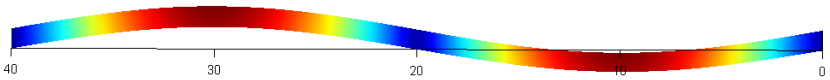
So the Error percentage for each mode is :
mode 1 : 0.026 %
mode 3 : 0.012 %
mode 5 : 0.013 %



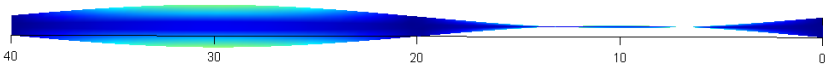
(a) Mode Shape 1



(b) Mode Shape 2

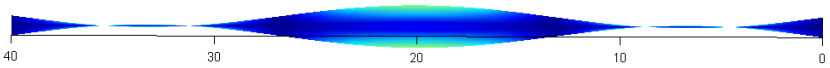


(c) Mode Shape 3



(d) Mode Shape 4

(e) Mode Shape 5



(f) Mode Shape 6

Figure 23: Natural Modes of a rectangular strip

8 Error analysis of strip with various thickness (NAS277)

Description :

It is the continuation of previous problem but the error of the solution is compared for different thicknesses.

Material and Geometric data :

Mesh and boundary condition :

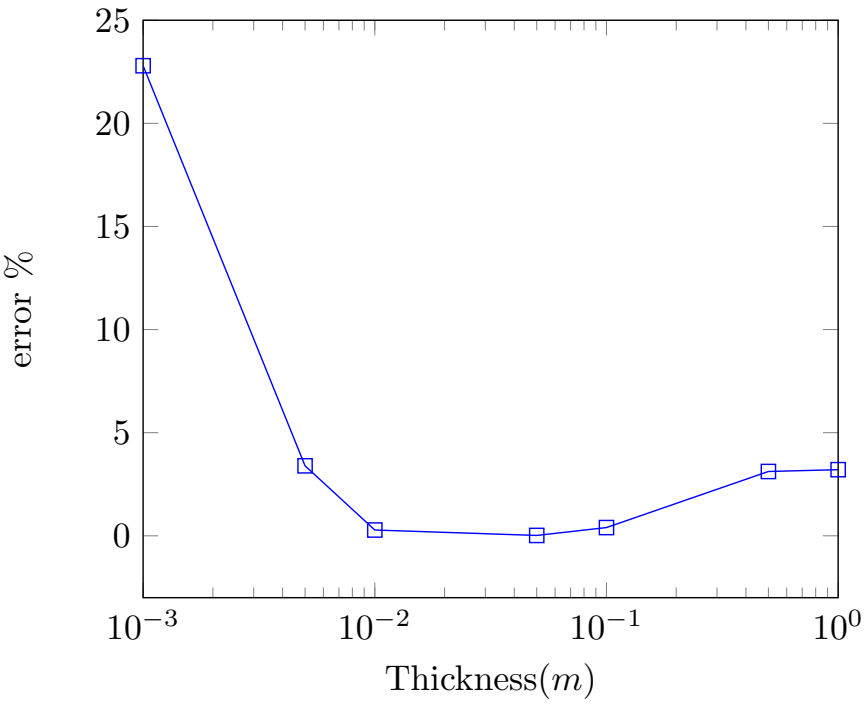
Result and error analysis :

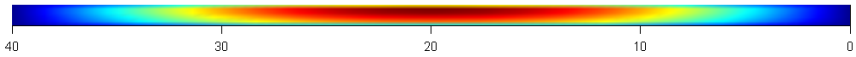
Table 15: Input Data

Material Property		Geometric Data		Loading Data	
Young's Modulus (E)	1E11 pa	Length (a)	1 m	N_2	$\frac{T}{t}$ N/m^2
Poission's Ratio (ν)	0.3	Breath (b)	40 m	Tension T	3E4 N/m
Density (ρ)	7810 Kg/m^3	Thickness(t)	{0.001,0.005,..,0.5,1} m		

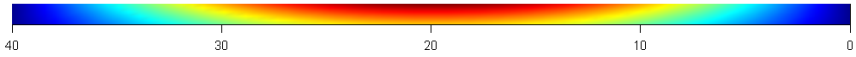
Table 16: FEM and Boundary condition data

Direchlet Boundary				Loading Conditions	
Geo -Entity	w	θ_x	θ_y	Geo -Entity	N_2
line {1,3}	Fixed	Free	Free	line {1,3}	$\frac{T}{t}$ N/m^2

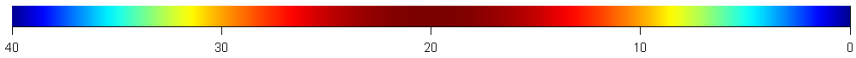




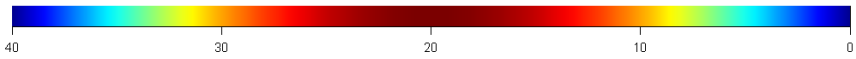
(a) Mode Shape 1 for thickness = 0.001 (m)



(b) Mode Shape 1 for thickness = 0.01 (m)



(c) Mode Shape 1 for thickness = 0.1 (m)



(d) Mode Shape 1 for thickness = 1 (m)

Figure 24: Natural Modes of a rectangular strip