Naive Simulation of a U(1) Gauge Field on a 2D Lattice

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1 Introduction

For the assignment I wrote a lattice simulation in C which

- generates an ensemble of U(1) gauge configurations on a 2 dimensional periodic lattice, via a metropolis algorithm, according to the Wilson gauge action.
- calculates the expectation value of two polyokov loops at varying spacial separations on the lattice, in the interest of studying confinement in the theory.

It also contains tools to;

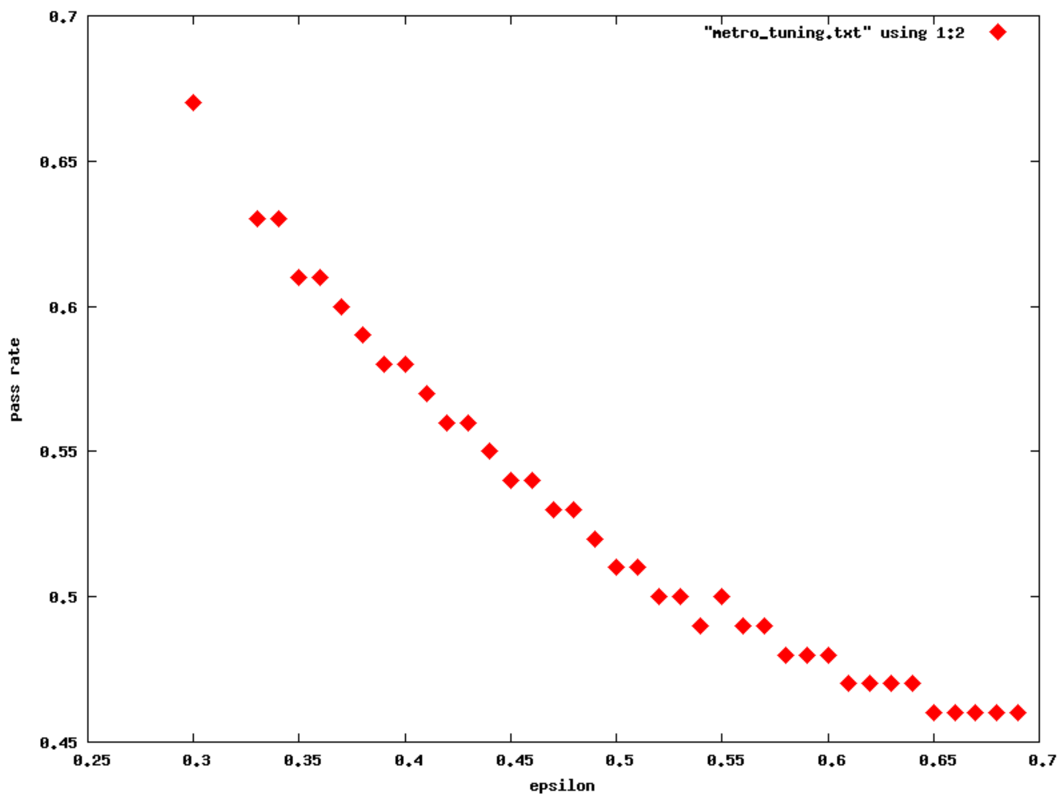
- test the Metropolis algorithm for different values of ϵ, the allowed range of random phase which is added to the links in each update, and see which delivers the optimal "pass rate".
- deduces the autocorrelations which arise in configurations of different separations in Monte Carlo time.

2 Metropolis Tuning

The Metropolis algorithm works by sweeping through the lattice, multiplying each link $U(x,t)$ by a pseudorandom phase between 0 and $2\pi\epsilon$. The change is accepted if

- $S[U_t] < S[U_{t-1}]$, where U_t is the t'th configuration generated and $S[U_t]$ is the action according to U_t.
- $\exp(-(S[U_t] - S[U_{t-1}])) > r$ where r is a pseudorandom number between 0 and 1.

One hopes the procedure accepts around half of the changes, so that there is sufficient fluctuation between configurations but they do not fluctuate too far from the minimum of S. The pass rate is strongly dependant on ϵ. So I tested the algorithm for a range of ϵ and studied pass rate:



The results above were deduced from a 6^2 lattice and an ensemble size of $N_{cfg} = 1000$. These results coupled with the above discussion lead to the decision of $\epsilon = 0.51$ for the simulation.

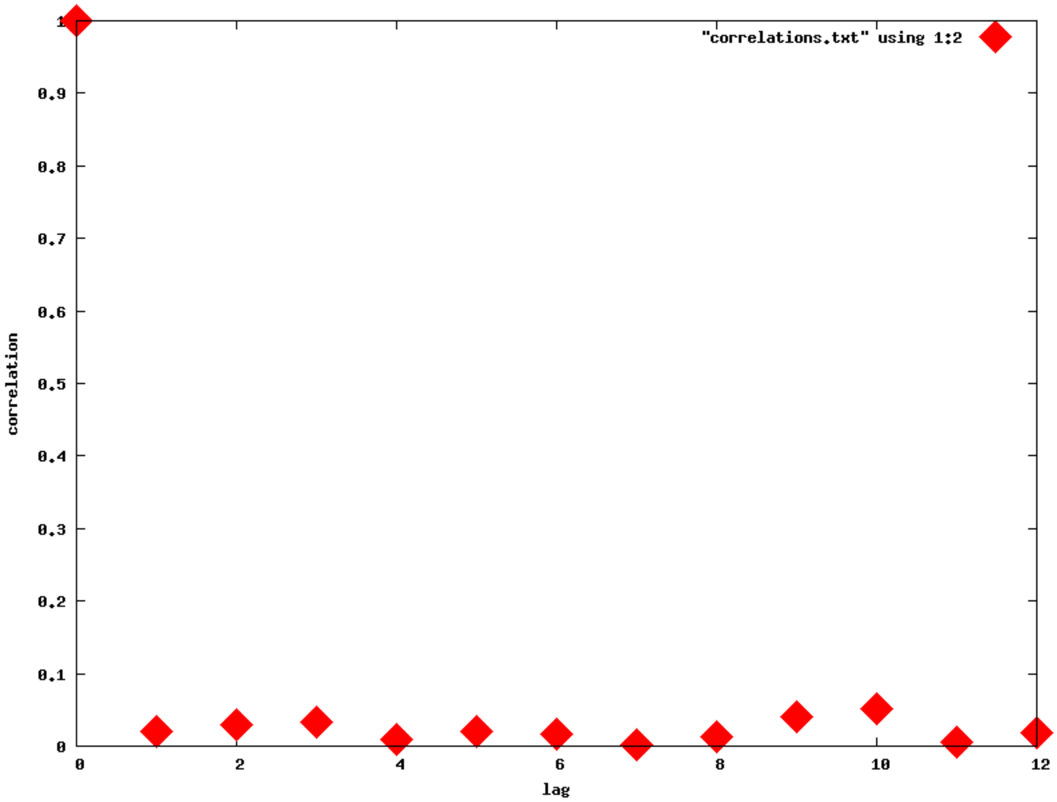
3 Correlations in the Ensemble

Correlations between configurations adjacent in the ensemble can cause statistical errors. There is therefore motivation to study the "correlation time" (separation between configurations in the ensemble where their correlation shrinks to $\approx 1/e$), and discard all configurations except those separated by the correlation time.

The average correlation between configurations separated by τ is

$$C(\tau) = N \sum_{t=\tau}^{N} (\ O[U_t] - \langle O \rangle\ )(\ O[U_{t-\tau}] - \langle O \rangle\ )\,, \tag{3.1}$$

where O is some observable, $\langle O \rangle$ is the average of O over the ensemble, and N is chosen such that $C(0) = 1$.



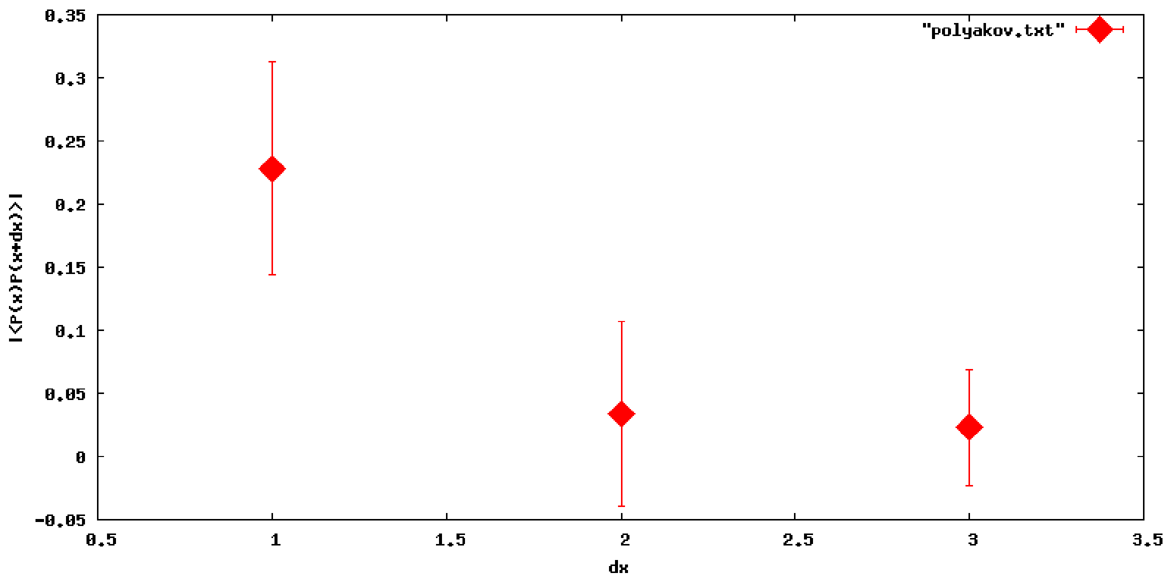
The plot above shows $C(\tau)$ against τ with O as a Polyakov loop, on a 6^2 lattice with $N_{cfg} = 2000$. The configurations between each sweep of the lattice were kept for the ensemble. As one can see, there is very little correlation within this set, so the complete ensemble can be kept.

4 Polyokov Loops

A Polyokov loop is the product of links all in the time direction on a fixed spacial index

$$P[U](x) = \prod_t U_t(x,t), \tag{4.1}$$

where now t is a time index on the lattice as opposed to Monte Carlo time. It has the physical interpretation of a stationary charge. The averaged over ensemble product of two P's is an estimate of the amplitude for such a configuration of charges. For a strongly coupled theory, the amplitude should decrease exponentially with spacial separation.



The above is a plot of the absolute value of the expectation value of two loops against their spacial separation. The measurements were taken on a 6^2 lattice with $N_{cfg} = 2000$, and an action with coupling $\beta = 0.2$. The deviation of the measurements were estimated using a Bootstrap analysis.