Vectors and Matrices

$X = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ $\quad y = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad z = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

1. $y \cdot z = 11$

2. $Xy = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 14 \\ 10 \end{pmatrix}$

3. $\det(X) = 2$, hence it is invertible

$$X^{-1} = \frac{\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}^T}{2} = \frac{1}{2}\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$$

4. since $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ are linearly independent, they span a vector space of dimension $= 2$.

$\therefore$ The rank of the matrix formed by these two column vectors is 2.

Calculus

1. $y = x^3 + x - 5$

$y' = 3x^2 + 1$

2. $f(x_1, x_2) = x_1 \sin(x_2)\, e^{-x_1}$

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{pmatrix} = \begin{pmatrix} \sin(x_2)\, e^{-x_1}(1 - x_1) \\ x_1 e^{-x_1} \cos(x_2) \end{pmatrix}$$

Probability and statistics

1. mean: $\frac{\Sigma x_i}{N} = \frac{3}{3}$

2. variance: $\frac{\Sigma (x_i - \bar{x})^2}{N} = \frac{\Sigma (x_i^2 + \bar{x}^2 - 2 x_i \bar{x})}{N}$

$\cancel{= \frac{1}{N}(\Sigma x_i^2 + N\bar{x}^2}$

$\cancel{= \frac{1}{N}(\Sigma x_i^2 -}$

$= \frac{1}{N}\left(\Sigma x_i^2 + N\bar{x}^2 - 2\bar{x}N\bar{x}\right)$ $\qquad \bar{x} = \frac{\Sigma x_i}{N}$

$= \cancel{\frac{1}{N}} \cdot \frac{\Sigma x_i^2}{N} - \bar{x}^2$

$= \left(\frac{3}{5}\right) - \left(\frac{3}{5}\right)^2$

$= \frac{3}{5} \cdot \frac{2}{5}$

$= \frac{6}{25}$

3. HHHTT.

$\left(\frac{1}{2}\right)^5 \frac{5!}{3!\,2!}$ ← assuming order does not matter

$\left(\frac{1}{2}\right)^5$ ← if order matters.

4. ~~assuming order matters, (eh some ... analysis~~

$P(p) = p^3(1-p)^2$

↳ probability for heads

maximize $p^3(1-p)^2$ for $p \in [0,1]$.

$P'(p) = \frac{d}{dp}\left(p^3 + p^5 - 2p^4\right) = 0$

$\Rightarrow 3p^2 + 5p^4 - 8p^3 = 0$

$\Rightarrow 5p^2 - 8p + 3 = 0$

$\Rightarrow (5p-3)(p-1) = 0$

$\boxed{p = \frac{3}{5}}$

5. $p(z=T \text{ and } y=b) = 0.1$

~~$p(z=T \text{ and } y=b) = \Sigma p(z=T) + \Sigma p(y=b)$~~

~~$= 0.5 + 0.25$~~

$$p(z = T \text{ give } y = b) = \frac{p(z = T \ \& \ y = b)}{\sum_z p(y = b)}$$

$$= \frac{0.1}{0.1 + 0.15}$$

$$= 0.4$$

Big O Notation

1. $f(n) = \ln(n)$, $g(n) = \lg(n)$

since they both only ~~of~~ differ by a factor, they are ~~of~~ the order of each other.

$$f(n) = \ln(n) = \frac{\lg(n)}{\lg(e)} = c\, g(n)$$

2. $f(n) = 3^n$, $g(n) = n^{100}$

for a large enough n,

$3^n \rightarrow$ ~~3x + 3x~~ $3 \times 3 \times 3 \ldots$ n times (~~exponential~~)

$n^{100} \rightarrow n \times n \times n \times \ldots$ 100 times. (

~~expo~~

$3^n > n^{100}$ for large enough n.

$\therefore g(n) = O(f(n))$

but $f(n) \neq O(g(n))$

3. $f(n) = 3^n$, $g(n) = 2^n$

clearly $g(n) = o(f(n))$.

$\nexists c \mid \forall k > k_0$ for some k_0,

$2^n > c\,3^n$.

↳ has to ~~that~~. a function of n.

4. $f(n) = o(n^2)$, $g(n) = O(n^3)$

$f(n) = o(g^n)$

~~TA~~ These answers can be proved ~~mor~~ with mathematical rigour but ~~I'm I'm just~~ -

Algorithms

$[0, 0, 0, \ldots \underbrace{0, 1}_{?}, 1, 1 \ldots, 1]$

1. found = False, position = 0. if $Ar[pos] = 1$, print ~~return~~ pos.

2. while found is false:

   position ~~th~~ += $len[Ar] * (\frac{1}{2})^{iteration}$.

   if $Ar[pos] = 1$:

   print pos,

   found = true

Since we are "halfing" our dataset ~~each~~ in each iteration, this $O(\ln(n))$

probability and random variables

i) $P(A \cup B) = P(A \cap (B \cap A^c))$ → 0.

False.

ii) $P(A \cup B) = P(A) - P(B) - P(A \cap B)$

True

iii) $P(A) = P(A \cap B) + P(A^c \cap B)$

False.

iv) $P(A|B) = P(B|A)$

False.

v) $P(A \cap B \cap C) = P(C|(A \cap B))\, P(B|A)\, P(A)$

$$= \frac{P(C \cap A \cap B)}{\cancel{P(A \cap B)}} \cdot \frac{\cancel{P(B \cap A)}}{\cancel{P(A)}} \cdot \cancel{P(A)}$$

True.

uniform → $\frac{1}{b-a}$ - - -

N.G → $\frac{1}{\sqrt{2 -}}$ exp - - -

Binomial → $\binom{n}{m} p$ - - -

Bernoulli → $p^n (1-p)^{1-n}$ - -

Mean, variance & Entropy

a) $var(x) = EX$

$= E[(x - EX)^2]$

$= E[x^2 + (Ex)^2 - 2(Ex)x]$

$= E(x^2) + (Ex)^2 - 2(Ex)(Ex)$

$= E(x^2) - (Ex)^2$

b) mean $\to \sum x\, p(x) = p.$ $\qquad p(1) = p$

var $\to \sum x^2 p(x) - (\sum x\, p(x))^2$ $\qquad p(0) = 1-p.$

$= p - p^2$

entropy $\to -\sum_p p \ln p$

$= -p \ln p - (1-p) \ln(1-p)$

Law of Large numbers

a) $\because p(3) = \frac{1}{6}$

in 6000 observations, we will see ~ 1000 3's.

~~probability~~

probability of seeing 1000 3's -

$$\left(\frac{1}{6}\right)^{1000} \frac{6000!}{1000!}$$

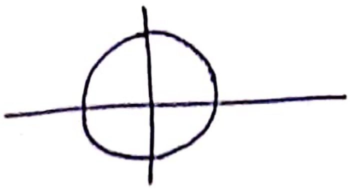
b) average no. of heads $= \bar{X} = \dfrac{n}{2}$

$$\bar{X} - \frac{1}{2} = \frac{1}{2}(n-1)$$

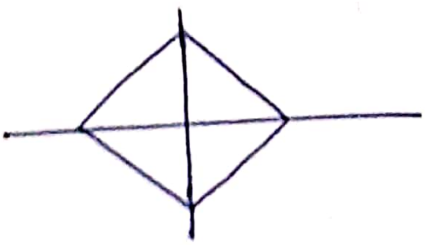
for large n, distribution will be gaussian.
with $E\cancel{\phi} = 0$, $\sigma = p(1-p) = \frac{1}{4}$.

L.A

a)



b)



c)

Geometry

~~a) w. (w...~~

a) $w \cdot (x_1 - x_2)$

$= w \cdot x_1 - w \cdot x_2$

$= -b + b$

$= 0$

$\therefore w \perp x_1 - x_2$

$\therefore$ w is $\perp$ to the line

b) $\perp$ distance $= \dfrac{|w \cdot (x - 0)|}{|w|} = \dfrac{|w \cdot x|}{|w|} = \dfrac{|b|}{|w|}$