MSc Human and Biological Robotics

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Human Neuro-Mechanical Control and Learning: Tutorial 3: Dynamics and Control

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May 5, 2018

1 Question 1

1.1 Part A

From the question,

$$\tau = \begin{bmatrix} \tau_s \\ \tau_e \end{bmatrix} = \Psi P \tag{1}$$

where

$$\Psi = \begin{bmatrix} \ddot{q_s} + \ddot{q_e}, & (2\ddot{q_s} + \ddot{q_e})cos(q_e) - \dot{q_e}(2\dot{q_s} + \dot{q_e})sin(q_e), & \ddot{q_e} \\ \ddot{q_s} + \ddot{q_e}, & \ddot{q_s}cos(q_e) - \dot{q_s}^2 sin(q_e), & 0 \end{bmatrix} \tag{2}$$

$$P = \begin{bmatrix} J_e + m_e l_m e^2 \\ m_e l_s l_m e \\ J_s + m_S l_m s^2 + m_E l_s^2 \end{bmatrix} = \begin{bmatrix} 0.1004 \\ 0.1200 \\ 0.2630 \end{bmatrix} \tag{3}$$

when the shoulder is fixed, $\ddot{q_s} = \dot{q_s} = 0$, therefore,

$$\Psi = \begin{bmatrix} \ddot{q_e}, & \ddot{q_e}cos(q_e) - \dot{q_e}^2 sin(q_e), & \ddot{q_e} \\ \ddot{q_e}, & 0, & 0 \end{bmatrix} \tag{4}$$

.

Hence,

$$\tau_e = \begin{bmatrix} \ddot{q_e}, & 0, & 0 \end{bmatrix} \begin{bmatrix} 0.1004 \\ 0.1200 \\ 0.2630 \end{bmatrix} \tag{5}$$

$$= 0.1004\ddot{q_e} \tag{6}$$

.

Equation 6 is the dynamic equation for the single elbow joint movement and is linear w.r.t. joint acceleration since it contains no nonlinear terms. Similarly, it can be shown that the torque, τ_s required to keep the shoulder fixed is given by equation 7. This equation is nonlinear due to the squared and trigonometric terms.

$$\tau_s = 0.1004\ddot{q_e} + 0.1200(\ddot{q_e}cos(q_e) - \dot{q_e}^2 sin(q_e)) + 0.2630\ddot{q_e} \tag{7}$$

Figure 1 shows the shoulder and elbow angle over time when the shoulder is fixed at 0^o and the elbow is subject to a torque given by

$$\tau_e(t) = \begin{cases} 0.02 - 0.1\dot{q_e}, & \text{if } 0s \leq t < 2s. \\ -0.1\dot{q_e}, & \text{if } 2s \leq t \leq 20s. \end{cases} \tag{8}$$

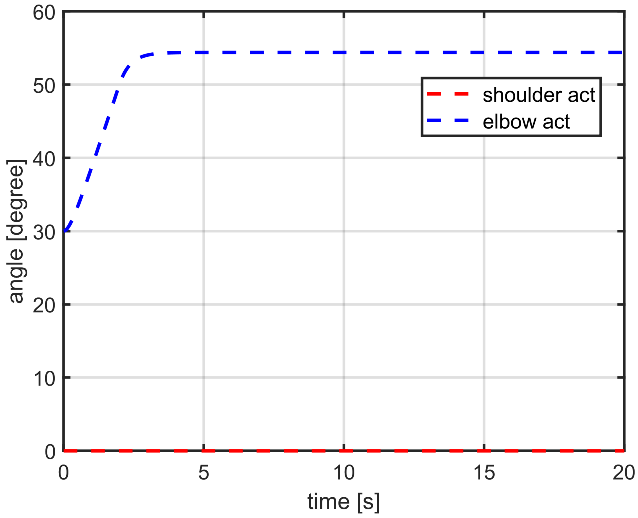
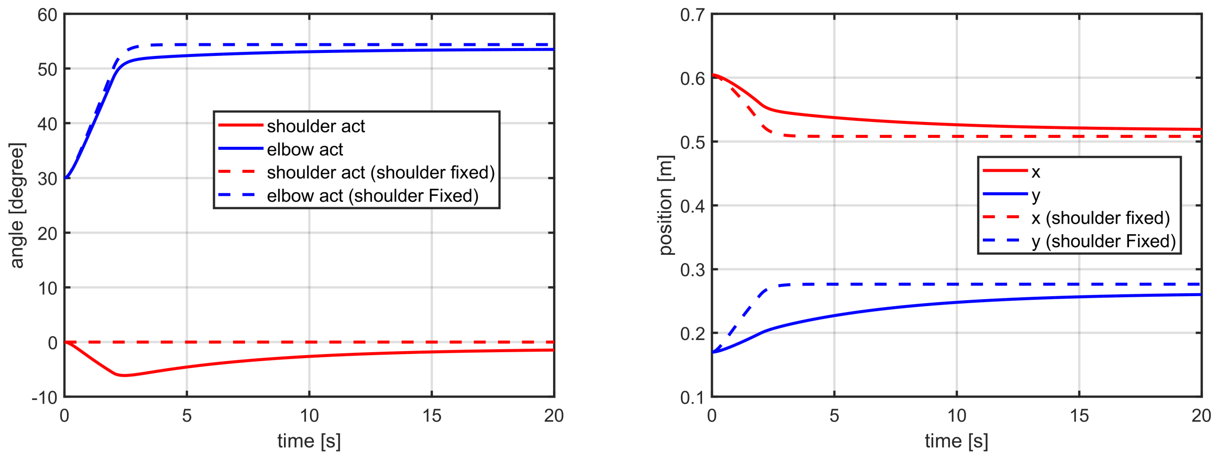


Figure 1: Elbow angle over time when the shoulder joint is fixed.

1.2 Part B

Figure 2 shows the shoulder and joint angles as well as the end-effector position in x and y for the two boundary conditions of the shoulder joint: fixed and free. In figure 2(a), it can be seen that the elbow must sweep a larger angle initially when the shoulder is fixed compared to when the shoulder is free. As the end-effector approaches its desired end point, the shoulder angle tends back to zero, thus the elbow tends to the same final angle in both cases. Figure 2(b) shows the x and y position of the end-effector for both simulations. In the simulation where the shoulder is free, the x- and y- position profiles are much smoother compared with when the shoulder if free to move. As with the joint angles, the x and y position of the end-effector tend to the same values in both cases.



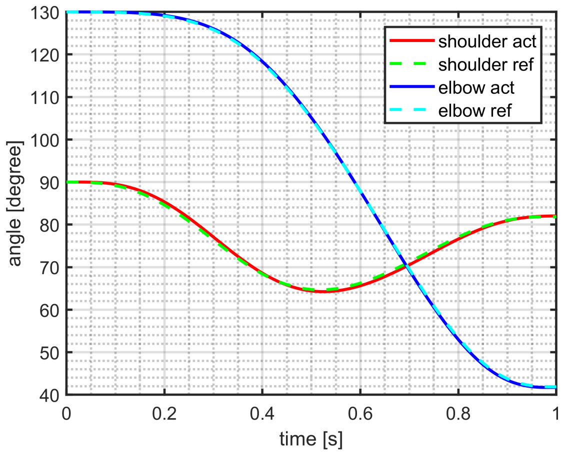
(a) Joint Angle (b) End-effector Position

Figure 2: Shoulder and elbow joint angles and end-effector position: comparison between when the shoulder is fixed and when it is free to move.

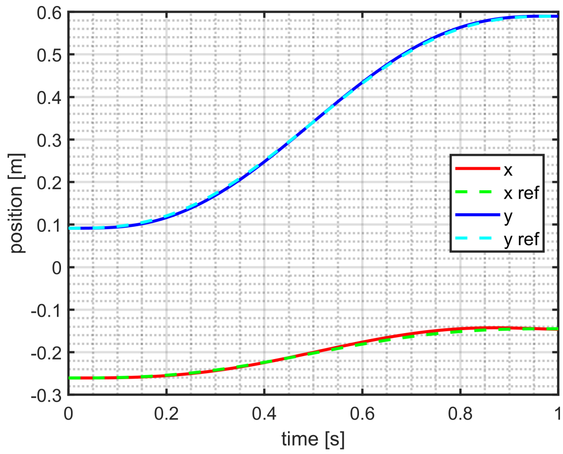
2 Question 2

Figure 3 shows the results from implementing the linear feedback controller. The desired, or reference, joint angles, position and end-effector trajectories are shown as well as the actual values when $K_p = 100Nm/rad$ and $K_d = 10Nms/rad$. The controller effectively tracks the reference joint angles (figure 3(a)) and x- and y-position profiles (3(b)) to achieve the desired trajectory (3(c)).

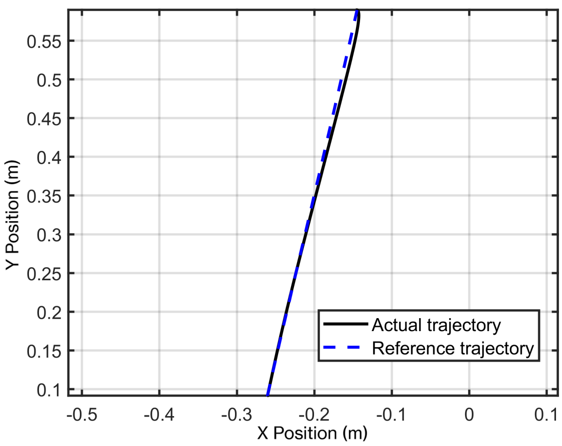
2.1 Part A



(a) Shoulder and Elbow joint angles



(b) End-effector position



(c) End-effector trajectory

Figure 3: Results from linear feedback controller: $K_p = 100Nm/rad$ and $K_d = 10Nms/rad$.

2.2 Part B

2.2.1 Varying K_p

Figures 4 and 5 show the effect of changing K_p on the end-effector trajectory as well as shoulder and elbow angles. The marginal difference as you increase the parameter from 50 to 150 NM/rad is that the actual trajectory gets closer to the reference trajectory. It is evident that the system is quite stable to perturbations in the proportional gain element of the feedback control parameter. This is most likely due to the feedforward control elements which take into consideration system dynamics including the mass and lengths of the links of the system.

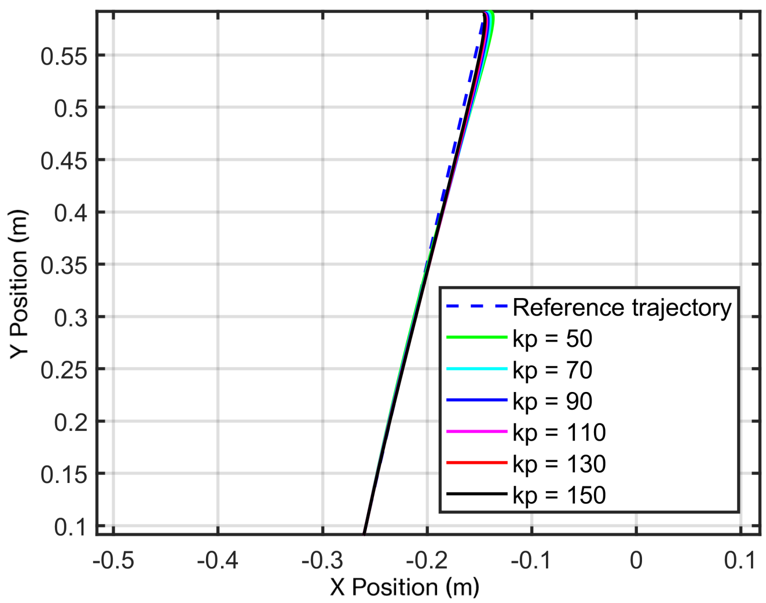


Figure 4: End-effector trajectory as K_p is varied from 50 to 150 Nm/rad.

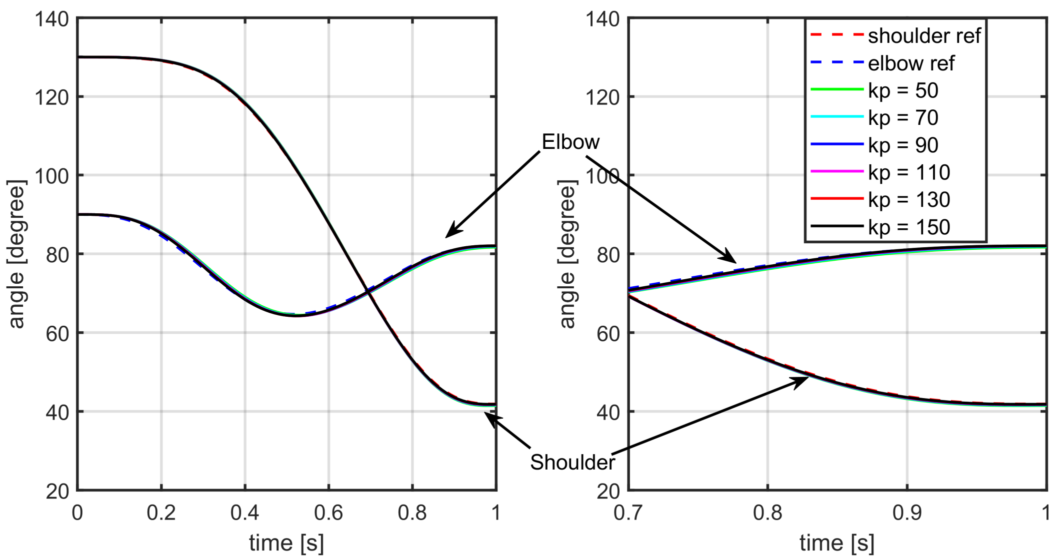


Figure 5: Elbow and Shoulder angles as K_p is varied from 50 to 150 Nm/rad.

2.2.2 Varying K_d

Conversely, varying K_d has a large effect on the performance of the controller. The effects of increasing K_d from 10 to 20 Nms/rad on shoulder and elbow angles, the x- and y- positions and the trajectory are shown in figures 6 to 8(a). When K_d = 10, the joint angles, position and hence trajectory are in line with the reference profiles. However, as K_d is increased the controller does not sufficiently track the reference profiles; the larger the value of K_d, the earlier in the movement the arm loses control. Once the derivative gain is increased above 12, the arm can no longer follow the desired trajectory, as can be seen in figure 8(a).

Figure 8(b) shows the maximum value of the inverse mass matrix. The magnitude of this gets larger as the system approaches singularities. These singularities occur earlier in the simulation for higher values of K_d - this is portrayed in the figure by the lines falling to 0, or undefined, which is highlighted by black circles. It is these large values in the inverse mass matrix which cause the system to lose stability and hence cause the joint angles to deviate from their desired profiles throughout the arm movement.

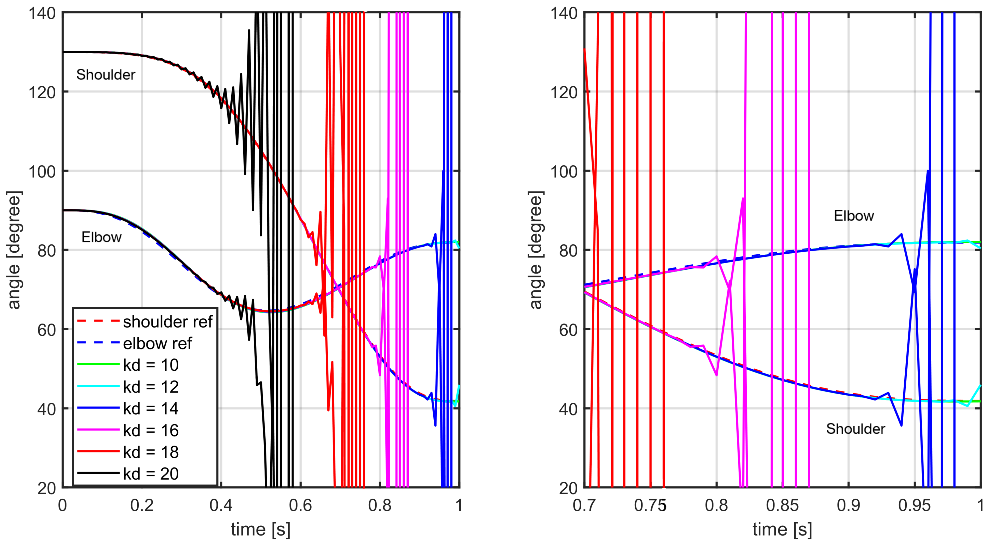


Figure 6: Shoulder and elbow angle over time as K_d is varied from 10 to 20 Nms/rad.

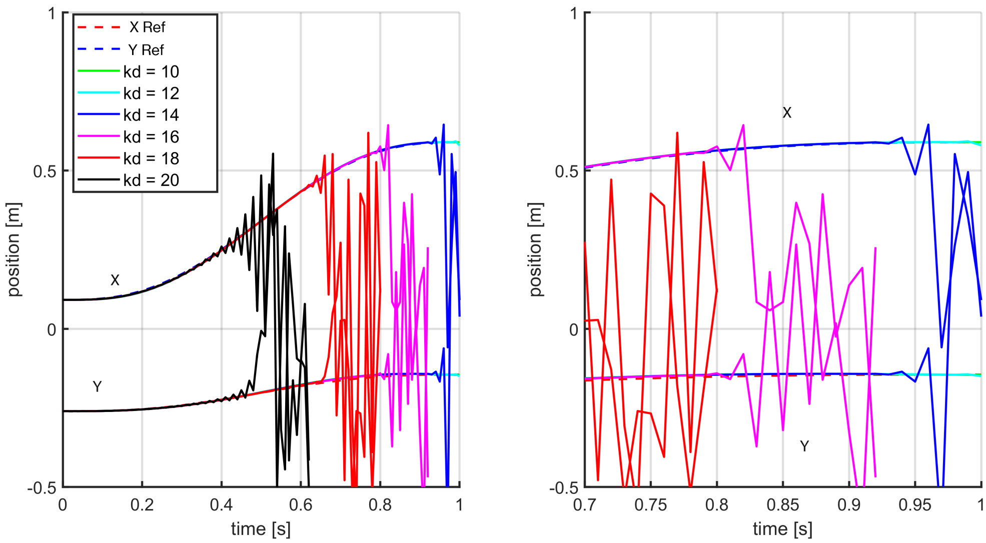
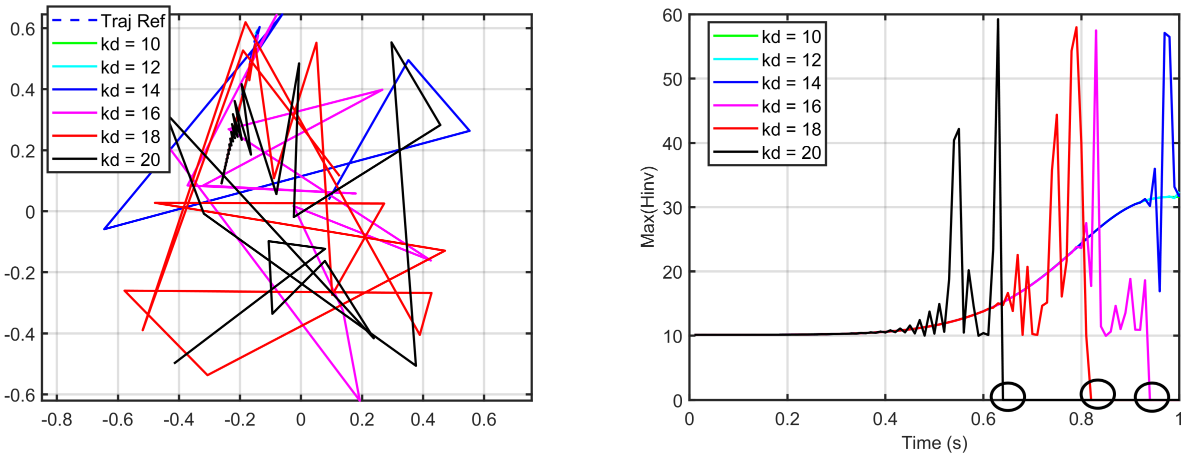


Figure 7: End-effector position over time as K_d is varied from 10 to 20 Nms/rad.



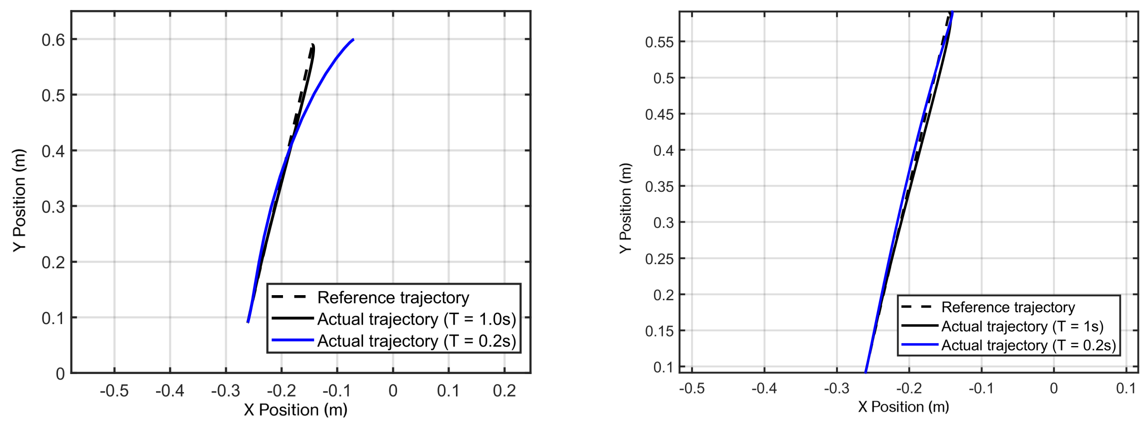
(a) End-effector trajectory as K_d is varied from 10 to 20 Nms/rad.

(b) Maximum magnitude of the inverse H matrix as K_d is varied from 10 to 20 Nms/rad.

Figure 8: Varying K_d and its effect of performance.

2.3 Part C

Figure 9(a) shows the trajectory of the end-effector when the movement is made in 0.2s as oppose to in 1.0s. It is shown that this results in insufficient control of the end-effector. This is due to the fact that the controller is effectively monitoring the system at a frequency five times slower than before, hence the control becomes inadequate. Figure 9(b) shows the trajectory of the system when the movement is made over 0.2s and again compares it to the reference trajectory and the trajectory for 1.0s. The difference here is that the time step (inversely proportional to the controller sampling frequency) is reduced by 5x, and the derivative gain is increased to 60Nms/rad. This renders good results in terms of control of the system. The reason that the derivative gain is increased but not the proportional is that the derivative gain is dependant on time (visible from looking at its units) whereas the proportional gain is not.



(a) Trajectory of the end-effector when the movement is made over 1.0s and 0.2s.

(b) Trajectory of the end-effector when the movement is made over 1.0s and 0.2s, the time step, dt, is reduced to 0.002s and K_d = 60Nms/rad.

Figure 9: The effects of making the movement 5x faster, and a possible adaptation of the controller to allow for good control of this movement.