Probability Theory and Statistics

CLASS 01

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1 Introduction

According to the *Encyclopaedia Britannica*, **probability theory** is "a branch of mathematics concerned with the analysis of random phenomena" [1]. This science is used to measure the likeliness that an event will occur. **Statistics** is defined by *The Oxford Dictionary of Statistical Terms* as "the study of the collection, analysis, interpretation, presentation and organization of data" [2].

A good example of probability and statistics as a tool is in election polls. Researchers use the statistics of the voting population (schooling, region and etc.) to know the probability of victory for each candidate, by taking a sample, generalizing it and calculating how similar it is to reality. Machine learing algorithms and insurance companies are also good examples of probability and statistics used to extract information.

This first class will approach combinatorial analysis and axioms of probability, mainly using [3] as reference (chapters 1 and 2). The intention is to be a concise and introductory explanation. If the reader desires to go deeper, further readings are recommended.

2 Combinatorial Analysis

A communication system consisting of a transmitter, four antennas (signal repeaters) and a receiver is organized in a straight line. The system is called *functional* as long as two consecutive antennas are not defective. If we have exactly m of n defective antennas, what is the probability that the resulting system will be functional? For the particular case where we have $n = 4$ and $m = 2$, there are 6 possible configurations, namely,

0 1 1 0
0 1 0 1
1 0 1 0
0 0 1 1
1 0 0 1
1 1 0 0

where 1 means *working* and 0, *defective*. As the resulting system is functional in the first 3 arrangements and not functional in the remaining 3, it is clear that the probability of have a functional system is $\frac{3}{6} = \frac{1}{2}$. So, if we generalize it to undefined m and n values, our probability problem reduces to a counting problem. The mathematical theory of counting is formally known as *combinatorial analysis*.

2.1 The Generalized Basic Principle of Counting

If r experiments that are to be performed are such that the first one may result in any of n_1 possible outcomes; and if, for each of these n_1 possible outcomes, there are n_2 possible outcomes of the second experiment; and if, for each of the possible outcomes of the first two experiments, there are n_3 possible outcomes of the third experiment; and if $\ldots$, then there is a total of $n_1 n_2 \cdots n_r = \prod_{i=1}^{r} n_i$ possible outcomes of the r experiments [3].

Example. How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?
Solution. By the generalized basic principle of counting, the answer is $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 175,760,000$.

2.2 Permutations

In mathematics, **permutation** means "rearranging the elements of a set in a sequence". The number of possible sequences is a function of the set's size. Given a set of n elements, we can *permute* it's elements according to the formula:

$$P_n = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1 = n!$$

As we are arranging the n elements in n positions, the first place can be chosen from n possibilities, the second from $(n-1)$ and so on, until the last with just one remaining option. We use the ! after a arbitrary number n to denote the **factorial** of n.

Example. In a group of 6 men and 4 women, how many possible arrangments can we have if ***(a)*** put all in the same group, and ***(b)*** divide by sex?
Solution. (a) We have 10 people to rearrange into 10 positions, so we just do $P_{10} = 10! = 3,628,800$.
(b) Using the **basic principle** we have two different experiments (rearranging men and rearranging women). So, our result is just $P_6 P_4 = 6!4! = 720 \cdot 24 = 17,280$ possibilities.

If our permutation do not preserve order (called **circular permutation**), the formulta is no longer $n!$ (simple permutation), but $(n-1)!$. Permuting abc circularly, the results abc, cab and bca are equal. The same to acb, bac and cba. For n elements, as the first n possibilities of the previous "first position" are the same, we do $\frac{P_n}{n} = \frac{n!}{n} = (n-1)!$.

2.3 k-permutations of n

2.4 Combinations

2.5 Multinomial Coefficients

2.6 The Number of Integer Solutions of Equations

3 Axioms of Probability

References

[1] "Probability theory, Encyclopaedia Britannica". Britannica.com. Retrieved 2014-10-11

[2] Dodge, Y. (2006) *The Oxford Dictionary of Statistical Terms*, OUP. ISBN 0-19-920613-9

[3] Ross, Sheldon M., 2010, "A First Course in Probability", 8th ed., Prentice Hall