

DS-GA.3001

Embodied Learning and Vision

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embodied-learning-vision-course.github.io



Lecture Slides for Note Taking



Multi-Sensor Fusion

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- Camera provides high resolution 2D view and good for long distance but lacks 3D. Can we achieve the best of both worlds?

Multi-Sensor Fusion

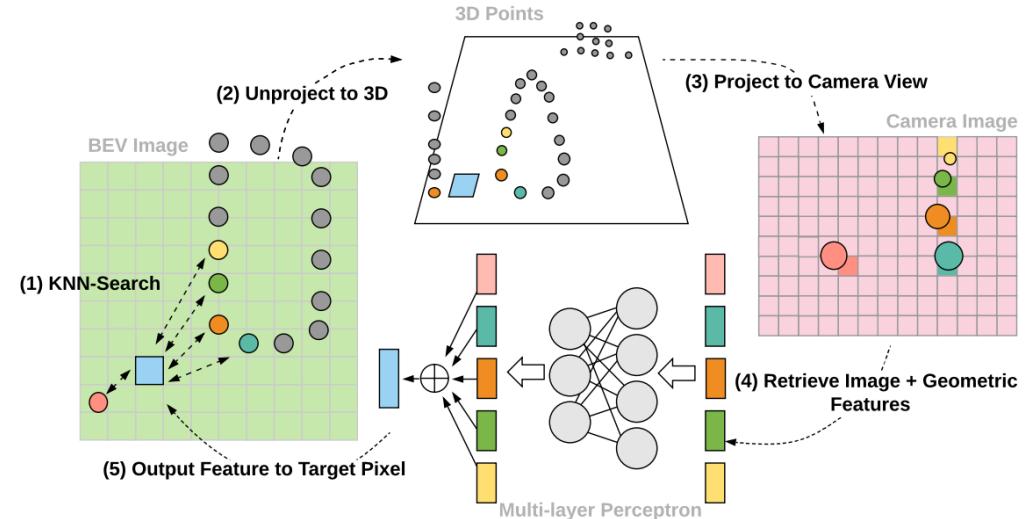
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- Late fusion: Generate proposals from one branch (e.g. LiDAR) and refine (e.g. using Camera).

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- Late fusion: Generate proposals from one branch (e.g. LiDAR) and refine (e.g. using Camera).
- Is there a way to combine the features from both modality in lower layers?

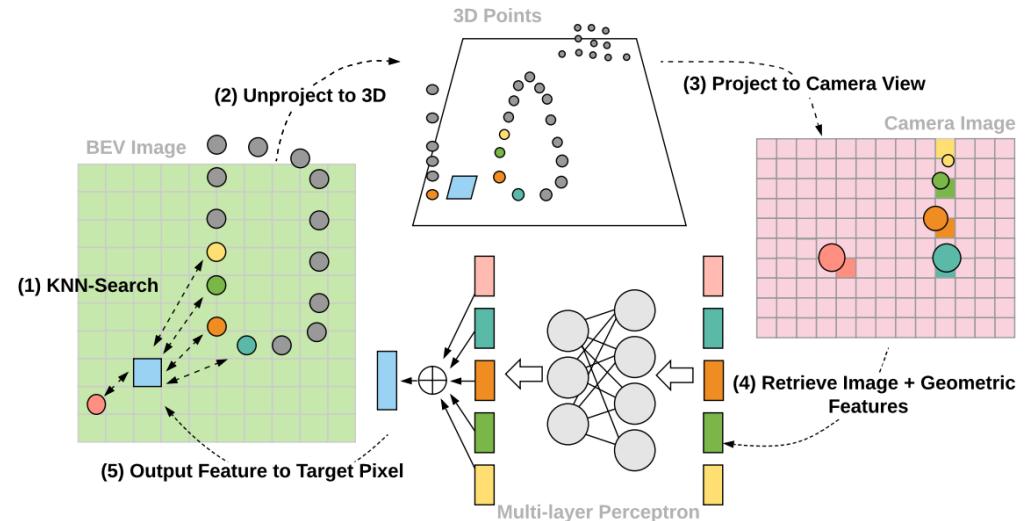
Camera-LiDAR Projection

- Unproject LiDAR points to camera view (i.e. Range View)



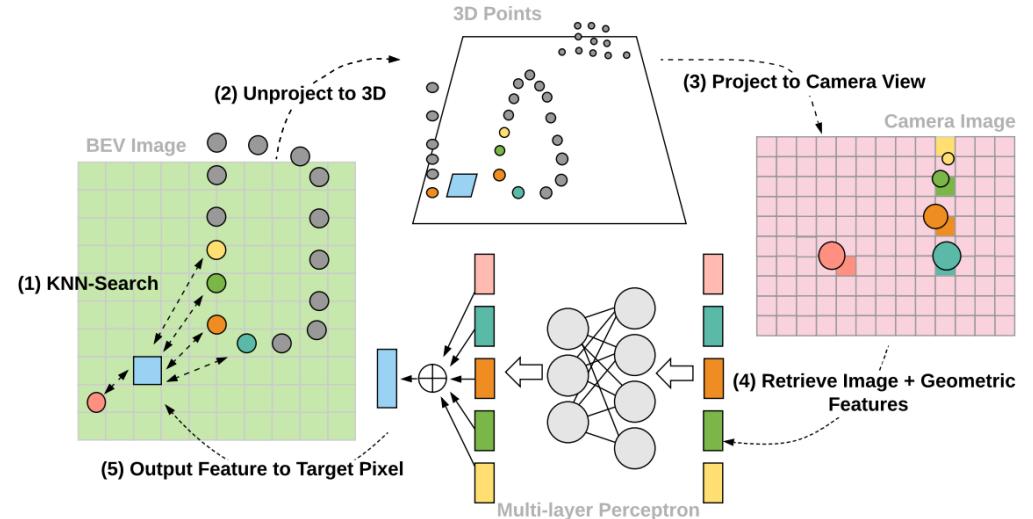
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Camera-LiDAR Projection

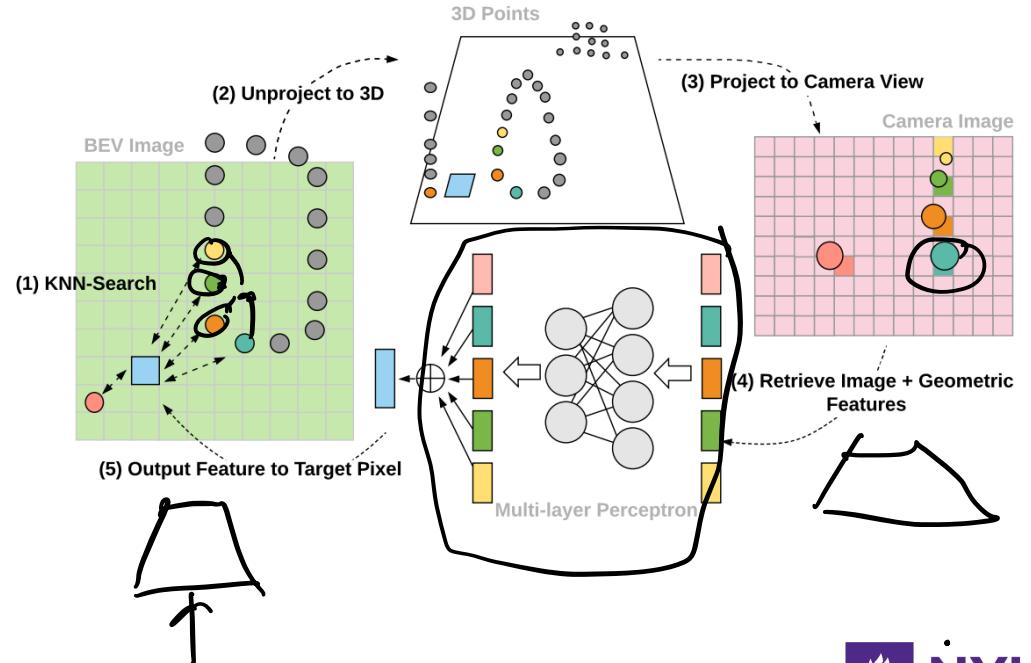
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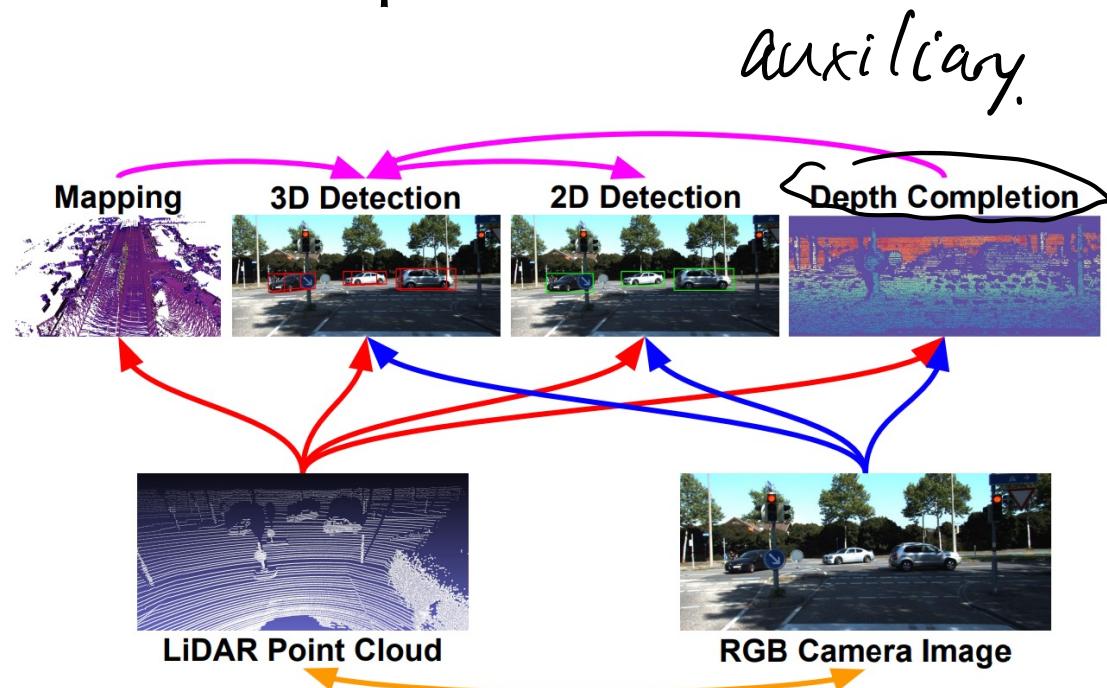
- Unproject LiDAR points to camera view (i.e. Range View)
- Query the closest camera RGB features for each LiDAR point.
- For empty space in BEV, we can interpolate from neighboring points using kNN.
- Continuous Fusion: $h_i = \sum_j \text{MLP}([f_j, x_j - x_i])$.

kernel



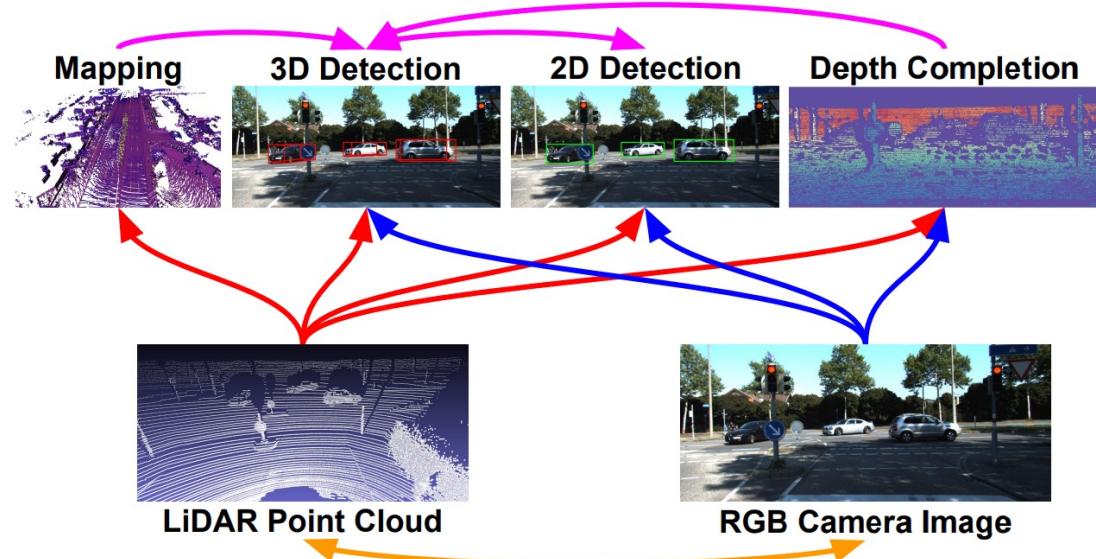
Supervised Dense Depth

- Drawback of continuous fusion: Sparse LiDAR can cause the fusion process to be less accurate. Relies on kNN.



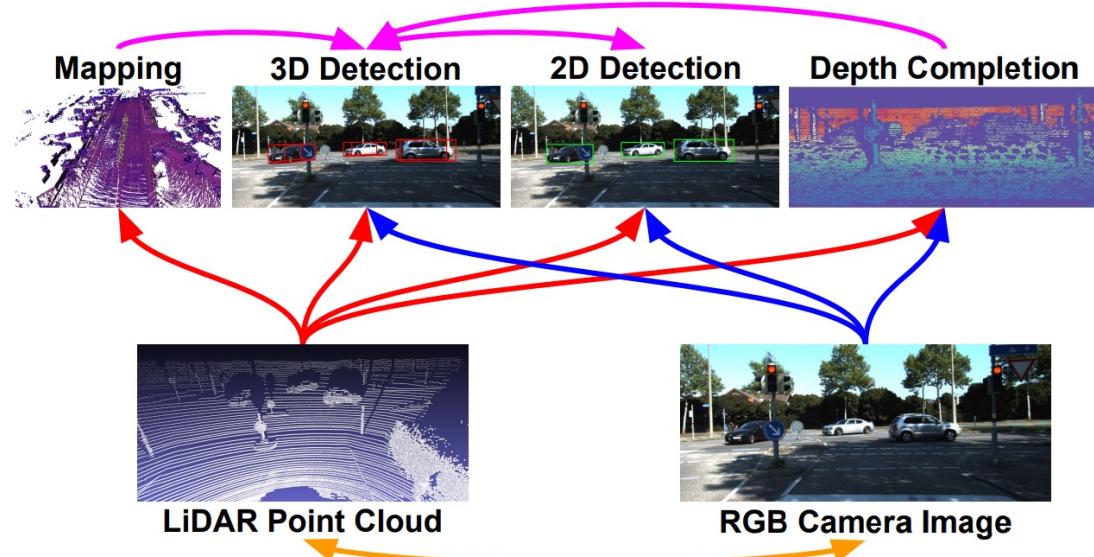
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Supervised Dense Depth

- Drawback of continuous fusion: Sparse LiDAR can cause the fusion process to be less accurate. Relies on kNN.
- Why not predict a dense depth to pair with the camera image?
- Depth completion module is supervised by sparse LiDAR and is used for dense fusion.



3D Perception

- With the ease of use of automatic differentiation libraries, we can compose a computation graph in millions of ways.

3D Perception

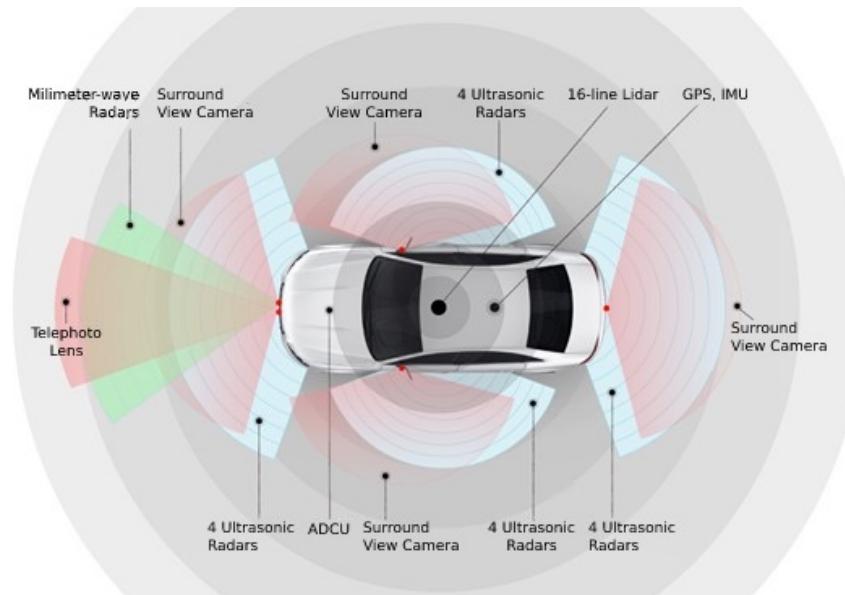
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- We can design layers and operators to accomodate different types of inputs and outputs. 3D, point cloud, sparse data, etc.

3D Perception

- With the ease of use of automatic differentiation libraries, we can compose a computation graph in millions of ways.
- We can design layers and operators to accomodate different types of inputs and outputs. 3D, point cloud, sparse data, etc.
- We can fuse different modalities together too, by leveraging geometric relationships.

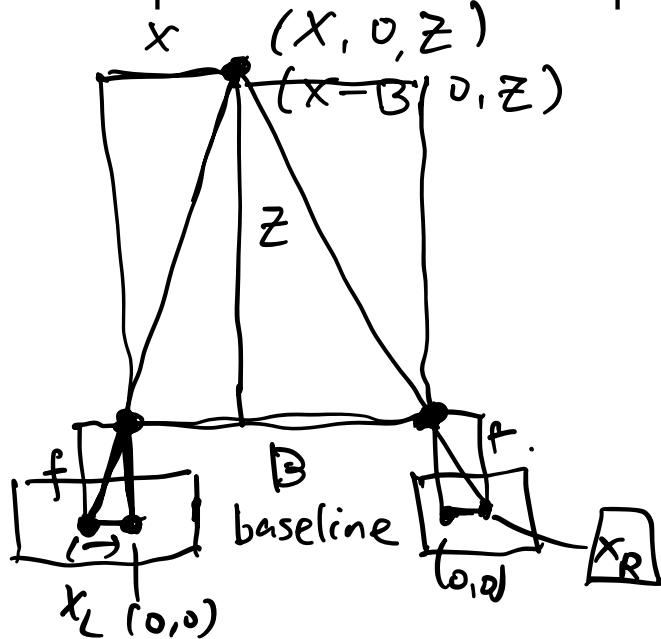
2D to 3D

- Not all embodied agents have the luxury to have a full set of sensors.
- Can we infer the geometric structure with 2D perception?



Classic Vision on Depth and Disparity

- One source of depth is from the displacement of pixels in a stereo setup.



$$\delta = x_L - x_R$$

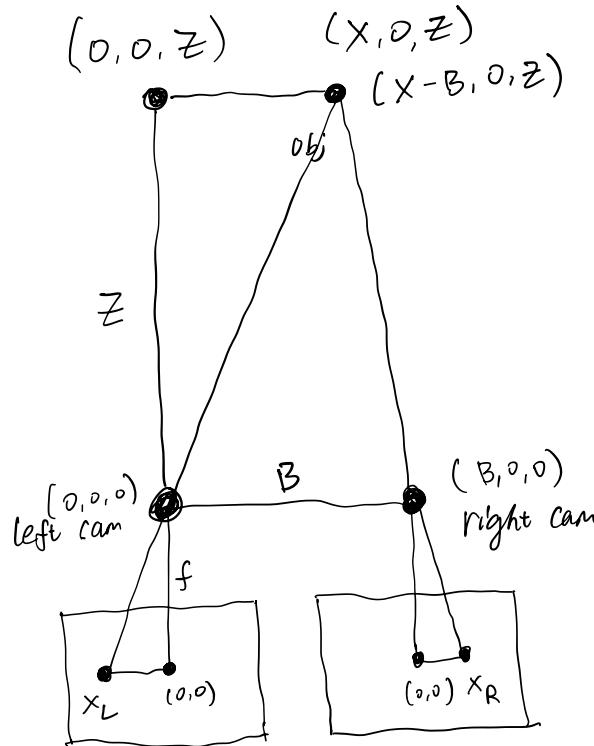
$$x_L = \frac{xf}{z}$$

$$x_R = \frac{(x-B)f}{z}$$

$$\delta = x_L - x_R = \frac{Bf}{z}$$

Classic Vision on Depth and Disparity

- One source of depth is from the displacement of pixels in a stereo setup.
- But we need to estimate disparity.



$$x_L = \frac{xf}{Z}$$

$$x_R = \frac{(x-B)f}{Z}$$

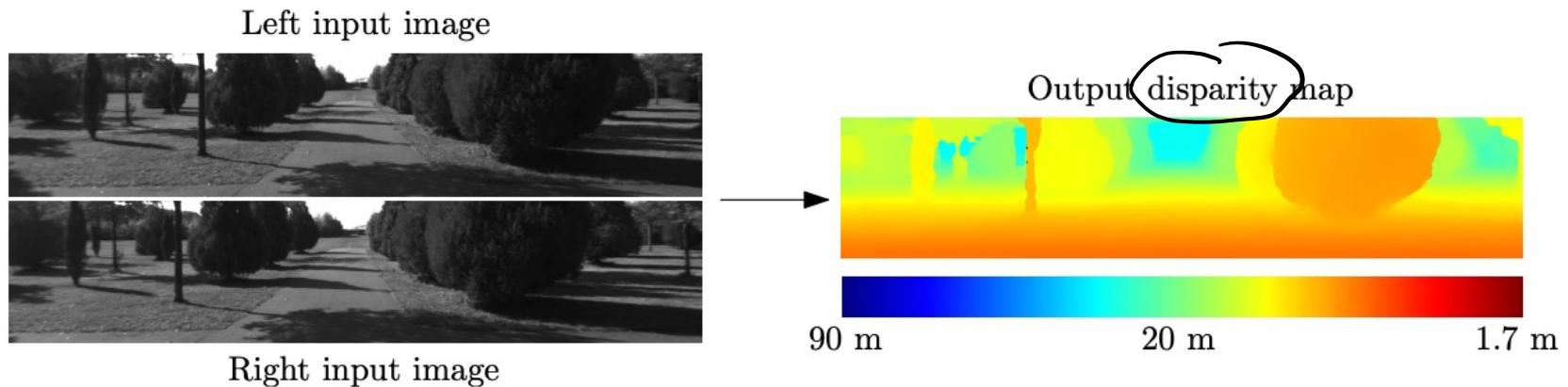
$$\begin{aligned}\delta &= x_L - x_R \\ &= \frac{[x - (x-B)]f}{Z}\end{aligned}$$

$$= \frac{Bf}{Z}$$

$$\boxed{Z = \frac{Bf}{\delta}}$$

From 2D to 3D: Depth Network

- A network that can output disparity.
- Using LiDAR or depth camera as groundtruth supervision.

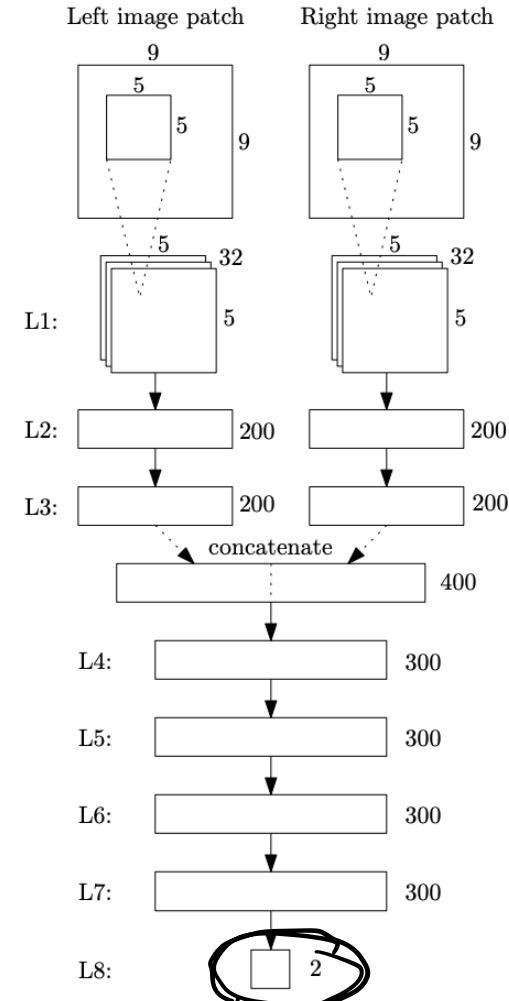


The Energy-Based Approach

- The energy penalize matching with high cost (unary), and when neighboring pixels have disparity differences greater or equal to one (pairwise).
- Cost network: Train with binary classification

Energy $E(D) = \sum_{\mathbf{p}} \left(C_{\text{CBCA}}^4(\mathbf{p}, D(\mathbf{p})) + \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_1 \times 1\{|D(\mathbf{p}) - D(\mathbf{q})| = 1\} + \sum_{\mathbf{q} \in \mathcal{N}_{\mathbf{p}}} P_2 \times 1\{|D(\mathbf{p}) - D(\mathbf{q})| > 1\} \right)$,
Smoothness

$$D(\mathbf{p}) = \underbrace{\arg\min_d C(\mathbf{p}, d)}_{\text{displacement prediction}} \quad d^*$$

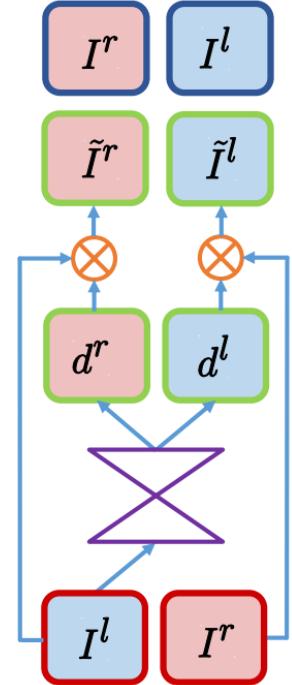
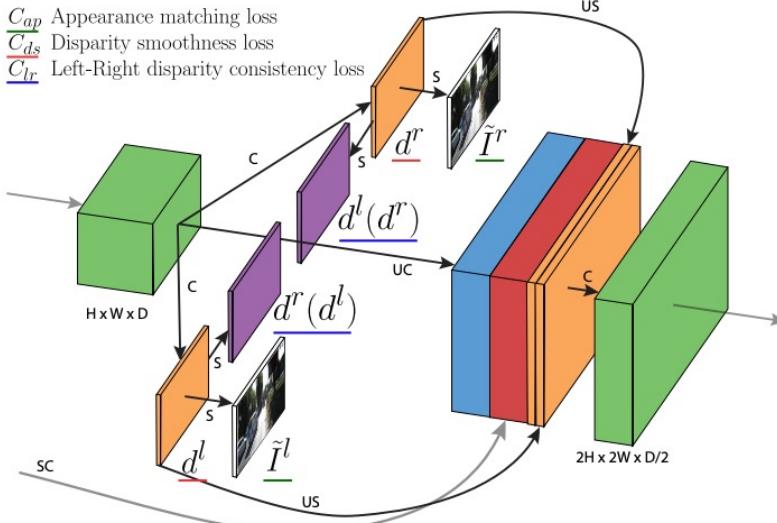


Self-Supervised Depth

- Appearance matching loss

$$C_{ap}^l = \frac{1}{N} \sum_{i,j} \underbrace{\alpha \frac{1 - \text{SSIM}(I_{ij}^l, \tilde{I}_{ij}^l)}{2}}_{\text{SSIM}} + (1-\alpha) \underbrace{\|I_{ij}^l - \tilde{I}_{ij}^l\|}_{\text{pixel}}$$

warped.



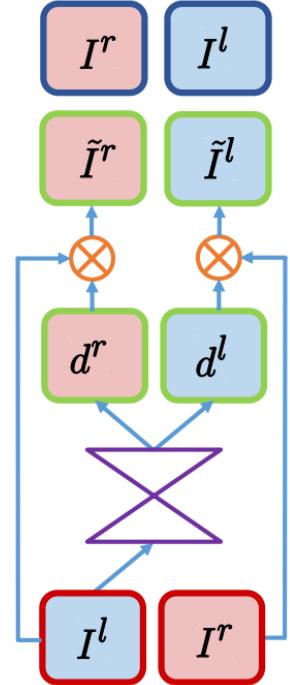
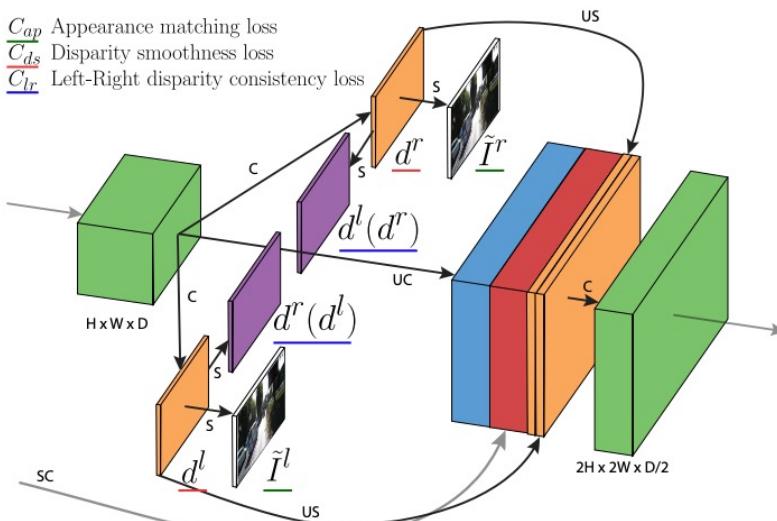
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- Disparity smoothness loss

$$C_{ds}^l = \frac{1}{N} \sum_{i,j} |\partial_x d_{ij}^l| e^{-\|\partial_x I_{ij}^l\|} + |\partial_y d_{ij}^l| e^{-\|\partial_y I_{ij}^l\|}.$$



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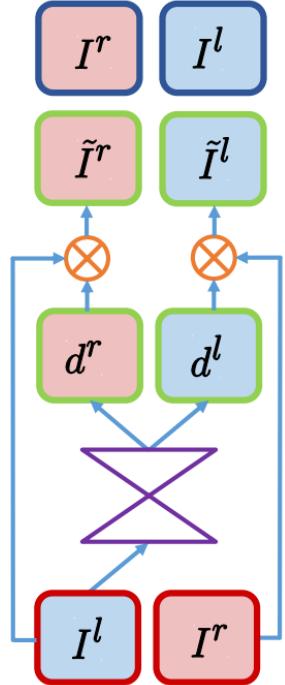
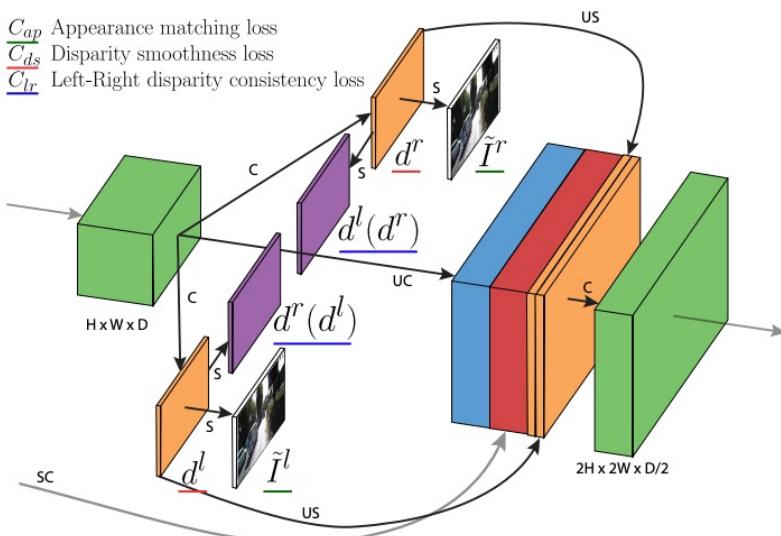
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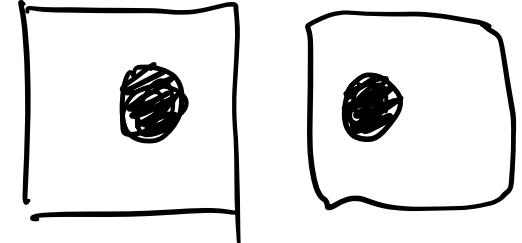
- Left-right disparity consistency loss

$$C_{lr}^l = \frac{1}{N} \sum_{i,j} |d_{ij}^l - d_{ij}^r + d_{ij}^l|.$$

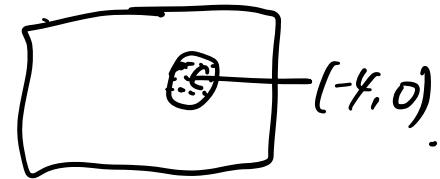
Godard et al. Unsupervised Monocular Depth Estimation with Left-Right Consistency. CVPR 2017.
 Wang et al. Image Quality Assessment: From Error Visibility to Structural Similarity. TIP 2004.



Motion, Optical Flow



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$$\begin{aligned} I_x &= \frac{\partial I}{\partial x} & \frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y + \frac{\partial I}{\partial t} \Delta t &= 0. \\ I_y &= \frac{\partial I}{\partial y} \end{aligned}$$

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$$I_y = \frac{\partial I}{\partial y}$$

$$I_x u + I_y v + I_t = 0.$$

x direction y direction

Classical Approach

- Under-constrained system

$$I_x u + I_y v + I_t = 0.$$

Classical Approach

2 unk.

- Under-constrained system
- Use a local patch and assume smooth motion

$$\boxed{I_x u + I_y v + I_t = 0.}$$

$$\mathbf{A}\mathbf{u} = \mathbf{b}$$

$$\begin{pmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{N^2}) & I_y(\mathbf{p}_{N^2}) \end{pmatrix} \begin{pmatrix} u \\ v \\ \text{unknown} \end{pmatrix} = - \begin{pmatrix} I_t(\mathbf{p}_1) \\ \vdots \\ I_t(\mathbf{p}_{N^2}) \end{pmatrix}$$

3x3.

Classical Approach

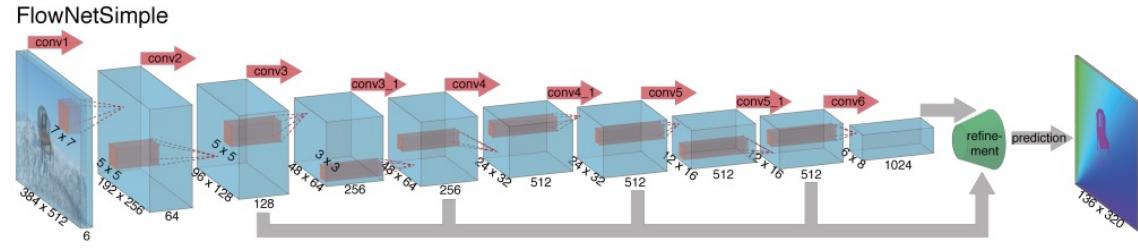
- Under-constrained system $I_x u + I_y v + I_t = 0.$
- Use a local patch and assume smooth motion
- Rigid, contains many assumptions

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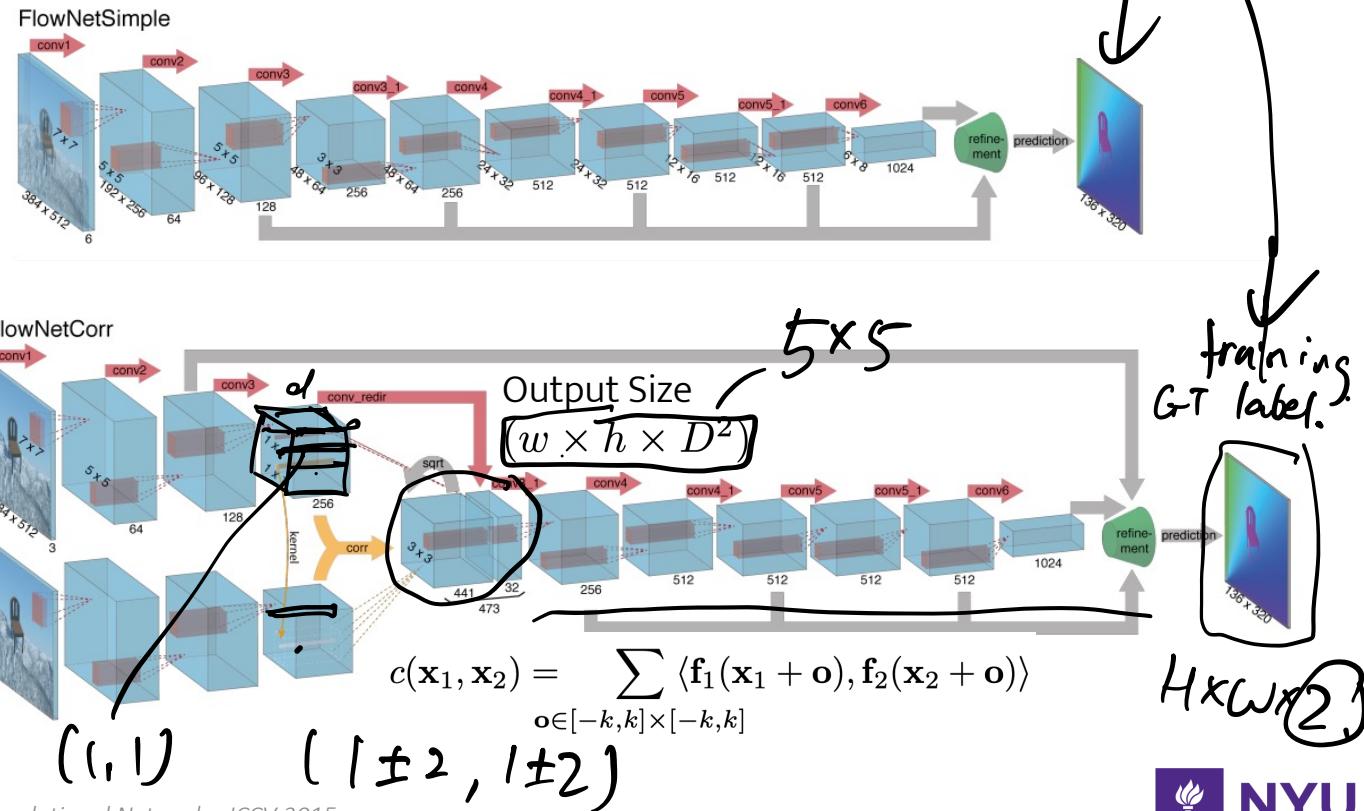
Correlation Volume Approach

- Simple Approach:
Concatenate the
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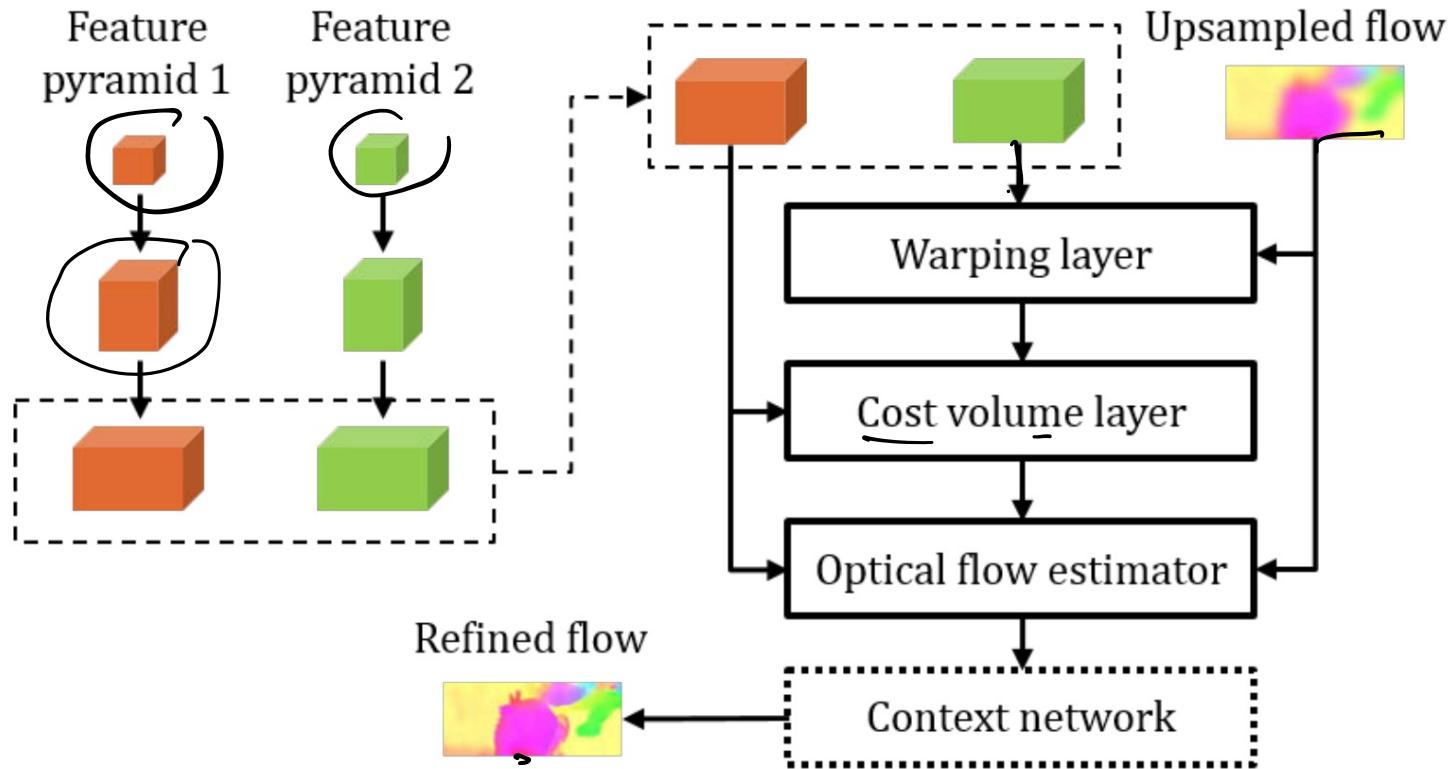


Correlation Volume Approach

- Simple Approach:
Concatenate the two images together.
- Correlation:
Extract some levels of features, and convolve one feature on top of another.



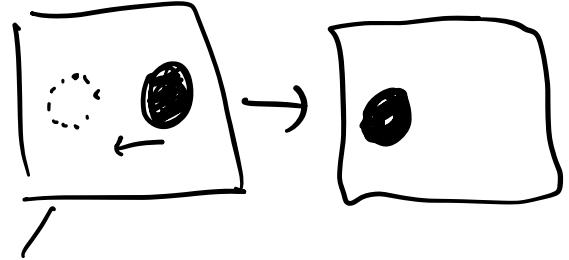
Iterative Refining through Feature Pyramid



Unsupervised Flow

- Photometric Consistency (Appearance)

Unsupervised Flow



- ✓
- Photometric Consistency (Appearance)
- Occlusion Estimation
 - Forward-backward consistency

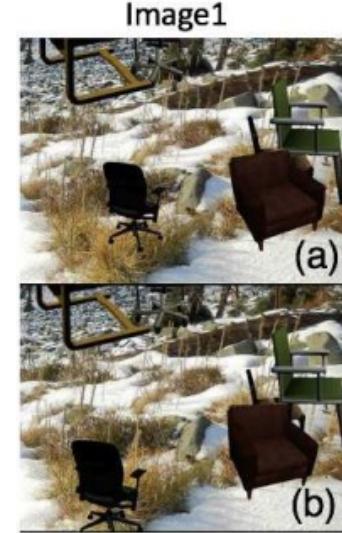
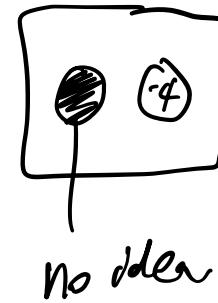


Image2

Wang et al., 2018

Unsupervised Flow

- Photometric Consistency (Appearance)
- Occlusion Estimation
 - Forward-backward consistency
- Smoothness

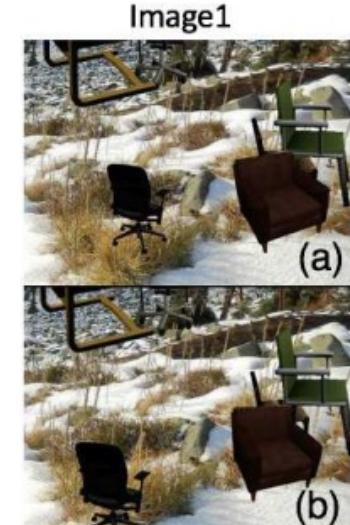


Image1

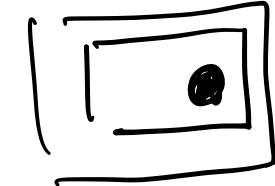
(a)

(b)

Image2

Wang et al., 2018

Unsupervised Flow



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- Occlusion Estimation
 - Forward-backward consistency
- Smoothness
- Self-supervision: Ensure consistent flow at different augmentation (e.g. crops)

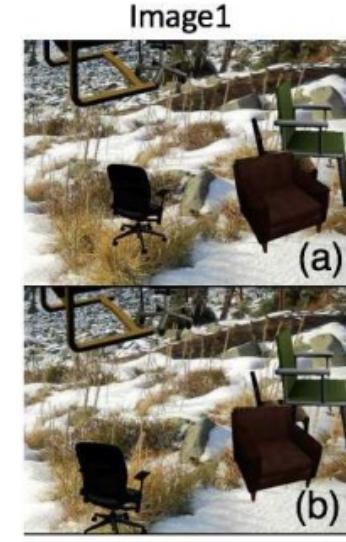
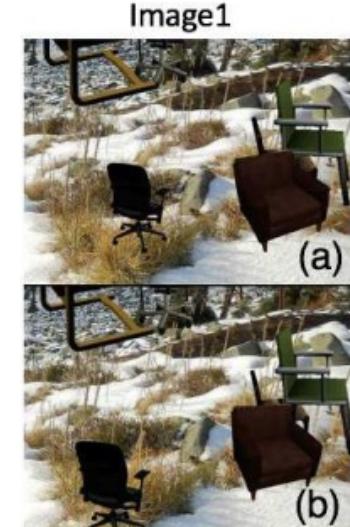


Image2

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Unsupervised Flow

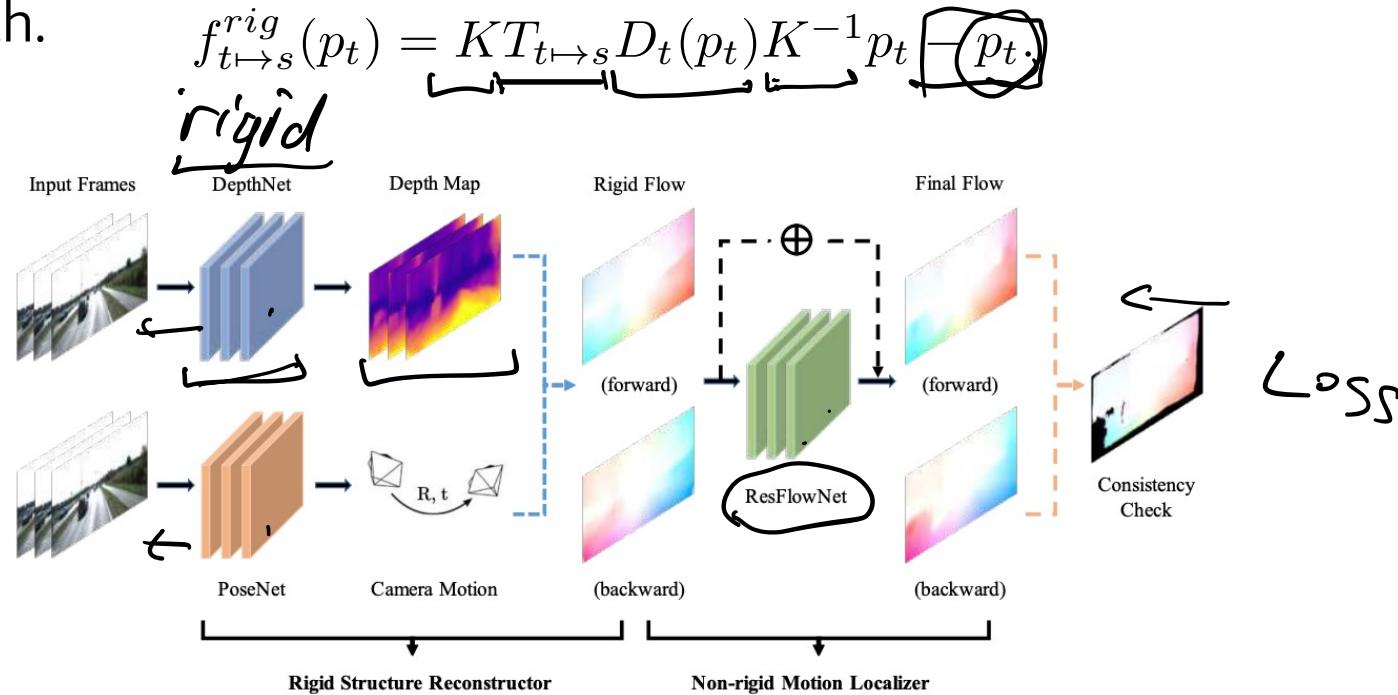
- Photometric Consistency (Appearance)
- Occlusion Estimation
 - Forward-backward consistency
- Smoothness
- Self-supervision: Ensure consistent flow at different augmentation (e.g. crops)
- Can 3D information help us reason about motion?



Wang et al., 2018

Depth, Flow, and Pose Movement

- The static objects follow rigid flow: determined by camera motion and depth.



Training Losses

- Appearance Loss (Warping):

$$\mathcal{L}_{rw} = \alpha \frac{1 - SSIM(I_t, \tilde{I}_s^{rig})}{2} + (1 - \alpha) \|I_t - \tilde{I}_s^{rig}\|_1.$$

$$\mathcal{L}_{fw} = \alpha \frac{1 - SSIM(I_t, \tilde{I}_s^{full})}{2} + (1 - \alpha) \|I_t - \tilde{I}_s^{full}\|_1.$$

SSIM . pixel

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$$\mathcal{L} = \sum_{p_t} |\underbrace{\nabla D(p_t)}_{\cdot} \cdot (\exp(\underbrace{-|\nabla I(p(t))|}_{})^T.$$

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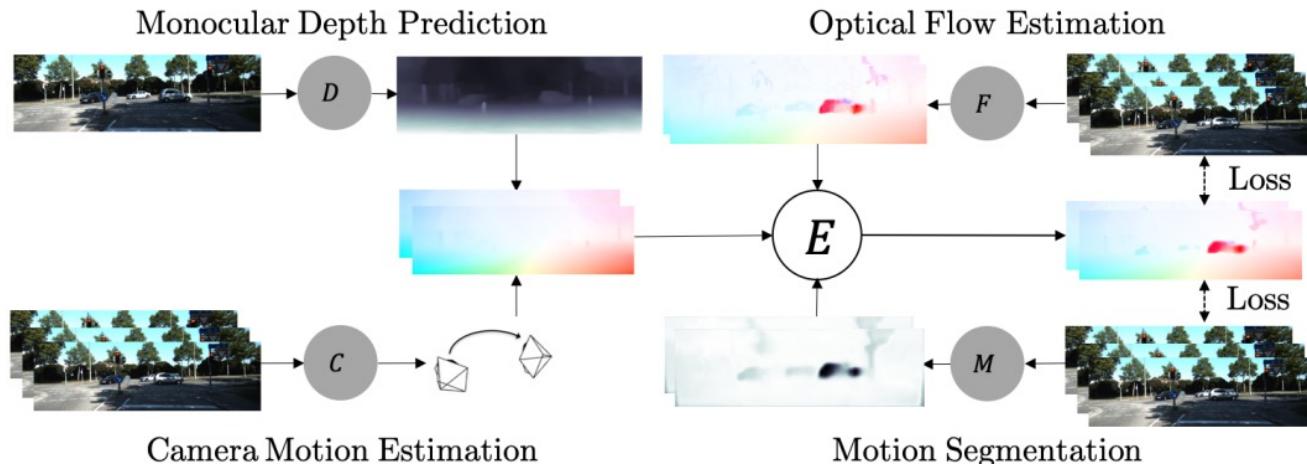
- Forward-Backward Consistency:

$$\mathcal{L} = \sum_{p_t} [\delta(p_t)] \|\Delta f_{t \rightarrow s}^{full}(p_t)\|_1.$$

$$\delta(p_t) = \|f_{t \rightarrow s}^{full}(p_t)\|_2 \max\{\underline{\alpha}, \underline{\beta} \|f_{t \rightarrow s}^{full}(p_t)\|_2\}.$$

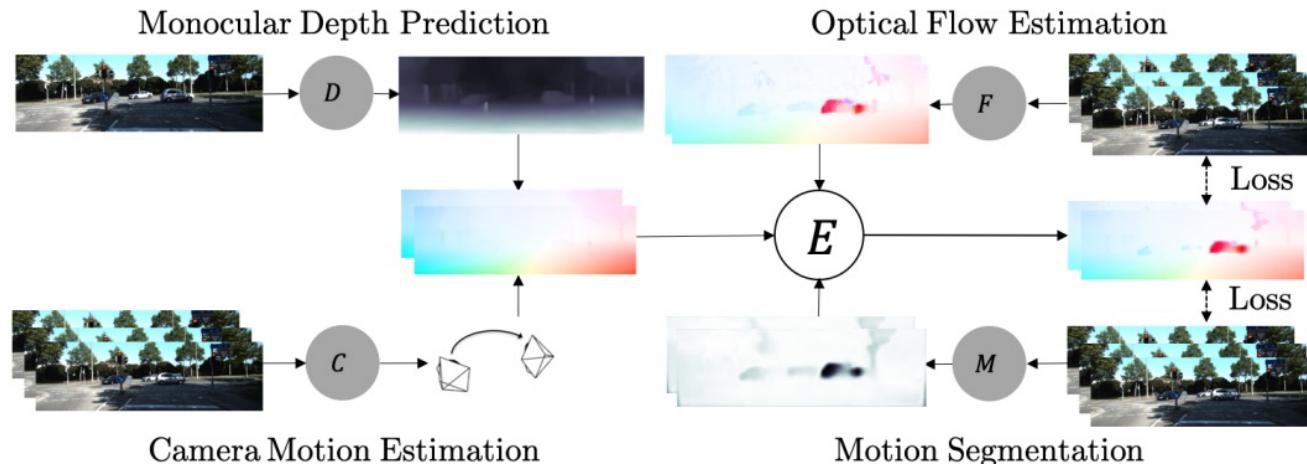
Summary

- Leverage cross correlation structure for spatial similarity matching.



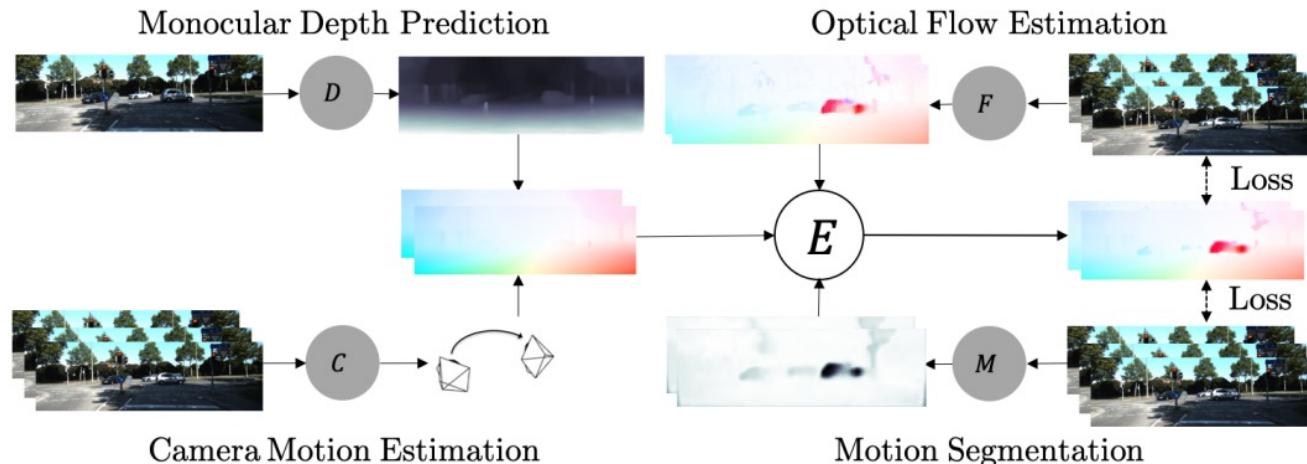
Summary

- Leverage cross correlation structure for spatial similarity matching.
- Can be used towards: depth, flow, and pose prediction.



Summary

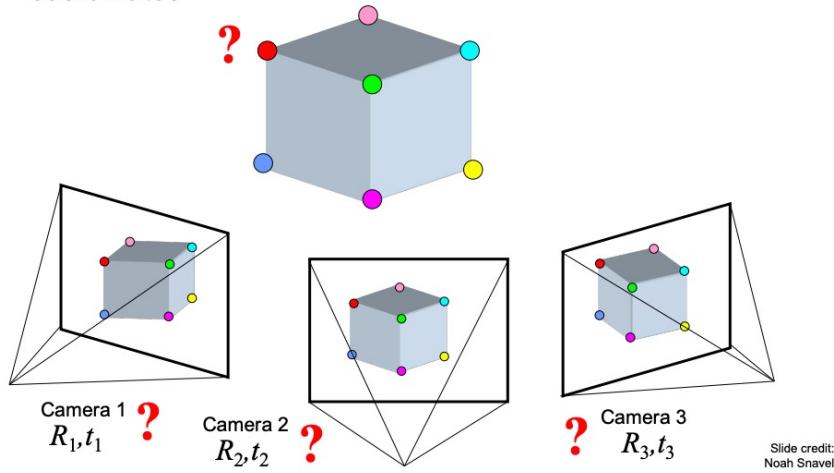
- Leverage cross correlation structure for spatial similarity matching.
- Can be used towards: depth, flow, and pose prediction.
- Can form triangulation for self-supervision.



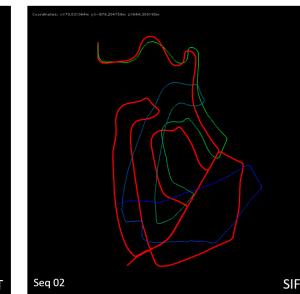
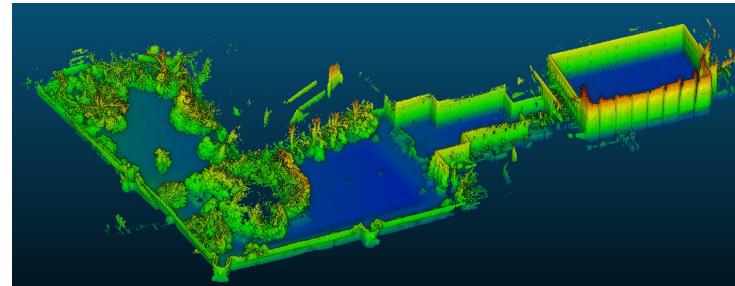
Classical Mapping

- Estimating 3D structure and location from 2D observations.

- Given a set of corresponding points in two or more images, compute the camera parameters and the 3D point coordinates



Slide credit:
Noah Snavely

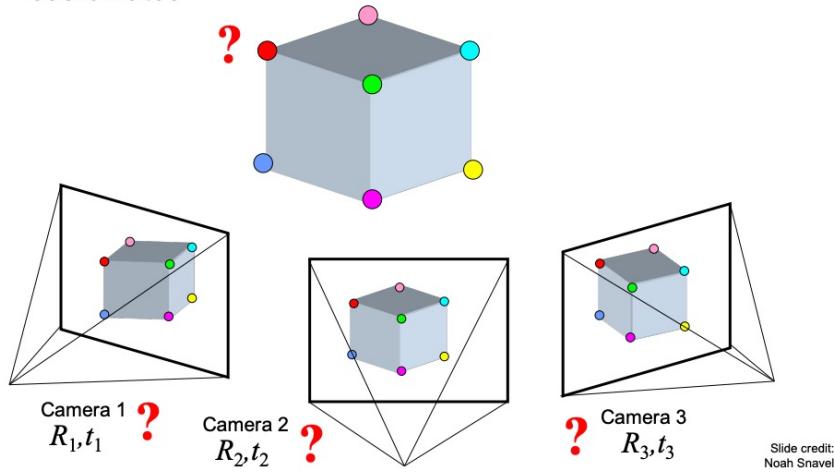


Garg & Jain

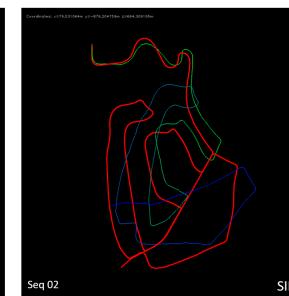
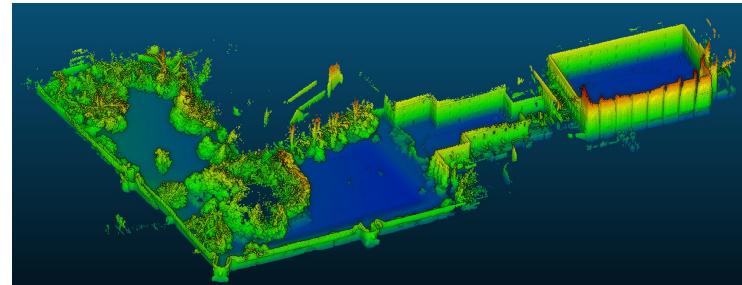
Classical Mapping

- Estimating 3D structure and location from 2D observations.
- Simultaneous Localization and Mapping.

- Given a set of corresponding points in two or more images, compute the camera parameters and the 3D point coordinates

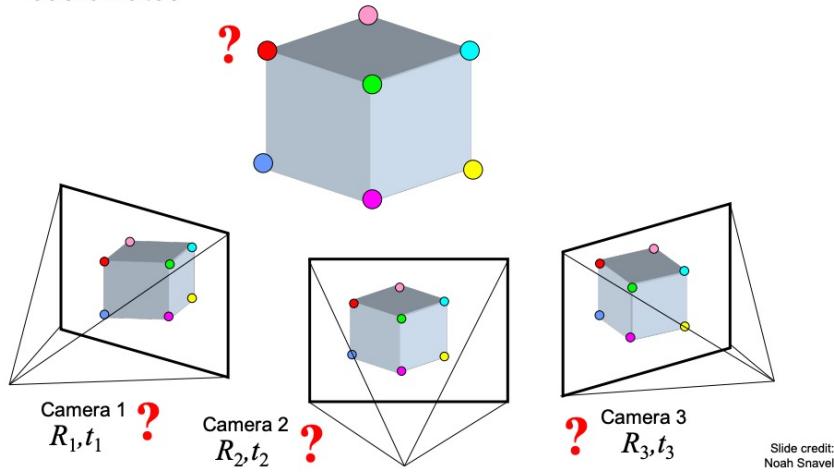


Slide credit:
Noah Snavely

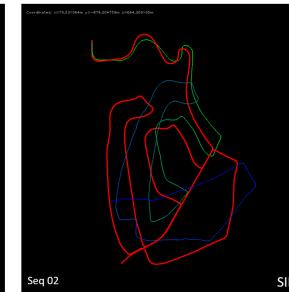
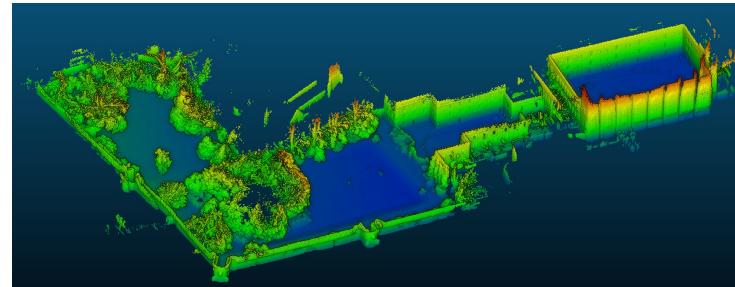


Classical Mapping

- Estimating 3D structure and location from 2D observations.
 - Simultaneous Localization and Mapping.
 - Common Techniques: Extended Kalman Filter, GraphSLAM
- Given a set of corresponding points in two or more images, compute the camera parameters and the 3D point coordinates



Slide credit:
Noah Snavely



Garg & Jain

Common Drawbacks

- Probabilistic inference can take long to compute, and mapping takes a large memory storage.

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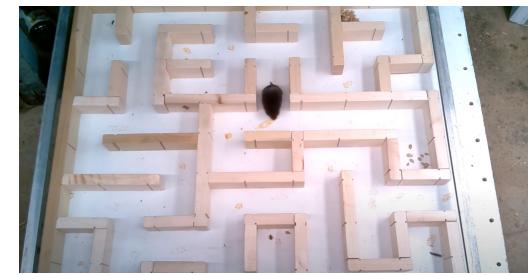
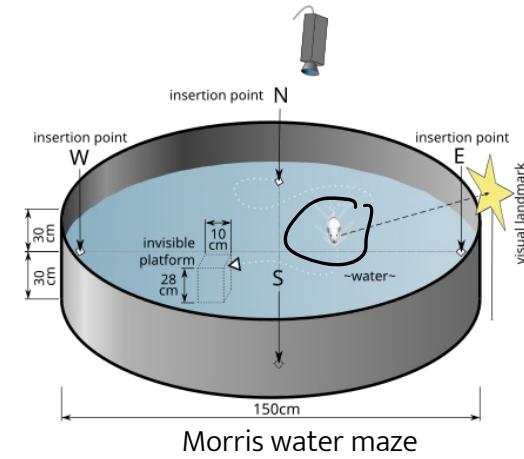
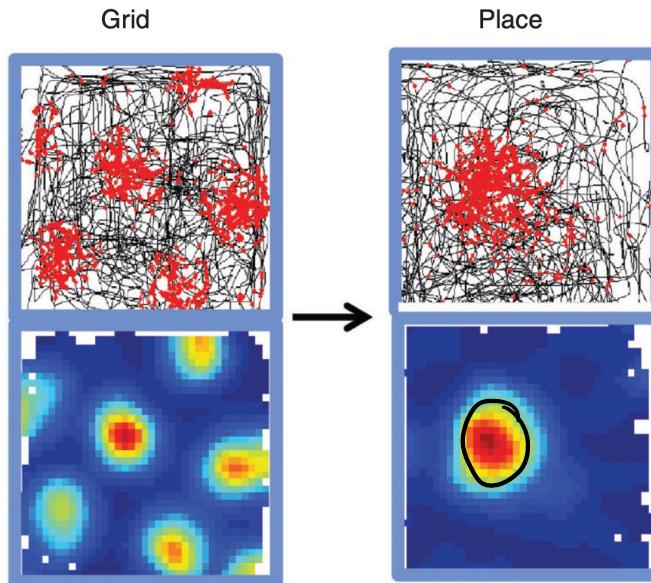
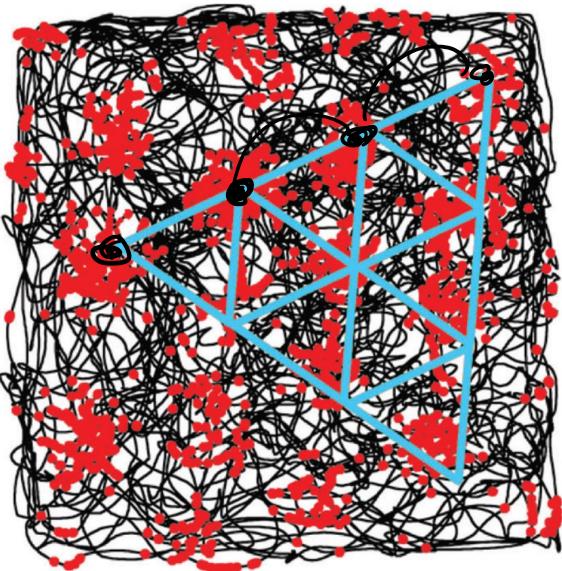
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- Probabilistic inference can take long to compute, and mapping takes a large memory storage.
- Great for 3D reconstruction but downstream tasks may not need a full precision explicit map.
- May not fully understand dynamic objects (averaging across multiple scans).
- Is there a more end-to-end version?

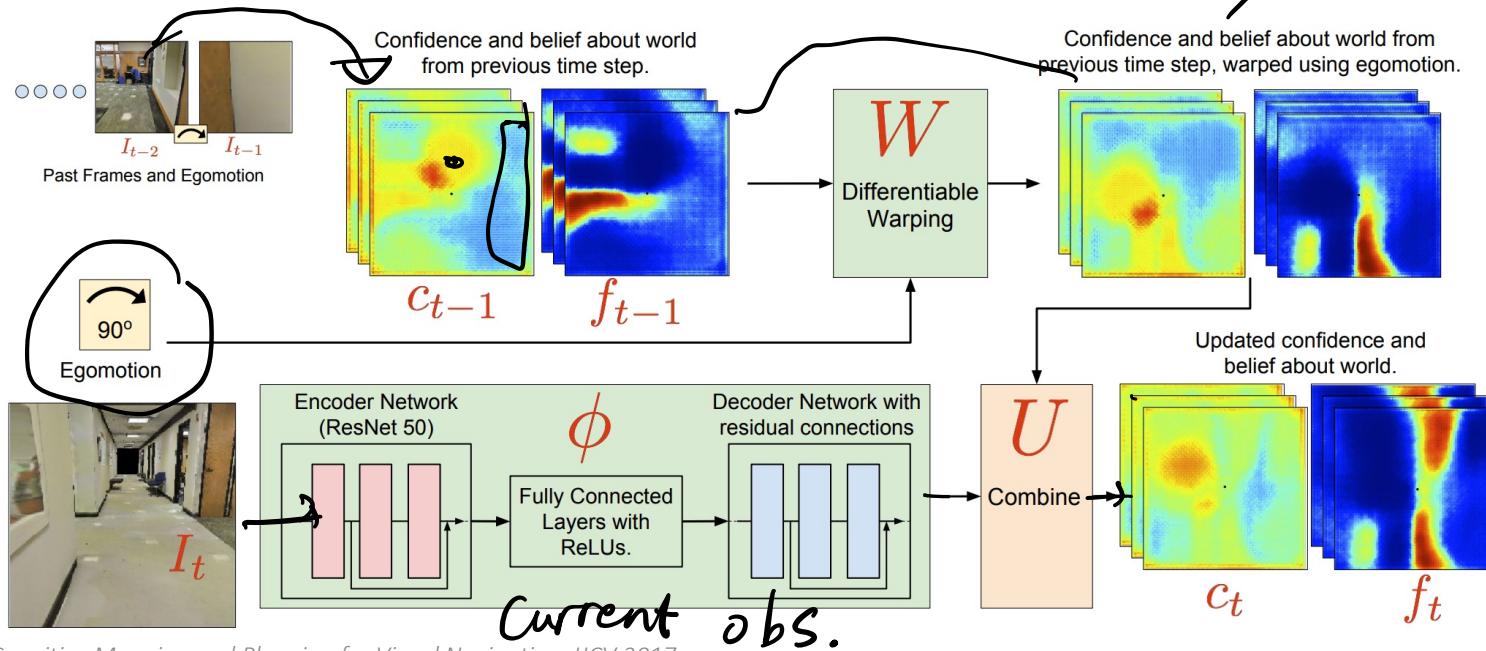
Mapping in the Brain: Grid and Place Cells



Matthias Wandel, 2018

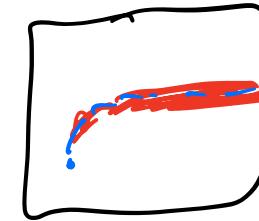
Neural Mapping

- Can we learn a mapping representation?
- Metric space, top-down warping (known egomotion).

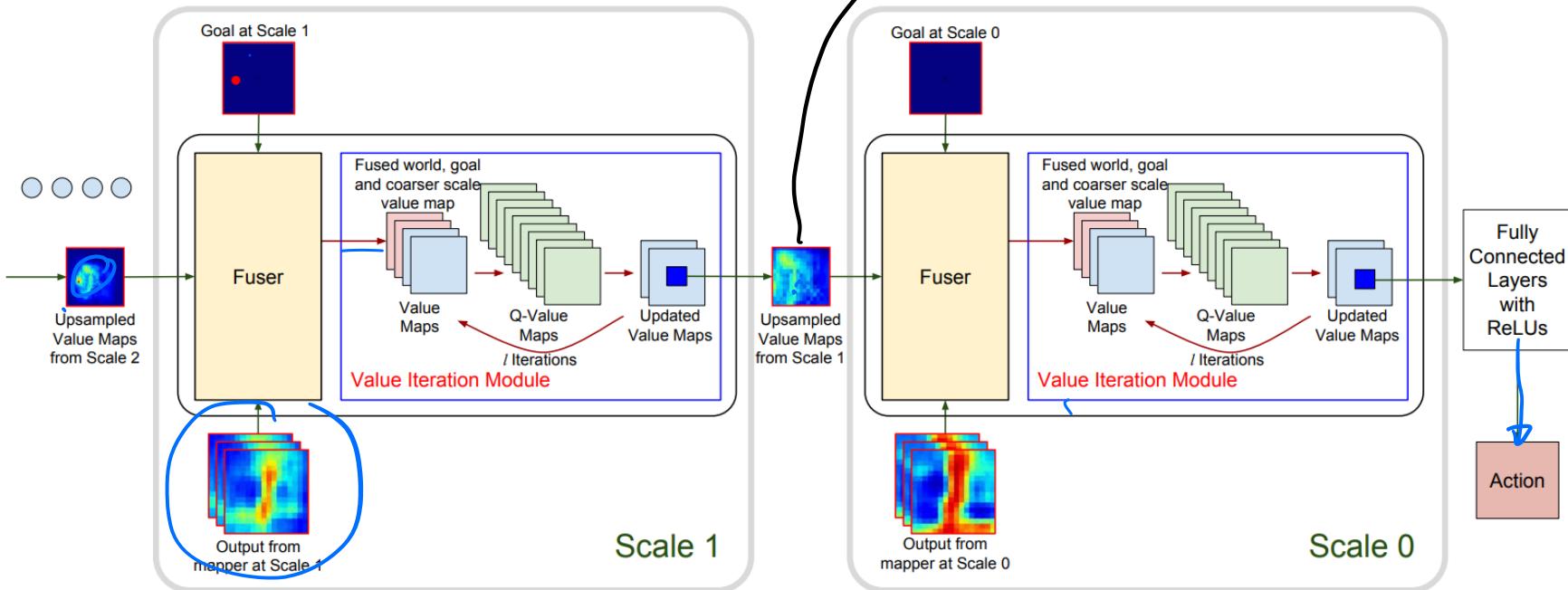




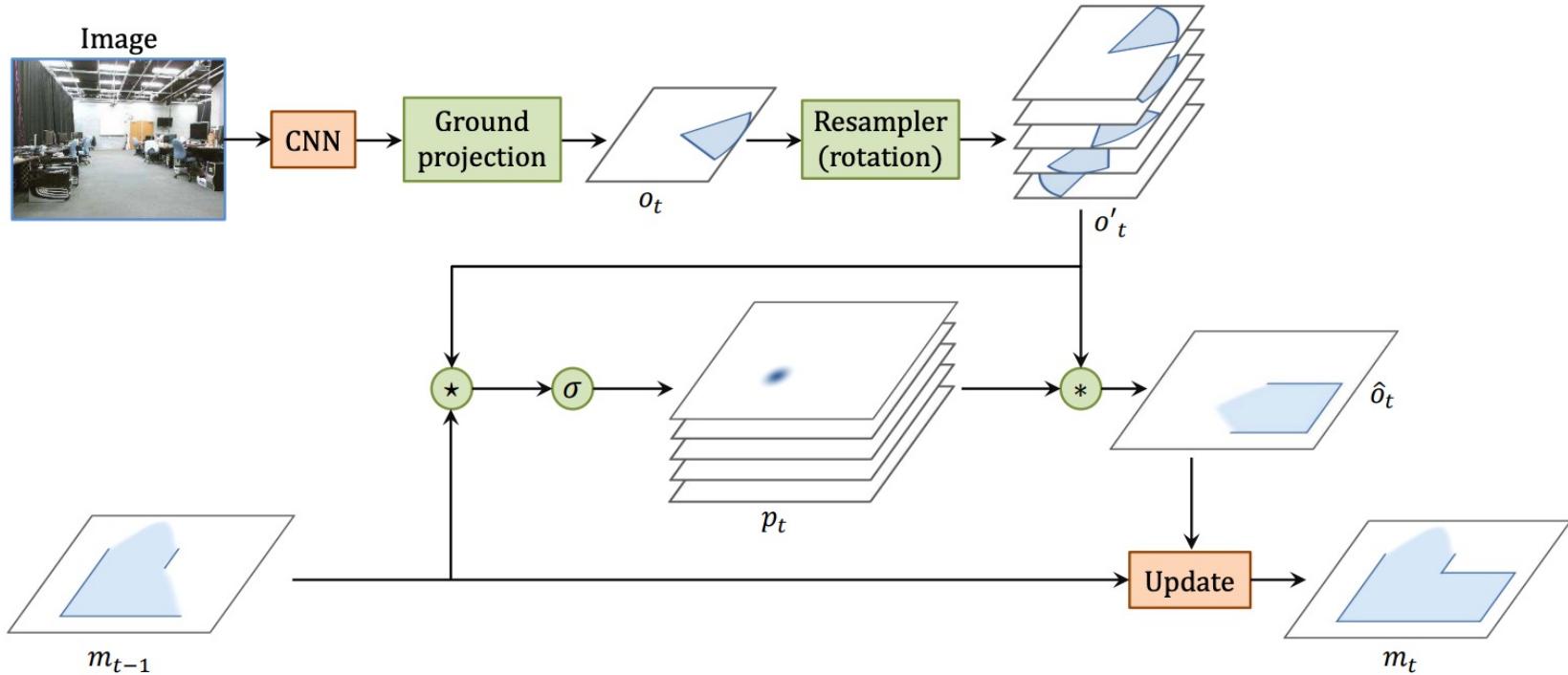
Hierarchical Planning



- How do we use the learned map (allocentric) feature of the world?



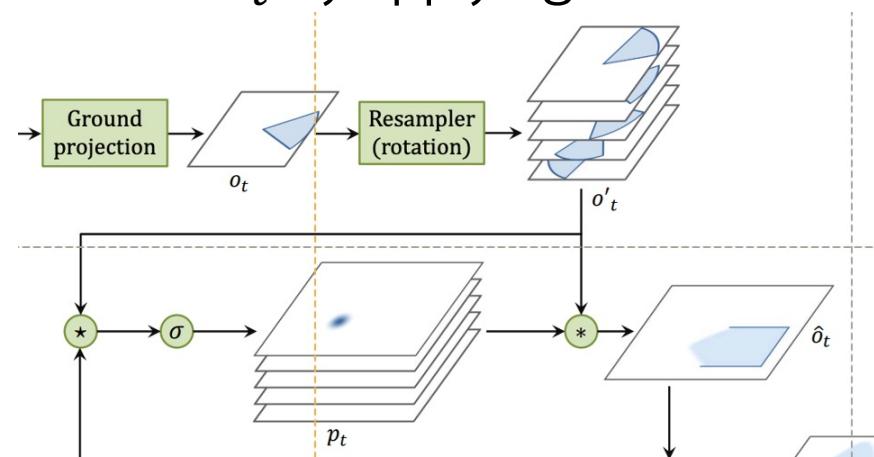
Simultaneous Localization and Registration



Simultaneous Localization and Registration

- The observations o_t are transformed into a stack o'_t by applying a rotation resampler.

$$o'_{ijkl} = [R(o, 2\pi l/r)]_{ijk}.$$



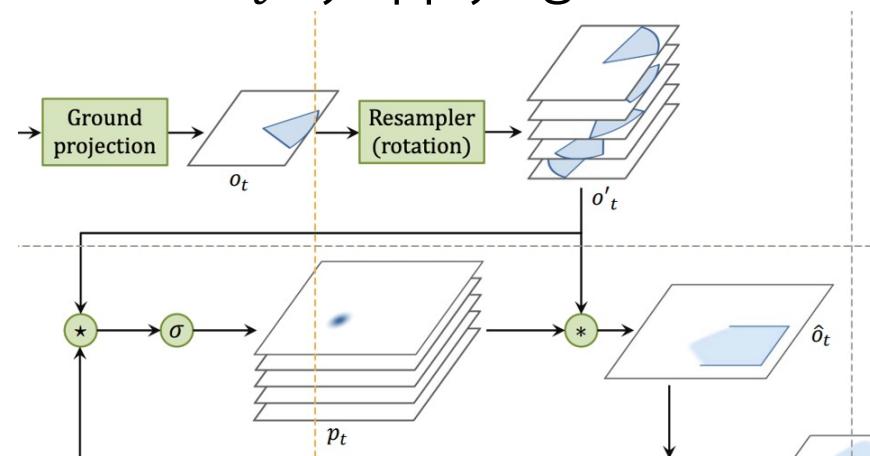
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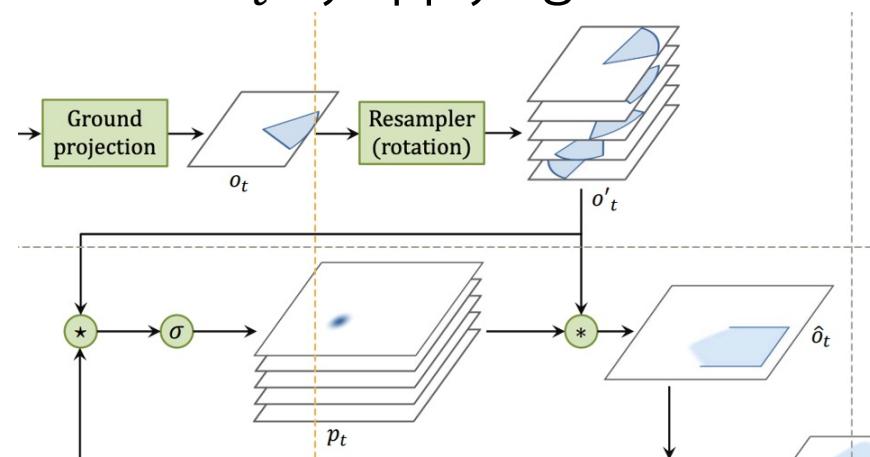
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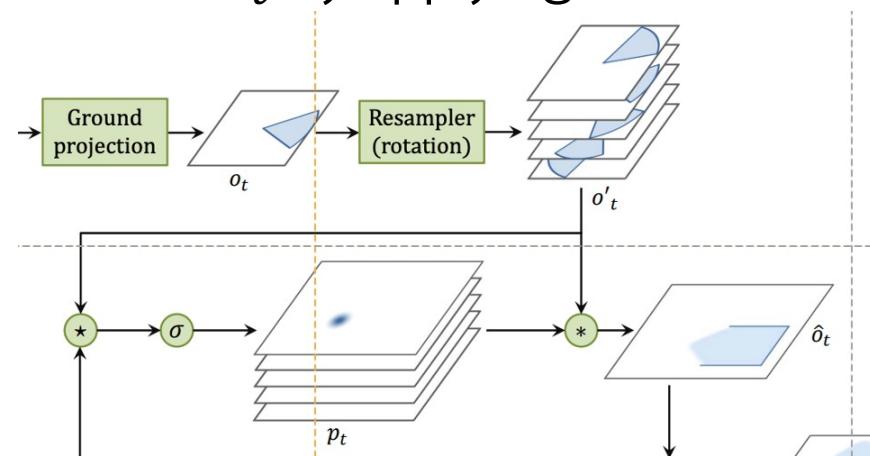
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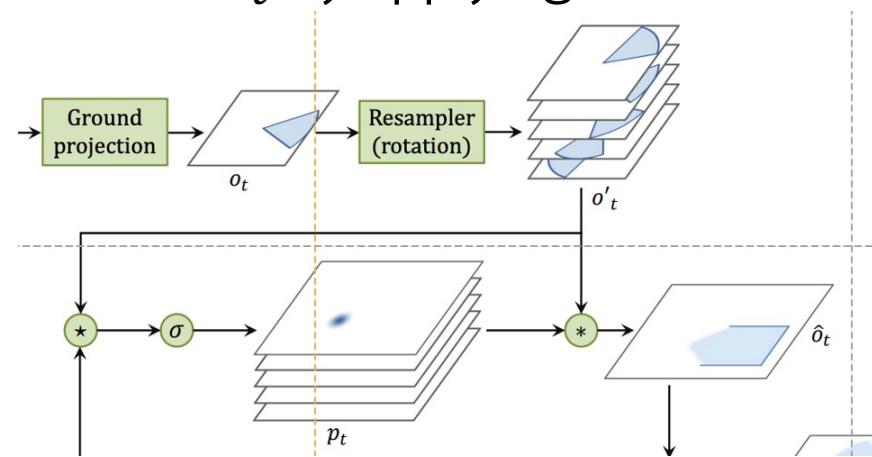
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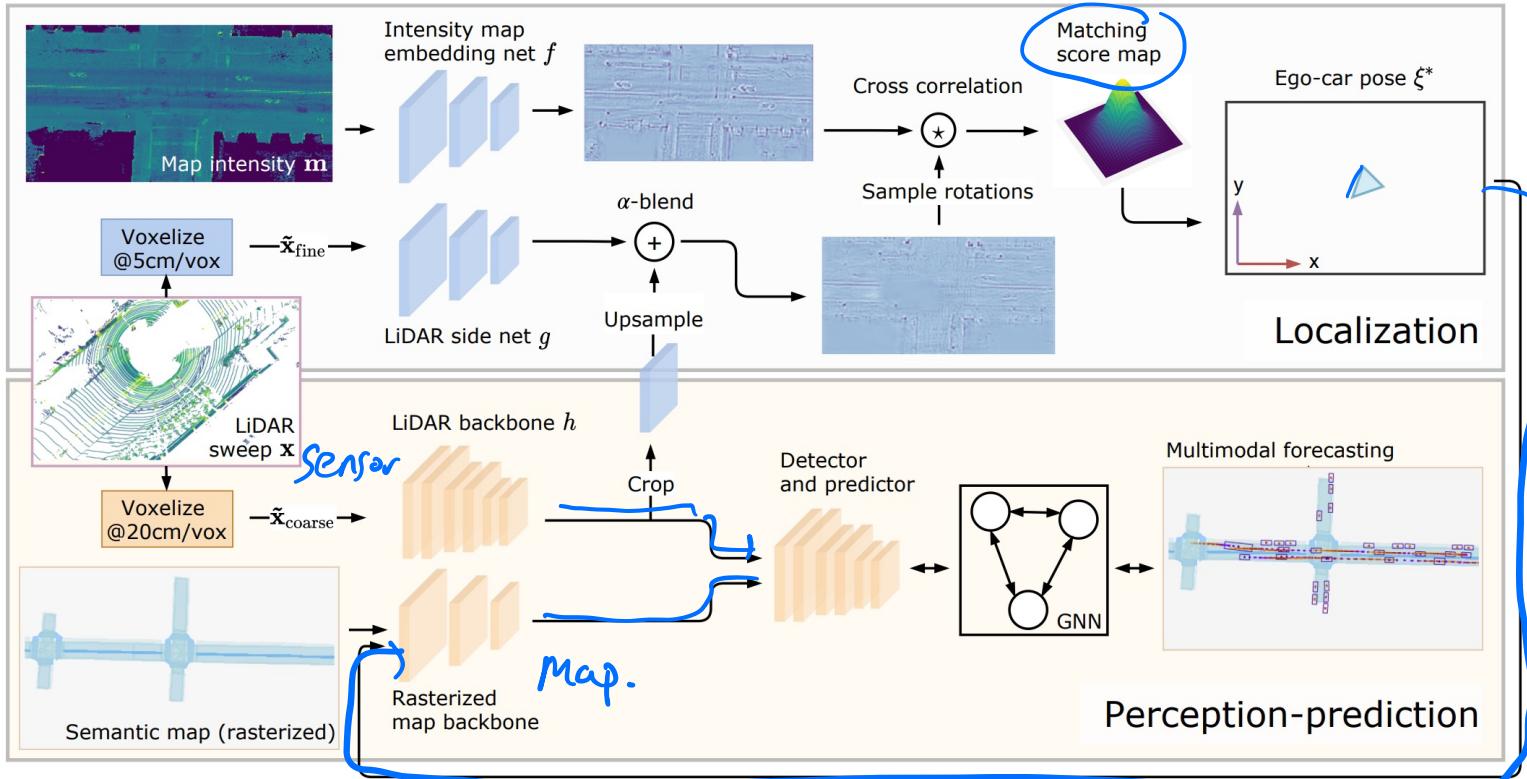
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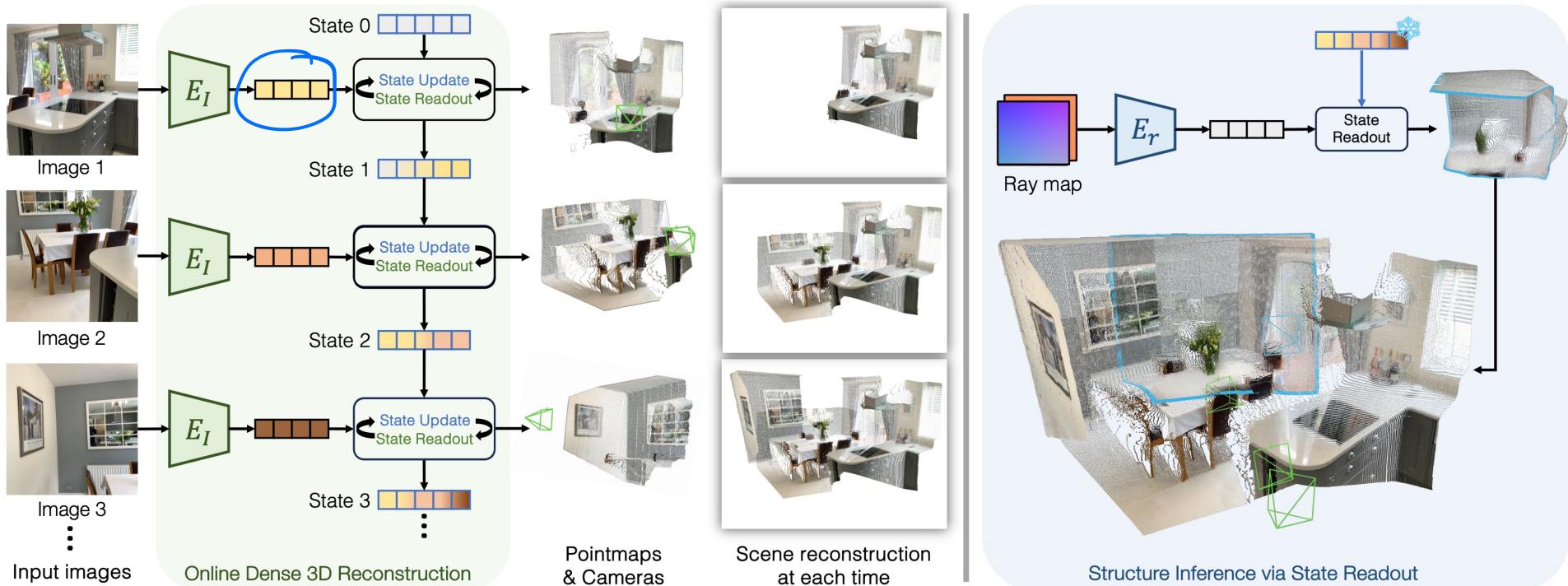
Loss:

$$\mathcal{L}(p) = -\log \sum_t p_{H_t W_t R_t t}.$$

Joint Localization, Perception and Prediction



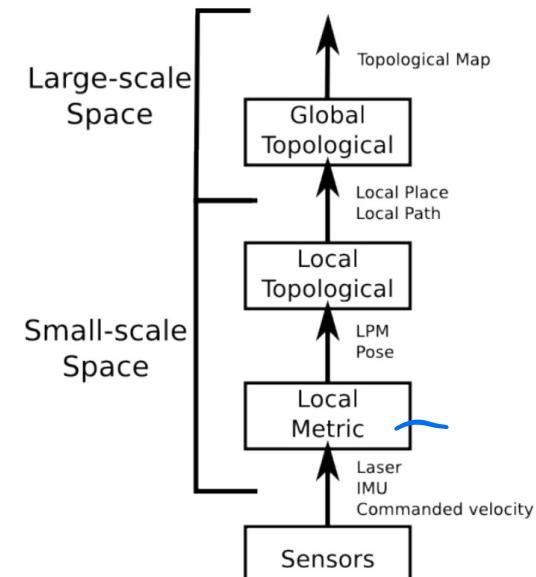
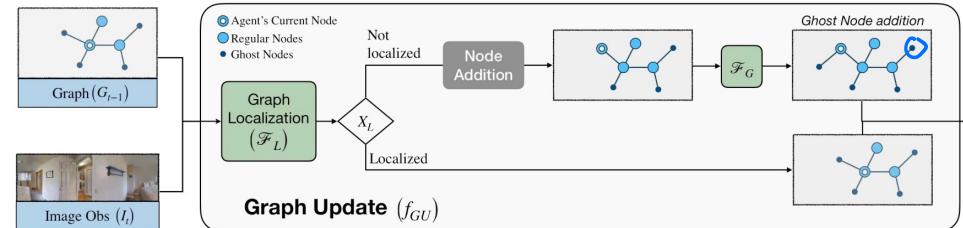
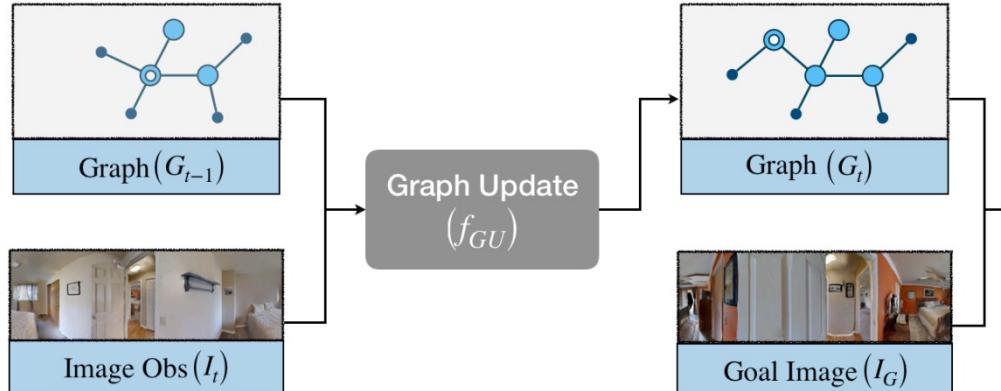
Continuous 3D Perception and Mapping





Topological Mapping

- High-level graph representation
- Each node contains more summarized information
- Enables global planning



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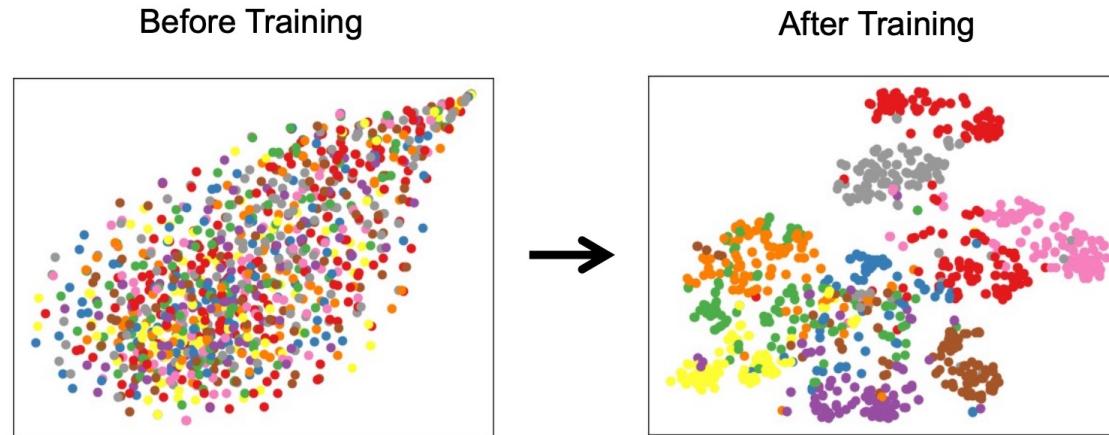
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- Useful for planning (a few weeks from now).

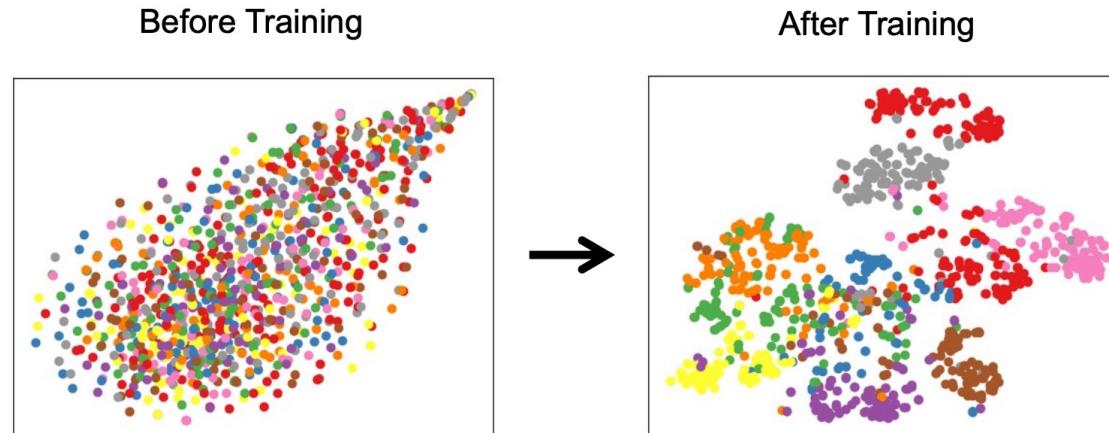
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- Efficient encoding of the world that can help us recognize semantic concepts (high-level cognition).



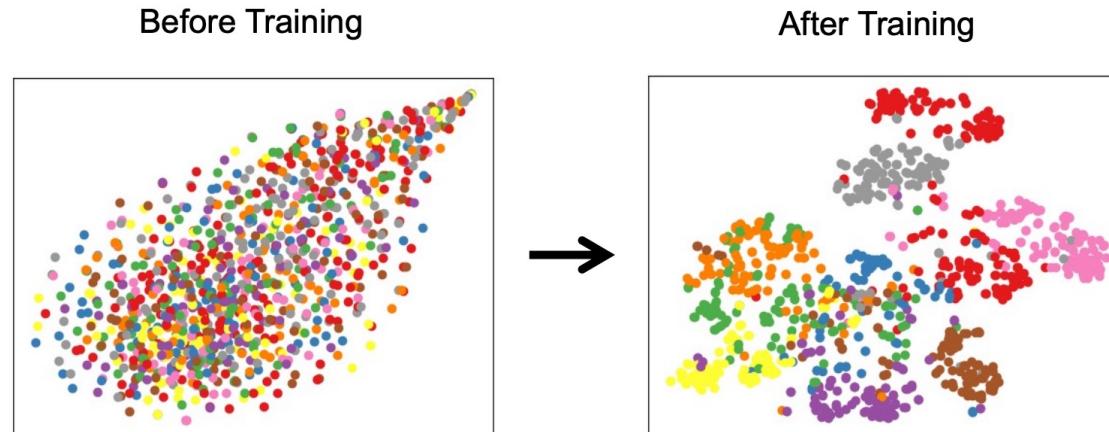
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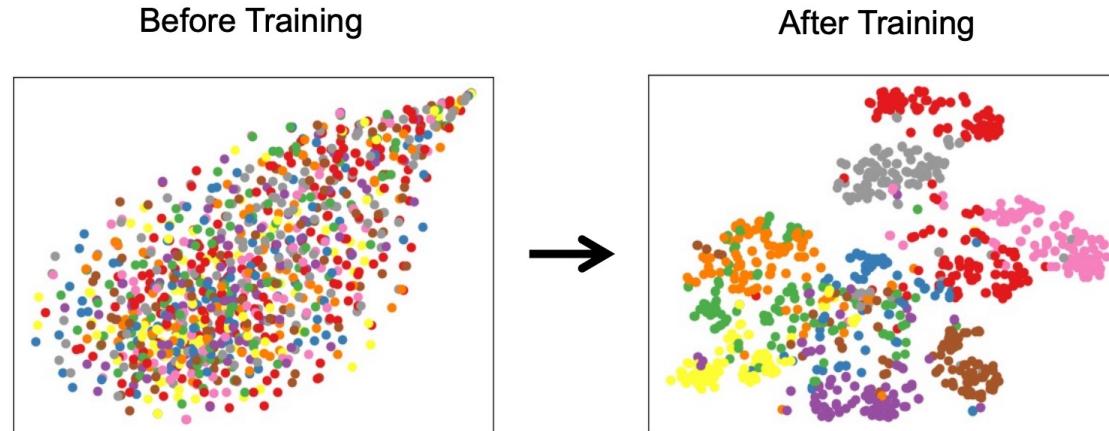
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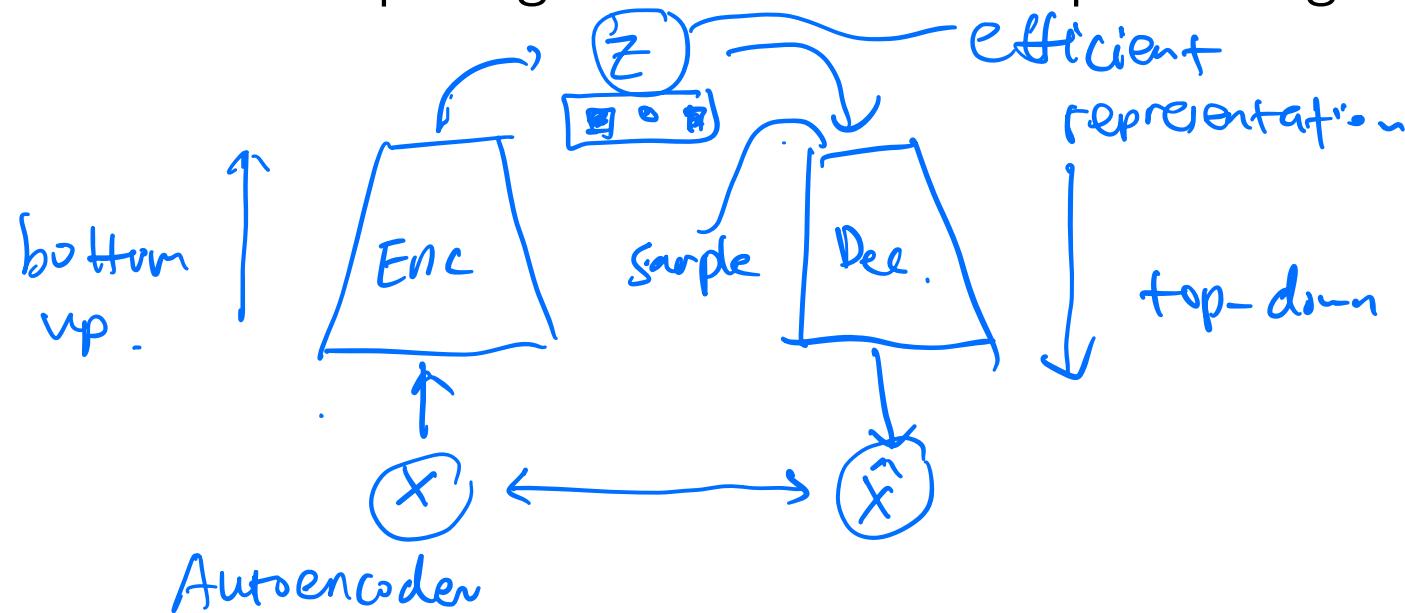
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- Historically, largely driven by supervised classification.



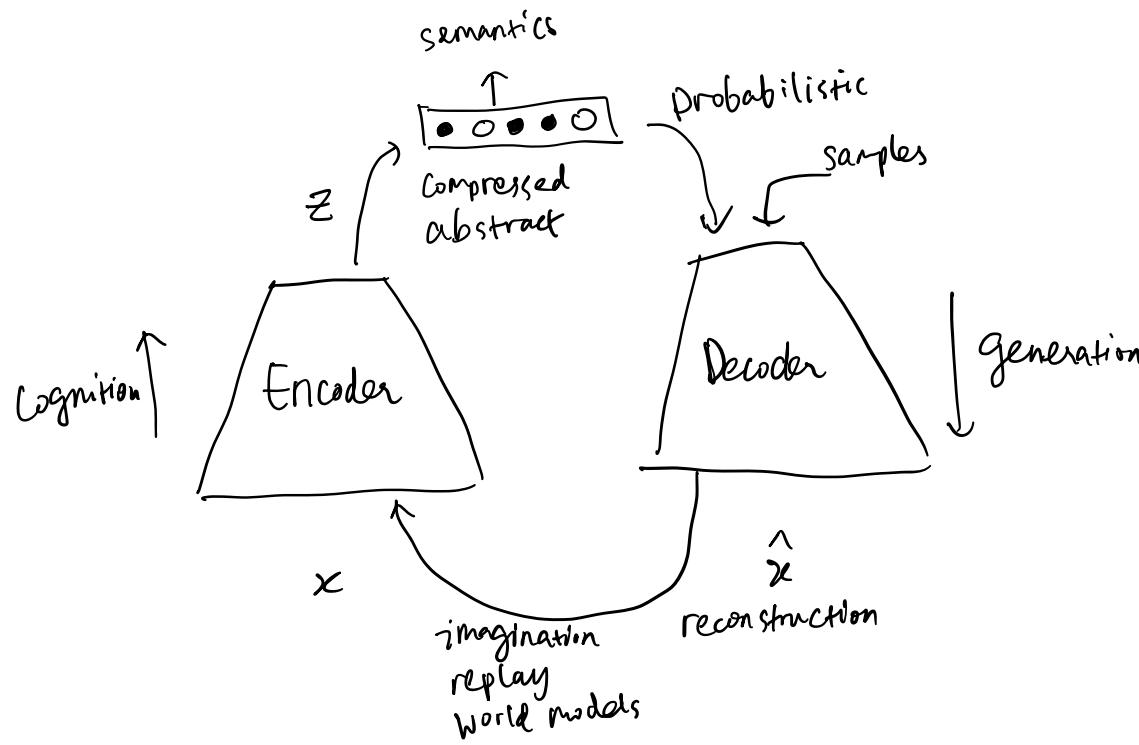
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Denoising Autoencoder (DAE)

- Making representations robust to partial corruption

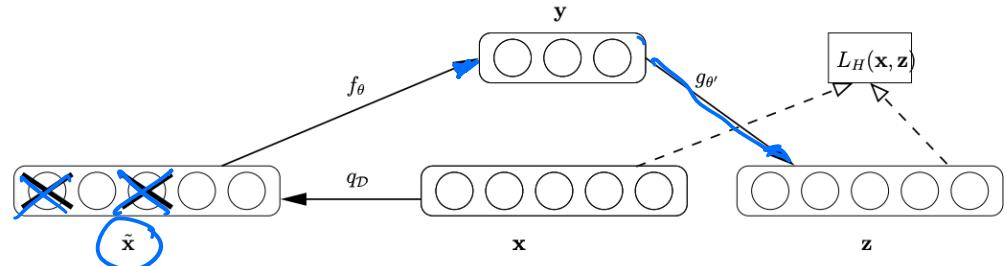
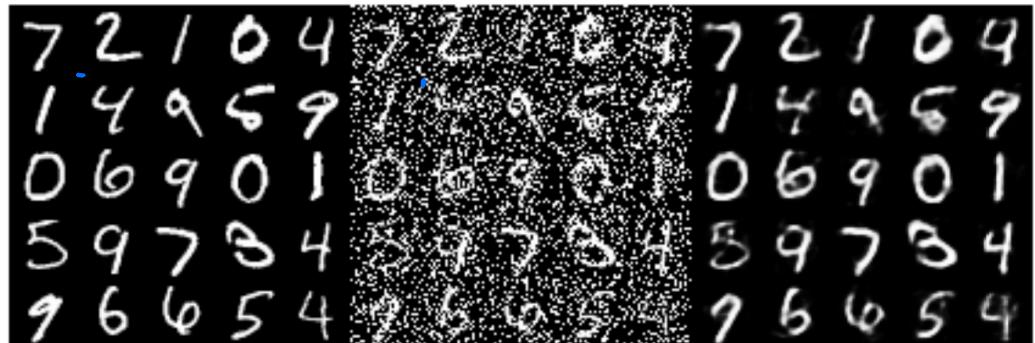


Figure 1. An example x is corrupted to \tilde{x} . The autoencoder then maps it to y and attempts to reconstruct x .

A noise



Denoising Autoencoder (DAE)

- Making representations robust to partial corruption
- Low-dimensional manifold near which the data concentrate:
 $p(x|\tilde{x}) = B_{g_{\theta'}(f_{\theta})(x)}.$

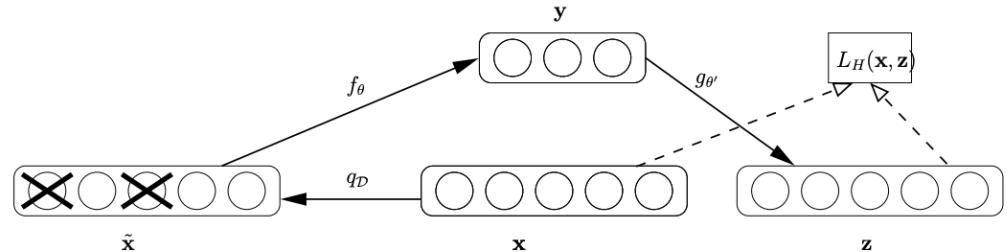
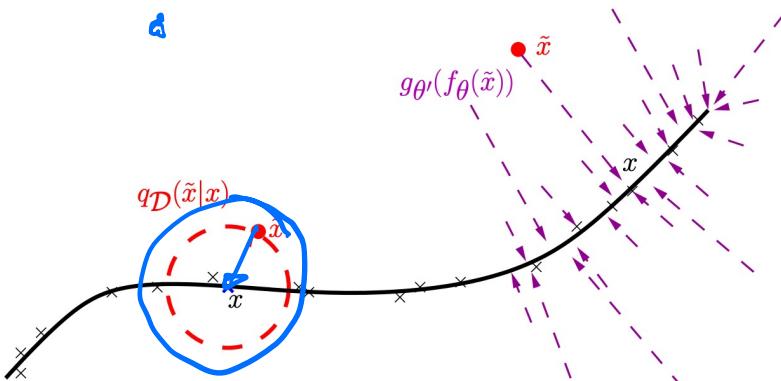
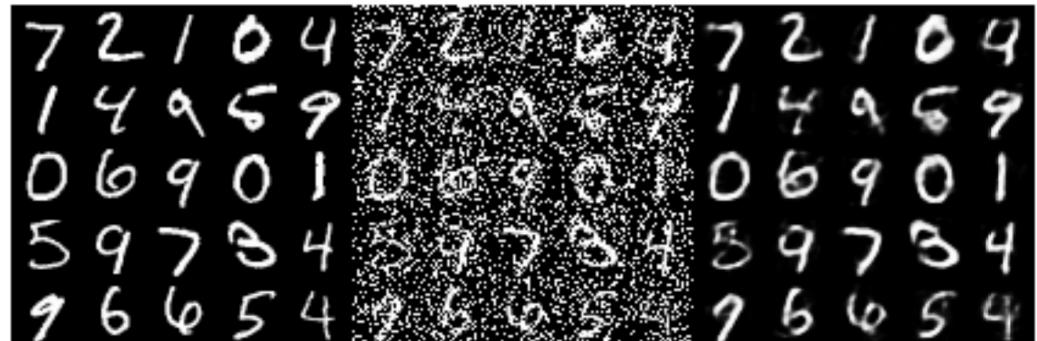
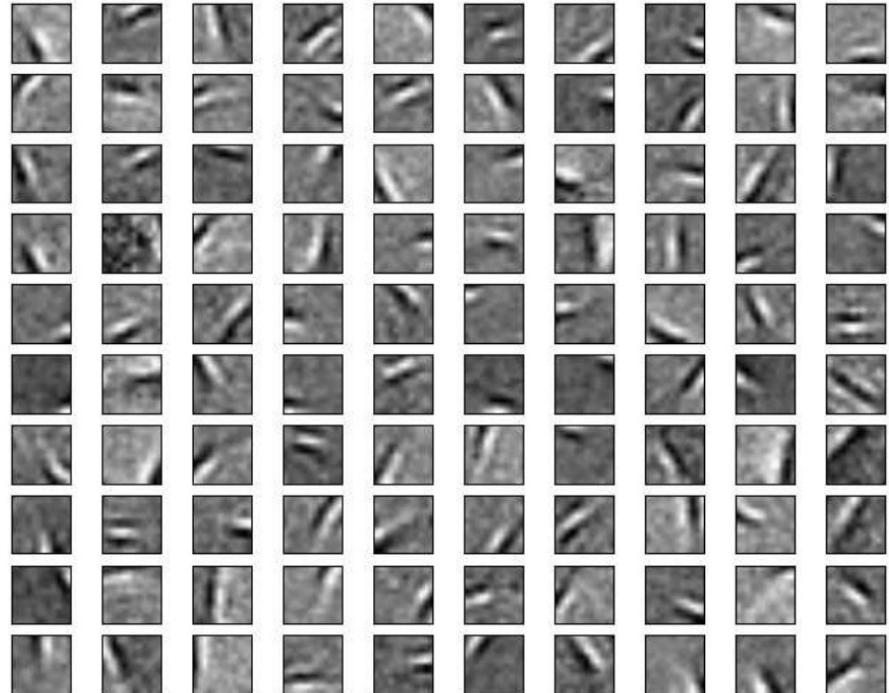
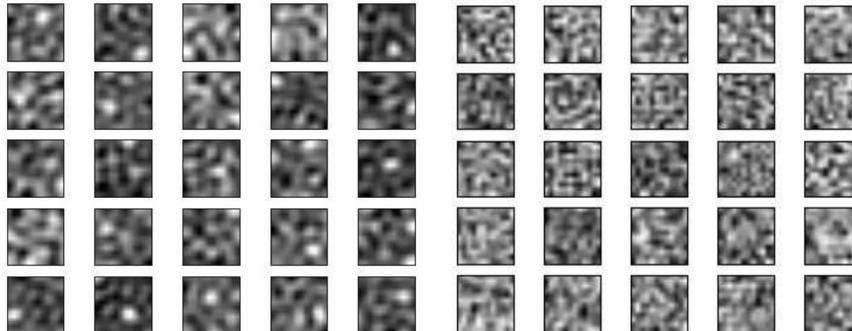


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Denoising Autoencoder (DAE)

- Regular autoencoders do not learn good filters.



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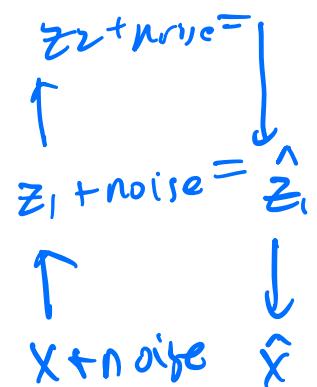
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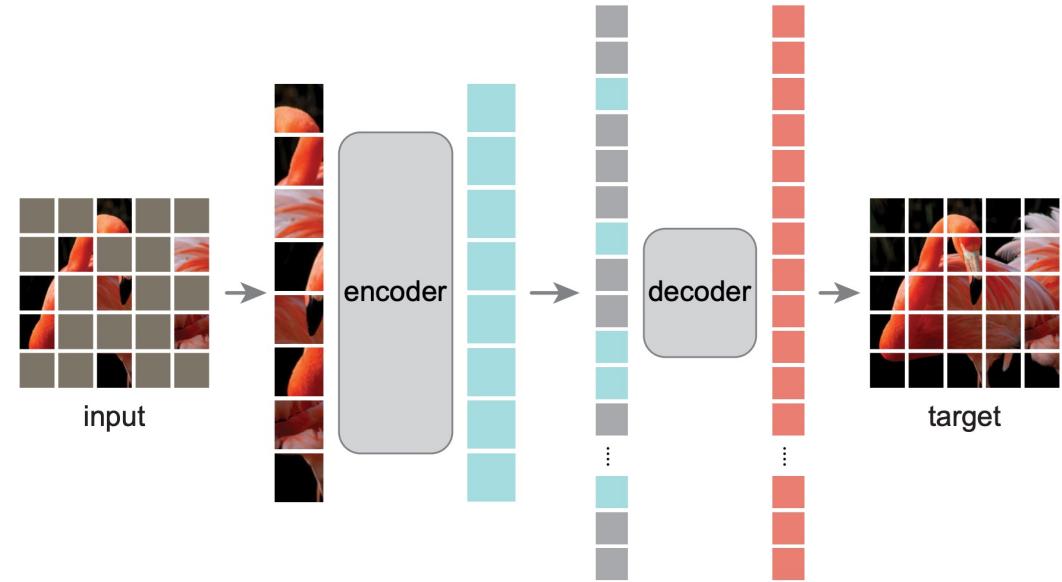
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- Stacked DAE: Stacked layerwise noise-denoise mechanism. Used to “pretrain” deep networks.

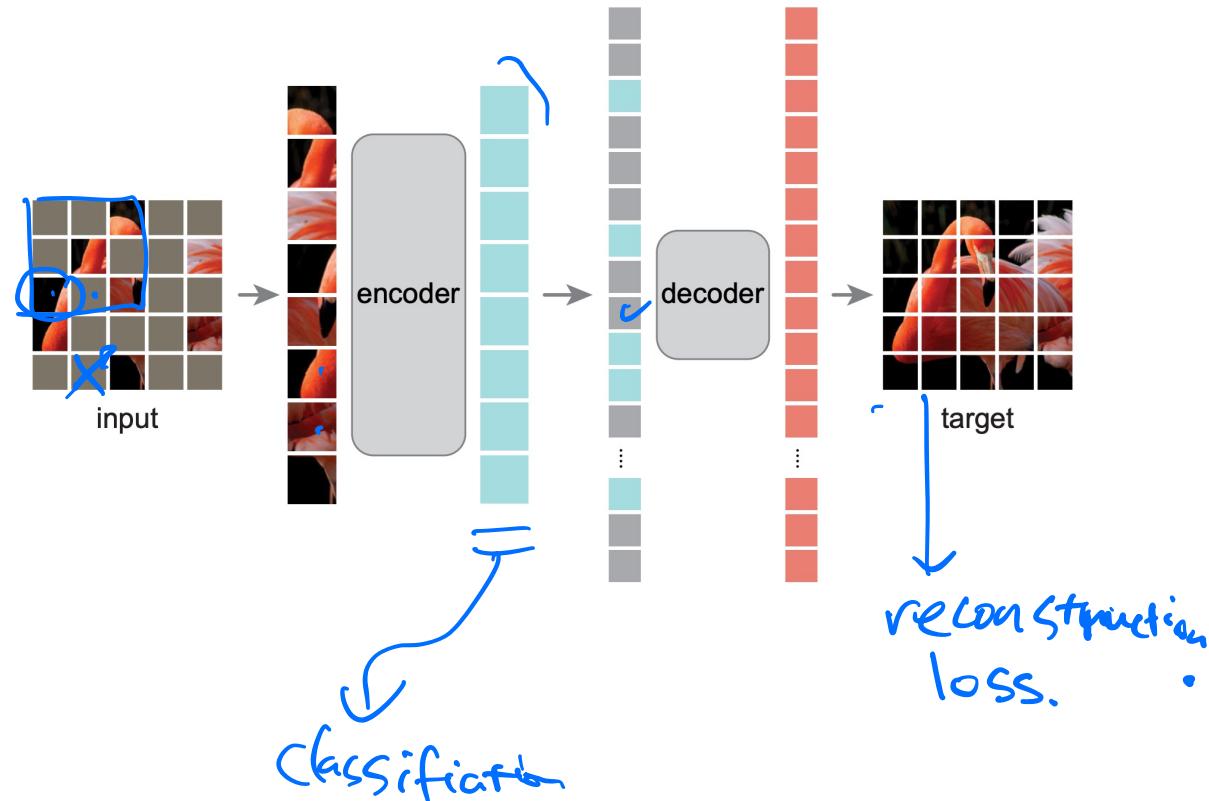
Masked Autoencoder (MAE)

- Modernized version of denoising autoencoder.



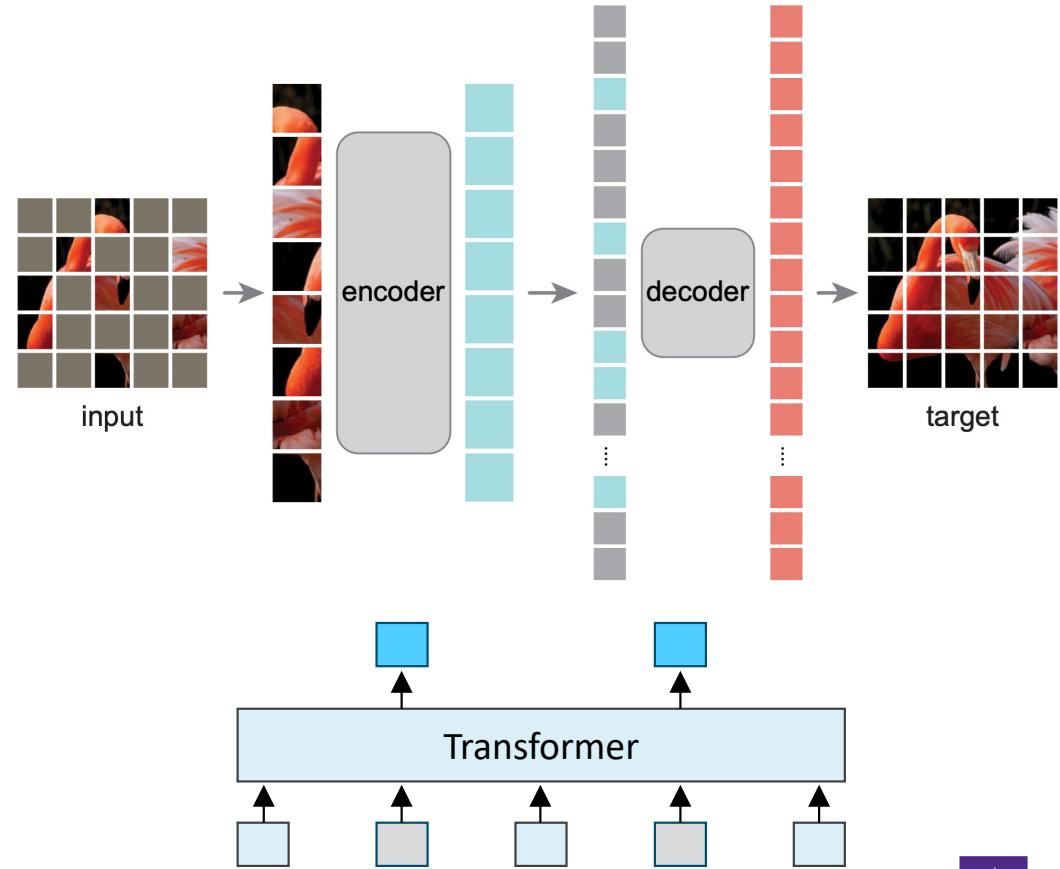
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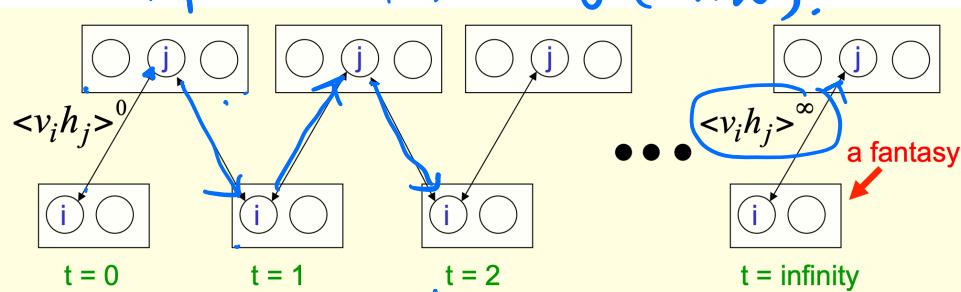
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- Idea also came from masked language models.



Energy-Based Learning

x input *v_{visible}* *h_{hidden}* *z*

- Example: RBMs
- Energy: $E(v, h) = -\sum_{i,j} v_i h_j w_{ij}$.
- $p(h_j = 1 | v_i) = \sigma(\sum_{ij} v_i w_{ij})$.
- $p(v=1 | h) = \sigma(h w)$.



$$\frac{\partial \log p(v)}{\partial w_{ij}} = \underbrace{< v_i h_j >^0}_{\text{data}} - \underbrace{< v_i h_j >^\infty}_{\text{model distribution}}$$

Hinton. Restricted Boltzmann Machines.



General EBMs

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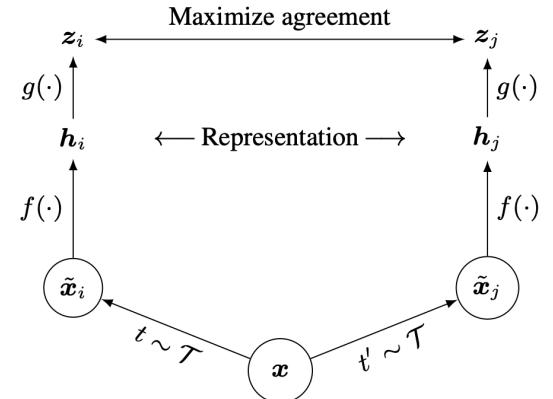
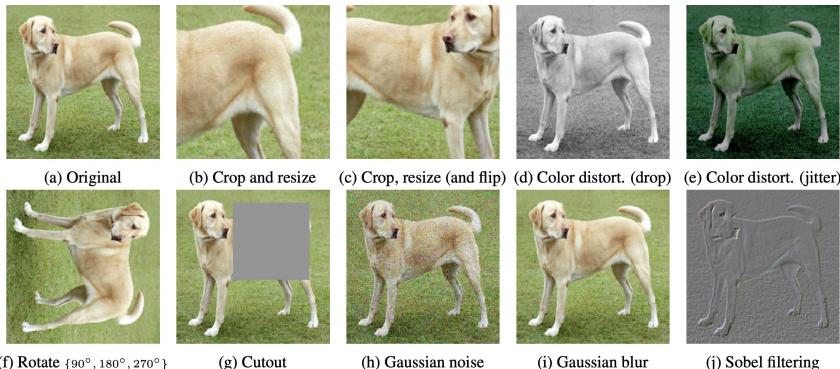
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- Can be applied on hand manipulation trajectory generation.
- Good results in generation but still not a generalized representation learning algorithm.

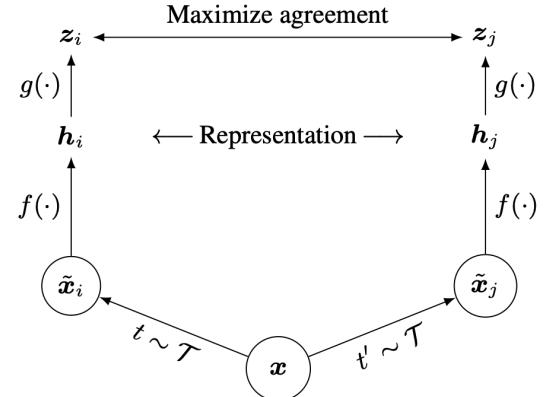
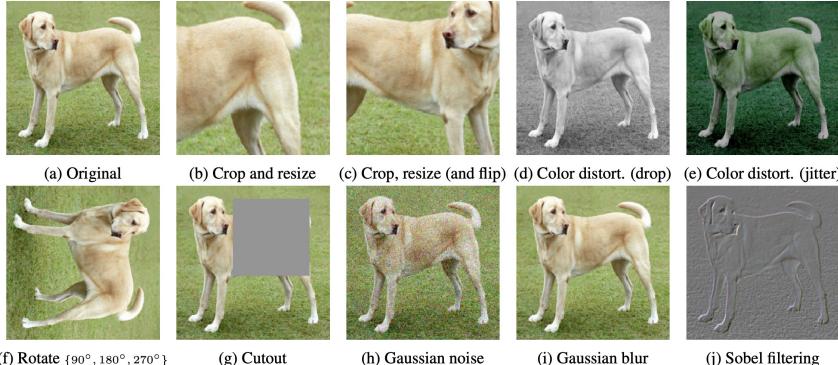
Self-Supervised Visual Learning

- Match the same image (with severe augmentation)



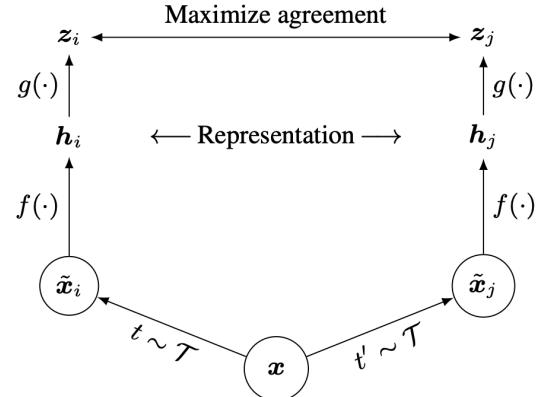
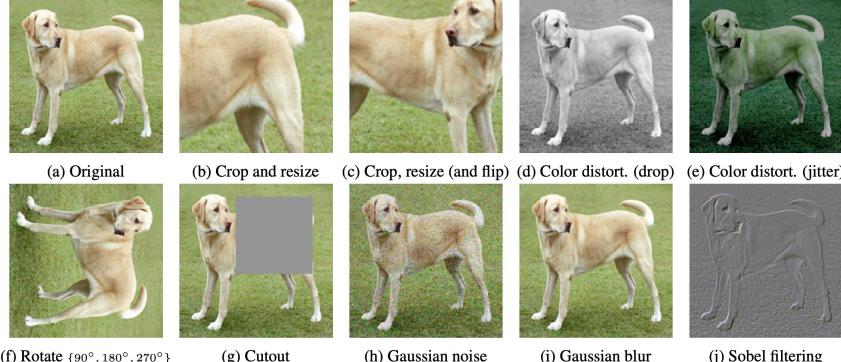
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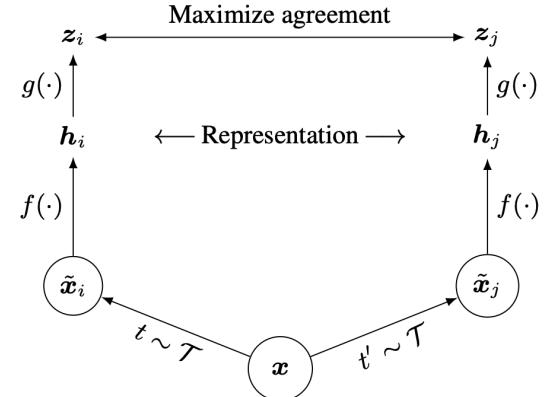
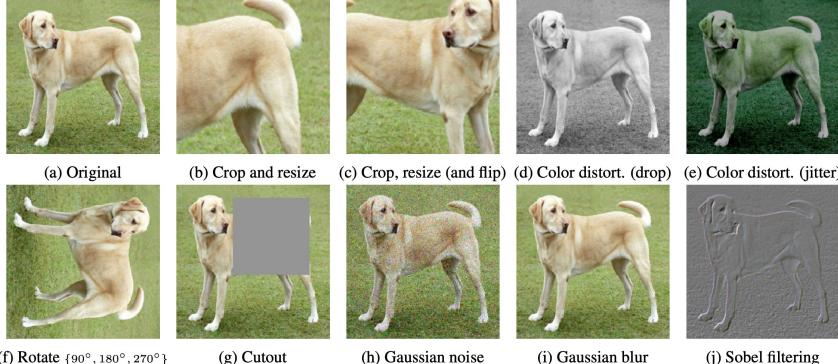
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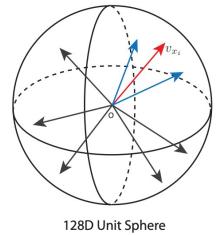
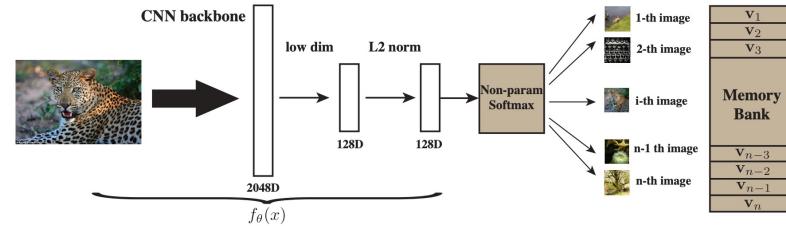
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- Energy is defined between a pair of images.



Several Embedding Loss Formulations

Wu et al., 2018

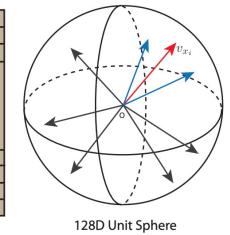
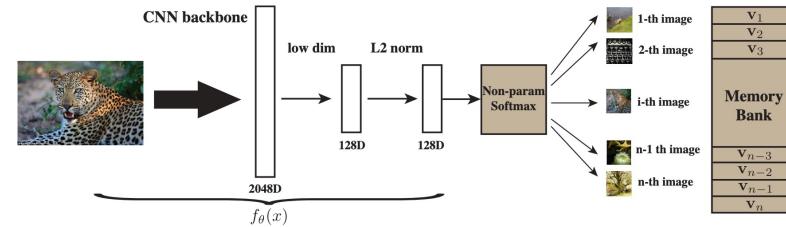
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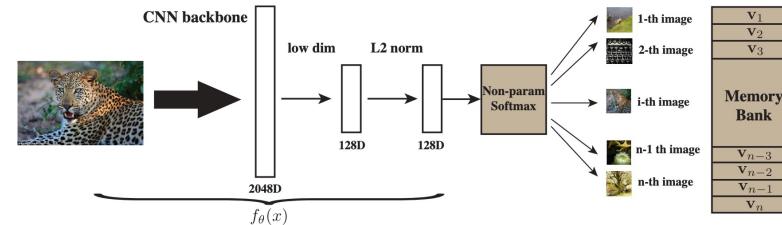
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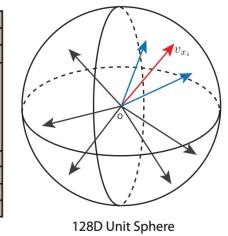
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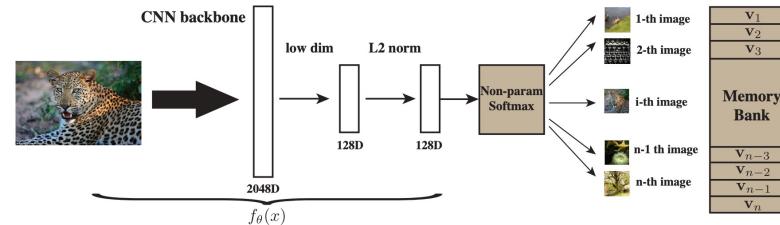
- Non-contrastive Learning (Positive Only)

- Moving Average [Grill et al., 2020]
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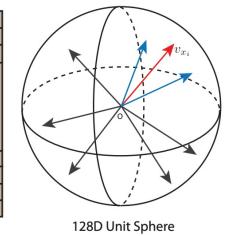


Several Embedding Loss Formulations

- Instance Classification:



Wu et al., 2018



- Contrastive Learning: Cross entropy on pairs

$$\ell_{i,j} = -\log \frac{\exp(\text{sim}(\mathbf{z}_i, \mathbf{z}_j)/\tau)}{\sum_{k=1}^{2N} \mathbb{1}_{[k \neq i]} \exp(\text{sim}(\mathbf{z}_i, \mathbf{z}_k)/\tau)}$$

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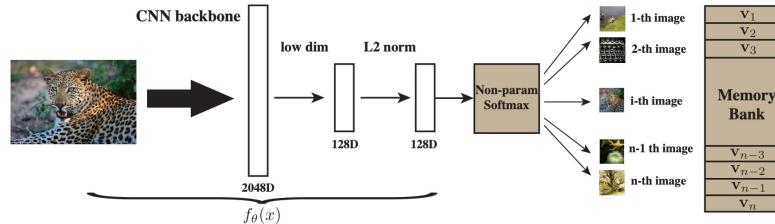
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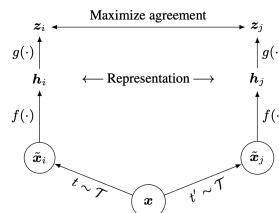
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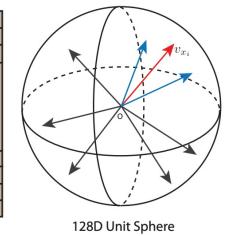
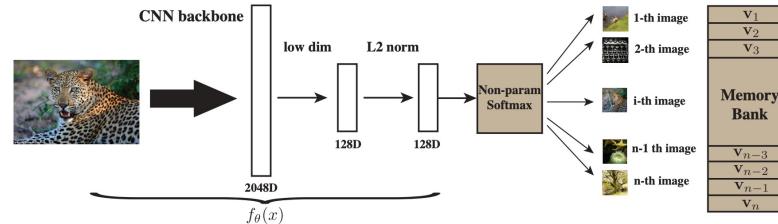


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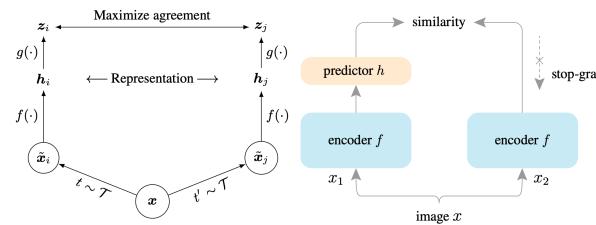
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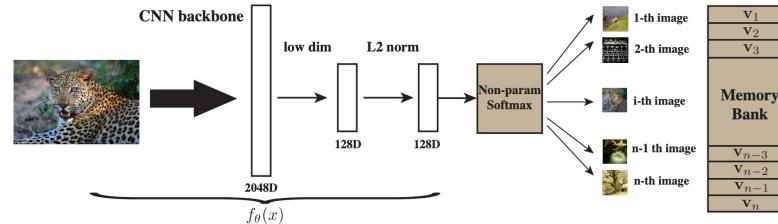
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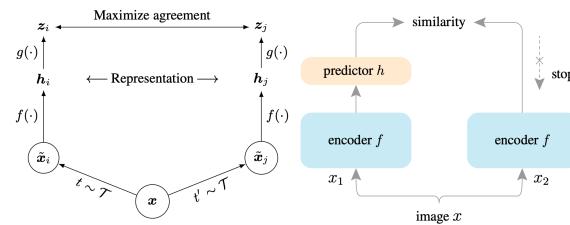
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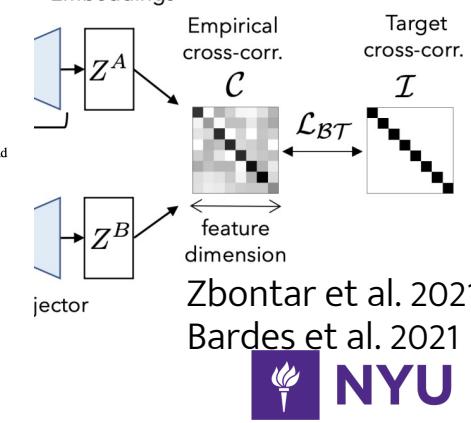
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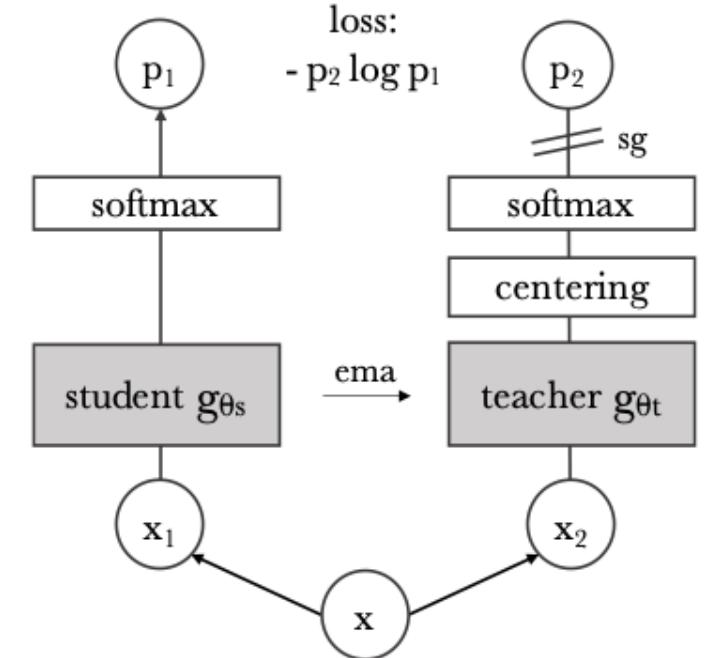
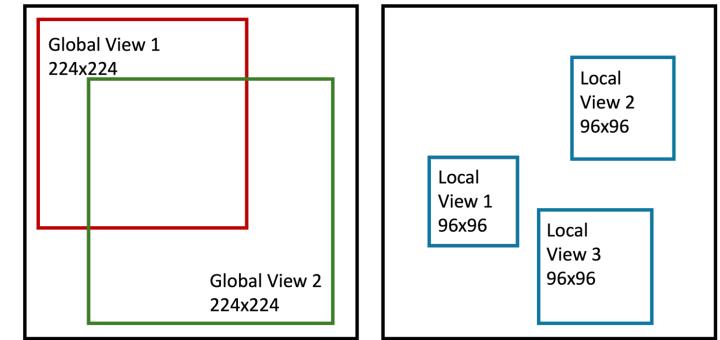
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Zbontar et al. 2021
Bardes et al. 2021

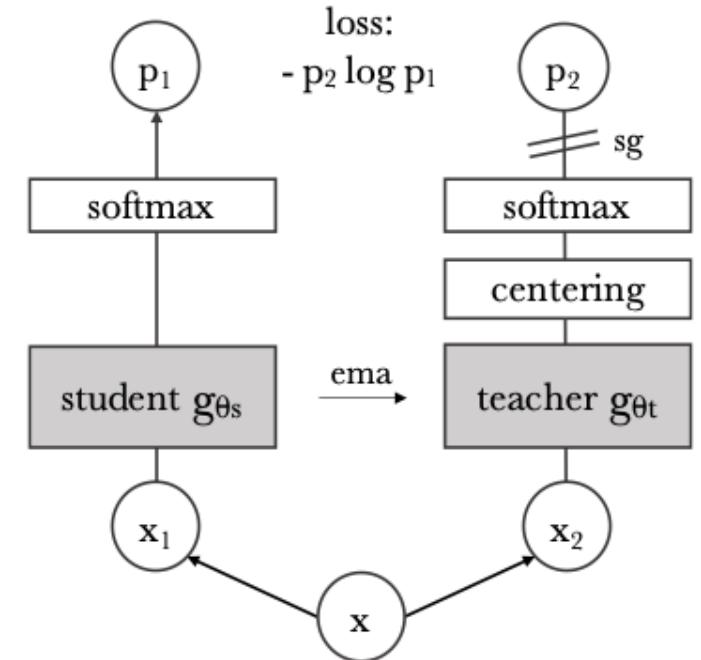
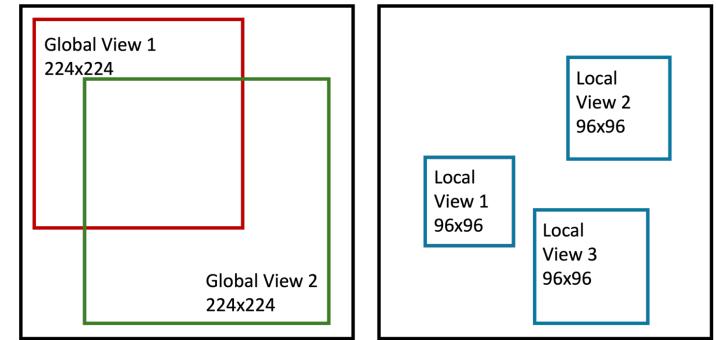
DINO

- Knowledge distillation between a student and a teacher network.



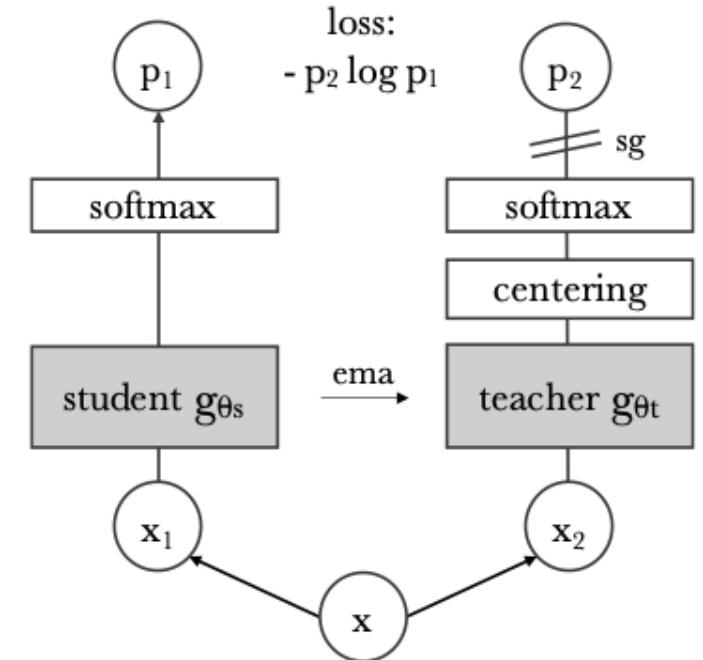
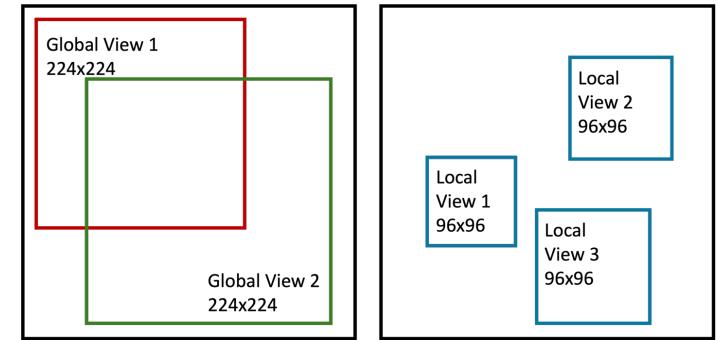
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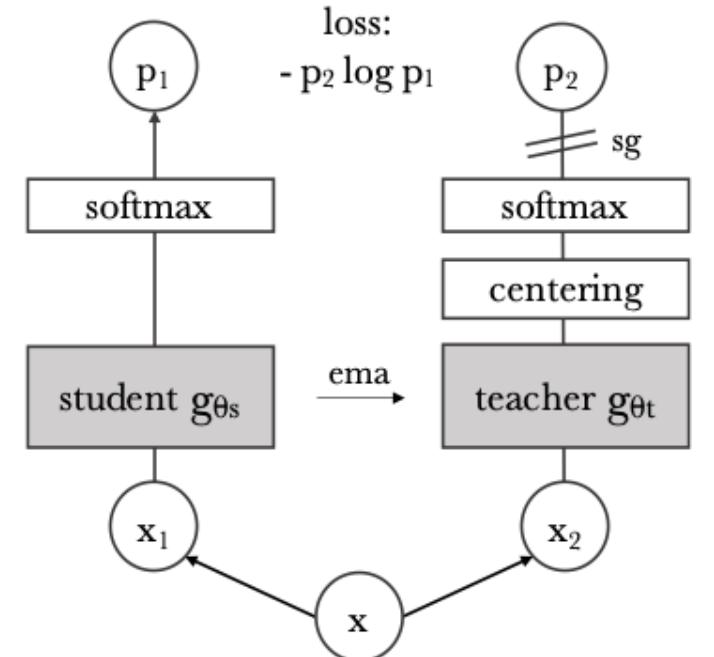
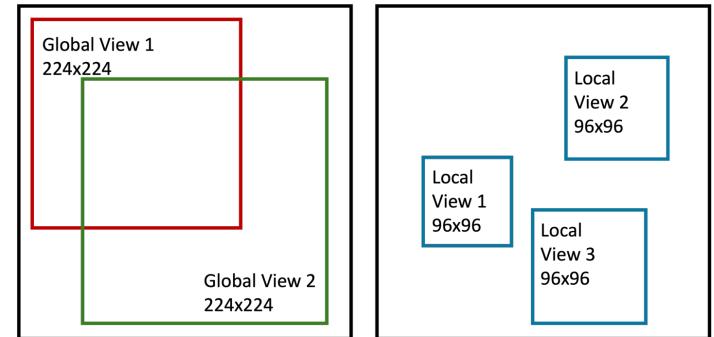
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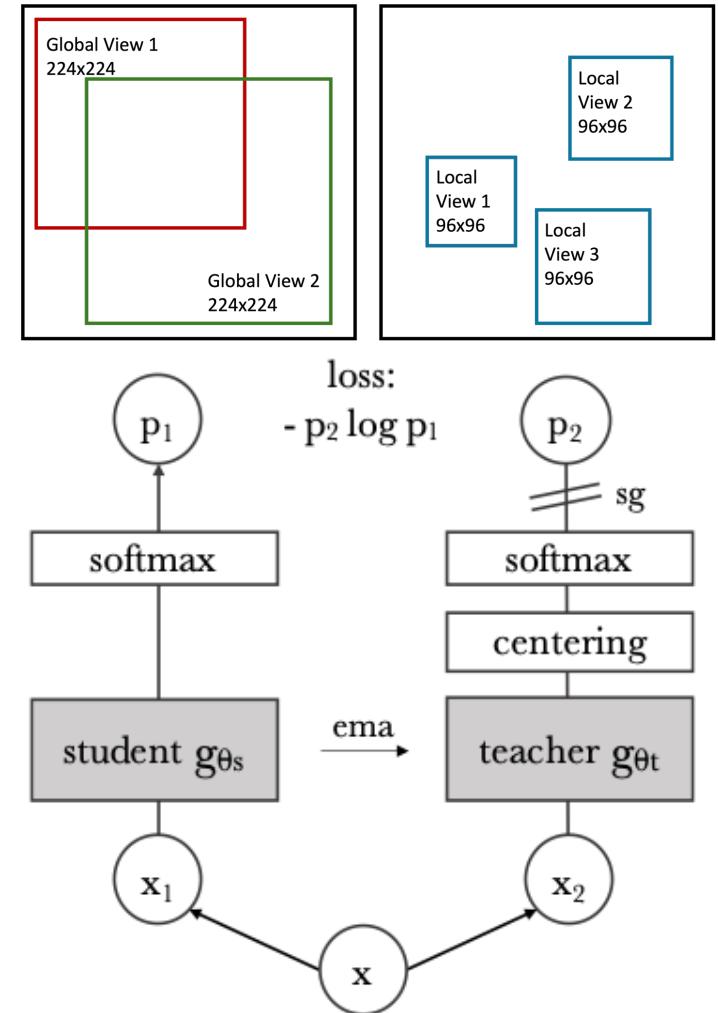
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- Stop gradient on the teacher (no true label).
- Teacher network has EMA weights copied from student (prevent collapse).



Preventing Collapse

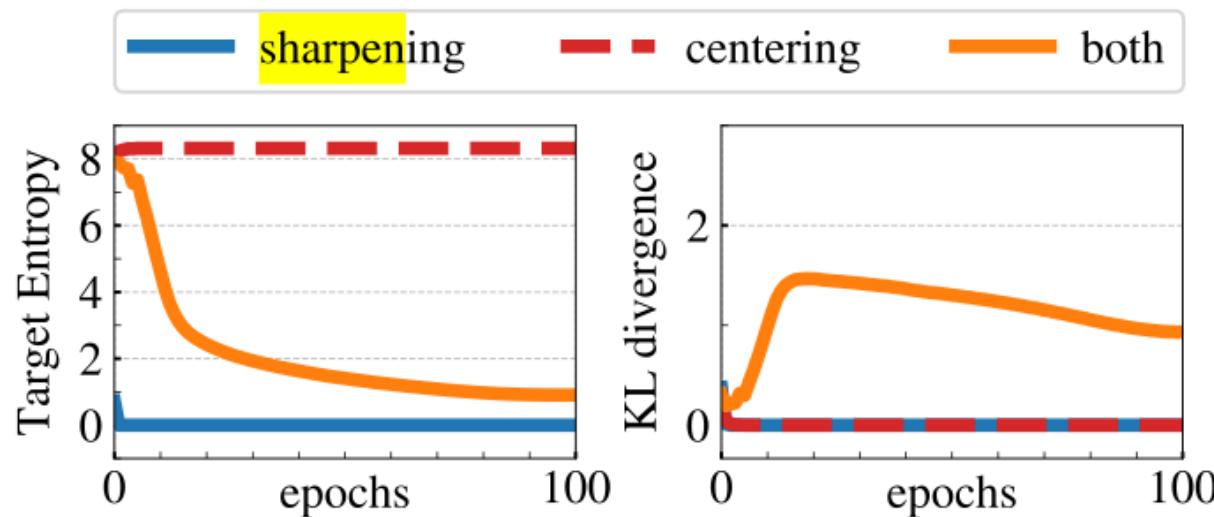
- Cross entropy objective can make both sides collapse to uniform distribution.
 - Apply sharpening, apply a temperature term on both teacher and student.
 - $\text{softmax}(g/\tau)$ The higher the temperature, the more uniform.

Preventing Collapse

- Cross entropy objective can make both sides collapse to uniform distribution.
 - Apply sharpening, apply a temperature term on both teacher and student.
 - $\text{softmax}(g/\tau)$ The higher the temperature, the more uniform.
- It can also collapse into always activating a single unit.
 - Mean statistics: $c_t = mc_{t-1} + (1 - m) \frac{1}{B} \sum_{i=1}^B g_{\theta_t}(x_i)$
 - Center teacher prediction: $p_t(x) = \frac{\exp((g_{\theta_t}(x)_i - c_t)/\tau_t)}{\sum_k \exp((g_{\theta_t}(x)_k - c_t)/\tau_t)}.$

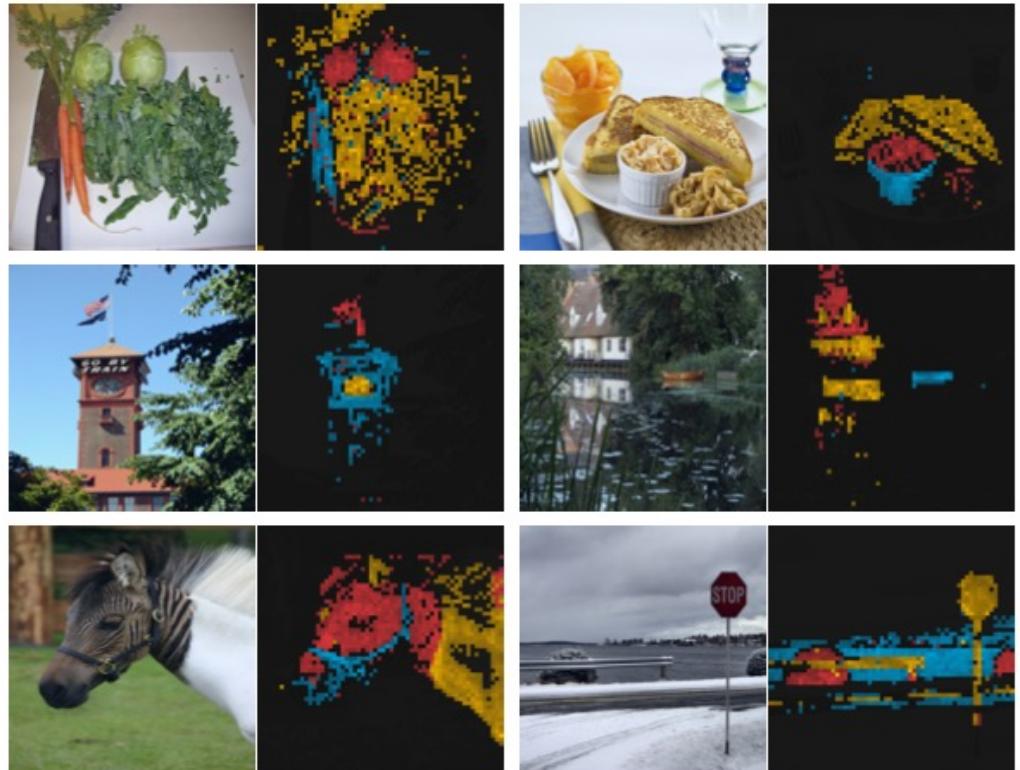
Centering and Sharpening

- Only centering: Always uniform distribution, high entropy, easy to guess.
- Only sharpening: Collapsed into one unit, easy to guess, low loss, but no real learning.



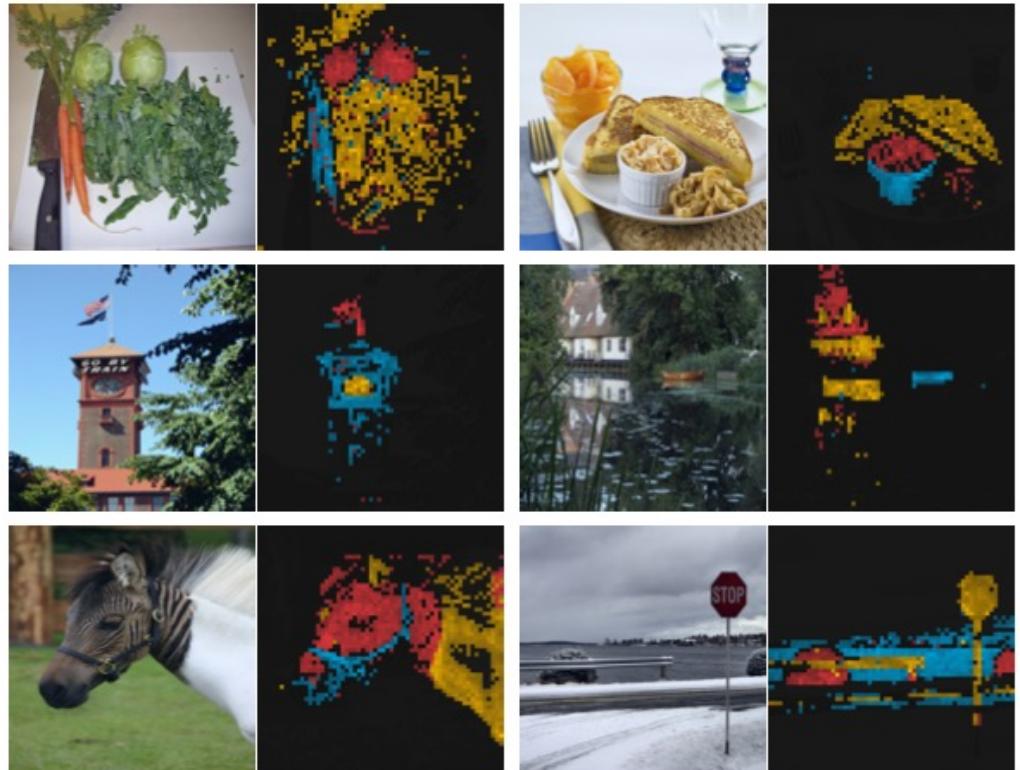
Visualizing Attention

- The [CLS] token is an extra token added to summarize the whole image into a vector.



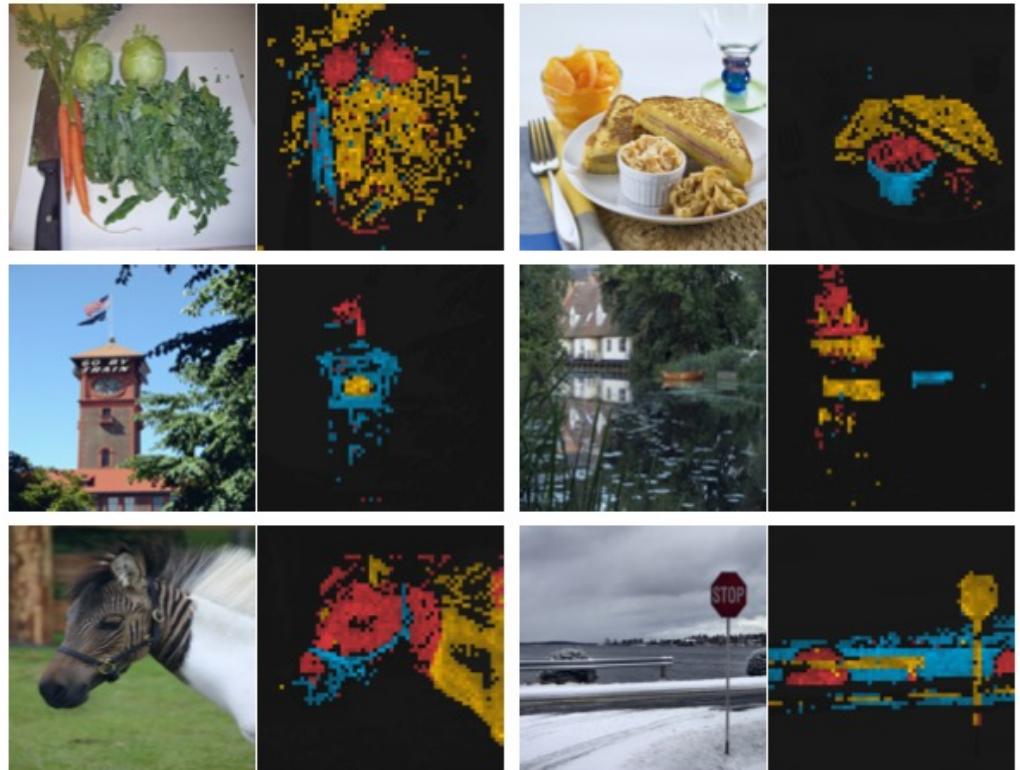
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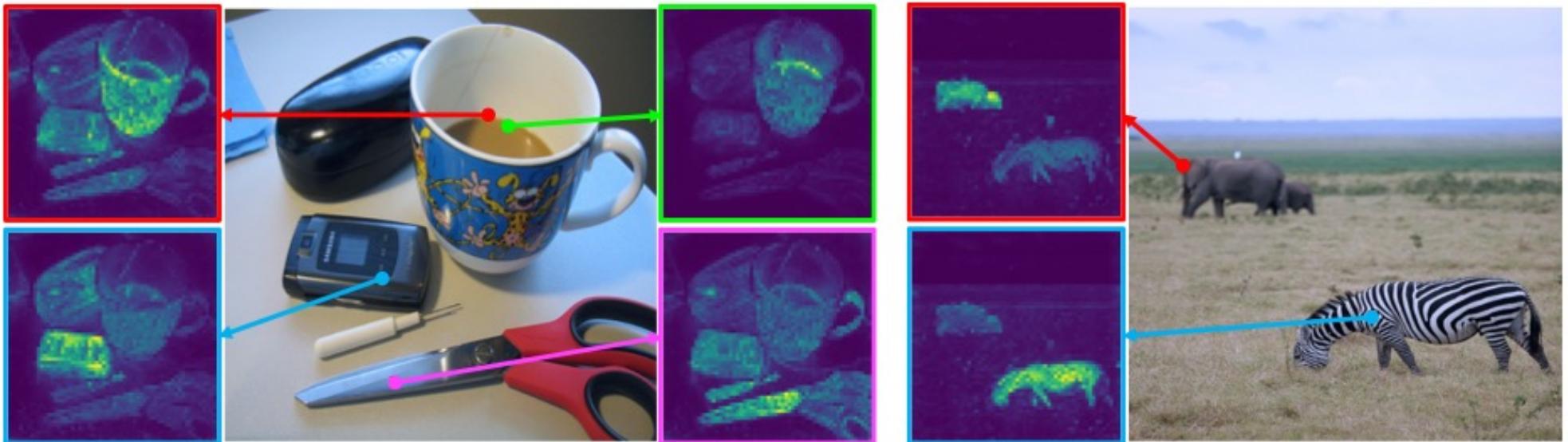
Visualizing Attention

- The [CLS] token is an extra token added to summarize the whole image into a vector.
- Visualize the attention map of different attention heads using different colors.
- Showing understanding of different objects and parts.



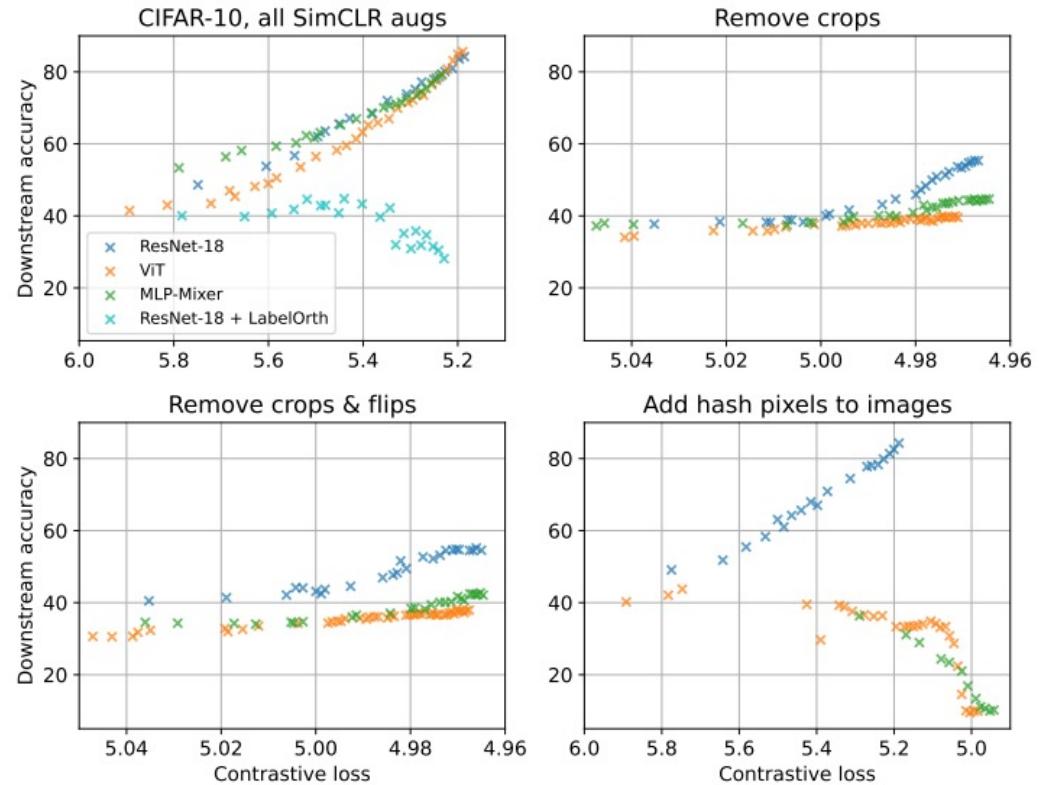
Visualizing Attention

- We can also visualize the attention by querying from a location.
- Weak separation of objects.



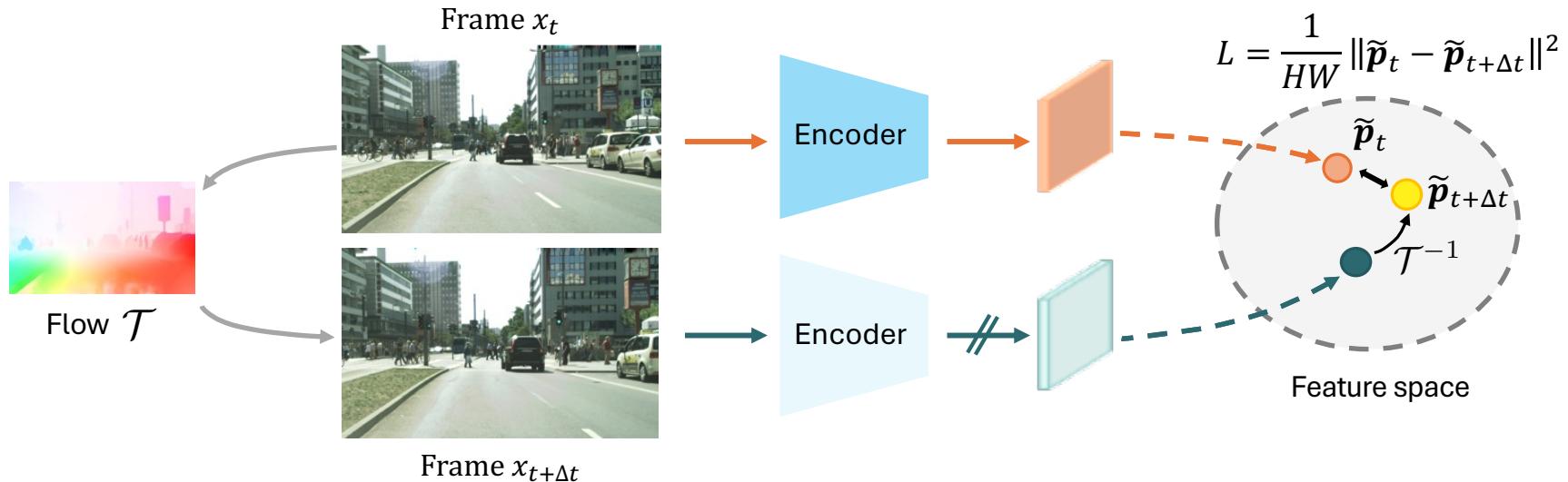
Why Does SSL Work?

- The unsupervised loss is a surrogate. If an image belongs to a similarity class, it also belongs to the same semantic class.
- The choice of similarity class matters.



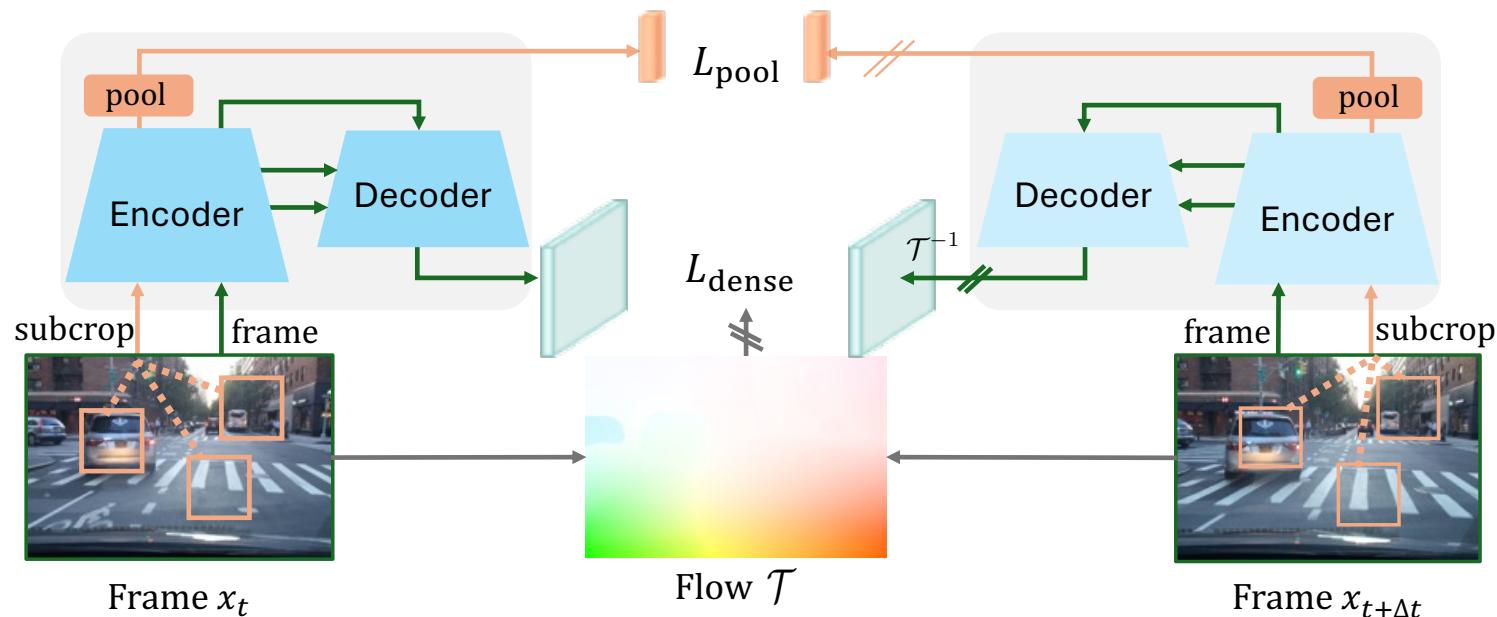
SSL with Motion

- Can we use adjacent frames as self-supervision?
- Objects move densely throughout the image.



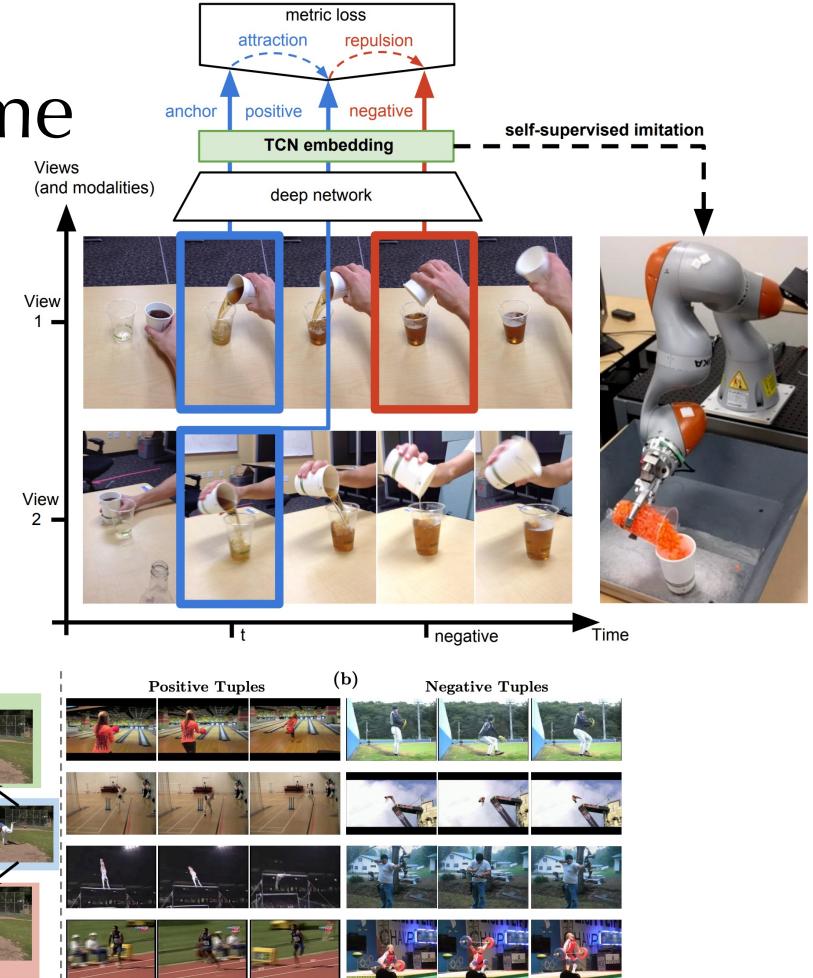
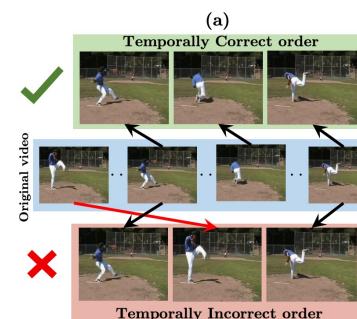
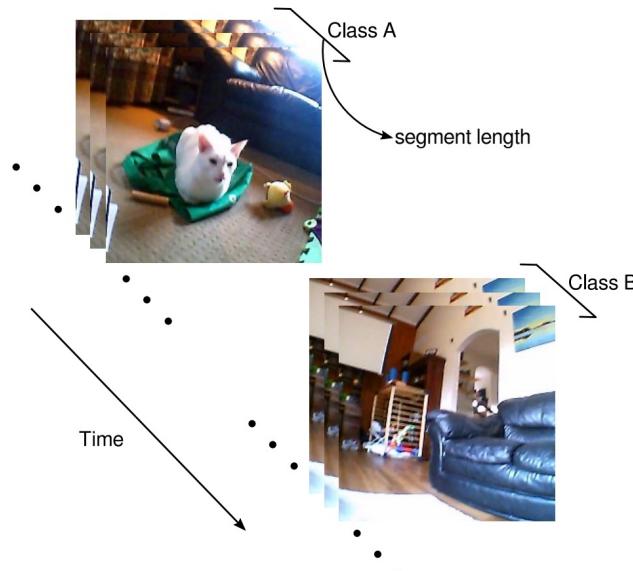
SSL with Motion

- Perform SSL in multiple scales (small objects vs. big regions).



SSL with Time

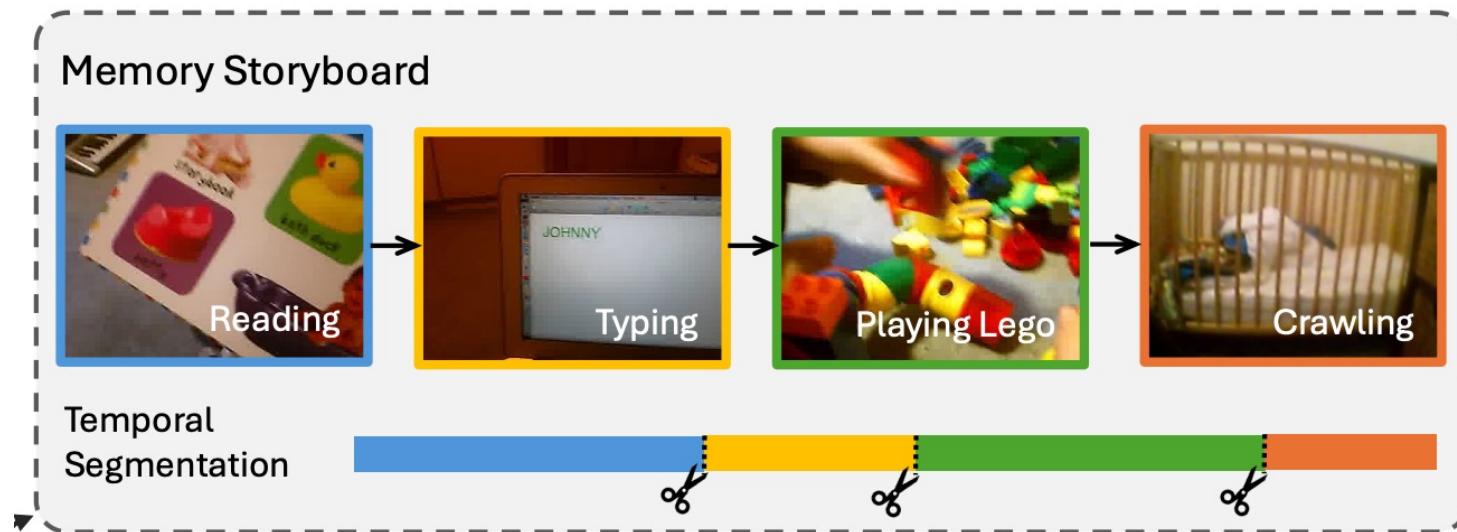
- Use time as an additional source of supervision.



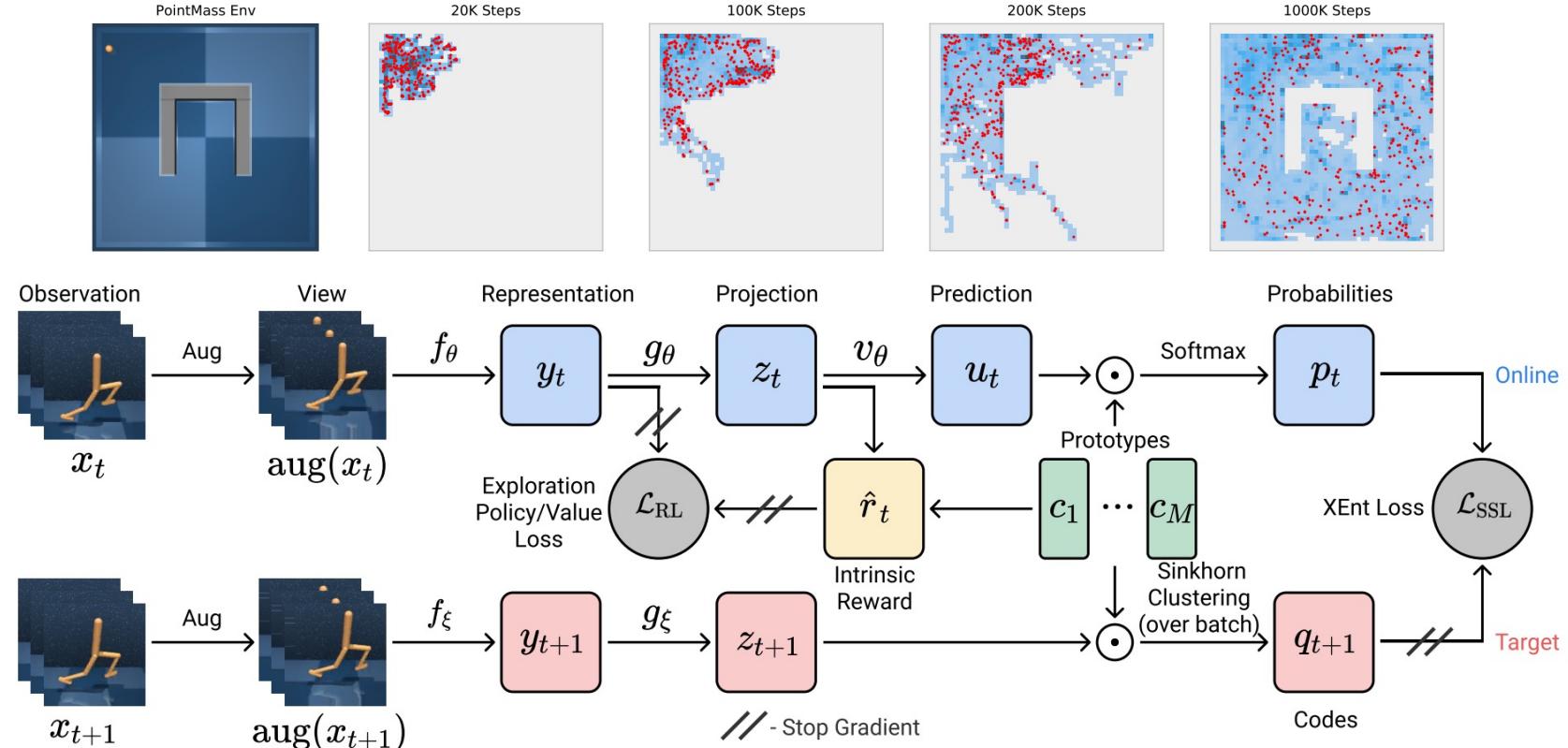
Misra et al. *Shuffle and Learn: Unsupervised Learning using Temporal Order Verification*. ECCV 2016.
Sermanet et al. *Time-Contrastive Networks: Self-Supervised Learning from Video*. ICRA 2018.
Orhan et al. *Self-Supervised Learning through the Eyes of a Child*. NeurIPS 2020.

SSL with Time

- We can segment videos into meaningful events.
- Leverage the spatiotemporal continuity structure.



SSL for Visual Control



SSL for Visual Control

Observations



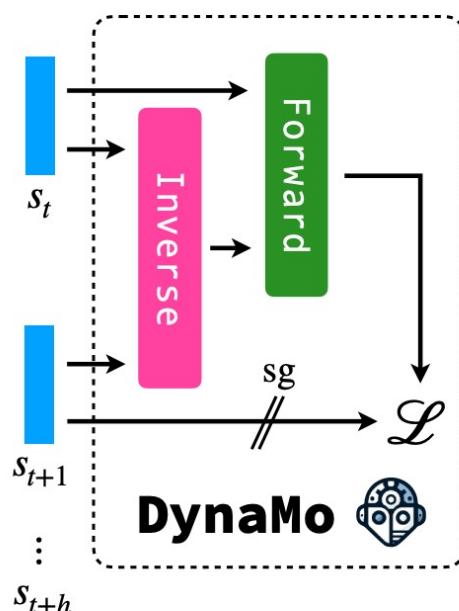
o_t



o_{t+1}

\vdots

o_{t+h}



(a) Representation learning

Embeddings

s_t

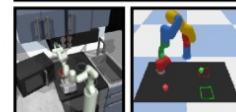
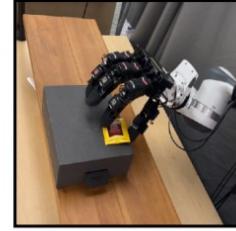
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\vdots

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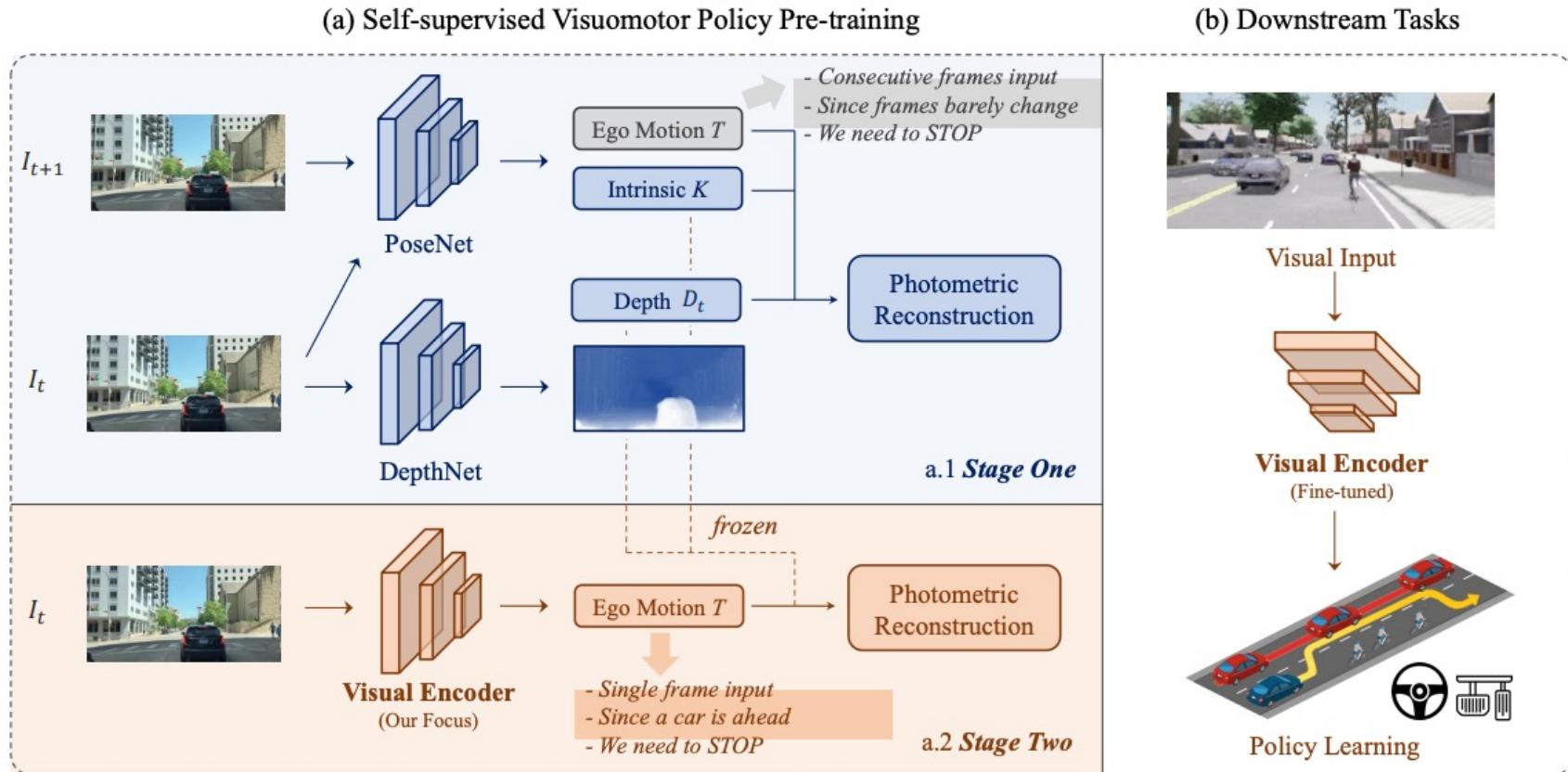


Environments



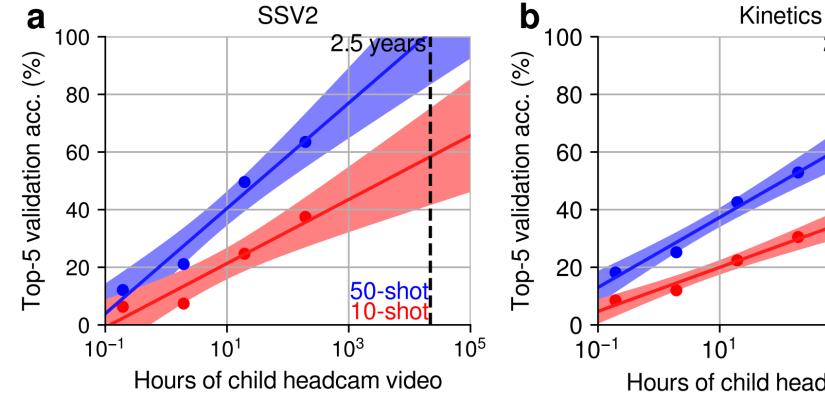
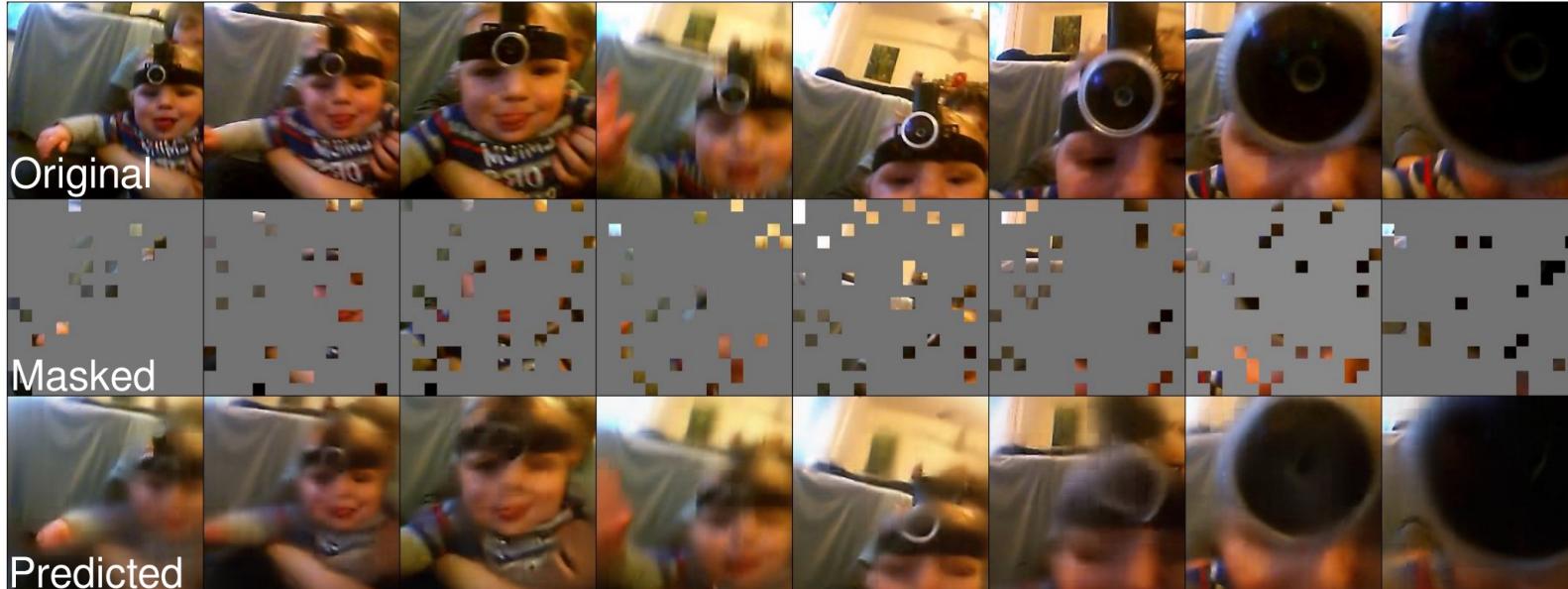
(b) Policy on pretrained representations

SSL for Visual Control



BabyCam

- Run visual learning algorithms on baby headcam videos.



Summary

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- Possible learning objectives for egocentric videos.
- Incorporate 3D vision and actions for downstream planning.

Emergent Attention, Object Discovery

- SSL representations show awareness of object classes and instance identities.

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- Why does attention show awareness of objects?
- The network is encouraged to associate different parts of the objects together in order to identify whether two inputs belong to the same image or not.
- Attending to semantically similar parts facilitates the process.
- The network is a hierarchical information processing pipeline – Lower layers integrate more granular and smaller neighborhood.

Weak-to-Strong Supervision

- General idea: Use self-supervised learning to learn good features, which allow us to generate low-quality masks.

Weak-to-Strong Supervision

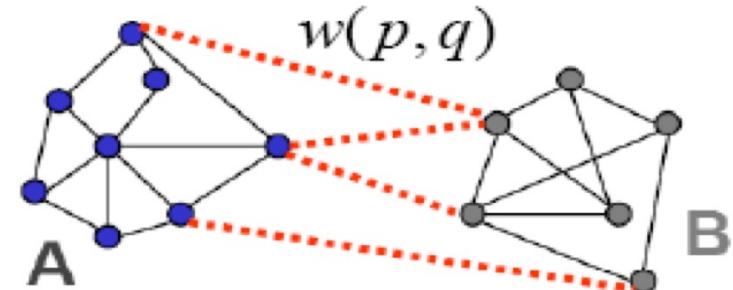
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- Then use these masks as pseudo labels and supervise the network to predict these low-quality masks.

Weak-to-Strong Supervision

- General idea: Use self-supervised learning to learn good features, which allow us to generate low-quality masks.
- Then use these masks as pseudo labels and supervise the network to predict these low-quality masks.
- Question: how do we come up with masks? What loss is used to supervise the network?

Graph Cut

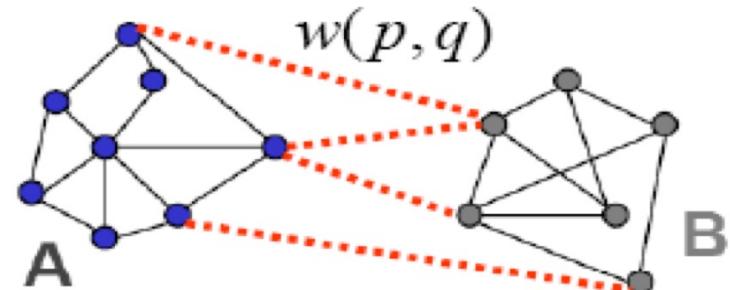
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$$cut(A, B) = \sum_{p \in A, q \in B} w(p, q)$$

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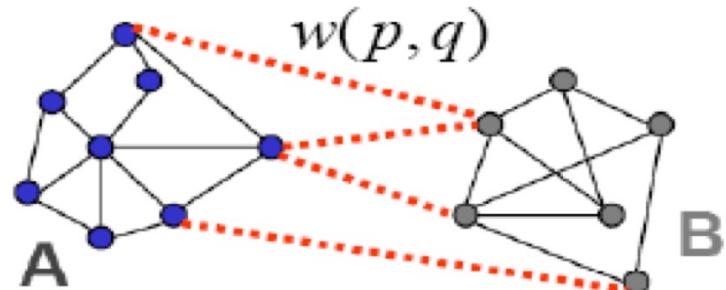
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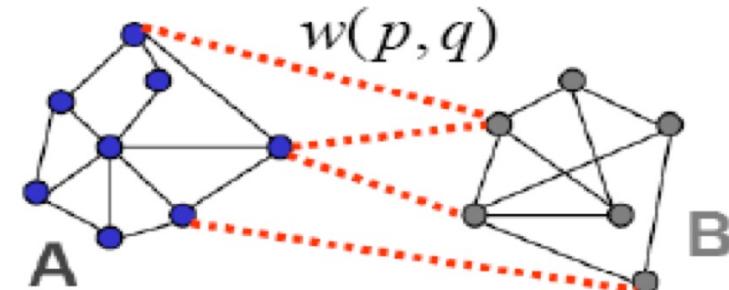
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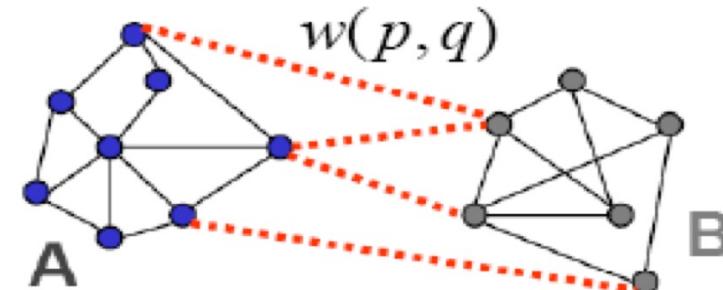
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Graph Cut

- Segmentation is essentially a clustering problem.
- We can transform the clustering problem with the graph cut problem.
- Pixel = node.
- Affinity between the two pixels = edge value (flow).
- Objective: Cut the graph into disconnected components with a minimum sum of edge values.

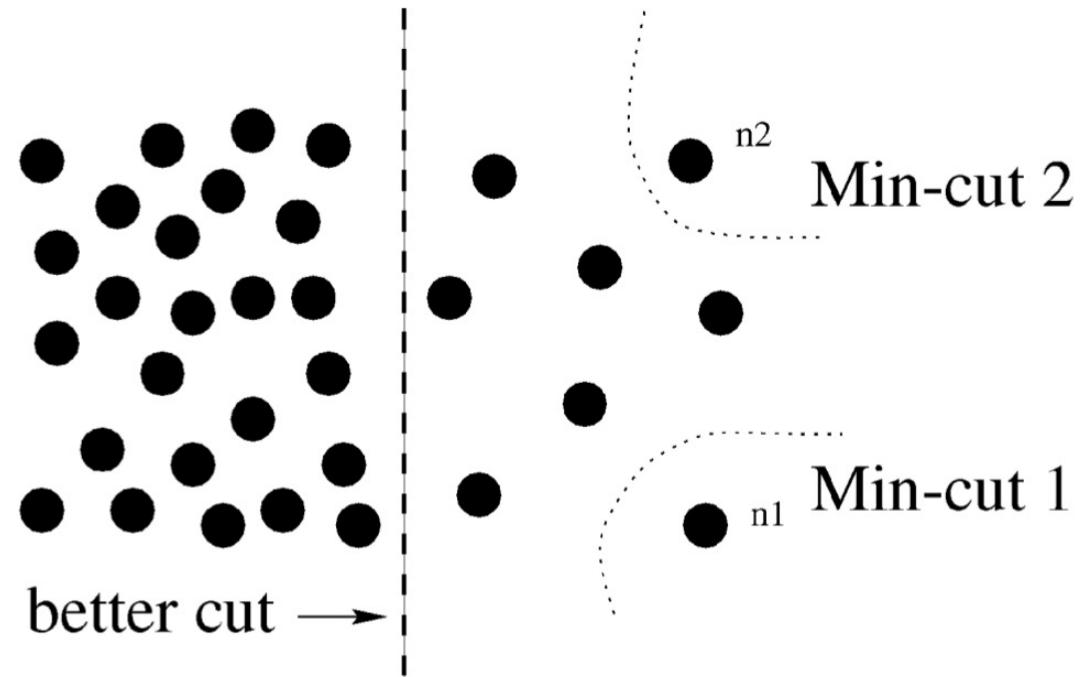


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Normalized Graph Cut (NCut)

- How to prevent cutting small isolated nodes?

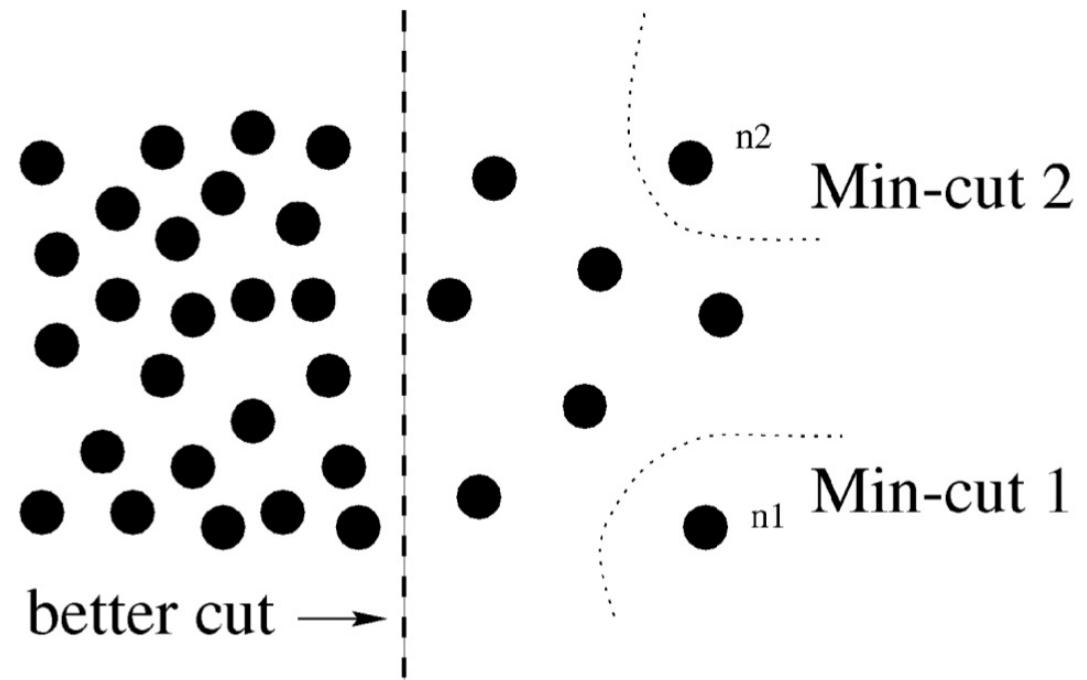
$$Ncut(A, B) = \frac{cut(A, B)}{assoc(A, V)} + \frac{cut(A, B)}{assoc(B, V)}$$



Normalized Graph Cut (NCut)

- How to prevent cutting small isolated nodes?
- Normalize by the total edge connections of a group to all the nodes.

$$Ncut(A, B) = \frac{cut(A, B)}{assoc(A, V)} + \frac{cut(A, B)}{assoc(B, V)}$$



NCut Details (Optional)

- A form of spectral clustering.
- Degree matrix D $N \times N$ with d_i on the diagonal.
- Weight matrix W $N \times N$ symmetric w_{ij} .
- Selection vector $x_i = 1$ if $i \in A$ otherwise -1 .
- Solve the minimization: $\min_y \frac{y^\top (D - W)y}{y^\top D y} \quad y = (1 + x) - \frac{\sum_{i|x_i > 0} d_i}{\sum_{i|x_i < 0} d_i} (1 - x)$.
- Generalized eigenvalue system: $(D - W)y = \lambda Dy$.
- Let $z = D^{1/2}y \quad D^{-\frac{1}{2}}(D - W)D^{-\frac{1}{2}}z = \lambda z$.

NCut

- Sort the eigenvectors from the smallest to the largest.



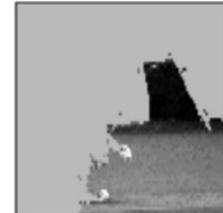
(a)



(b)



(c)



(d)



(e)



(f)



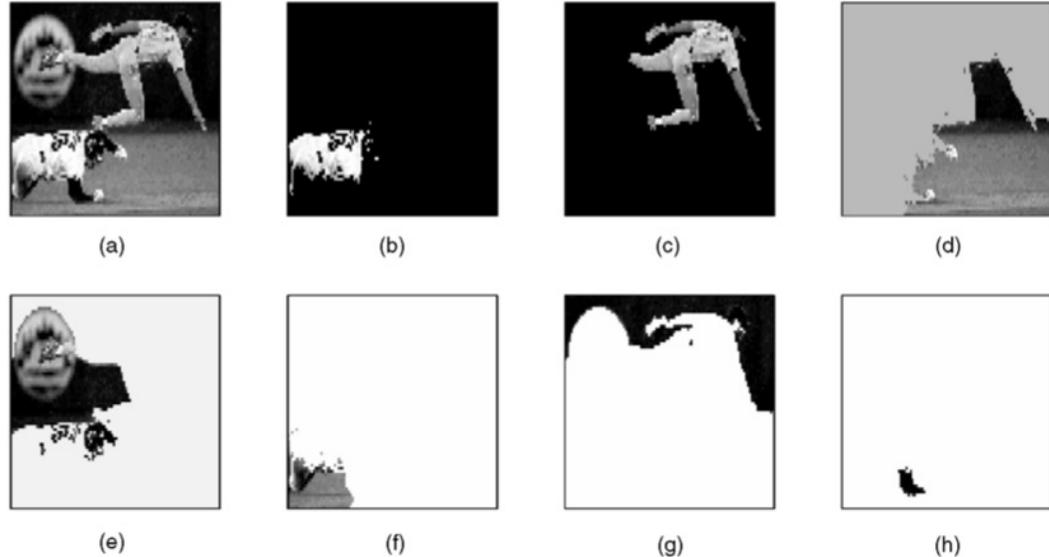
(g)



(h)

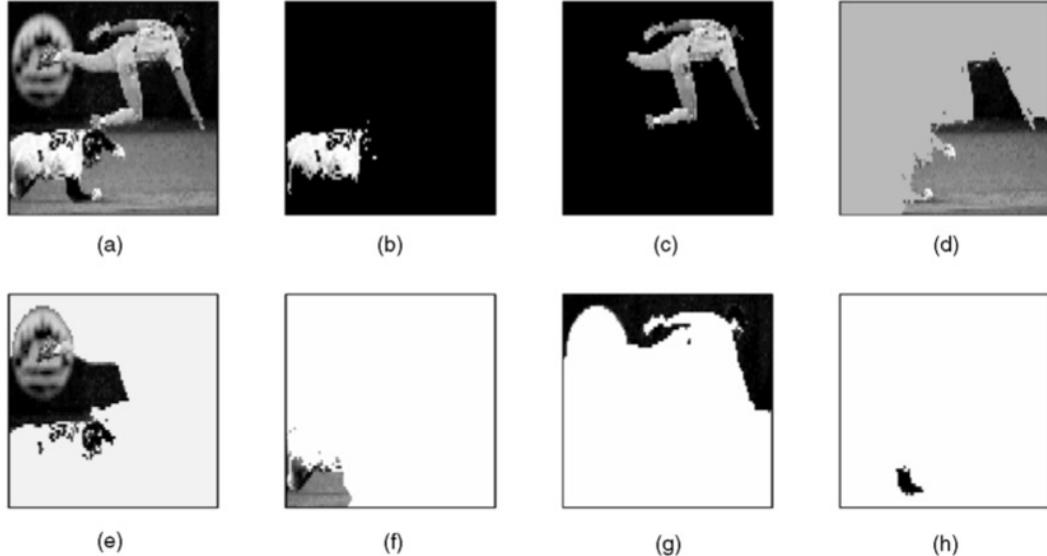
NCut

- Sort the eigenvectors from the smallest to the largest.
- This was a classic image segmentation technique operating directly on image intensity.



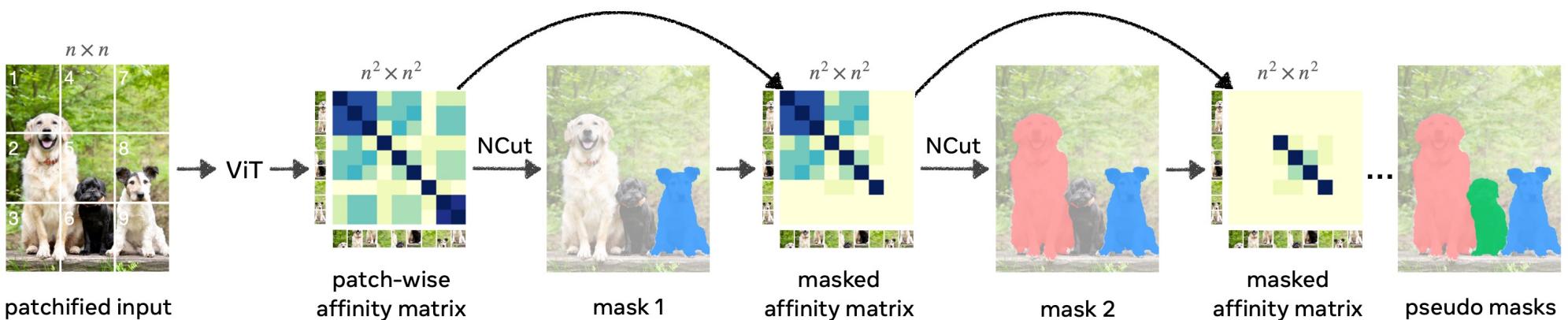
NCut

- Sort the eigenvectors from the smallest to the largest.
- This was a classic image segmentation technique operating directly on image intensity.
- Now, instead of segmenting pixels, we can directly segment semantically meaningful representations from self-supervision.



MaskCut

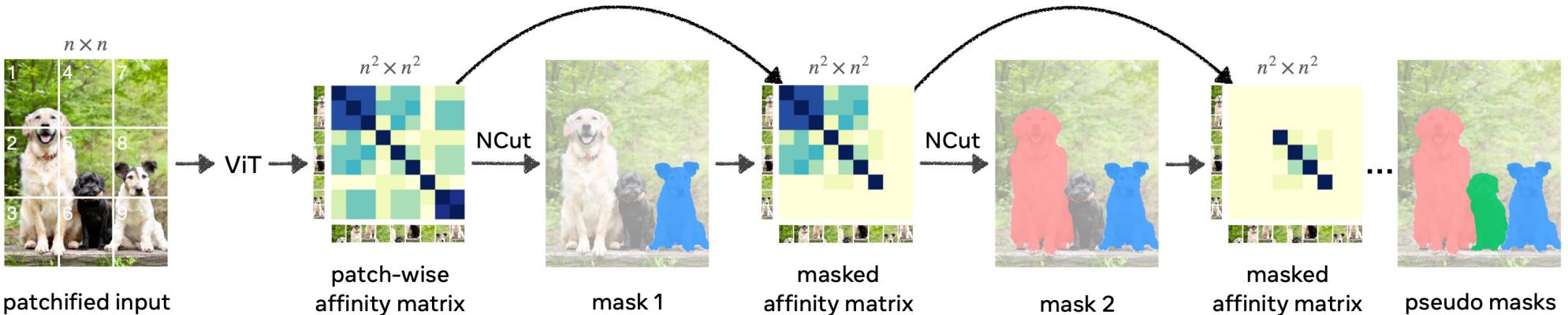
- Use a pretrained DINO ViT network.



Wang et al. Self-supervised transformers for unsupervised object discovery using normalized cut. CVPR 2022.
Wang et al. Cut and Learn for Unsupervised Object Detection and Instance Segmentation. CVPR 2023.

MaskCut

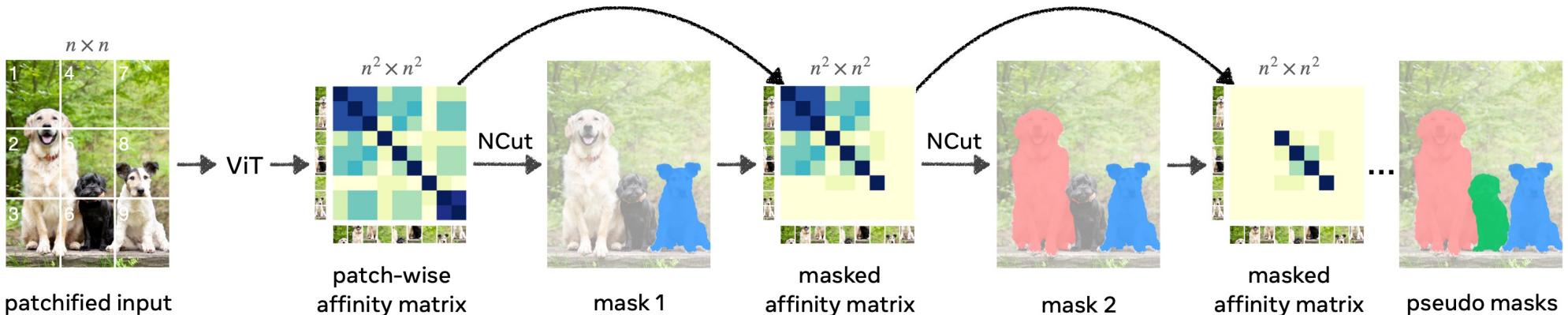
- Use a pretrained DINO ViT network.
- Use the “key” features from the last attention layer: $W_{ij} = \frac{K_i K_j}{\|K_i\|_2 \|K_j\|_2}$



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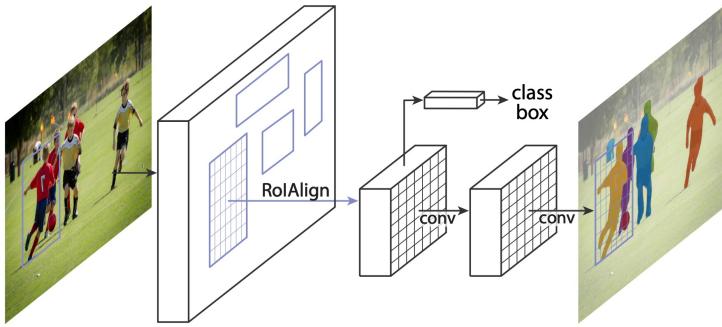
MaskCut

- Use a pretrained DINO ViT network.
- Use the “key” features from the last attention layer: $W_{ij} = \frac{K_i K_j}{\|K_i\|_2 \|K_j\|_2}$
- Iterative NCut on the pairwise matrix by masking out the regions from previous stages.



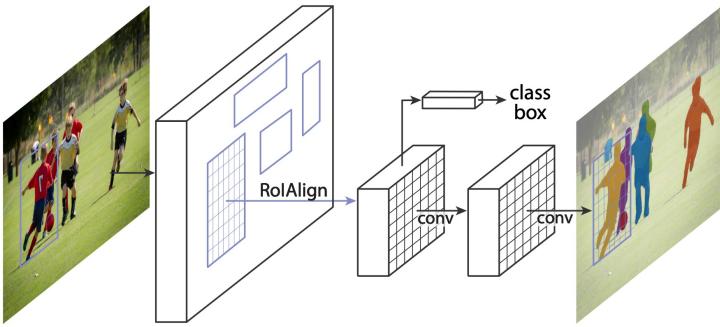
Iterative Self-Training

- Now add a MaskRCNN structure on top of the pretrained network.



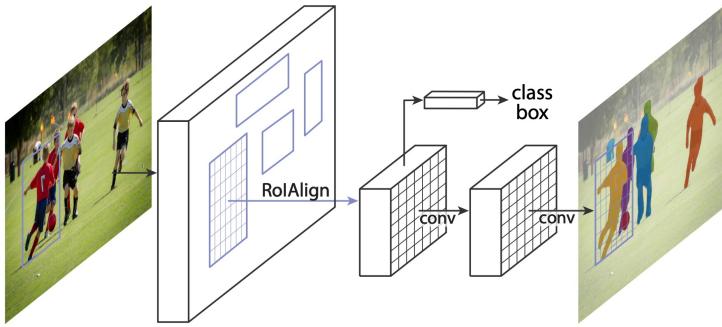
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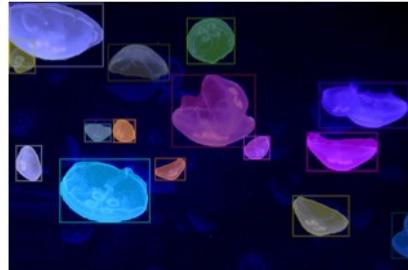
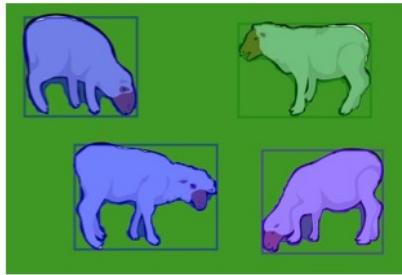


Iterative Self-Training

- Now add a MaskRCNN structure on top of the pretrained network.
- Select the predictions with the highest confidence score and use them as labels.
- Neural networks can learn from the noisy labels and output smoother predictions.

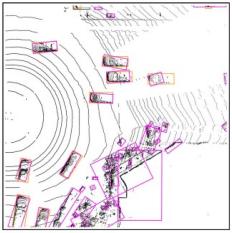


More Visualization

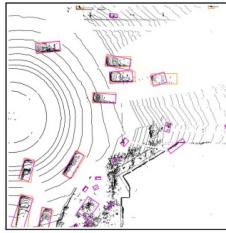


Pseudo Labels in 3D

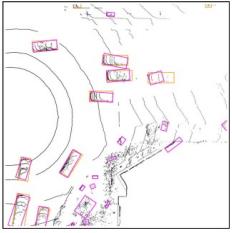
- 1 Point clustering pseudo-labels



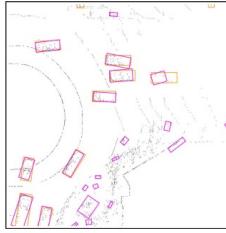
- 2 Filter out temporally inconsistent tracks



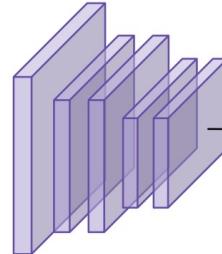
- 3 Randomly drop lidar beams



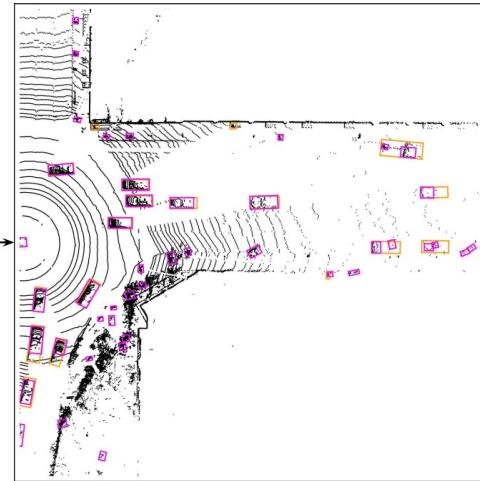
- 4 Randomly drop spherical rows/cols



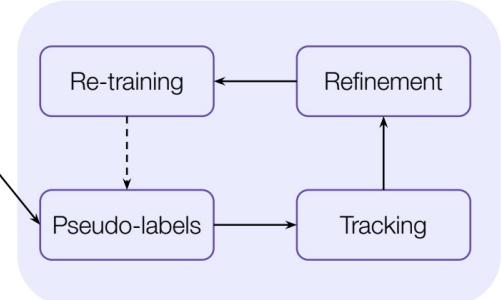
- 5 Train CNN in short range



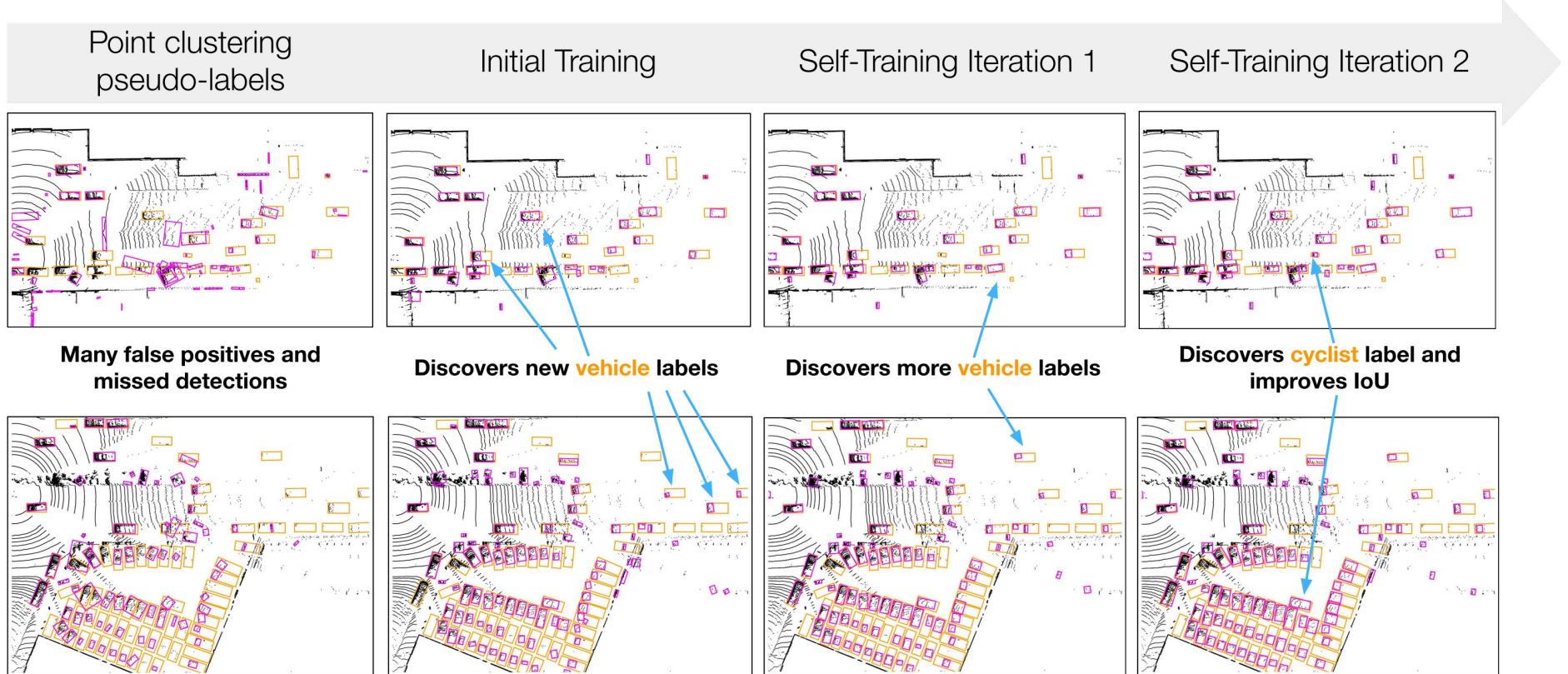
- 6 Zero-shot generalization to long-range



- 7 Self-training in long-range

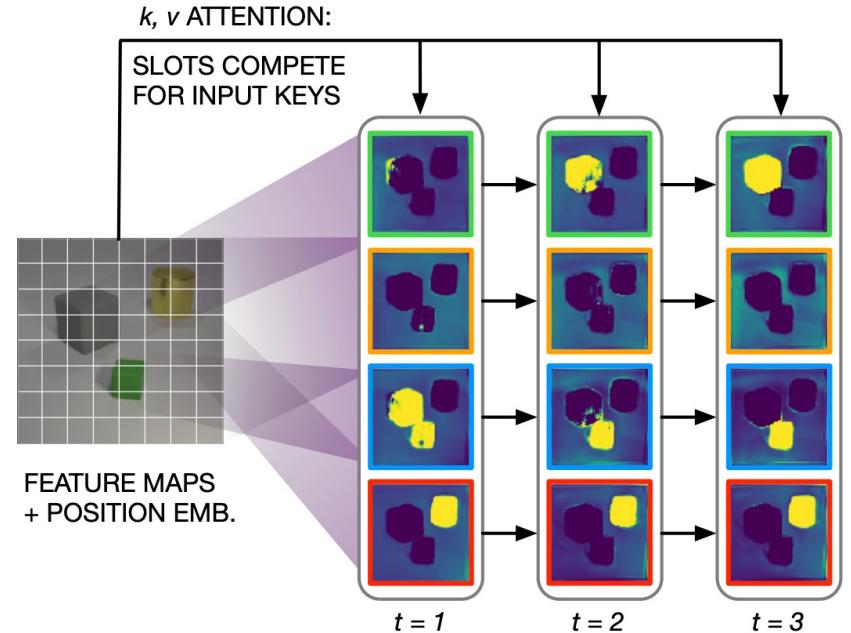


Iterative Refinement of Pseudo Labels



Slot Attention Networks

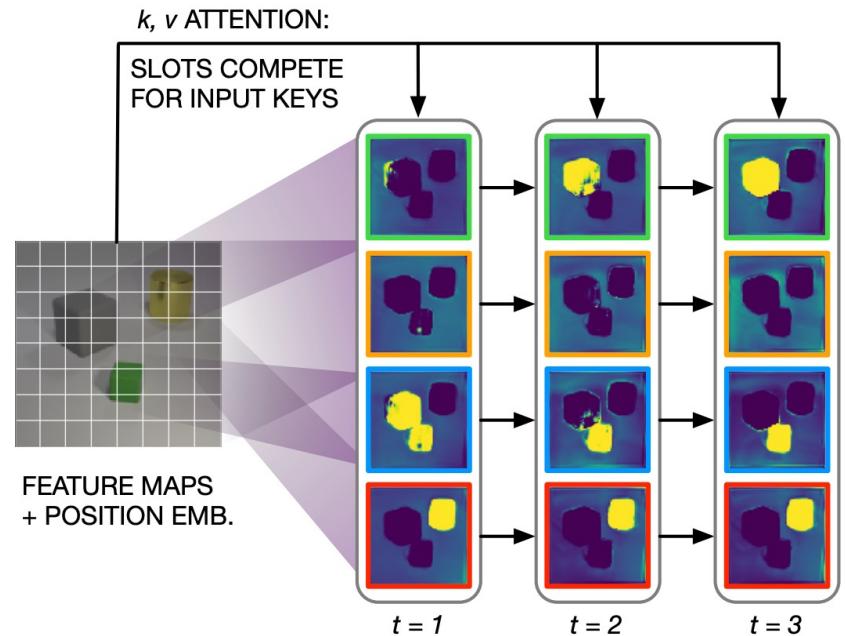
- Can we learn clustering as an end-to-end operation?



(a) Slot Attention module.

Slot Attention Networks

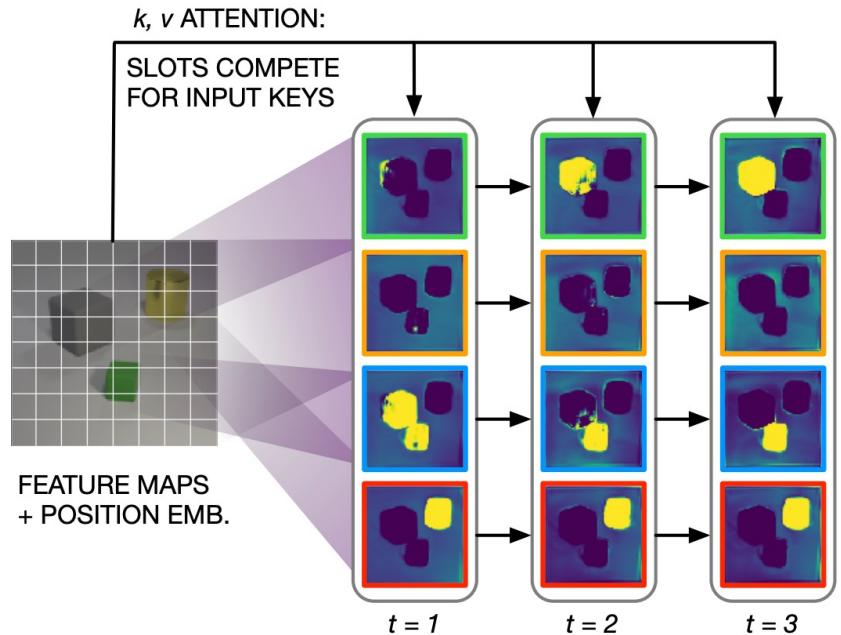
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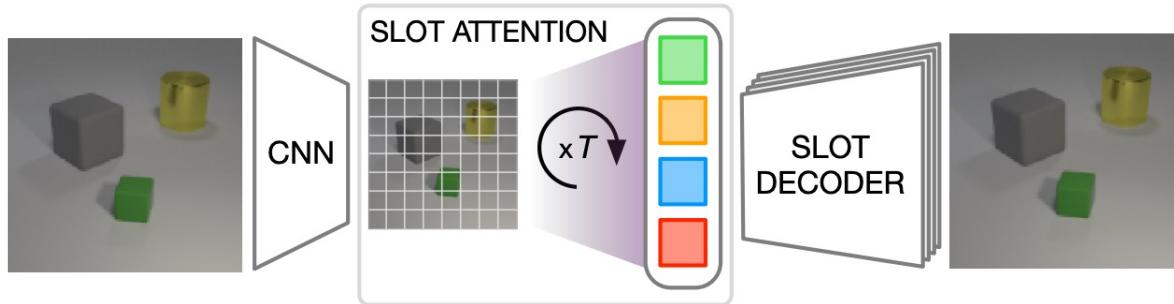
- Can we learn clustering as an end-to-end operation?
- Slot attention is inspired by the success of the attention mechanism.
- Each “slot” attends to a region of the image and stores an object centric representation.



(a) Slot Attention module.

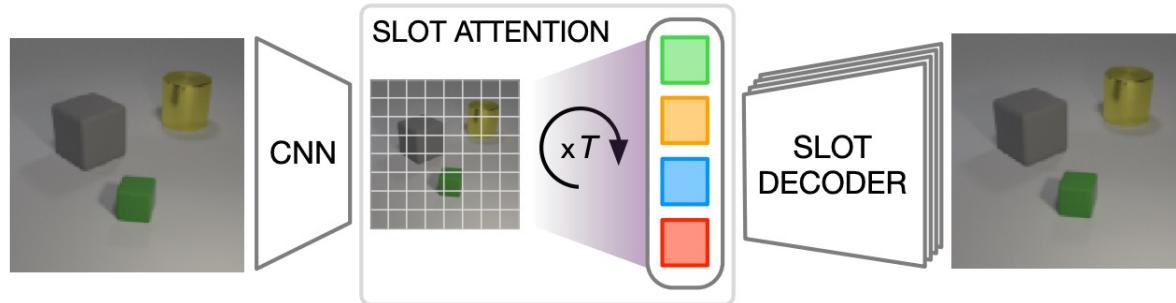
Slot Attention Networks

- Goal: Reconstruct the image with a concise slot-based representation.



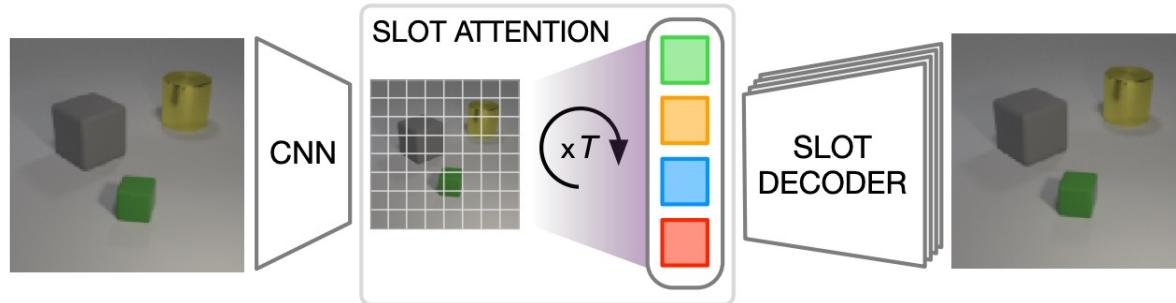
Slot Attention Networks

- Goal: Reconstruct the image with a concise slot-based representation.
- Input: $x \in \mathbb{R}^{N \times D}$ (after encoder), Slots: $m \in \mathbb{R}^{M \times D}$. Normalize:
 $\tilde{m}_{t-1} = LN(m_{t-1})$.



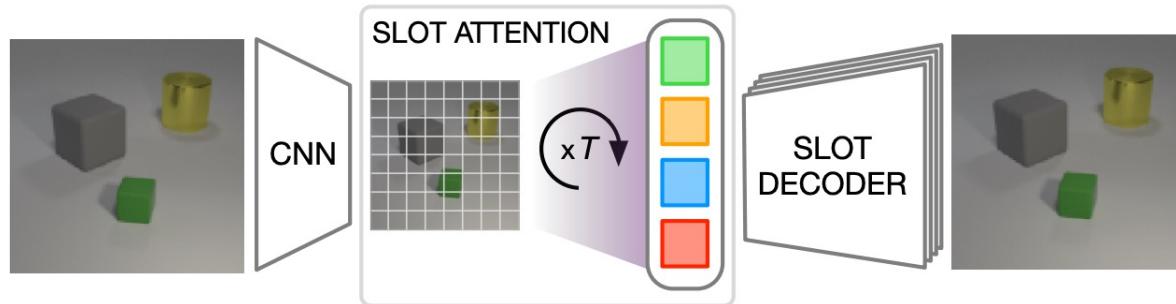
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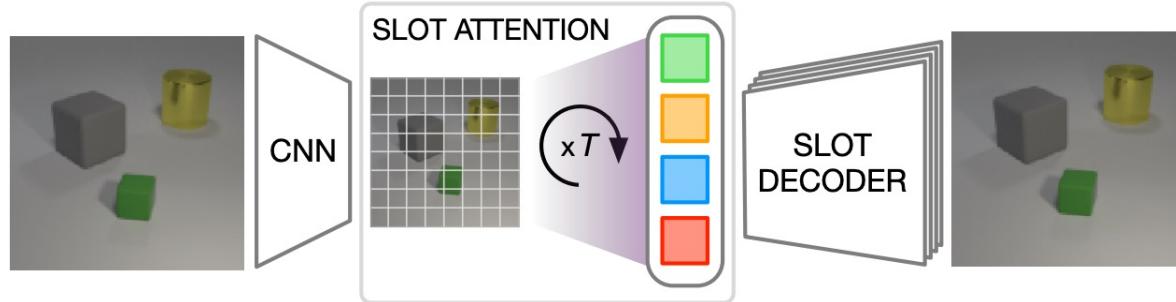
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- Updates: $u_{tj} = \sum_i a_{tij} v(x_i).$

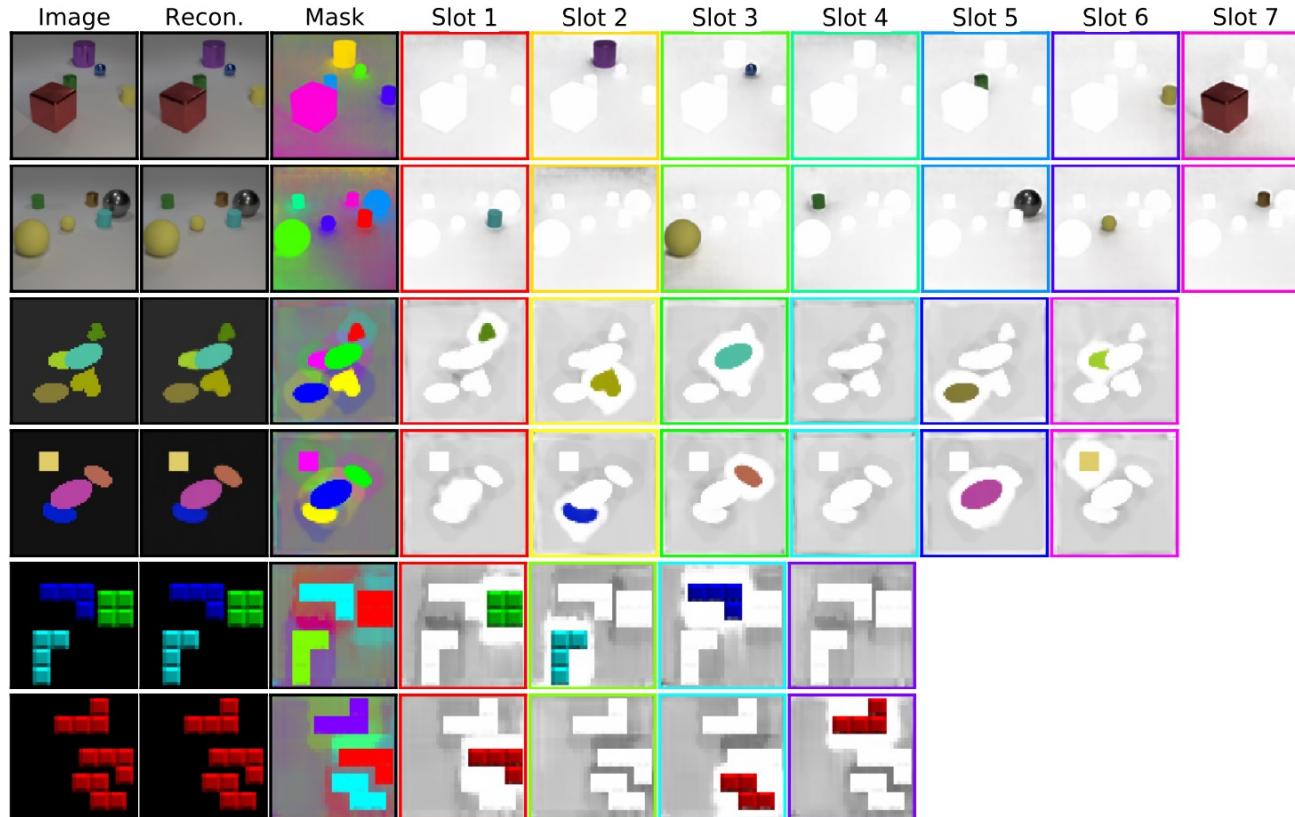


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- Write into slots: $m_t = GRU(m_{t-1}, u_t) + MLP(\tilde{m}_{t-1})$.

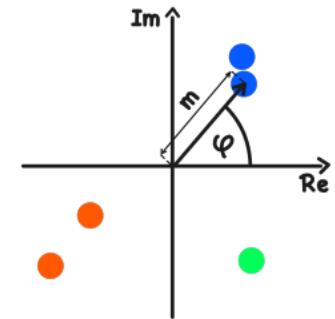
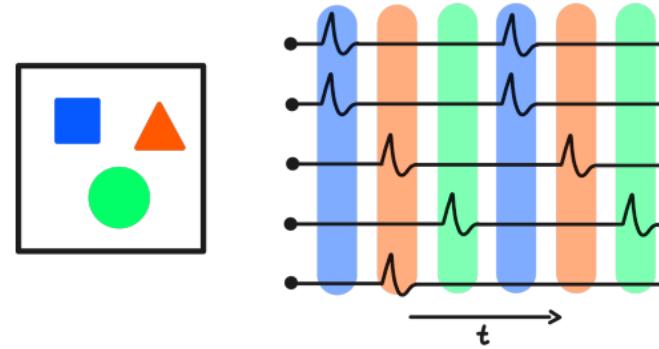
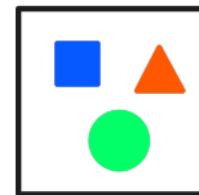
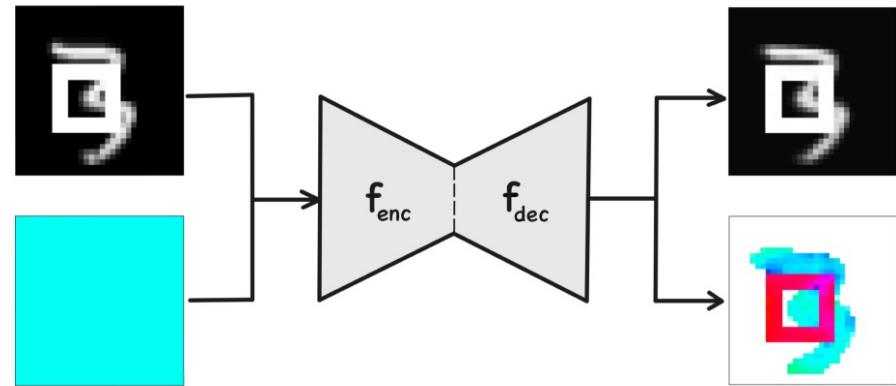


Slot Attention Networks



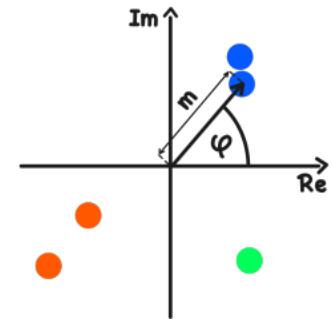
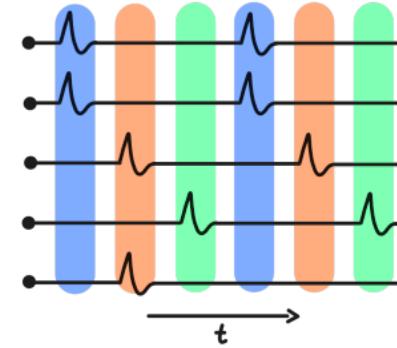
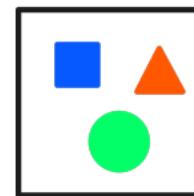
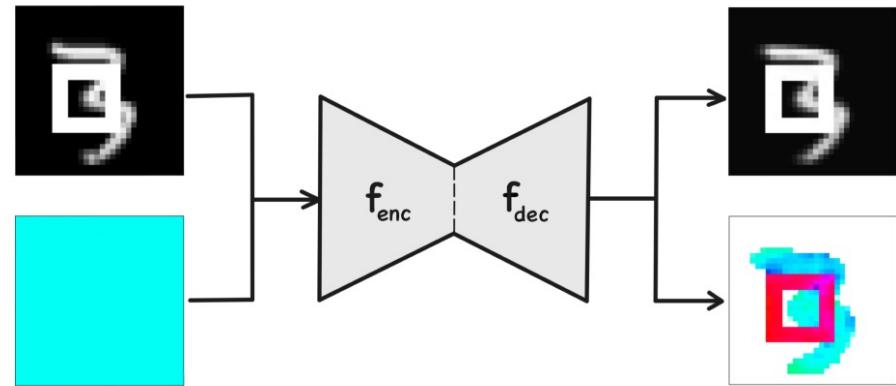
Complex-Valued Autoencoders (CAEs)

- The complex number can represent magnitude and phase: $z = m \cdot e^{i\varphi} \in \mathbb{C}$.



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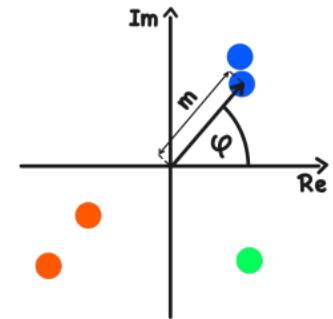
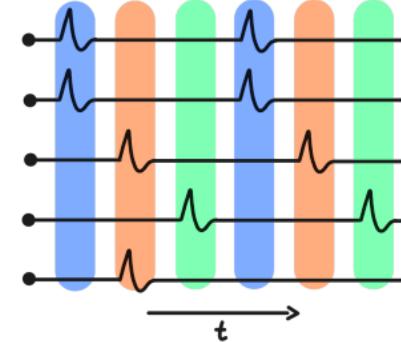
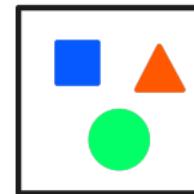
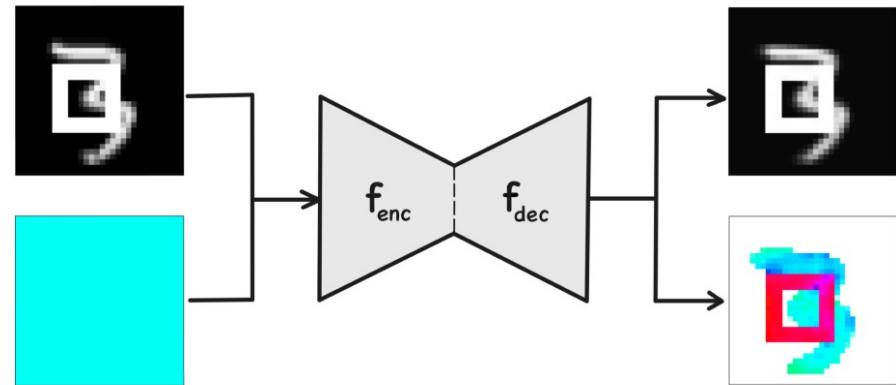
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$$\hat{\mathbf{z}} = f_{\text{dec}}(f_{\text{enc}}(\mathbf{x})) \in \mathbb{C}^{h \times w}.$$

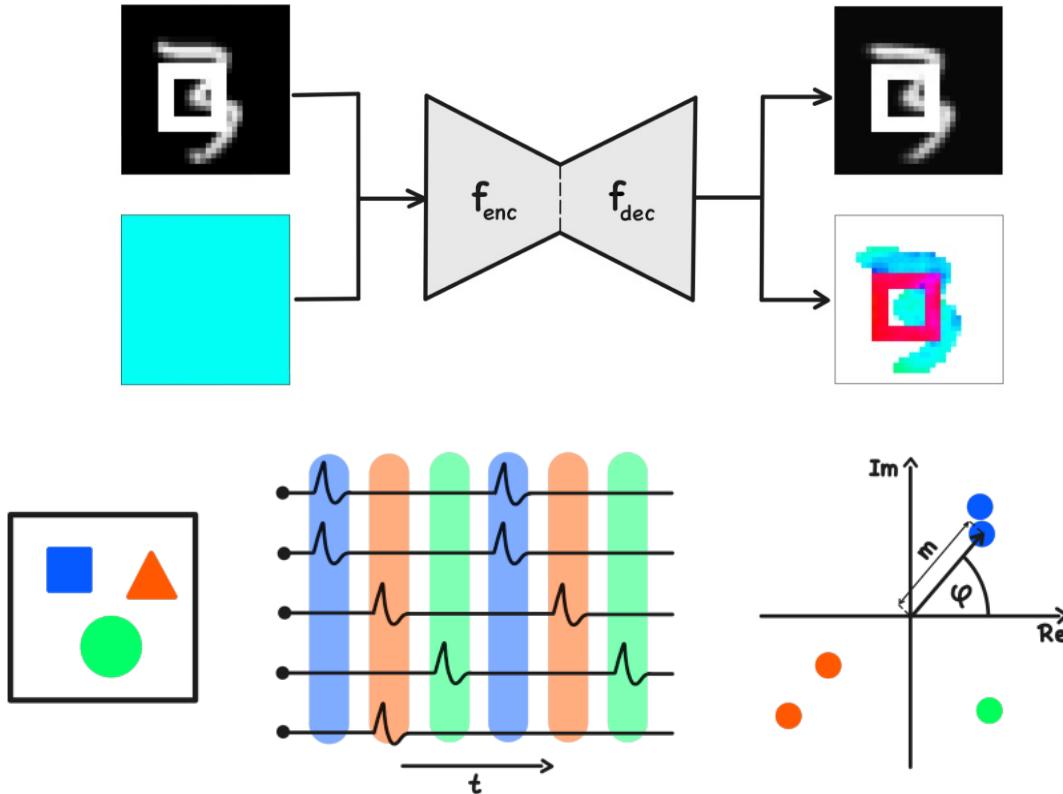


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CAE: More Details

- Apply weights separately to real and imaginary:

$$\psi = f_{\mathbf{w}}(\mathbf{z}) = f_{\mathbf{w}}(\text{Re}(\mathbf{z})) + f_{\mathbf{w}}(\text{Im}(\mathbf{z})) \cdot i \in \mathbb{C}^{d_{\text{out}}}$$

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$$m_\psi = |\psi| + b_m \quad \in \mathbb{R}^{d_{\text{out}}} \quad \varphi_\psi = \arg(\psi) + b_\varphi \quad \in \mathbb{R}^{d_{\text{out}}}$$

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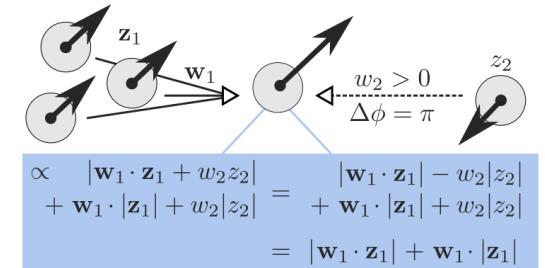
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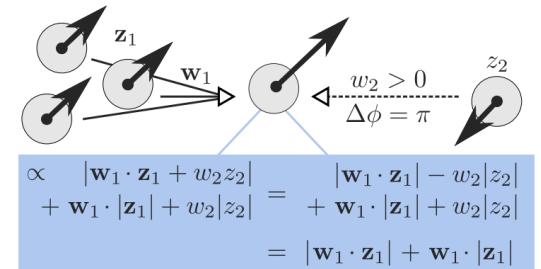
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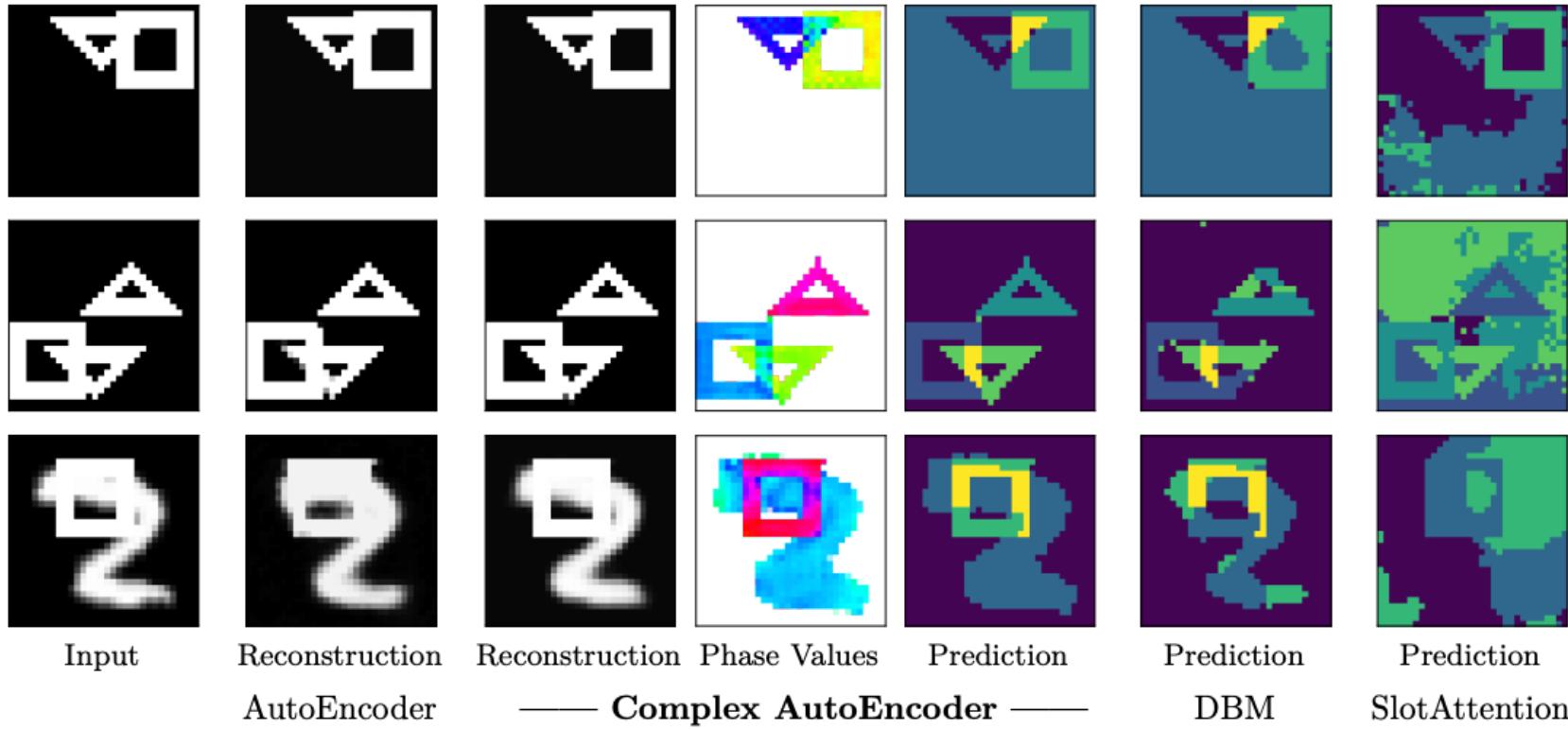
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- Activation $\mathbf{z}' = \text{ReLU}(\text{BatchNorm}(\mathbf{m}_{\mathbf{z}})) \circ e^{i\varphi_{\psi}} \in \mathbb{C}^{d_{\text{out}}}$



Complex-Valued Autoencoders



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- What do we make use of the discovered objects? Is it better to keep the awareness in the latent space?