Notation

- $E = \{e_1, \dots, e_n\}$: the set of individuals
- $\mathbf{x}_i = (x_{i1}, \dots, x_{ip})$ where $x_{ij} \in \Re \; (j = 1, \dots, p)$: description of individual e_i
- $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$: the data matrix
- $\mathcal{D} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$: interval-valued data set
- $\mathbf{w}_k = (w_{k1}, \dots, w_{kp})$ where $w_{kj} \in \Re \; (j = 1, \dots, p)$: description of the prototype of the cluster k $(k = 1, \dots, K)$
- $\mathbf{W} = (\mathbf{w}_1, \dots, \mathbf{w}_K)$: the matrix of prototypes
- $\mathcal{P} = (P_1, \dots, P_K)$: partition of E into K clusters
- $\mathbf{\Lambda} = [\lambda_{kj}]$: matrix of weights
- $\mathbf{u}_i = (u_{i1}, \dots, u_{ic}) \, (i = 1, \dots, n)$: the vector of membership degrees of individual e_i in the clusters
- $\mathbf{U} = (\mathbf{u}_1, \dots, \mathbf{u}_c)$: the matrix of membership degrees

Notation

- objective function

$$J = \sum_{k=1}^{K} \sum_{i=1}^{n} (u_{ik})^m \, \Delta_{\boldsymbol{\lambda}_k}(\mathbf{x}_i, \mathbf{w}_k)$$

$$u_{ik} \in [0, 1]; \; \sum_{k=1}^{K} u_{ik} = 1; \, m > 1$$

$$\Delta_{\boldsymbol{\lambda}_k}(\mathbf{x}_i, \mathbf{w}_k) = \sum_{j=1}^{p} \lambda_{kj}(x_{ij} - w_{kj})^2$$

$$\boldsymbol{\lambda}_k = (\lambda_{k1}, \ldots, \lambda_{kp}); \; \prod_{j=1}^{p} \lambda_{kj} = 1; \; \lambda_j > 0$$

$\boldsymbol{\Lambda} = [\lambda_{kj}]_{K \times p}$: matrix of weights

INPUT:

- the data set $\mathcal{D} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$;
- the data matrix $\mathbf{X}$
- the number K of clusters;
- T (maximum number of iterations); $\varepsilon << 0$;
- the parameter $m (m > 1)$;

OUTPUT:

- the matrix of prototypes $\mathbf{W}$;
- the matrix of relevance weights $\boldsymbol{\Lambda}$
- the matrix of membership degrees $\mathbf{U} = [u_{il}]$;

INITIALIZATION

- Set $t \leftarrow 0$;
- Randomly select K distinct prototypes $\mathbf{w}_k^{(t)} \in \mathcal{D}$ $(k = 1, \dots, K)$ to obtain the matrix of prototypes $\mathbf{W}^{(t)} = (\mathbf{w}_1^{(t)}, \dots, \mathbf{w}_K^{(t)})$;
  Set $\boldsymbol{\lambda}_k^{(t)} = (1, \dots, 1)\,(k = 1, \dots, K)$;
  Compute the membership degree of e_i dans le cluster k:
  $$u_{ik}^{(t)} = \left[\sum_{h=1}^{K} \left(\frac{\Delta_{\boldsymbol{\lambda}_k^{(t)}}(\mathbf{x}_i, \mathbf{w}_k^{(t)})}{\Delta_{\boldsymbol{\lambda}_h^{(t)}}(\mathbf{x}_i, \mathbf{w}_h^{(t)})} \right)^{\frac{1}{m-1}} \right]^{-1} = \left[\sum_{h=1}^{K} \left(\frac{\sum_{j=1}^{p} \lambda_{kj}^{(t)} (x_{ij} - w_{kj}^{(t)})^2}{\sum_{j=1}^{p} \lambda_{hj}^{(t)} (x_{ij} - w_{hj}^{(t)})^2} \right)^{\frac{1}{m-1}} \right]^{-1}$$
- Compute $J^{(t)} = \sum_{k=1}^{K} \sum_{i=1}^{n} (u_{ik}^{(t)})^m \Delta_{\boldsymbol{\lambda}_k^{(t)}}(\mathbf{x}_i, \mathbf{w}_k^{(t)})$

2) *Step 1: computation of the best prototypes.*
Set $t = t + 1$;
$\mathbf{\Lambda}^{(t-1)}$ and $\mathbf{U}^{(t-1)}$ are kept fixed.
Compute the prototype $\mathbf{w}_k^{(t)} = (w_{k1}^{(t)}, \ldots, w_{kp}^{(t)})$ according to:

$$w_{kj}^{(t)} = \frac{\sum_{i=1}^{n} \left(u_{ik}^{(t-1)}\right)^m x_{ij}}{\sum_{i=1}^{n} \left(u_{ik}^{(t-1)}\right)^m}$$

3) Step 2: Weighting

$\mathbf{W}^{(t)}$ and $\mathbf{U}^{(t-1)}$ are kept fixed

Compute the vector of relevance weight of the variable j into cluster k, $\boldsymbol{\lambda}_k^{(t)} = (\lambda_{k1}^{(t)}, \ldots, \lambda_{kp}^{(t)})$ as

$$\lambda_{kj}^{(t)} = \frac{\left\{\prod_{h=1}^{p}\left[\sum_{i=1}^{n}(u_{ik}^{(t-1)})^m \, (x_{ih} - w_{kh}^{(t)})^2\right]\right\}^{\frac{1}{p}}}{\sum_{i=1}^{n}(u_{ik}^{(t-1)})^m \, (x_{ij} - w_{kj}^{(t)})^2}$$

4) Step 3: Allocation

$\mathbf{W}^{(t)}$ and $\boldsymbol{\Lambda}^{(t)}$ are kept fixed

For $i = 1$ to n do compute $\mathbf{u}_i^{(t)} = (u_{i1}^{(t)}, \ldots, u_{il}^{(t)}, \ldots, u_{iK}^{(t)})$

$$u_{ik}^{(t)} = \left[\sum_{h=1}^{K} \left(\frac{\Delta_{\boldsymbol{\lambda}_k^{(t)}}(\mathbf{x}_i, \mathbf{w}_k^{(t)})}{\Delta_{\boldsymbol{\lambda}_h^{(t)}}(\mathbf{x}_i, \mathbf{w}_h^{(t)})} \right)^{\frac{1}{m-1}} \right]^{-1} =$$

$$\left[\sum_{h=1}^{K} \left(\frac{\sum_{j=1}^{p} \lambda_{kj}^{(t)} (x_{ij} - w_{kj}^{(t)})^2}{\sum_{j=1}^{p} \lambda_{hj}^{(t)} (x_{ij} - w_{hj}^{(t)})^2} \right)^{\frac{1}{m-1}} \right]^{-1}$$

- Compute $J^{(t)} = \sum_{k=1}^{K} \sum_{i=1}^{n} (u_{ik}^{(t)})^m \Delta_{\boldsymbol{\lambda}_k^{(t)}}(\mathbf{x}_i, \mathbf{w}_k^{(t)})$

5) *Stopping criterion.*
If $|J^{(t)} - J^{(t-1)}| < \varepsilon$ or $t > T$ then STOP; otherwise go to 2 (Step 1).