HNCL Tutorial 2: Redundancy

In this tutorial you will examine the kinematics of a three-joint arm composed of the hand, forearm and upper arm. MATLAB will be required for the plots, which should be clearly labeled.

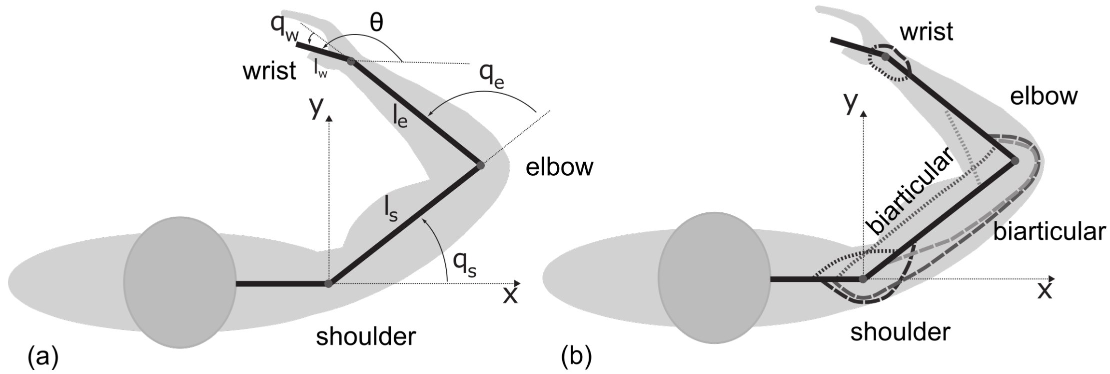

Figure 1: **Three-joint system of planar arm movements: (a) the angle convention, (b) 8 representative muscles acting on the limbs.**

Question 1

(a) Derive the direct kinematics, using the notation given in Figure 1. Assuming that the wrist orientation with respect to the body (θ) is fixed, derive the inverse kinematics. (q_s, q_e, and q_w are the shoulder, elbow and wrist angles respectively.)

[25 marks]

(b) Derive the differential kinematics by finding the relevant Jacobian matrices to transform between task, joint and muscle space velocities. Compute the stiffness matrix in joint space explicitly by assuming that the matrix is completely diagonal in muscle space and that all moment arms are equal.

[25 marks]

(c) Based on the example given in the notes, use the following trajectory for a smooth point-to-point reaching movement:

$$\begin{aligned} x(t) &= -0.1334 + 0.1289\, g(t/T)\,, \quad 0 \le t \le T\,, \\ y(t) &= -0.0077 + 0.7355\, g(t/T)\,, \quad g(t_n) \equiv t_n^3\left(6\,t_n^2 - 15\,t_n + 10\right) \end{aligned}$$

where $T = 2s$ is the movement duration.
Simulate this example in MATLAB and show the endpoint trajectory in the task space. Also, present graphs of the endpoint position, endpoint velocity, joint angles and joint velocities as functions of time for the duration of the movement.
The initial joint angles of the 3-joint system are $\{q_s(0) = 90°,\ q_e(0) = 130°,\ q_w(0) = 90°\}$ and the length of the different segments are $\{l_s = l_e = 0.3m, l_w = 0.15m\}$.
List and explain the different steps (of the MATLAB code) involved in solving the problem.

[50 marks]