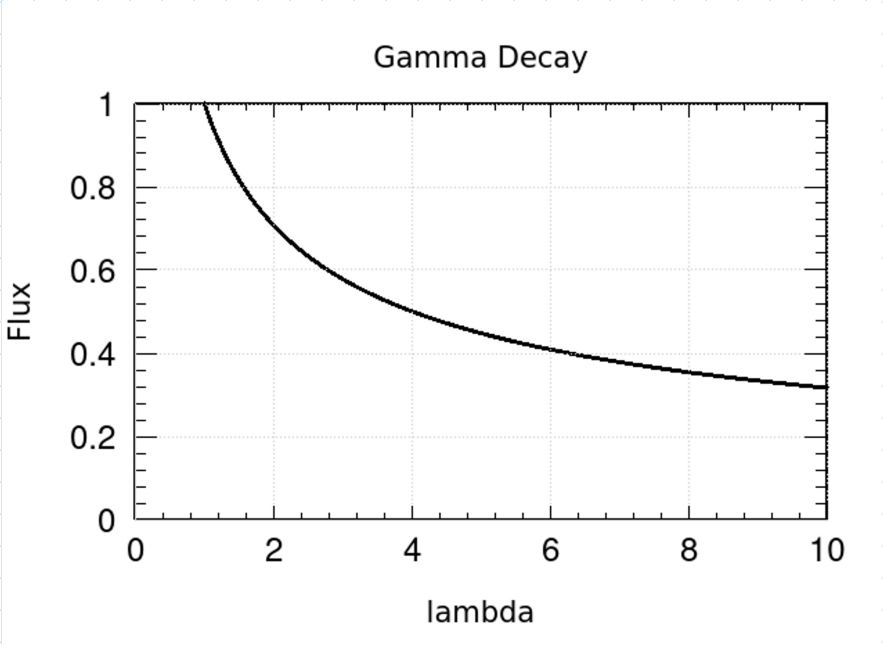
Biermann CompPhys HW8

Friday, October 16, 2020 6:52 PM

7.8] I had a hard time getting these ones to work but I uploaded my code.

7.9]



7.15]

(a) $\sigma = \sqrt{\frac{\bar{h}^2 - (\bar{h})^2}{N-1}} \approx \sqrt{\frac{\bar{h}^2 - (\bar{h})^2}{N}}$ for large N

$$I = \int f(x)\,dx = \int w(x)\frac{f(x)}{w(x)}\,dx, \quad \frac{f(x)}{w(x)} = h(x)$$

Uniform: $w(x) = 1$

$$\langle h \rangle = \int_0^1 \frac{e^x - 1}{e-1}\,dx = \frac{e-2}{e-1}$$

$$\langle h^2 \rangle = \int_0^1 \frac{1}{(e-1)^2}(e^x - 1)^2\,dx$$

$$\langle h^2 \rangle = \int_0^1 \frac{1}{(e-1)^2}(e^x-1)\,dx$$

$$= \int_0^1 \frac{1}{(e-1)^2}(e^{2x}-2e^x+1)\,dx$$

$$= \frac{e^2-e+1}{2(e-1)^2}$$

$$\Rightarrow \sigma^2 = \frac{1}{N}\left(\frac{e^2-e+1}{2(e-1)^2} - \frac{2(e-2)^2}{2(e-1)^2}\right)$$

$$= \frac{1}{N}\left(\frac{-e^2+7e-7}{2(e-1)^2}\right)$$

$$\Rightarrow \sigma = \frac{0.78}{\sqrt{N}}$$

Sample size to achieve 1% accuracy:

$$\frac{0.286}{\sqrt{N}} = 0.01\left(\frac{e-2}{e-1}\right)$$

$$\Rightarrow N = \left(\frac{0.286}{0.01}\left(\frac{e-1}{e-2}\right)\right)^2 \quad \text{(to nearest integer, rounding up)}$$

$$\boxed{N = 4681}$$

(b) $$h(x) = \frac{f(x)}{w(x)} = \frac{1}{x}\,\frac{e^x-1}{e-1}$$

$$\langle h \rangle = \frac{1}{e-1}\int_0^1 \frac{e^x-1}{x}\,dx = 0.767 \quad \text{(wolfram alpha)}$$

$$\langle h^2 \rangle = \frac{1}{(e-1)^2}\int_0^1 \frac{(e^x-1)^2}{x^2}\,dx = 0.603 \quad \text{(wolfram alpha)}$$

$$\sigma^2 = \frac{(0.603) - (0.767)^2}{N}$$

$$\Rightarrow \quad \sigma = \frac{0.12}{\sqrt{N}}$$

(c) $f = \Pi_i f(x_i)$

$$I = \int \Pi_i f(x_i)\, dx_i$$

$$\overline{h} = \int \Pi_i \frac{f(x_i)}{x_i}\, dx_i$$

$$(\overline{h^2}) = \int \Pi_i \frac{f^2(x_i)}{x_i^2}\, dx_i$$

$$\sigma^2 = \frac{1}{N}\left(\int \left(\Pi_i \frac{f^2(x_i)}{x_i^2} - \Pi_i \frac{f(x_i)}{x_i} \right) dx_i \right)$$