MSc Human and Biological Robotics

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Robotics: Tutorial 1 - 2D Kinematics

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1 Question 1

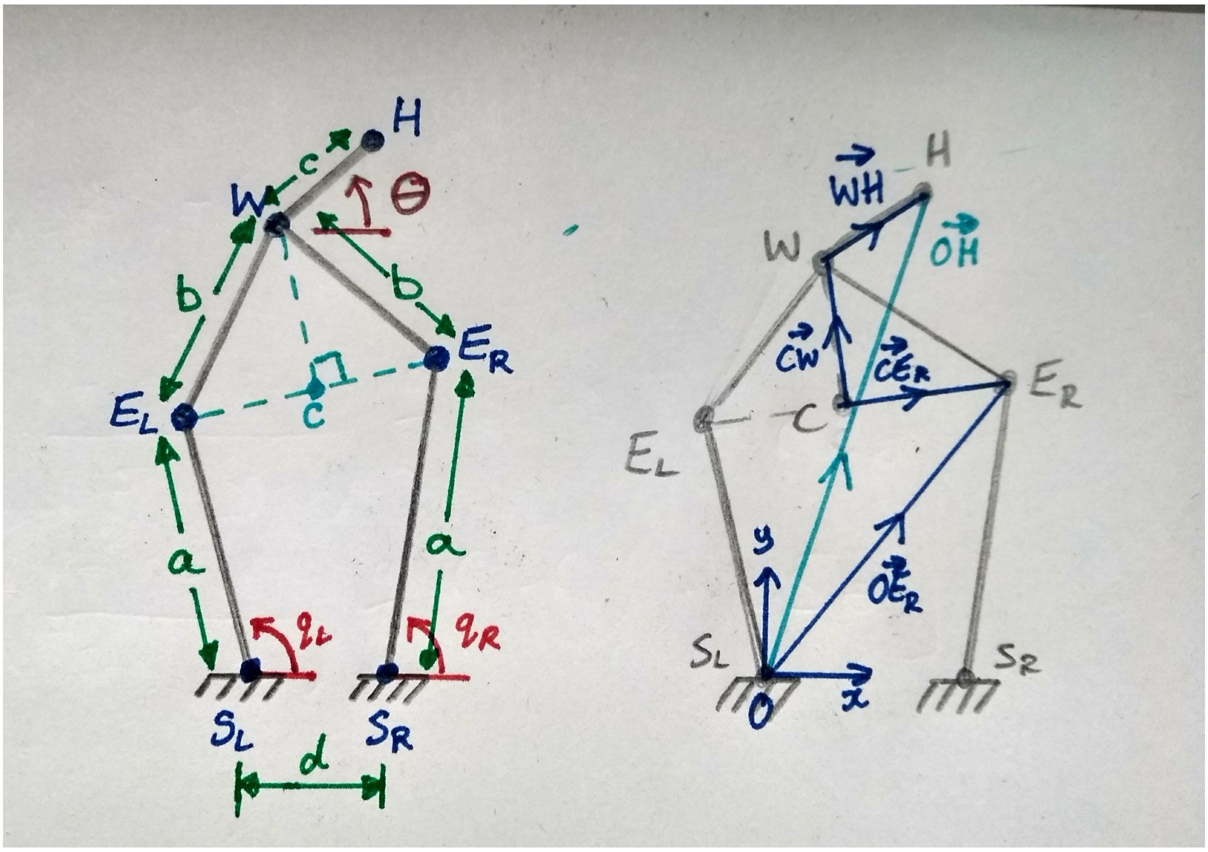


Figure 1.1

Coordinates of points E_L and E_R are given by equations 1 and 2 respectively. By definition these are also the vectors $\vec{OE_L}$ and $\vec{OE_R}$, where O denotes the origin at point S_L.

$$E_L = [a\cos q_L), a\sin q_L] \tag{1}$$

$$E_R = [d + a\cos q_R, a\sin q_R] \tag{2}$$

Subtracting vector $\vec{OE_L}$ from $\vec{OE_R}$ gives the vector $\vec{E_LE_R}$ passing through centre point C. Hence, the vector $\vec{CE_R}$ is half of $\vec{E_LE_R}$.

$$\vec{E_LE_R} = [d + a(\cos q_R - \cos q_L), a(\sin q_R - \sin q_L] \tag{3}$$

$$\vec{CE_R} = [\frac{d}{2} + \frac{a}{2}(\cos q_R - \cos q_L), \frac{a}{2}(\sin q_R - \sin q_L] \tag{4}$$

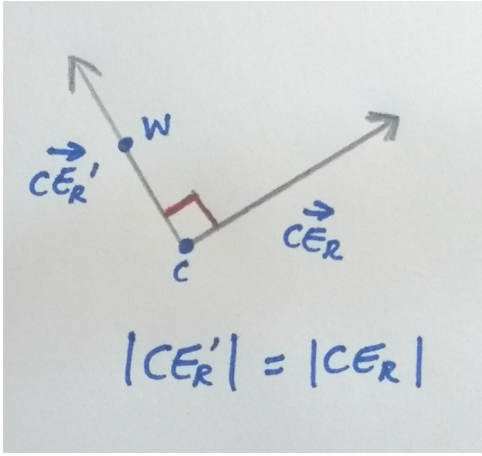


Figure 1.2

From figure 1.2, the vector $C\vec{E}R'$ which lies perpendicular to $C\vec{E}R$, can be defined as:

$$C\vec{E}_R' = [\frac{a}{2}(\sin q_L - \sin q_R), \frac{d}{2} + \frac{a}{2}(\cos q_R - \cos q_L)] \tag{5}$$

Since C is the midpoint of $E_L\vec{E}_R$ and point W lies on a perpendicular line to this vector, the vector $C\vec{W}$ is given by:

$$C\vec{W} = \frac{|CW|}{|CE_R|} \times C\vec{E}_R' \tag{6}$$

where:

$$|CE_R| = \gamma \tag{7}$$

$$= \sqrt{\frac{a^2}{4}(S_L - S_R)^2 + \frac{d^2}{4} + \frac{a^2}{4}(C_R - C_L)^2 + \frac{ad}{4}(C_R - C_L)} \tag{8}$$

$$= \sqrt{\frac{a^2}{2}(1 - C_{R-L}) + \frac{d^2}{4} + \frac{ad}{4}(C_R - C_L)} \tag{9}$$

$$|CW| = \sqrt{b^2 - \gamma^2} \tag{10}$$

C and S denoting cos and S respectively and the subscript signifying the angle e.g. $C_R = \cos q_R$ and $C_{R-L} = \cos(q_R - q_L)$. It can also be shown that $W\vec{H}$ is given by eqn 11.

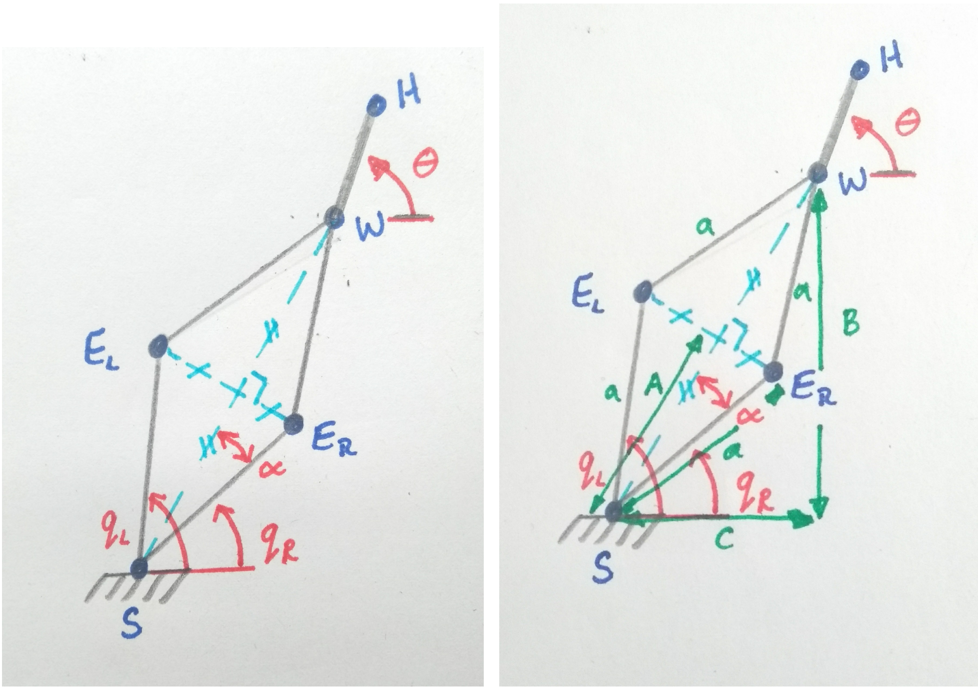
$$W\vec{H} = [c\cos\theta, c\sin\theta] \tag{11}$$

Hence, the vector $O\vec{H}$ (figure 1.1) is given by eqn 12.

$$
\begin{aligned}
\vec{OH} &= \vec{ER} - \vec{CER} + \vec{CW} + \vec{WH} && (12)\\
&= [d + aC_R, aS_R] - [\frac{d}{2} + \frac{a}{2}(C_R - C_L), \frac{a}{2}(S_R - S_L] && (13)\\
&+ \frac{\sqrt{b^2 - \sqrt{\frac{a^2}{2}(1 - C_{R-L}) + \frac{d^2}{4} + \frac{ad}{4}(C_R - C_L)}^2}}{\sqrt{\frac{a^2}{2}(1 - C_{R-L}) + \frac{d^2}{4} + \frac{ad}{4}(C_R - C_L)}}[\frac{a}{2}(S_L - S_R), \frac{d}{2} + \frac{a}{2}(C_R - C_L)] && (14)\\
&+ [cC_\theta, cS_\theta] && (15)
\end{aligned}
$$

2 Question 2

Part A



(a) (b)

Figure 2.1: NOT TO SCALE

The problem given in Question 1 is redrawn in figure 2.1 with the following parameters: $a = b = 0.3m$, $c = 0.2m$, $d = 0m$, $\alpha = \frac{q_L - q_R}{2}$. In figure 1(b), B and C represent the y and x coordinates of the end-effector with respect to point S. From trigonometry the following relationships are found:

$$A = a\cos\alpha \tag{16}$$
$$B = 2A\sin(q_R + \alpha) \tag{17}$$
$$C = 2A\cos(q_R + \alpha) \tag{18}$$

$\therefore$

$$B = 2a\cos(\frac{q_L - q_R}{2})\sin(\frac{q_L + q_R}{2}) = a(\sin q_L + \sin q_R) \tag{19}$$
$$C = 2a\cos(\frac{q_L - q_R}{2})\cos(\frac{q_L + q_R}{2}) = a(\cos q_L + \cos q_R) \tag{20}$$

thus the x and y coordinates of H are given by eqn 21 below.

$$H = [a(\cos q_L + \cos q_R) + c\cos\theta, a(\sin q_L + \sin q_R) + c\sin\theta] \tag{21}$$

Therefore, the jacobian matrix is given by J(ψ)such that:

$$\dot{H} = J(\psi)\dot{\psi} \tag{22}$$

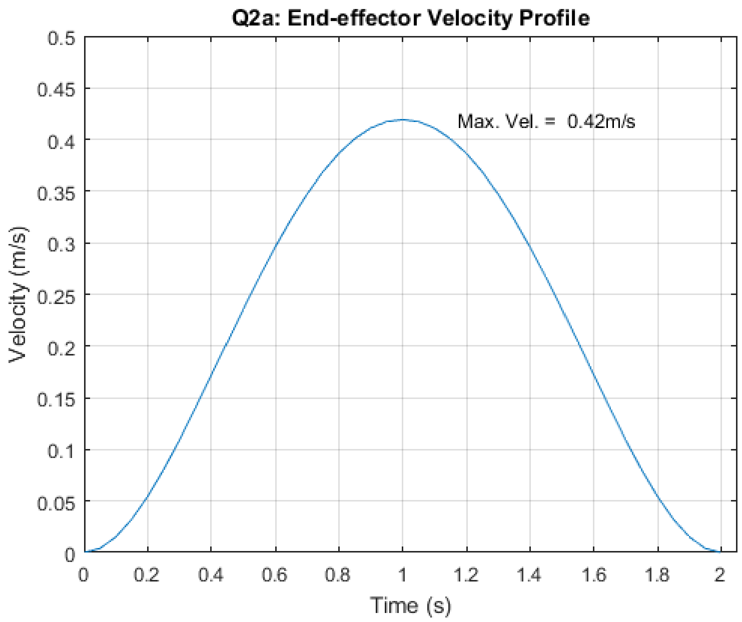
where:

$$\psi = \begin{bmatrix} q_R \\ q_L \\ \theta \end{bmatrix} \tag{23}$$
$$J(\psi) = \begin{bmatrix} -a\sin q_L & -a\sin q_L & -c\sin\theta \\ a\cos q_L & a\cos q_L & c\cos\theta \end{bmatrix} \tag{24}$$

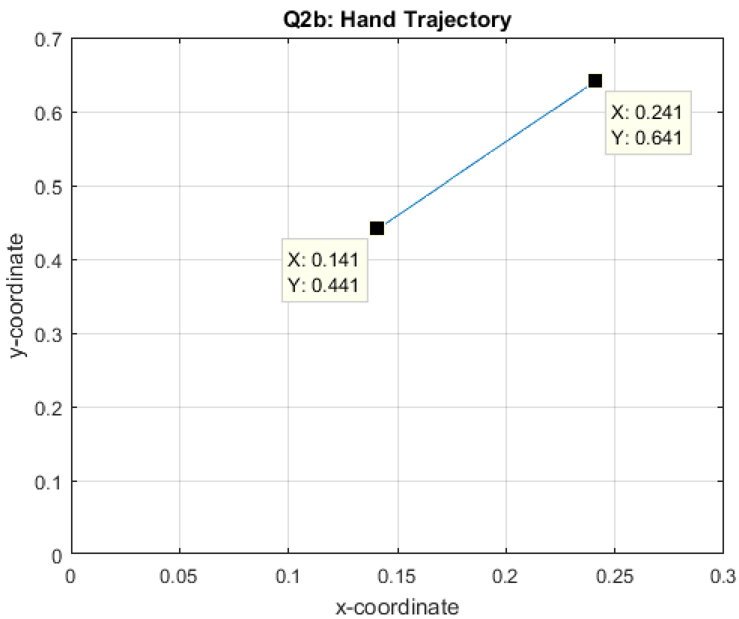
Part B

$$\sigma(\tau) = 30\tau^2(tau^2\tau + 1); \tau = t/T \tag{25}$$

Using the formula given in equation 25, where t is between 0 and T, and knowing the start ([0.141, 0.441]) and end ([0.241, 0.641]) points of the straight line trajectory, the hand velocity profile as well as its trajectory and x- and y- position profile were found. These are shown below in figures 2.2-2.3.

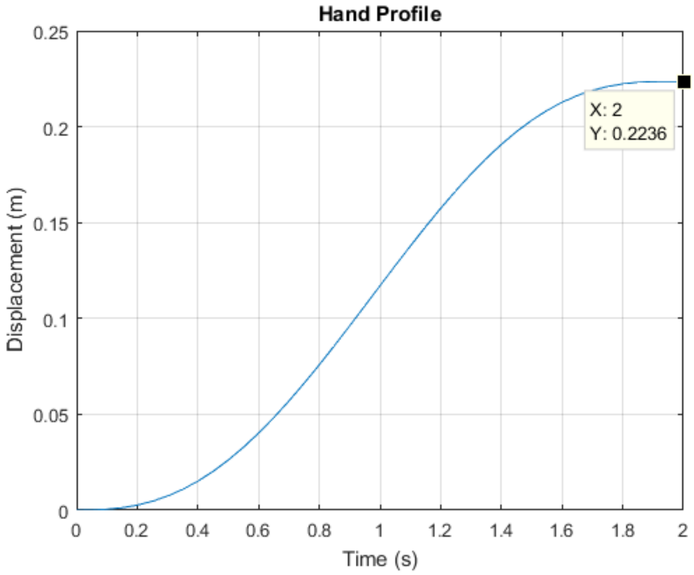


(a) Hand velocity profile

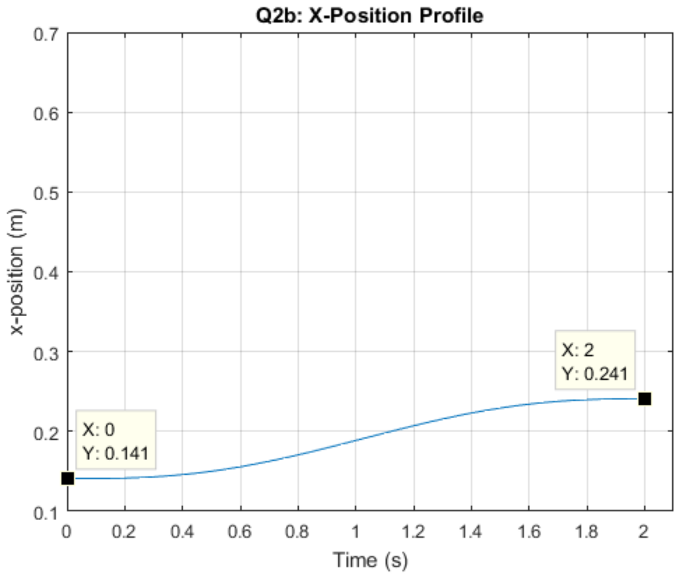


(b) Hand trajectory vs time

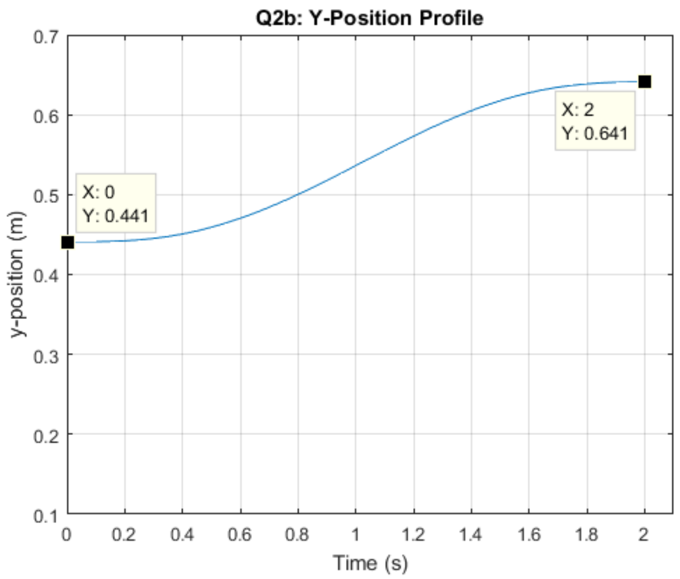
Figure 2.2



(a) Hand profile



(b) Hand x-profile



(c) Hand y-profile

Figure 2.3

Part C

Figure 2.4 shows the profile of the angles q_R, q_L and θ in order to minimise the cost function $\sqrt{\dot{q_R} + \dot{q_L} + \dot{\theta}}$.

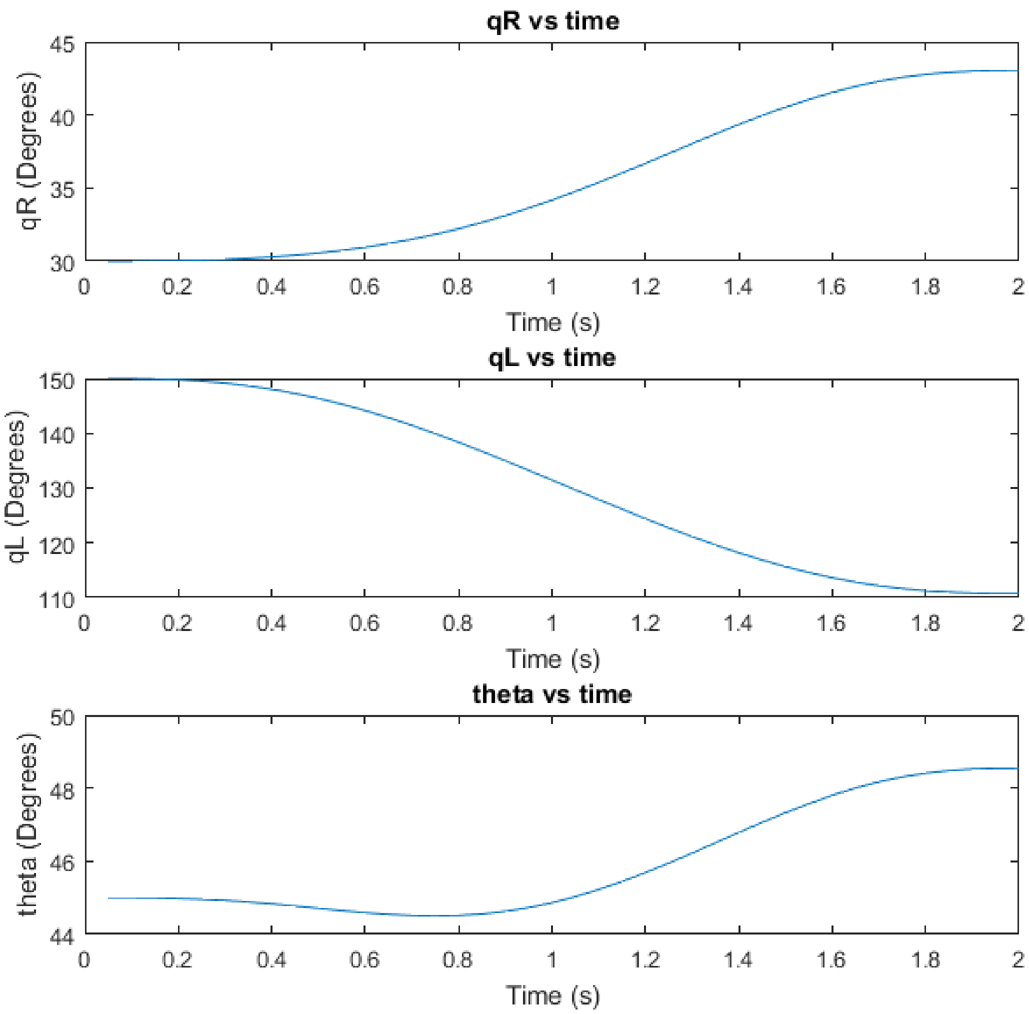


Figure 2.4