Finite Element Simulation of 2D Metal Strip Vibration in Hot-Dip Galvanization Process

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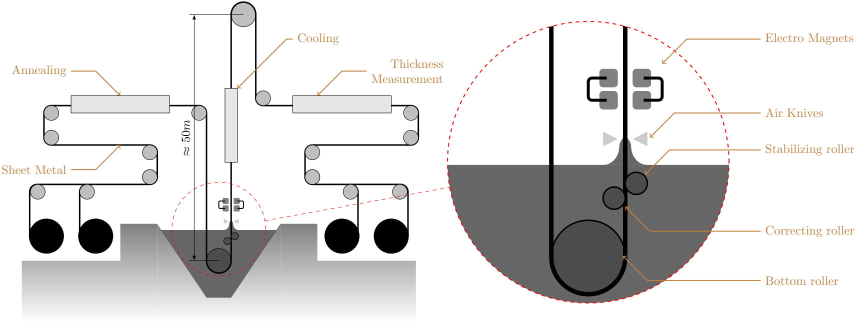
Objective of the internship

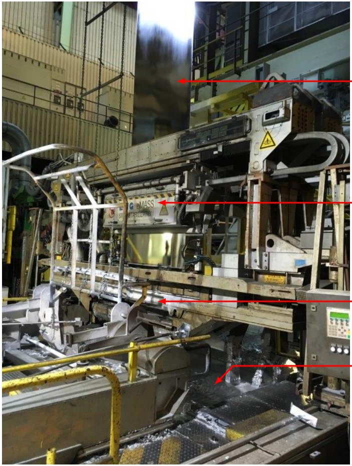
- To understand the Hot-Dip Galvanization process to have an idea about the loads and boundary condition.
- To find existing research about finite element of strip used in this process.
- To formulate plate equation of motion
- To code accurate and efficient finite element program from scratch
- To test the finite element code with analytical and existing numerical methods
- To find a way to integrate with existing control algorithms.

ArcelorMittal Maizières research SA

- ArcelorMittal is the global leader in steel production and mining activity.
- Formed in 2006 by merging MittalSteel and Arcelor.The headquarters is in Luxembourg.
- ArcelorMittal spends hundreds of millions of dollars in research and development.
- ArcelorMittal Maizières research SA is the research center where this thesis is undertaken under the department of measurement and control.
- The main task of this department was to explore and fine turn the new measurement techniques in profit of increasing the quality of the steel production.
- The control team of the department is specialized in developing advanced control strategies (Model Predictive control, Model - based control etc,.) to continuously improve the comfort of operators and the product quality.



(a) Schematics of the Hot-Dip Galvanization Line



(b) Hot-Dip Galvanization Line

- A thin Layer of Zinc is coated to Increase the corrosion resistance of steel
- Air knives control the thickness of the Zinc layer
- Excessive Vibration results in uneven coating.
- Electromagnets are used to control the vibration of the strip.

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Axially Moving Plate

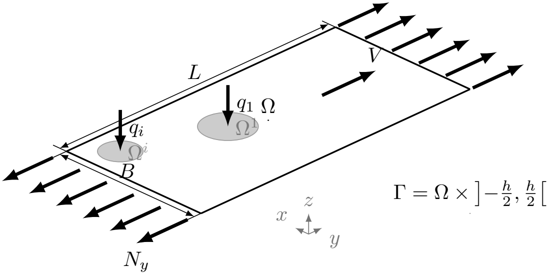

Figure: Description of domain

- Ω is the two dimensional domain strictly in xy plane
- h is the thickness of the plate
- $\Omega_1 ... \Omega_i$ are the sub-domains where pressure forces $q_1 .. q_i$ are applied
- V is the Line speed and Nx is the tension on the line
- L is the length and B is the width

Hamilton Principle

The modified form of the Hamilton principle is taken as.

$$\delta H = \int_{t_0}^{t_1} (\delta U - \delta K + \delta W + \delta M)\, dt = 0 \tag{1}$$

$$\delta \mathbf{u}\Big|_{t_0}^{t_1} = 0 \tag{2}$$

δ is the variation, t_0 and t_1 are any arbitrary temporal points, U is the total potential energy , K is the kinematic energy and W is the work performed by external forces on the system. $\mathbf{u}$ is the total displacement of the plate. M is the momentum transports at boundaries y=0 and y = L.

$$\delta M = \int_0^W \int_{-h/2}^{h/2} L\rho\mathbf{v}\delta\mathbf{u}\Big|_{y=0}^{y=L} dzdx = 0 \tag{3}$$

Here, $\mathbf{v}$ is the total velocity vector of the plate. M becomes zero because the line speed is equal at the boundaries. So, there is no overall change in the mass of the plate.

Plate assumptions

- The plate thickness does not change after deformation. $\epsilon_{zz} = 0$
- In the absence of axial deformation, any point in the mid-plane only moves either in an upward or downward direction
- The flat plane normal to mid-plane will always be a flat plane, they won't distort.

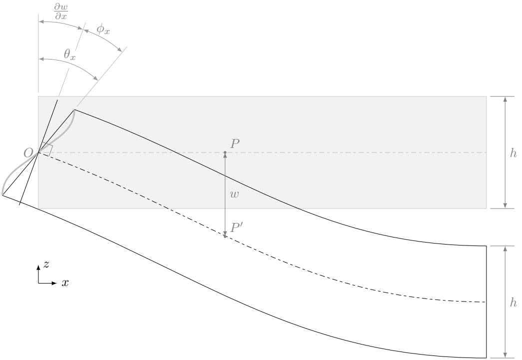

Figure: Description of domain

Plate displacement

$$\tilde{\mathbf{u}} = \left\{ \begin{array}{r} -z\theta_x\,(x, y) \\ -z\theta_y\,(x, y) \\ w\,(x, y) \end{array} \right\}$$

Kirchhoff Plate

Kirchhoff plate theory is well suitable for thin plates. A straight line normal to mid-plane stays normal and straight after deformation($\phi_x = 0$). Because of this assumption, the shear strains (ϵ_{23} and ϵ_{13}) are neglected.

$$\theta_x = \frac{\partial w}{\partial x} \qquad \theta_y = \frac{\partial w}{\partial y} \tag{4}$$

Reissner Mindlin plate theory

The Reissner Mindlin plate theory is developed for the thick plates but can be used for thin plates with caution. For Reissner Mindlin plate theory, the line normal to the middle plate will not necessarily be normal after deformation, but will be straight In equation.5, ϕ_x and ϕ_y are the angles between plane normal to middle plane and plane of actual deformation.

$$\theta_x = \frac{\partial w}{\partial x} + \phi_x \qquad \theta_y = \frac{\partial w}{\partial y} + \phi_y \tag{5}$$

Potential Energy

The total potential strain energy U is given as.

$$U = \frac{1}{2}\int\int\int_{\Gamma} (\epsilon)^{T} \boldsymbol{\sigma} d\Gamma = \frac{1}{2}\int\int\int_{\Gamma} \left(\epsilon^{B}\right)^{T} \boldsymbol{\sigma}^{B} + \left(\epsilon^{S}\right)^{T} \boldsymbol{\sigma}^{S} + \left(\epsilon^{A}\right)^{T} \boldsymbol{\sigma}^{A} d\Gamma$$

B,S,A on the superscript indicates bending, shear and axial components of the strain and stress. Using strain formula each of the strain is.

$$\epsilon^{B} = -z \begin{bmatrix} \frac{\partial w^2}{\partial x^2} \\ \frac{\partial w^2}{\partial y^2} \\ \frac{\partial w^2}{\partial x \partial y} \end{bmatrix} = z\boldsymbol{\kappa} \qquad \epsilon^{S} = \frac{1}{2}\begin{bmatrix} \frac{\partial w}{\partial x} - \theta_x \\ \frac{\partial w}{\partial y} - \theta_y \end{bmatrix} \qquad \epsilon^{A} = \left(\frac{\partial w}{\partial y}\right)^2 = (w_{,2})^2$$

For Kirchhoff Plate

$$\epsilon^{S} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For Reissner Mindlin Plate

$$\epsilon^{S} = \frac{1}{2}\begin{bmatrix} -\phi_x \\ -\phi_y \end{bmatrix}$$

The Hooke's law for the homogenous linear isotropic material is considered.

$$\boldsymbol{\sigma}^B = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \frac{1}{1-\nu^2} \begin{bmatrix} E & \nu E & 0 \\ \nu E & E & 0 \\ 0 & 0 & (1-\nu^2)G \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{bmatrix} = \boldsymbol{D}\boldsymbol{\epsilon}^B \tag{6}$$

$$\boldsymbol{\sigma}^S = \begin{bmatrix} \sigma_{31} \\ \sigma_{32} \end{bmatrix} = KG \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{31} \\ \epsilon_{32} \end{bmatrix} = \boldsymbol{D_c}\boldsymbol{\epsilon}^S \qquad \boldsymbol{\sigma}^A = \begin{bmatrix} \sigma_{22} \end{bmatrix} = \begin{bmatrix} N_y \end{bmatrix}$$

E is Young's modulus, ν is the Poisson's ratio and G is the shear modulus which is given by $G = E/2(1+\nu)$. K is the shear correction factor. Shear correction factor value of 5/6 is used for this application. Substituting everything in strain energy formula and taking variation gives.

$$.U = \frac{1}{2} \int \int_\Omega \left[\int_{-h/2}^{+h/2} z^2 dz \right] \boldsymbol{\kappa}^T \boldsymbol{D} \boldsymbol{\kappa} + \left[\int_{-h/2}^{+h/2} dz \right] (\left(\boldsymbol{\epsilon}^S\right)^T \boldsymbol{D_c} \boldsymbol{\epsilon}^S + \left(\boldsymbol{\epsilon}^A\right)^T \boldsymbol{\sigma}^A) d\Omega$$

$$\delta U = \int \int_\Omega \boldsymbol{\kappa}^T \tilde{\boldsymbol{D}} \delta \boldsymbol{\kappa} + \left(\boldsymbol{\epsilon}^S\right)^T \tilde{\boldsymbol{D}}_{\boldsymbol{c}} \delta \boldsymbol{\epsilon}^S + w_{,2} \tilde{\boldsymbol{\sigma}}^A \delta w_{,2} d\Omega$$

Kinetic Energy

Based on Euler - Lagrange formulation the velocity is given as

$$\mathbf{v} = \{\dot{u}_1 + V_2 u_{1,2} \quad \dot{u}_2 + V_2 u_{2,2} \quad \dot{u}_3 + V_2 u_{3,2}\}^T$$

The kinetic energy is

$$K = \frac{1}{2}\int\int\int_\Gamma \mathbf{v}^T \rho \mathbf{v} d\Gamma = \frac{1}{2}\int\int_\Omega \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho \dot{\boldsymbol{u}}^T \dot{\boldsymbol{u}} + 2\rho V_2 \dot{\boldsymbol{u}}^T \boldsymbol{u}_{,2} + \rho V_2^2 (\boldsymbol{u}_{,2})^T \boldsymbol{u}_{,2} \quad dz d\Omega$$

$$K = \frac{1}{2}\int\int_\Omega \rho \dot{\tilde{\boldsymbol{u}}}^T \boldsymbol{Z} \dot{\tilde{\boldsymbol{u}}} + 2\rho V_2 \dot{\tilde{\boldsymbol{u}}}^T \boldsymbol{Z} \tilde{\boldsymbol{u}}_{,2} + \rho V_2^2 (\tilde{\boldsymbol{u}}_{,2})^T \boldsymbol{Z} \tilde{\boldsymbol{u}}_{,2} \quad d\Omega$$

$$\boldsymbol{Z} = \begin{bmatrix} h & 0 & 0 \\ 0 & \frac{h^3}{12} & 0 \\ 0 & 0 & \frac{h^3}{12} \end{bmatrix} \qquad (and) \qquad \tilde{\boldsymbol{u}} = \begin{bmatrix} w \\ \theta_x \\ \theta_y \end{bmatrix}$$

Finally the variation of the kinetic energy

$$\delta K = \int\int_\Omega \rho \dot{\tilde{\boldsymbol{u}}}^T \boldsymbol{Z} \delta \dot{\tilde{\boldsymbol{u}}} + \rho V_2 \delta \dot{\tilde{\boldsymbol{u}}}^T \boldsymbol{Z} \tilde{\boldsymbol{u}}_{,2} + \rho V_2 \dot{\tilde{\boldsymbol{u}}}^T \boldsymbol{Z} \delta \tilde{\boldsymbol{u}}_{,2} + \rho V_2^2 (\tilde{\boldsymbol{u}}_{,2})^T \boldsymbol{Z} \delta \tilde{\boldsymbol{u}}_{,2} \quad d\Omega$$

Final Weak form

The Transverse distributed forces q_j is applied in the regions in Ω^j is given as

$$\delta W = \sum_j^{nb} \int_{\Omega^j} q_j \delta \tilde{\boldsymbol{u}} \quad d\Omega^j$$

Substituting everything in the Hamilton principle and taking integration by parts gives the final weak form of the axially moving and axially tensed plate.

$$\int\int_{\Omega} \rho \ddot{\tilde{\boldsymbol{u}}}^T \boldsymbol{Z} \delta \tilde{\boldsymbol{u}} + \rho V_1 \delta \tilde{\boldsymbol{u}} \boldsymbol{Z} \dot{\tilde{\boldsymbol{u}}}_{,2} + \rho V_1 \tilde{\boldsymbol{u}} \boldsymbol{Z} \delta \dot{\tilde{\boldsymbol{u}}}_{,2} - \rho V_1^2 \tilde{\boldsymbol{u}}_{,2} \boldsymbol{Z} \delta \tilde{\boldsymbol{u}}_{,2}$$

$$+ \boldsymbol{\kappa}^T \tilde{\boldsymbol{D}} \delta \boldsymbol{\kappa} + \left(\boldsymbol{\epsilon}^S\right)^T \tilde{\boldsymbol{D}}_{\boldsymbol{c}} \delta \boldsymbol{\epsilon}^S + w_{,2} \tilde{\boldsymbol{\sigma}}^A \delta w_{,2} d\Omega = \sum_j^{nb} \int_{\Omega^j} q_j \delta \tilde{\boldsymbol{u}} d\Omega^j$$

First term in the equation corresponds to the local acceleration. second and third terms are the Coriolis acceleration and the fourth term is the centripetal acceleration. Because of theseterms this equation is gyroscopic.

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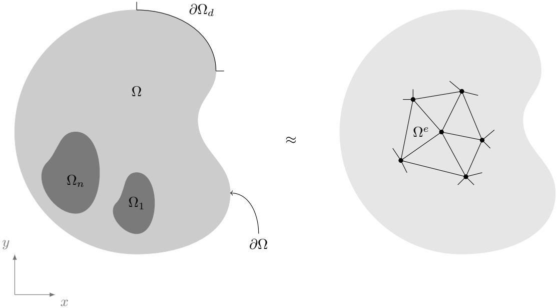


Figure: FEM domain

The description of the domain is given in the the figure.3. Ω is the total two dimensional domain in the x , y plane. The domain is discretized into small elements Ω^e. $\Omega^1 \cdots \Omega^n$ are the regions where transverse distributed loads $(q_1 \cdots q_i)$ are described. $\partial\Omega$ is the boundary of the domain. $\partial\Omega_d$ is part of the boundary where Dirichlet boundary condition is applied.

The Displacement field of the each element is the function of displacement of degree of freedom of each node, which lets us have a finite number of unknowns to denote the over all displacement field of the domain.
Since it is a plate element, three independent degrees of freedom are described for each node.

$$\tilde{\mathbf{u}} = [w, \theta_x, \theta_y]^T$$

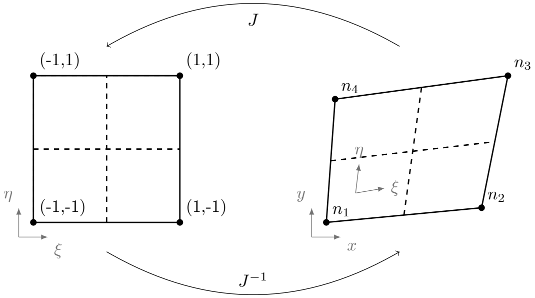
w represents the transverse displacement. θ_x and θ_y represents the rotations.

$$\theta_x = \frac{\partial w}{\partial x} \qquad \theta_y = \frac{\partial w}{\partial y} \tag{7}$$

The approximate displacement of the element is the given as sum of product of nodal degree of freedom and its corresponding shape functions (N,$\overline{N}$,$\overline{\overline{N}}$).

$$\tilde{\mathbf{u}} \approx \sum_{i=1}^{n} \left(N_i w_i + \overline{N}_i \theta_{x_i} + \overline{\overline{N}}_i \theta_{y_i} \right) \tag{8}$$

Reissner Mindlin Plate element



$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$
$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$
$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

Figure: Jacobian transformation

$$\left\{ \begin{matrix} \frac{\partial N}{\partial \xi} \\ \frac{\partial N}{\partial \eta} \end{matrix} \right\} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \left\{ \begin{matrix} \frac{\partial N}{\partial x} \\ \frac{\partial N}{\partial y} \end{matrix} \right\} \quad J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \tag{9}$$

The FE approximation

$$\tilde{\mathbf{u}} \approx \sum_{i=1}^{n} \left(N_i w_i + \overline{N}_i \theta_{x_i} + \overline{\overline{N}}_i \theta_{y_i} \right)$$

is written in matrix format as

$$\tilde{\mathbf{u}} \approx \begin{bmatrix} N_1 & 0 & 0 & \cdots & N_{nN} & 0 & 0 \\ 0 & \overline{N}_1 & 0 & \cdots & 0 & \overline{N}_{nN} & 0 \\ 0 & 0 & \overline{\overline{N}}_1 & \cdots & 0 & 0 & \overline{\overline{N}}_{nN} \end{bmatrix} \begin{Bmatrix} w_1 \\ \theta_{x_1} \\ \theta_{y_1} \\ \vdots \\ w_{nN} \\ \theta_{x_{nN}} \\ \theta_{y_{nN}} \end{Bmatrix} = \mathbf{N}\tilde{\mathbf{u}}^e$$

Similarly other terms of the Finite Element Matrices are

$$\dot{\tilde{\mathbf{u}}} \approx \mathbf{N}\dot{\tilde{\mathbf{u}}}^e \qquad \ddot{\tilde{\mathbf{u}}} \approx \mathbf{N}\ddot{\tilde{\mathbf{u}}}^e \qquad \kappa \approx \mathbf{B}\tilde{\mathbf{u}}^e \qquad \tilde{\epsilon}^S \approx \mathbf{B_S}\tilde{\mathbf{u}}^e$$

$$\tilde{u}_{1,\alpha} \approx \mathbf{H_A}\tilde{\mathbf{u}}^e \qquad \tilde{u}_{\alpha,1} \approx \mathbf{H_v}\tilde{\mathbf{u}}^e \qquad \tilde{w} \approx \mathbf{N_f}\tilde{\mathbf{u}}^e$$

Kirchhoff Plate Element (PAT)

The triangle element with three nodes, is given here.

$$N = \begin{bmatrix} P(1) - P(4) + P(6) + 2 * (P(7) - P(9)) \\ -b(2) * (P(9) - P(6)) - b(3) * P(7) \\ -c(2) * (P(9) - P(6)) - c(3) * P(7) \\ P(2) - P(5) + P(4) + 2 * (P(8) - P(7)) \\ -b(3) * (P(7) - P(4)) - b(1) * P(8) \\ -c(3) * (P(7) - P(4)) - c(1) * P(8) \\ P(3) - P(6) + P(5) + 2 * (P(9) - P(8)) \\ -b(1) * (P(8) - P(5)) - b(2) * P(9) \\ -c(1) * (P(8) - P(5)) - c(2) * P(9) \end{bmatrix}$$

The element is based on a polynomial expression of nine terms.

$$\begin{aligned} \mathbf{P} = [&L_1 \quad L_2 \quad L_3 \quad L_1L_2 \quad L_2L_3 \quad L_3L_1 \\ &L_1^2L_2 + \tfrac{1}{2}L_1L_2L_3(3(1-\mu_3)L_1 - (1+3\mu_3)L_2 + (1+3\mu_3)L_3)) \\ &L_2^2L_3 + \tfrac{1}{2}L_1L_2L_3(3(1-\mu_1)L_2 - (1+3\mu_1)L_3 + (1+3\mu_1)L_1)) \\ &L_3^2L_1 + \tfrac{1}{2}L_1L_2L_3(3(1-\mu_2)L_3 - (1+3\mu_2)L_1 + (1+3\mu_2)L_2))] \end{aligned}$$

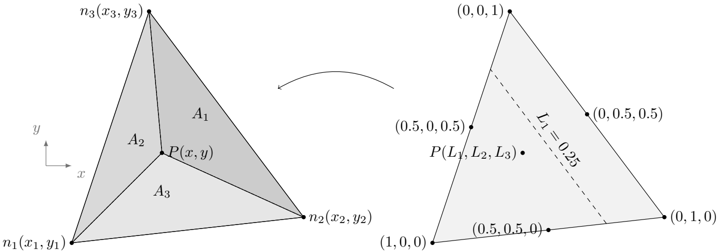


Figure: Area coordinate

$$L_1 = \frac{A_1}{A} \quad L_2 = \frac{A_2}{A} \quad L_3 = \frac{A_3}{A}$$

$$L_1 + L_2 + L_3 = 1$$

$$\left\{ \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right\} = \frac{1}{4A} \begin{bmatrix} y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_2 - x_3 & x_3 - x_1 & x_1 - x_2 \end{bmatrix} \left\{ \begin{array}{c} \frac{\partial}{\partial L_1} \\ \frac{\partial}{\partial L_2} \\ \frac{\partial}{\partial L_3} \end{array} \right\}$$

$$\tilde{\mathbf{u}} \approx \begin{Bmatrix} w \\ \theta_x \\ \theta_y \end{Bmatrix} = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 & \cdots & N_9 \\ N_{1,1} & N_{2,1} & N_{3,1} & N_{4,1} & \cdots & N_{9,1} \\ N_{1,2} & N_{2,2} & N_{3,2} & N_{4,2} & \cdots & N_{9,2} \end{bmatrix} \begin{Bmatrix} w_1 \\ \theta_{x_1} \\ \theta_{y_1} \\ w_2 \\ \vdots \\ \theta_{y_3} \end{Bmatrix} = \mathbf{N}\tilde{\mathbf{u}}^{\mathbf{e}}$$

the curvature term is given as

$$\boldsymbol{\kappa} \approx \begin{bmatrix} N_{1,11} & N_{2,11} & N_{3,11} & \cdots & N_{9,11} \\ N_{1,22} & N_{2,22} & N_{3,22} & \cdots & N_{9,22} \\ N_{1,12} & N_{2,12} & N_{3,12} & \cdots & N_{9,12} \end{bmatrix} \begin{Bmatrix} w_1 \\ \theta_{x_1} \\ \theta_{y_1} \\ \vdots \\ \theta_{y_3} \end{Bmatrix} = \mathbf{B}\tilde{\mathbf{u}}^{\mathbf{e}} \tag{10}$$

similarly all other terms are converted similar to QUAD4 element

The Finite Element Matrix equation is given as

$$\int\int_{\Omega}\left(\rho\,[\mathbf{N}]\,[\mathbf{Z}]\,[\mathbf{N}]\,\{\ddot{\tilde{\mathbf{u}}}^{e}\}\right)\delta\tilde{\mathbf{u}}^{e}+\left(2\rho V_1\,[\mathbf{N}]\,[\mathbf{Z}]\,[\mathbf{H_v}]\,\{\dot{\tilde{\mathbf{u}}}^{e}\}\right)\delta\tilde{\mathbf{u}}^{e}$$
$$-\left(\rho V_1^2\,[\mathbf{H_v}]\,[\mathbf{Z}]\,[\mathbf{H_v}]\,\{\tilde{\mathbf{u}}^{e}\}\right)\delta\tilde{\mathbf{u}}^{e}+\left([\mathbf{B}]\left[\tilde{\mathbf{D}}\right][\mathbf{B}]\,\{\tilde{\mathbf{u}}^{e}\}\right)\delta\tilde{\mathbf{u}}^{e}$$
$$+\left([\mathbf{B_S}]\left[\tilde{\mathbf{D}}_\mathbf{S}\right][\mathbf{B_S}]\,\{\tilde{\mathbf{u}}^{e}\}\right)\delta\tilde{\mathbf{u}}^{e}+\left([\mathbf{H_A}]\left[\tilde{\mathbf{N}}_\mathbf{A}\right][\mathbf{H_A}]\,\{\tilde{\mathbf{u}}^{e}\}\right)\delta\tilde{\mathbf{u}}^{e}d\Omega$$
$$=\sum_{i}^{nb}\int\int_{\Omega_i}\left(q_i\left[\tilde{\mathbf{N}}_\mathbf{f}\right]\right)\delta\tilde{\mathbf{u}}^{e}d\Omega_i$$

After rearranging them to their respective groups we get.

$$[\mathbf{M}^e]\,\{\ddot{\mathbf{u}}^e\}+[\mathbf{C}^e]\,\{\dot{\mathbf{u}}^e\}+[\mathbf{K}^e]\,\{\mathbf{u}^e\}=\{\mathbf{F}^e\}$$

All the Element mass Matrices $[\mathbf{M^e}]$ are assembled in the final Mass Matrix $[\mathbf{M}]$.Doing same for other matrices gives us the ODE in terms of FE matrices.

$$[\mathbf{M}]\,\{\ddot{\mathbf{u}}\}+[\mathbf{C}]\,\{\dot{\mathbf{u}}\}+[\mathbf{K}]\,\{\mathbf{u}\}=\{\mathbf{F}\}$$

Solution Procedure

Dynamic Analysis:
To solve the dynamic system

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\} + [\mathbf{C}]\{\dot{\mathbf{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{F}\}$$

Newmark time integration scheme is employed.

Newmark algorithm

$$R = F_t + \mathbf{M}\left(a_0 u_t + a_2 \dot{u}_t + a_3 \ddot{u}_t\right) + \mathbf{C}\left(a_1 u_t + a_4 \dot{u}_t + a_5 \ddot{u}_t\right)$$

$$\begin{aligned}
u_{t+1} &= \left[a_0\mathbf{M} + a_1\mathbf{C} + \mathbf{K}\right]^{-1} R \\
\dot{u}_{t+1} &= a_1\left(u_{t+1} - u_t\right) - a_4\dot{u}_t - a_5\ddot{u}_t \\
\ddot{u}_{t+1} &= a_0\left(u_{t+1} - u_t\right) - a_2\dot{u}_t - a_3\ddot{u}_t
\end{aligned}$$

a_0 .. a_5 are the integration variables which depends on the Integration parameters θ, α and time step size h. u_t,$\dot{u}_t$ and $\ddot{u}_t$ are the displacement , velocity and acceleration of current time step. u_{t+1},$\dot{u}_{t+1}$ and $\ddot{u}_{t+1}$ are the displacement , velocity and acceleration of next time step.

Unconditionally Stable for

$$\theta \geq \frac{1}{2}$$

$$\alpha \geq \frac{1}{4}\left(\frac{1}{2} + \theta\right)^2$$

Code Features

- MATLAB is used as the language to program the FEM from scratch.
- GMSH open source tool is used for pre processing and post processing.
- Object oriented programming style is adopted.

Listing: Example input script

```
FEM=DYNAMIC('Dynamic_WC1');
FEM.ReadMesh('strip.msh');
FEM.Mat(1)=MATDat("Mat.dat.txt");

t=linspace(0,10,2001);
x1=sin(25*t);

FEM.Pp(1)=ProbeOnPhyEn([21 22 23 24],[1 0 0]);
FEM.T=FEMTime(0.01,3);
FEM.TS(1)=TimeSeries(t,x1);


FEM.Up(1)=DirichletOnPhyEn(FEM, [12], [1 0 0], 0 );
FEM.Up(2)=DirichletOnPhyEn(FEM, [11], [1 0 0], 0.01 );
FEM.Fp(1)=NeumannOnPhyEn(FEM, [22 23 24 21], [1 0 0], 20000 );
```

```
FEM.ImposeU( FEM. Up(2)  *  FEM.TS(1)  );
FEM.ImposeU( FEM. Up(1)   );

K1=10;
K2=2;
K3=10;
P1 = K1 * FEM.Pp(1).forT(0);
P2 = K2 *(FEM.Pp(1).forT(0)-FEM.Pp(1).forT(-1))/(FEM.T.forT(0)-FEM.T.forT
    (-1));
P3 = K3 * Int(FEM.Pp(1));
 FEM.ApplyF( -1 *  FEM.Fp(1)  * (P1 + P2 + P3) );


 FEM.SetDomain([1 21 22 23 24],[1],'MITC4');
%
 FEM.InitialX('zero');
 FEM.InitialV('zero');
%
 tic;
 FEM.SolveLU();
 toc;
%
 FEM.WritePos();
 FEM.WriteProbe();
 FEM.PlotProbe();
 FEM.PlotF();
 FEM.WriteF();
```

State space format

State - Space form is the widely used format for control study of a dynamic system. State - state form is represented as first order ordinary differential equation.

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}(t)$$

$x(t)$ is the state variable.

$$\boldsymbol{x} = \begin{Bmatrix} \boldsymbol{u}(t) \\ \dot{\boldsymbol{u}}(t) \end{Bmatrix}$$

using this the second order ODE is reprented in state - space form as

$$\dot{\boldsymbol{x}} = \frac{d}{dt}\begin{Bmatrix} \boldsymbol{u}(t) \\ \dot{\boldsymbol{u}}(t) \end{Bmatrix} = \begin{bmatrix} 0 & \boldsymbol{I} \\ -[\boldsymbol{M}]^{-1}[\boldsymbol{K}] & -[\boldsymbol{M}]^{-1}[\boldsymbol{C}] \end{bmatrix}\begin{Bmatrix} \boldsymbol{u}(t) \\ \dot{\boldsymbol{u}}(t) \end{Bmatrix} + \begin{bmatrix} 0 \\ [\boldsymbol{M}]^{-1}[\boldsymbol{F}] \end{bmatrix}$$

Unfortunately, the FEM discretization of the domain have huge number of nodes which means the system size will also be huge. To overcome this problem a the size of the model is reduced by using modal - superposition method.

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   - Reissner Mindlin plate element (QUAD4)
   - Kirchhoff Plate Element (PAT)
   - FEM Program Features
   - State Space form
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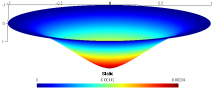
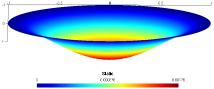
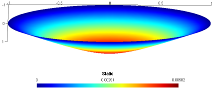


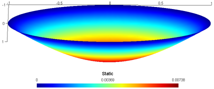
(a) BuiltIN point load



(b) BuiltIN distributed load



(c) SS point load



(d) SS distributed load

Figure: Solution plot

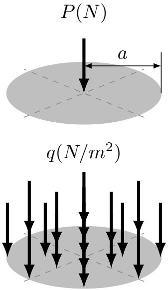


Figure: loads

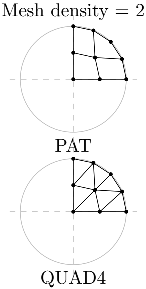


Figure: Mesh density

Comparison of elements for different loads and boundary conditions(2/2)

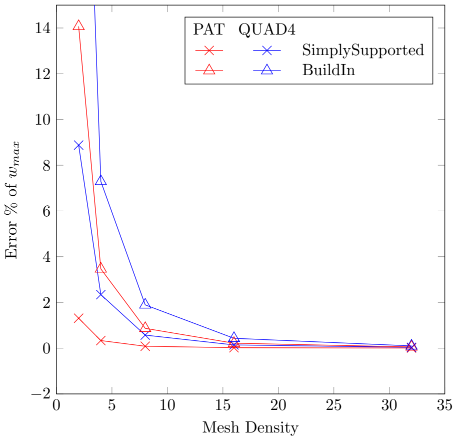


Figure: Distributed load(q)

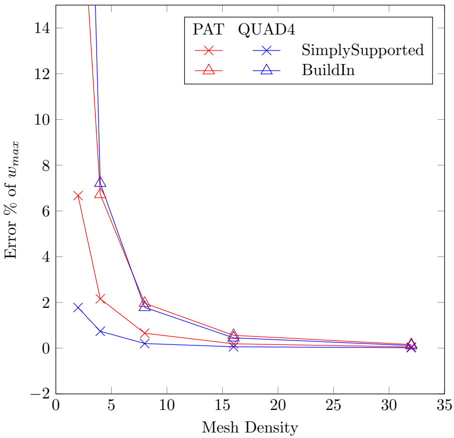
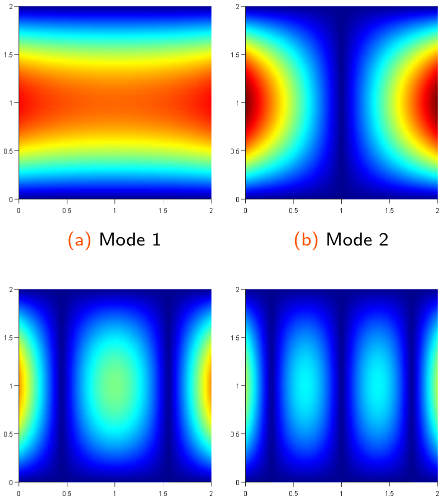


Figure: Point load(P)

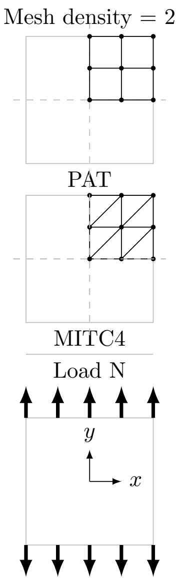
Comparison of elements with axial load (1/2)



(a) Mode 1 (b) Mode 2

(c) Mode 3 (d) Mode 4

Figure: Natural modes of square plate



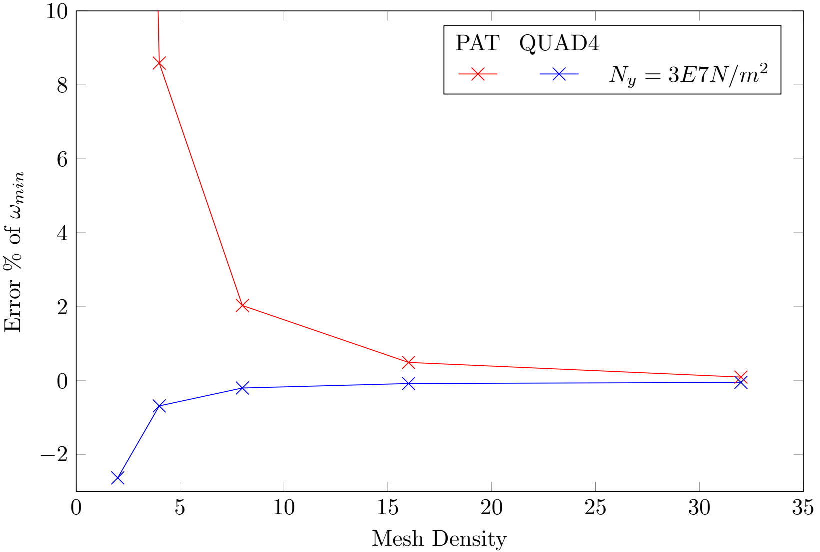


Figure: Convergence of dominant natural frequency of the square plate with axial load

Strip with distributed load and imposed displacement (1/2)

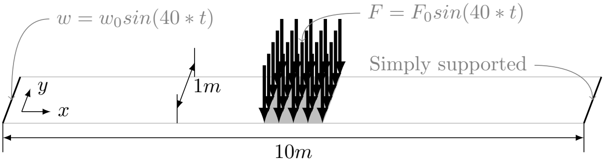


Figure: Strip with imposed displacement and transverse load

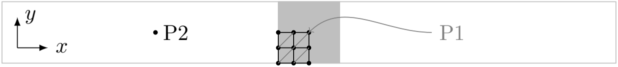
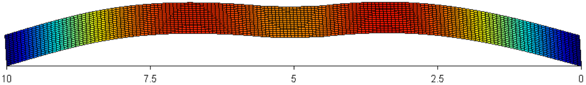


Figure: Mesh Density of Strip



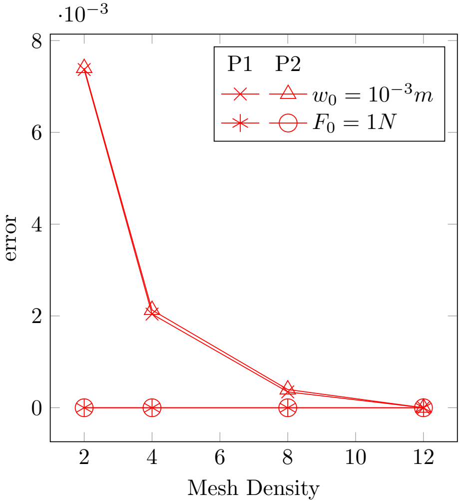


Figure: convergence of PAT element

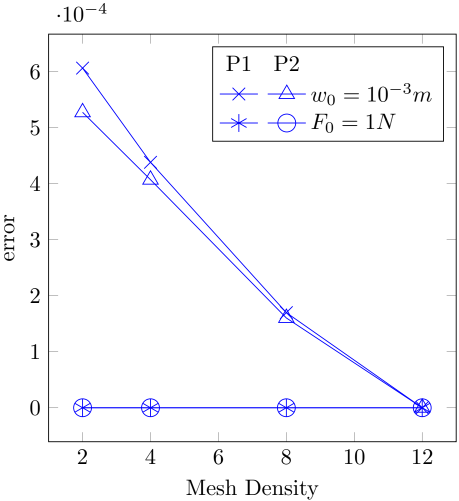
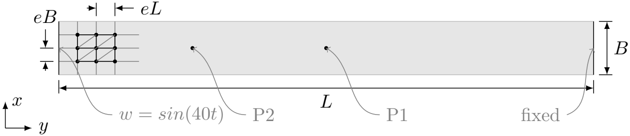
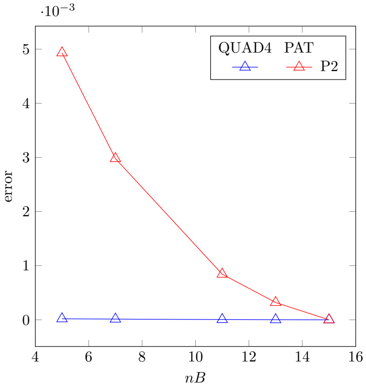


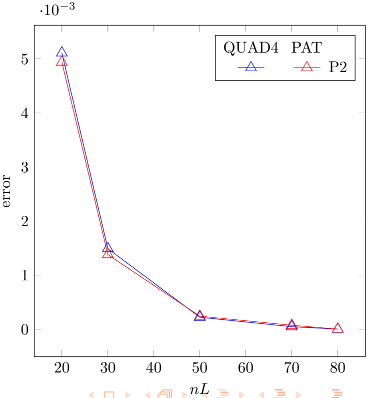
Figure: convergence of QUAD4 element

Study on Directional mesh density

$nL = \frac{L}{eL}$ $nB = \frac{B}{eB}$







Study on Mesh Density Skewness

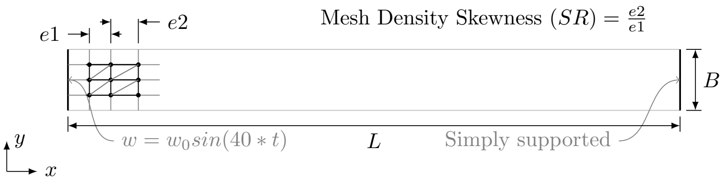


Figure: Mesh density skewness

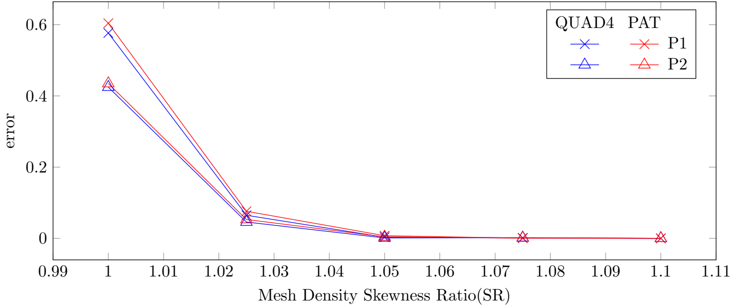
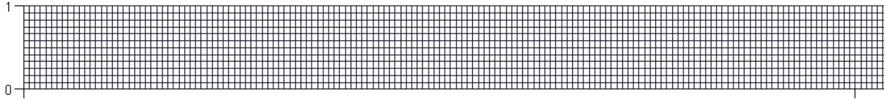
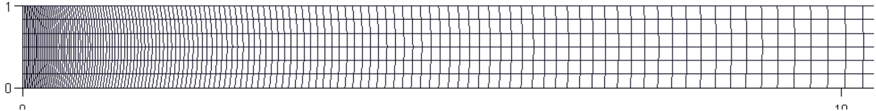


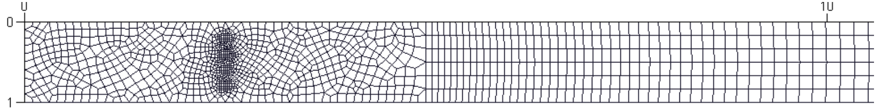
Figure: Comarison of PAT and MITC4 elements for different mesh density skewness



(a) Mesh : 4 , Nn = 7813, Solution Time = 929s

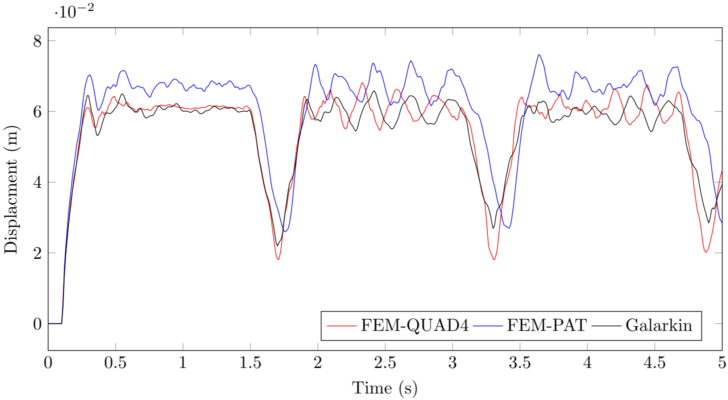


(b) Mesh : 2_3 , Nn = 1407, Solution Time = 13s

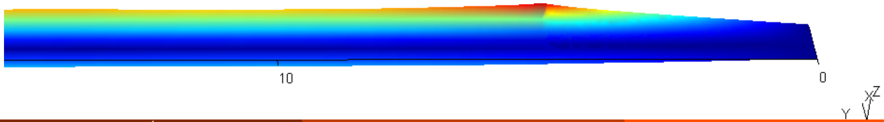


(c) Mesh : strip , Nn = 1886, Solution Time = 21s

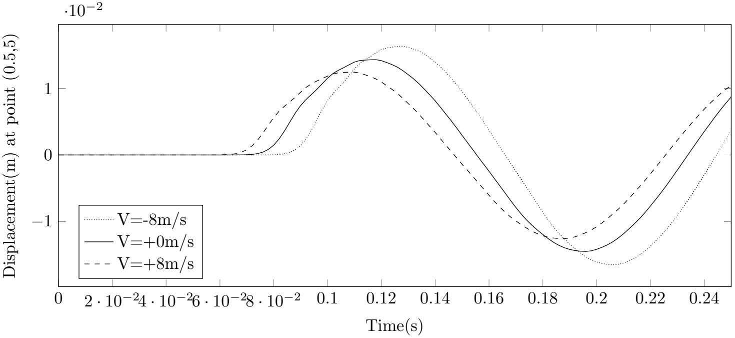
Comparison of FEM solution with Galarkin method



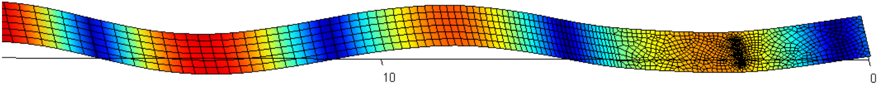
(d) Mesh : strip , Nn = 1886, Solution Time = 21s



Study with different axial velocity

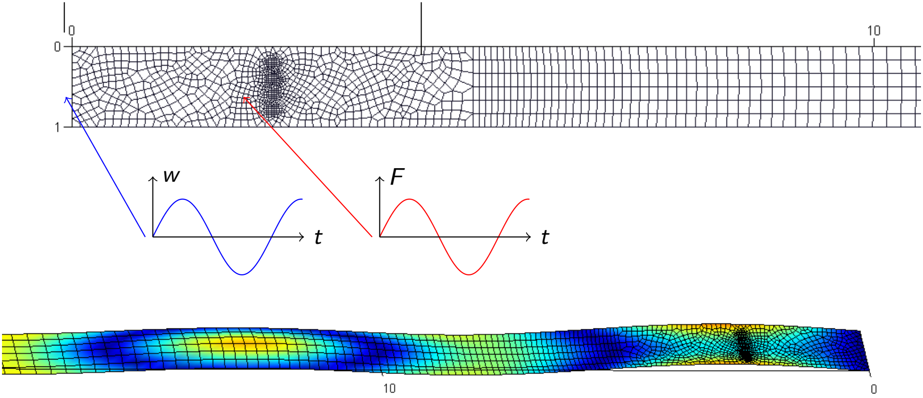


(e) Mesh : strip , Nn = 1886, Solution Time = 21s



Study with sinusoidal displacement and load

Along with previous loading, forces are applied at designated spots.

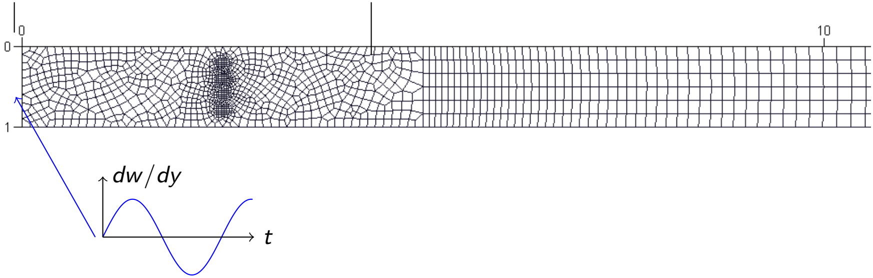


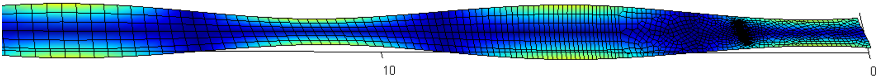
Analysis Statistics

nT=1000 , nN = 1886 , nE = 1836, nDOF = 5640, Solution Time =182 s

Study with twisting moment

Twisting deformation is applied at one end.

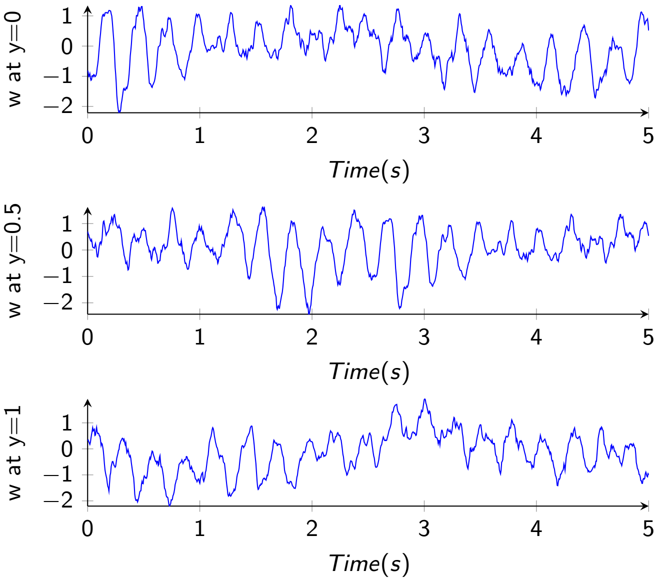


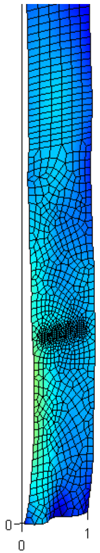


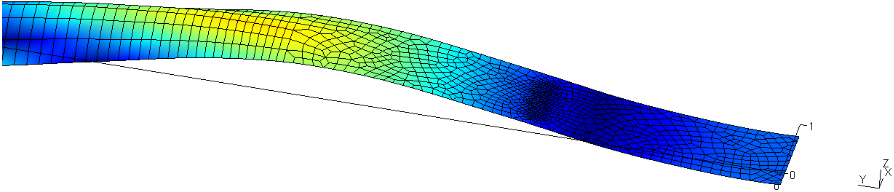
Analysis Statistics

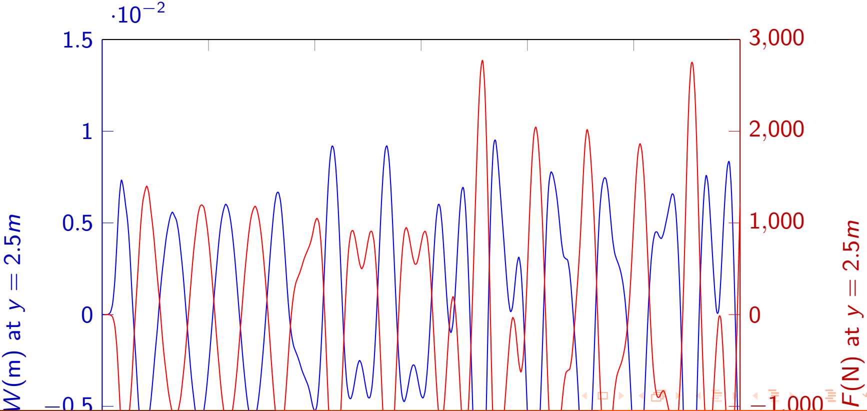
nT=200 , nN = 1886 , nE = 1836, nDOF = 5640, Solution Time = 39s

Strip with imposed displacement load using real data






1
0
0
Z
Y
X


$\cdot 10^{-2}$
1.5
1
0.5
0
−0.5
3,000
2,000
1,000
0
−1,000
W(m) at $y = 2.5m$
F(N) at $y = 2.5m$

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Conclusion

- In most of the cases the QUAD4 element performs better. Particular when there is point load and simply supported boundary condition.
- PAT element shows good convergence property for distributed load.
- Inclusion of axial velocity makes Coriolis and centripetal acceleration components appear in the equation which makes it gyroscopic.
- PAT element is unaffected by the low directional mesh density in direction perpendicular to the axial velocity.
- The element size has to be very small only on the boundary where the Dirichlet load is applied and also at the direction of the line speed.
- FEM program converges to the solution produced by the Galarkin method.
- Techniques like pre-factorization and Modal Superposition techniques drastically decreased the solution time.

Suggestions for future improvements

- Better Shape function(MITC family)
- Including non-linearity and Multi physics
- Including Contact between plate and rollers
- Better Linear algebra solver packages (LAPACK, cuBLAS ..)
- Modal order reduction techniques
- Creating a user friendly GUI

Thank you for your attention!!!