IMPULSE APPROXIMATION IMPROVED

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Abstract. Improved formulas of impulse approximation method for stellar perturbations are derived. The method proposed involves a deflection of the stellar path. It is also applicable to an arbitrary time interval. A comparison of the classical vs improved method is presented both in qualitative discussion and numerical results for Oort cloud cometary orbits.

Key words: Impulse approximation, stellar perturbations, Oort cloud.

1. Introduction

The idea of the impulse approximation (known also as the impact one) was introduced by Öpik (1932), as a method of calculating stellar perturbations on a very elongated, heliocentric orbit of a 'meteor'. Oort (1950) adopted it to estimate stellar perturbations on cometary orbits at great (of order 10^5 AU) heliocentric distance. The method is based on an intuitive simplification of the three body problem (Sun-star-comet) by splitting it into two, separate two body problems: (Sun-star and comet-star).

The main reason which justifies this approach is that a stellar perturber moves very fast when compared with the heliocentric motion of a comet at a great distance. In such a case the attraction of a comet by the Sun can be omitted during the relatively very short time of the stellar passage. However, assumptions necessary for this approximation to be valid, were not formulated in explicit form.

2. Classical Method

The only detailed paper on the impulse approximation method containing both explicitly stated simplifying assumptions and derivation of formulas from equations of motion, was presented by Rickman (1976). The heliocentric equation of motion in the three body problem (Sun-comet-star) is:

$$\ddot{\mathbf{r}}_c = -k^2 \frac{\mathbf{r}_c}{r_c^3} + k^2 M_* \left(\frac{\mathbf{r}_* - \mathbf{r}_c}{|\mathbf{r}_* - \mathbf{r}_c|^3} - \frac{\mathbf{r}_*}{r_*^3} \right) , \tag{1}$$

where $\mathbf{r}_c$ and $\mathbf{r}_*$ are the heliocentric positions of a comet and a star respectively. Rickman stated, that the total change in the velocity of a comet in the time interval $\langle t_1, t_2 \rangle$ due to the star's attraction can be found by integrating the second term of the Equation (1):

$$\Delta\mathbf{v} = \Delta\mathbf{v}_c - \Delta\mathbf{v}_s = k^2 M_* \int_{t_1}^{t_2} \frac{\mathbf{r}_* - \mathbf{r}_c}{|\mathbf{r}_* - \mathbf{r}_c|^3}\,\mathrm{d}t - k^2 M_* \int_{t_1}^{t_2} \frac{\mathbf{r}_*}{r_*^3}\,\mathrm{d}t\,. \tag{2}$$

He evaluated these two integrals analytically, making three simplifying assumptions:

(1) The comet's heliocentric radius vector $\mathbf{r}_c$ is constant during the stellar passage.

(2) The time interval of the star path taken into account is large enough to be treated as symmetric with respect to the time of the closest approach both to the Sun and to the comet.

(3) The stellar velocity vector $\mathbf{v}_*$ is also constant.

He obtained, that:

$$|\Delta\mathbf{v}_c| = \frac{2k^2 M_*}{v_* d_c} \sin\left[\arctan\left(\frac{v_* T}{d_c}\right)\right] \tag{3}$$

and

$$|\Delta\mathbf{v}_s| = \frac{2k^2 M_*}{v_* d_s} \sin\left[\arctan\left(\frac{v_* T}{d_s}\right)\right]\,, \tag{4}$$

where $\Delta\mathbf{v}_c$ and $\Delta\mathbf{v}_s$ are the velocity impulses on the comet and the Sun respectively, d_c and d_s are the closest stellar distances to the comet and the Sun, M_* is the star mass in solar masses, k is Gaussian gravitational constant and $T = (t_2 - t_1)/2$. Additionally, he stated that for large T:

$$v_* T \gg d_c \quad \text{and} \quad v_* T \gg d_s \tag{5}$$

the trigonometric coefficient in Equations (3) and (4) can be omitted as it equals one. According to Rickman, the final, vectorial formula is:

$$\Delta\mathbf{v} = \frac{2k^2 M_*}{v_*} \left[\frac{\hat{\mathbf{d}}_c}{d_c} - \frac{\hat{\mathbf{d}}_s}{d_s}\right]\,, \tag{6}$$

where $\hat{\mathbf{d}}_c$ is the unit vector directed from the comet to the star at their closest approach, $\hat{\mathbf{d}}_s$ is the unit vector pointing from the Sun to the star at their closest approach.

This formula, equivalent to that proposed by Oort (1950) has been widely used by many authors, see for example: Weissman (1980, 1982, 1986), Fernandez (1980, 1981), Bailey (1983, 1986), Scholl *et al.* (1982), Yabushita *et al.* (1982), Remy and Mignard (1985), Rickman and Froeschlé (1988).

Only in a paper by Morris and O'Neill (1988) there appeared a first, limited attempt to improve the impulse approximation method. The authors noticed that even for $T = \infty$ the coefficient rejected by Rickman is not equal to one when the deflection of the stellar path is taken into account.

In the next section we show that the integrals in Equation (2) can be obtained in an exact, simple manner, using the two body problem and reducing the number of assumptions to only the first one.

3. Improved Method

Consider again Equation (2). Its right side is in fact the difference between the barycentric velocity changes of the comet and the Sun due to the stellar passage. We evaluate this expression in a manner different from that proposed by Rickman (1976). First we notice that

$$\Delta \mathbf{v}_s = k^2 M_* \int_{t_1}^{t_2} \frac{\mathbf{r}_*}{r_*^3}\, \mathrm{d}t \tag{8}$$

is the velocity change of the Sun in the barycentric two body problem: Sun-star ($\mathbf{r}_*$ is still the position of a star with respect to the Sun). $\Delta \mathbf{v}_s$ can be easily obtained without any simplifying assumptions as shown in the Appendix.

Assuming that $\mathbf{r}_c$ is constant and omitting the influence of the Sun on the two body motion: star-comet, one can also obtain the first integral of Equation (2) in the same manner. Both results are expressed in terms of relative star orbit parameters with respect to the appropriate orbital reference frames.

To obtain the final, heliocentric velocity impulse gained by a comet we introduce the appropriate common reference frame connected with the heliocentric motion of the star. We define a stellar orbit with two, perpendicular vectors: $\mathbf{V}_\infty$ – the heliocentric velocity of the star at infinity prior to the encounter and $\mathbf{b}_s$ – the vector from the Sun to the closest point of the straight line containing $\mathbf{V}_\infty$ (i.e. asymptote of the stellar hyperbola). We adopt the reference frame to be a heliocentric one with the x-axis anti parallel to the vector $\mathbf{V}_\infty$, the y-axis parallel to $\mathbf{b}_s$ and z-axis completing a right-handed system. In this frame the position of a comet is described by the radius vector $\mathbf{r}_c = (x_c, y_c, z_c)$.

Let M_*, $\mathbf{V}_\infty$, $\mathbf{b}_s$ and $M_s = 1$ be given. The star's heliocentric semimajor axis is (in our convention semimajor axis of hyperbola is positive):

$$a_s = k^2 (M_* + M_s)/V_\infty^2 \;\text{ and }\; c_s^2 = a_s^2 + b_s^2 \,. \tag{9}$$

According to our assumptions $\mathbf{r}_c$ is constant during the stellar passage so we can obtain the cometocentric star orbit parameters also:

$$a_c = k^2 M_*/V_\infty^2 \,,\;\; c_c^2 = a_c^2 + b_c^2 \,,\;\; b_c = \sqrt{(b_s - y_c)^2 + z_c^2} \,. \tag{10}$$

With the above data, the barycentric velocity changes for the Sun and a comet in some common interval $\langle t_1, t_2 \rangle$ can be obtained directly from formula (A.9) derived in the Appendix.

Then, after transformation to our common, heliocentric frame we obtain:

$$\Delta v_x = (a_c \Delta v_{c\xi} + b_c \Delta v_{c\eta}) \frac{1}{c_c} - (a_s \Delta v_{s\xi} + b_s \Delta v_{s\eta}) \frac{1}{c_s} \,,$$

$$\Delta v_y = (a_c \Delta v_{c\eta} - b_c \Delta v_{c\xi}) \frac{b_s - y_c}{c_c b_c} - (a_s \Delta v_{s\eta} - b_s \Delta v_{s\xi}) \frac{1}{c_s} \,, \tag{11}$$

$$\Delta v_z = (a_c \Delta v_{c\eta} - b_c \Delta v_{c\xi}) \frac{-z_c}{c_c b_c} \, .$$

where

$$\begin{aligned}
\Delta v_{s\xi} &= \frac{-k^2 M_*}{b_s V_\infty} (\sin \vartheta_{s2} - \sin \vartheta_{s1}) \, , \\
\Delta v_{s\eta} &= \frac{-k^2 M_*}{b_s V_\infty} (\cos \vartheta_{s1} - \cos \vartheta_{s2}) \, , \\
\Delta v_{c\xi} &= \frac{-k^2 M_*}{b_c V_\infty} (\sin \vartheta_{c2} - \sin \vartheta_{c1}) \, , \\
\Delta v_{c\eta} &= \frac{-k^2 M_*}{b_c V_\infty} (\cos \vartheta_{c1} - \cos \vartheta_{c2}) \, .
\end{aligned} \tag{12}$$

The final result, $\Delta \mathbf{v} = (\Delta v_x, \Delta v_y, \Delta \, v_z)$ is equal to the total heliocentric velocity change of the comet due to the stellar passage taking into account the gravitational action of the star in the interval $\langle t_1, t_2 \rangle$. It should be stressed that we did not make any assumption about this time interval or about the star velocity vector (in contrast with Rickman, 1976).

Formulae (11) and (12) for the full, improved impulse approximation for arbitrary time interval are necessary, for example, when comparing results with numerical integration in a finite time interval.

In other cases one should replace them with simpler formulae for an infinite time interval, derived in the same manner from equation (A.10):

$$\begin{aligned}
\Delta v_x &= -\frac{2k^2 M_*}{V_\infty} \left(\frac{a_c}{c_c^2} - \frac{a_s}{c_s^2} \right) \, , \\
\Delta v_y &= \frac{2k^2 M_*}{V_\infty} \left(\frac{b_s - y_c}{c_c^2} - \frac{b_s}{c_s^2} \right) \, , \\
\Delta v_z &= -\frac{2k^2 M_*}{V_\infty} \left(\frac{z_c}{c_c^2} \right) \, .
\end{aligned} \tag{13}$$

For the sake of comparison we write formulae for classical impulse method in the same notation and the same reference frame:

$$\begin{aligned}
\Delta v_x &= 0 \, , \\
\Delta v_y &= \frac{2k^2 M_*}{V_\infty} \left(\frac{b_s - y_c}{b_c^2} - \frac{b_s}{b_s^2} \right) \, , \\
\Delta v_z &= -\frac{2k^2 M_*}{V_\infty} \left(\frac{z_c}{b_c^2} \right) \, .
\end{aligned} \tag{14}$$

TABLE I
Values of the a_c/c_c ratio, for $M_* = 1M_s$.

$b_c \backslash V_\infty$	5 km s^{-1}	10 km s^{-1}	20 km s^{-1}	40 km s^{-1}
10 AU	0.9625	0.6636	0.2165	0.0554
100 AU	0.3344	0.0884	0.0222	0.0055
1000 AU	0.0335	0.0089	0.0022	0.0006
10000 AU	0.0035	0.0009	0.0002	0.0001
a_c	35.49 AU	8.871 AU	2.218 AU	0.5545 AU

4. Improved Impulse Method Versus Classical One

The improved impulse approximation derived in the previous section includes two essential, qualitative modifications when compared with the classical version. Firstly, it is fully applicable to finite and/or asymmetric time intervals. Secondly, it takes into account the deflection of the star's path which changes both the magnitude and the direction of the velocity impulse obtained. This improvement can be shown by a comparison of Equations (13) and (14). Below we perform such a comparison for solar and cometary parts of the impulse separately.

In the classical method the Δv_x component equals zero. The importance of this component can be shown by examination of the following simple ratios:

$$\frac{\Delta v_{sx}}{|\Delta \mathbf{v}_s|} = \frac{a_s}{c_s} \text{ and } \frac{\Delta v_{cx}}{|\Delta \mathbf{v}_c|} = \frac{a_c}{c_c}$$

where the index s denotes the solar and c the cometary part of the velocity impulse.

Some values of those ratios for different parameters of the problem are shown in Table I and II. Nonzero values of this ratio prove the advantage of the proposed method over the classical one.

It should be mentioned that the values of these ratios are significantly increased for larger stellar masses.

In the remaining components Δv_y and Δv_z the difference between the classical and improved methods reduces to the multiplication by coefficients:

$$\frac{c_s^2}{b_s^2} \text{ and } \frac{c_c^2}{b_c^2}$$

in the solar and cometary parts of the impulse, respectively. In Tables III to V we present some values of these coefficients for different values of the parameters. One should remember that the classical and improved methods will be equivalent when these coefficients are equal to unity.

TABLE II
Values of the a_s/c_s ratio, for $M_* = 1M_s$.

$b_s \backslash V_\infty$	5 km s^{-1}	10 km s^{-1}	20 km s^{-1}	40 km s^{-1}
10 AU	0.9902	0.8712	0.4055	0.1102
100 AU	0.5788	0.1747	0.0443	0.0111
1000 AU	0.0708	0.0177	0.0044	0.0011
10000 AU	0.0071	0.0018	0.0004	0.0001
a_s	70.97 AU	17.74 AU	4.436 AU	1.109 AU

TABLE III
Values of the c_c^2/b_c^2 coefficient for $M_* = 1M_s$.

$b_c \backslash V_\infty$	5 km s^{-1}	10 km s^{-1}	20 km s^{-1}	40 km s^{-1}
10 AU	13.5919	1.7870	1.0492	1.0031
50 AU	1.5037	1.0315	1.0020	1.0001
100 AU	1.1259	1.0079	1.0005	1.0000
200 AU	1.0315	1.0020	1.0001	1.0000
500 AU	1.0050	1.0003	1.0000	1.0000
1000 AU	1.0013	1.0001	1.0000	1.0000

TABLE IV
Values of the c_s^2/b_s^2 coefficient for $M_* = 1M_s$.

$b_s \backslash V_\infty$	5 km s^{-1}	10 km s^{-1}	20 km s^{-1}	40 km s^{-1}
10 AU	51.3677	4.1480	1.1967	1.0123
50 AU	3.0147	1.1259	1.0079	1.0005
100 AU	1.5037	1.0315	1.0020	1.0001
200 AU	1.1259	1.0079	1.0005	1.0000
500 AU	1.0201	1.0013	1.0001	1.0000
1000 AU	1.0005	1.0003	1.0000	1.0000

TABLE V
Values of the c_s^2/b_s^2 coefficient for $M_* = 5M_s$.

$b_s \backslash V_\infty$	5 km s^{-1}	10 km s^{-1}	20 km s^{-1}	40 km s^{-1}
10 AU	454.3096	29.3319	2.7707	1.1107
50 AU	19.1324	2.1333	1.0708	1.0044
100 AU	5.5331	1.2833	1.0177	1.0011
200 AU	2.1333	1.0708	1.0044	1.0003
500 AU	1.1813	1.0113	1.0007	1.0000
1000 AU	1.0453	1.0028	1.0002	1.0000

The impulse approximation method may be used to study stellar perturbation on cometary orbits in the Oort cloud of comets. In such a case very close stellar approaches to the Sun are usually excluded due to dynamical and statistical arguments. However, for the minimum distance between a comet and the star one should not introduce any restriction. As we have shown above, for small impact parameters b the improvements proposed in this paper are very important. We present an example of several very close stellar approaches to a comet. We use an impulse approximation to determine the perihelion distance of a comet after the stellar passage (solid line in the upper part of Figures 1 and 2) in comparison with the numerical integration (square dots).

Both plots show the dependence of the resulting perihelion distance on the initial comet position described by the mean anomaly. It should be stressed that each point of the solid line (as well as each square dot) represent a result of a full stellar passage for a specific value of the initial mean anomaly of the comet.

Figure 1 presents the results obtained using the classical impulse method. Figure 2 illustrates the advantage of the method proposed in this paper. In the middle part of both figures there is an additional plot of the final eccentricity and the lower part presents the geometrical configuration and all values of parameters.

5. Conclusions

We recommend the improved impulse approximation as a very good method both for analytical studies and numerical simulations of stellar perturbations on elongated cometary orbits. The improvement proposed in this paper is essential when one deals with close stellar approaches or with arbitrary time intervals. In those cases, the inaccuracies and restrictions of the classical method can be overcome. The necessity of the improvement of the impulse approximation is also stated in Staniucha and Banaszkiewicz (1988). The proposed method is still very fast

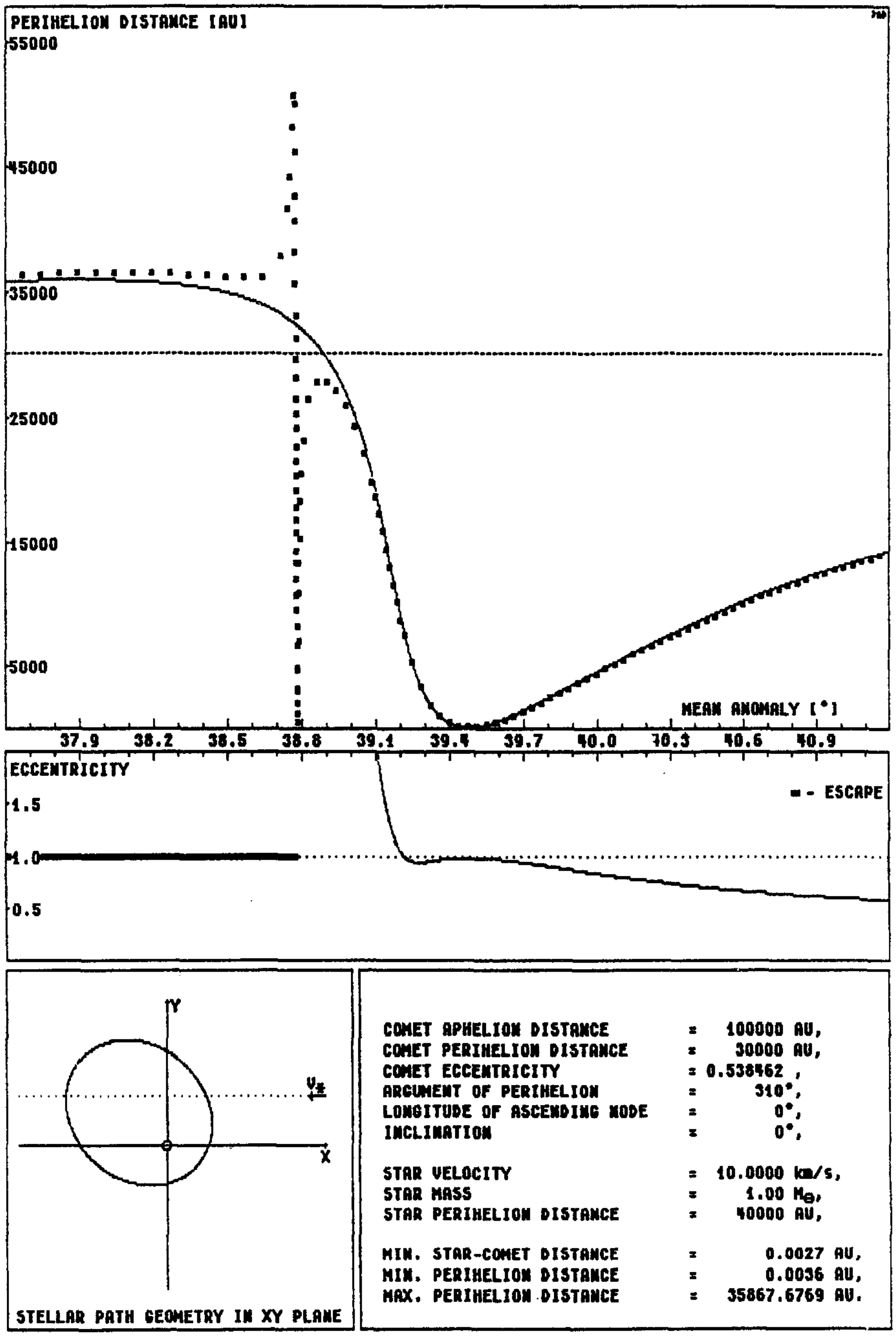

Fig. 1. Classical impulse approximation versus numerical integration: final perihelion distance in terms of initial mean anomaly.

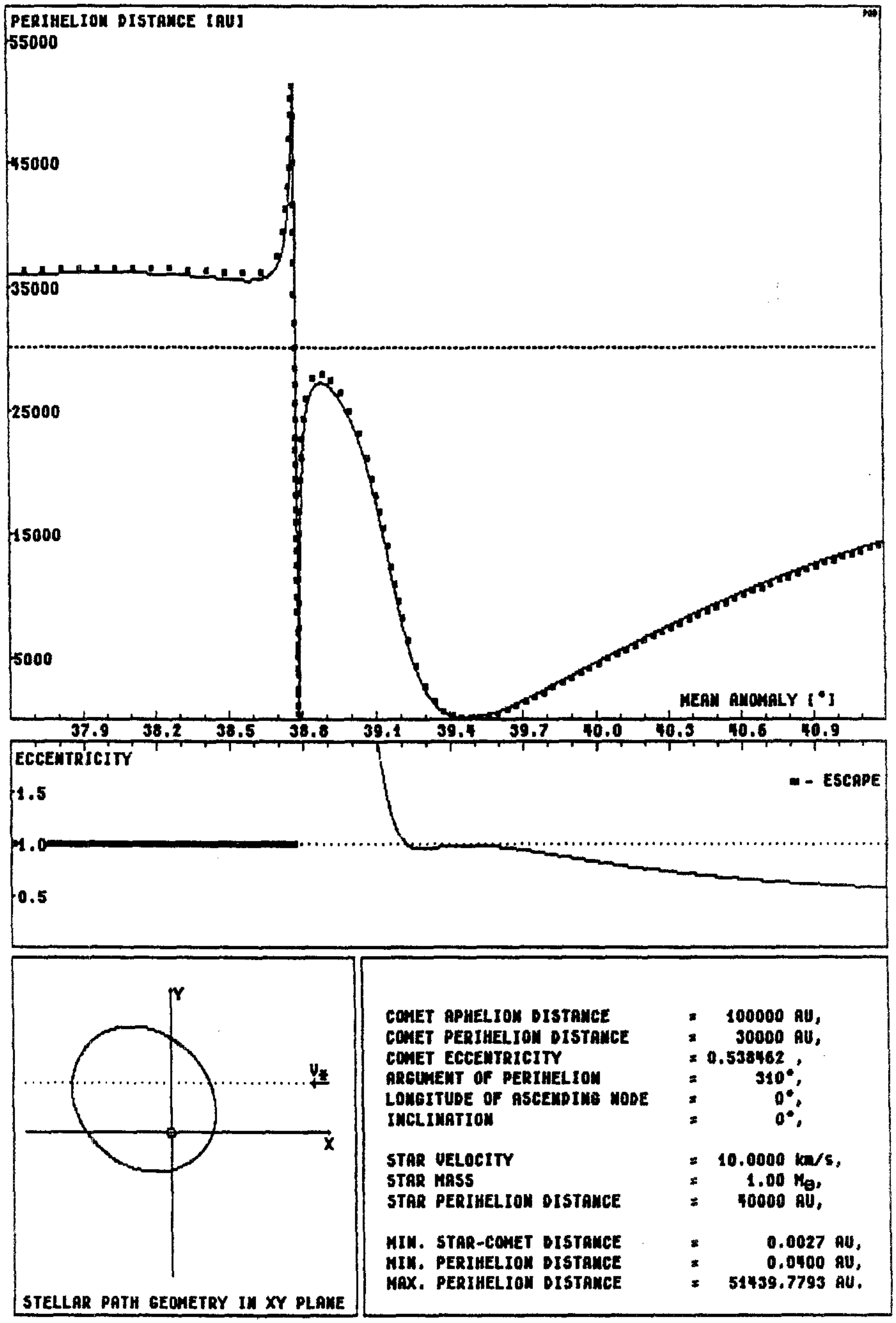

Fig. 2. Improved impulse approximation versus numerical integration: final perihelion distance in terms of initial mean anomaly.

when compared with numerical integrations. Our simulation program shows that the improved impulse method is 500 times faster than our best numerical integration procedure. Detailed derivation of all formulas and extended study of the applicability of the method proposed one can found in Dybczyński (1990).

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Appendix. The velocity change vector in hyperbolic two body problem

In the unperturbed two body problem, the equations of motion of both point masses with respect to the orbital frame are:

$$\ddot{\xi}_i = -\mu_i \frac{\xi_i}{\rho_i^3}\,, \quad \ddot{\eta}_i = -\mu_i \frac{\eta_i}{\rho_i^3}\,, \qquad \text{(A.1)}$$
$$\rho_i^2 = \xi_i^2 + \eta_i^2\,, \quad i = 1, 2\,,$$

where μ_i is a coefficient which is a function of the masses of two bodies. The solution of Equations (A.1) have a well known, classical form:

$$\xi = \rho \cos\vartheta\,, \quad \eta = \rho \sin\vartheta\,,$$

where ϑ is a true anomaly. The equations of motion may be rewritten as follows (omitting index i):

$$\ddot{\xi} = -\mu \frac{\cos\vartheta}{\rho^2}\,, \quad \ddot{\eta} = -\mu \frac{\sin\vartheta}{\rho^2}\,. \qquad \text{(A.2)}$$

Integrating Equations (A.2) in some interval of time $\langle t_1, t_2 \rangle$ we obtain the velocity change vector $\Delta\mathbf{v} = (\Delta v_\xi, \Delta v_\eta)$:

$$\Delta v_\xi = -\mu \int_{t_1}^{t_2} \frac{\cos\vartheta \,\mathrm{d}t}{\rho^2}\,, \quad \Delta v_\eta = -\mu \int_{t_1}^{t_2} \frac{\sin\vartheta \,\mathrm{d}t}{\rho^2}\,.$$

In hyperbolic motion (which is the only one considered here) the angular momentum integral is given by:

$$\rho^2 \frac{\mathrm{d}\vartheta}{\mathrm{d}t} = bV_\infty\,,$$

so we can write:

$$\frac{\mathrm{d}\vartheta}{bV_\infty} = \frac{\mathrm{d}t}{\rho^2}\,, \qquad \text{(A.3)}$$

where V_∞ is a hyperbolic velocity at infinity and b is the semi minor axis of a hyperbola.

Putting (A.3) into (A.2) we can exchange variables:

$$\Delta v_\xi = -\mu \int\limits_{\vartheta_1}^{\vartheta_2} \frac{\cos\vartheta \, d\vartheta}{bV_\infty} , \quad \Delta v_\eta = -\mu \int\limits_{\vartheta_1}^{\vartheta_2} \frac{\sin\vartheta \, d\vartheta}{bV_\infty} . \tag{A.4}$$

New limits ϑ_1 and ϑ_2 are values of the true anomaly for the moments t_1 and t_2 respectively. The above equations can be easily integrated, giving:

$$\begin{aligned} \Delta v_\xi &= \frac{-\mu}{bV_\infty} (\sin\vartheta_2 - \sin\vartheta_1) , \\ \Delta v_\eta &= \frac{-\mu}{bV_\infty} (\cos\vartheta_1 - \cos\vartheta_2) , \end{aligned} \tag{A.5}$$

and finally:

$$|\Delta \mathbf{v}| = \frac{-2\mu}{bV_\infty} \sin\left(\frac{\vartheta_2 - \vartheta_1}{2}\right) . \tag{A.6}$$

The geometrical interpretation of the Equations (A.5) brings us to the conclusion that in every case $\Delta \mathbf{v}$ vector is directed along the bisector of an angle between the radius vectors $\mathbf{r}_1(t_1)$ and $\mathbf{r}_2(t_2)$. In a symmetric case, for $\vartheta_1 = -\vartheta_2 = \vartheta_*$, Equations (A.5) and (A.6) may be significantly simplified:

$$|\Delta \mathbf{v}| = \Delta v_\xi = \frac{-2\mu}{bV_\infty} \sin\vartheta_* , \quad \Delta v_\eta = 0 . \tag{A.7}$$

In this case the velocity change vector $\Delta \mathbf{v}$ is directed along the line of apses. The value of ϑ_* can vary from zero to $\vartheta_{\max}$ which describes a position of a body in infinity.

Using Jeans equation it is possible to show that:

$$\sin\vartheta_{\max} = b/c$$

where $c^2 = a^2 + b^2$ and a is the semimajor axis of a hyperbola, we can rewrite the Equation (A.7) as follows:

$$|\Delta \mathbf{v}| = \Delta v_\xi = \frac{-2\mu}{cV_\infty} . \tag{A.8}$$

It must be stressed that this equation is valid only when one takes into account the whole hyperbolic path, putting $t_1 = -\infty$ and $t_2 = \infty$.

In this paper we need barycentric velocity changes of a body (a comet or the Sun) having given parameters of the relative stellar orbit, $\mathbf{V}_\infty$ and $\mathbf{b}$. Simple transformations allow us to rewrite Equations (A.5) in terms of these values:

$$\Delta v_{1\xi} = \frac{-k^2 M_*}{bV_\infty} (\sin \vartheta_2 - \sin \vartheta_1) ,$$
$$\Delta v_{1\eta} = \frac{-k^2 M_*}{bV_\infty} (\cos \vartheta_1 - \cos \vartheta_2) , \tag{A.9}$$

In the same manner one can transform Equation (A.8) which describes the case $\langle t_1, t_2 \rangle = \langle -\infty, \infty \rangle$:

$$|\Delta \mathbf{v}| = \Delta v_\xi = \frac{-2k^2 M_*}{cV_\infty} . \tag{A.10}$$

Equations (A.9) and (A.10) are used to derive the improved version of the impulse approximation method.

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