MULTI-JOINT DYNAMICS & CONTROL

ACTION DYNAMICS

- to perform an action, e.g. move an object on a table, the muscles need to accelerate this object and compensate for interaction forces, e.g. friction with the table, as well as to accelerate the limb

- joint torque produced by muscles to move along the **undisturbed movement** $\mathbf{q}_u$:

$$\mathbf{J}_\mu^T \boldsymbol{\mu}(\boldsymbol{\lambda}_u, \dot{\boldsymbol{\lambda}}_u, \mathbf{u}_u) = \boldsymbol{\tau}_B(\mathbf{q}_u, \dot{\mathbf{q}}_u, \ddot{\mathbf{q}}_u) - \mathbf{J}(\mathbf{q}_u)^T \mathbf{F}_E$$

$\boldsymbol{\lambda}_u(\mathbf{q}_u)$:	muscle lengths corresponding to $\mathbf{q}_u$ (slightly different for every trial)
$\boldsymbol{\tau}_B$:	torque to move the body
$\mathbf{F}_E$:	external force on hand

RIGID BODY DYNAMICS

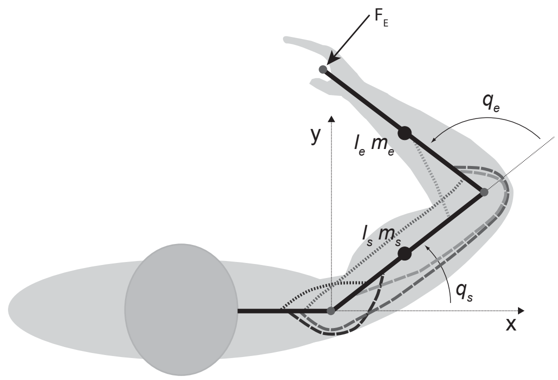
- the main forces arising when moving the bones and flesh are the inertia of the corresponding rigid body dynamics $\boldsymbol{\tau}_B$, elasticity due to muscles, tendons, etc., and reflexes

- neglecting joint elasticity and damping:

$$\boldsymbol{\tau}_B = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q})$$

$\mathbf{H}(\mathbf{q})$: mass or inertia matrix
$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$: velocity dependent forces
$\mathbf{G}(\mathbf{q})$: gravity

DYNAMICS OF 2-JOINT MODEL



$$\boldsymbol{\tau}_B = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}\,, \quad \mathbf{H}(\mathbf{q}) = \begin{bmatrix} H_{ss} & H_{se} \\ H_{es} & H_{ee} \end{bmatrix},$$

$$H_{ss} = I_s + I_e + M_s\, l_{ms}^2 + M_e\,(l_s^2 + l_{me}^2 + 2\, l_s\, l_{me}\, c_e),$$
$$H_{se} = I_e + M_e\,(l_{me}^2 + l_s\, l_{me}\, c_e) = H_{es},$$
$$H_{ee} = I_e + M_e\, l_{me}^2\,, \;\; c_e \equiv \cos(q_e)\,, \;\; s_e \equiv \sin(q_e),$$

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} = \begin{bmatrix} -M_e\, l_s\, l_{me}\, \dot{q}_e\,(2\,\dot{q}_s + \dot{q}_e)\, s_e \\ M_e\, l_s\, l_{me}\, \dot{q}_s^2 s_e \end{bmatrix}$$

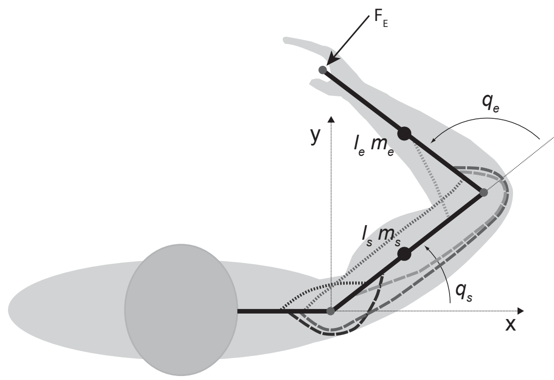
m_s, m_e : masses of the upper and lower arms

l_s, l_e : corresponding segment lengths

l_{ms}, l_{me} : distances to the centers of mass

I_s, I_e : moments of inertia

DYNAMICS OF 2-JOINT MODEL



$$\boldsymbol{\tau}_B = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} = \boldsymbol{\Psi}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\,\mathbf{p}$$

$$p_1 \equiv I_e + m_e\, l_{me}^2 \quad p_2 \equiv m_e\, l_s\, l_{me}$$
$$p_3 \equiv I_s + m_s\, l_{ms}^2 + m_e\, l_s^2$$

$$\Psi_{12} = c_e(2\ddot{q}_s + \ddot{q}_e) - s_e\dot{q}_e\,(2\dot{q}_s + \dot{q}_e)\ , \qquad \Psi_{13} = \ddot{q}_s$$
$$\Psi_{11} = \Psi_{21} = \ \ddot{q}_s + \ddot{q}_e\,,\ \Psi_{22} = c_e\ddot{q}_s + s_e\dot{q}_s^2\,,\ \Psi_{23} = 0$$

m_s, m_e : masses of the upper and lower arms

l_s, l_e : corresponding segment lengths

l_{ms}, l_{me} : distances to the centers of mass

I_s, I_e : moments of inertia

FORCE & IMPEDANCE IN MOTION

if disturbance $\triangle\mathbf{F}_E$ is small, then $\mathbf{q}$ is close to the undisturbed trajectory $\mathbf{q}_u$, and we linearise with $\mathbf{e} = \mathbf{q}_u - \mathbf{q}$:

$$\begin{aligned}
\mathbf{J}_\mu^T \boldsymbol{\mu}(\boldsymbol{\lambda}, \dot{\boldsymbol{\lambda}}, \mathbf{u}) &= \boldsymbol{\tau}_T(\mathbf{q}_u, \dot{\mathbf{q}}_u, \ddot{\mathbf{q}}_u) - \mathbf{J}(\mathbf{q})^T \triangle\mathbf{F}_E \\
&= \boldsymbol{\tau}_T(\mathbf{q}_u, \dot{\mathbf{q}}_u, \ddot{\mathbf{q}}_u) + Ke + D\dot{e} \\
\mathbf{K} &\equiv \left(-\frac{\partial \tau_i}{\partial q_j} - \sum_k \frac{\partial \tau_i}{\partial u_k} \frac{\partial u_k}{\partial q_j} \right) \\
\mathbf{D} &\equiv \left(-\frac{\partial \tau_i}{\partial \dot{q}_j} - \sum_k \frac{\partial \tau_i}{\partial u_k} \frac{\partial u_k}{\partial \dot{q}_j} \right)
\end{aligned}$$

$\boldsymbol{\tau}_T(\mathbf{q}_u, \dot{\mathbf{q}}_u, \ddot{\mathbf{q}}_u) \equiv \boldsymbol{\tau}_B(\mathbf{q}_u, \dot{\mathbf{q}}_u, \ddot{\mathbf{q}}_u) - \mathbf{J}^T \mathbf{F}_E$
unperturbed task dynamics

LINEAR ROBOT CONTROL

- it is not sufficient to send the torque required to move the robot along a desired trajectory to the motors

- unmodelled dynamics would prevent good tracking and may lead to instability

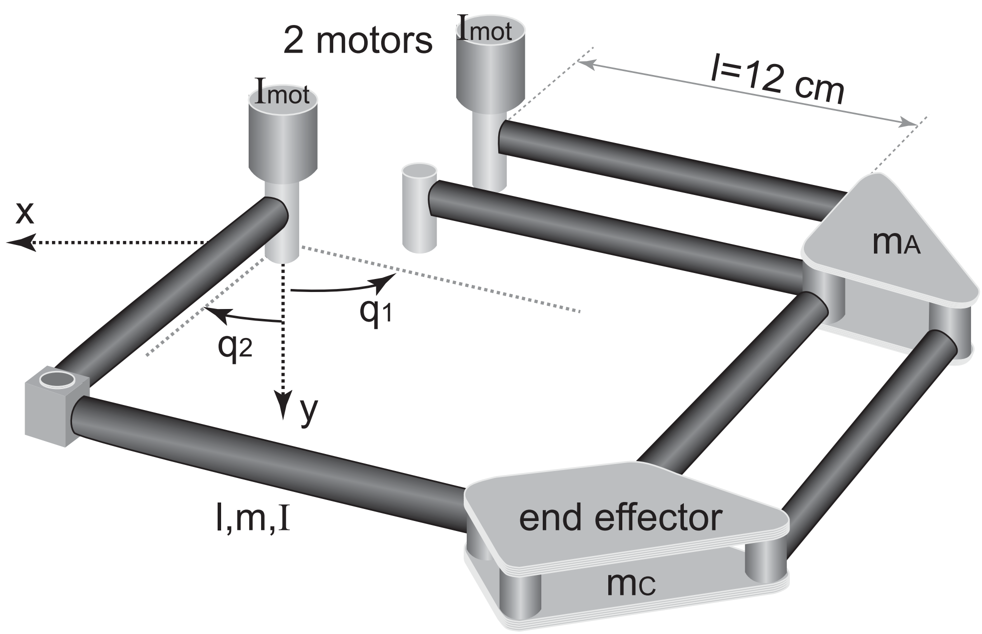
- **linear feedback control** to minimise the tracking error $\boldsymbol{\varepsilon}$ during each time interval:

$$\boldsymbol{\tau}_{FB} = \mathbf{K}\boldsymbol{\varepsilon}\,, \quad \boldsymbol{\varepsilon} \equiv \mathbf{e} + \kappa\dot{\mathbf{e}}\,, \quad \mathbf{e} \equiv \mathbf{q}_u - \mathbf{q}$$

- two functions of the feedback torque $\boldsymbol{\tau}_{FB}$:
  - to drive the plant along the desired trajectory
  - to stabilise it against disturbances

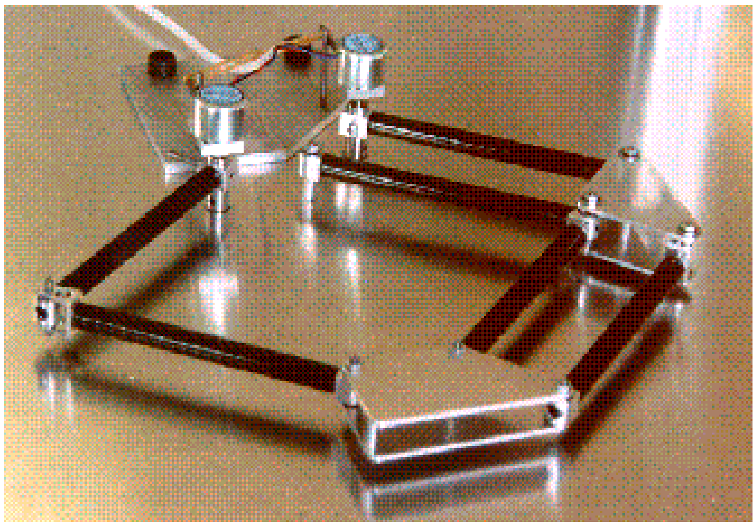
NONLINEAR ROBOT CONTROL

the dynamics of most multi-link mechanisms are nonlinear



the 6 links with length $l = 12cm$, mass m and moment of inertia I, joined by two metallic plates of masses m_A and m_C, are driven by two DC motors with moment of inertia I_{mot}

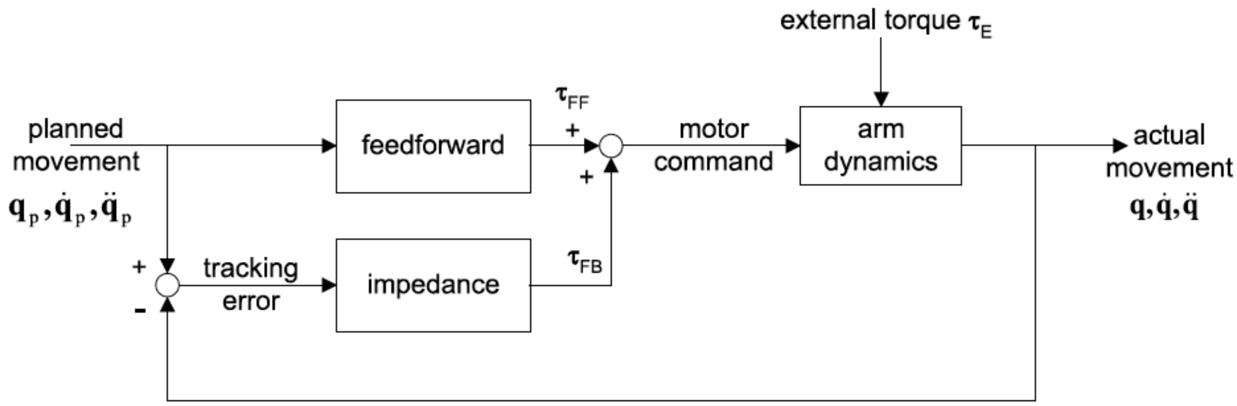
NONLINEAR ROBOT CONTROL



$$\begin{aligned}
\boldsymbol{\tau} &= \boldsymbol{\Psi}\,\mathbf{p} \\
\mathbf{p} &= (l^2 m_C, l^2 m, 3I + I_{\mathrm{mot}}, l^2 m_A, b_{11}, b_{12}, b_{21}, b_{22})^T \\
\boldsymbol{\Psi} &= \begin{bmatrix}
s_{12}\dot{q}_2^2 - c_{12}\ddot{q}_2 + \ddot{q}_1 & s_{12}\dot{q}_1^2 - c_{12}\ddot{q}_1 + \ddot{q}_2 \\
\frac{3}{2}\left(s_{12}\dot{q}_2^2 - c_{12}\ddot{q}_2\right) + \frac{11}{4}\ddot{q}_1 & \frac{3}{2}\left(s_{12}\dot{q}_1^2 - c_{12}\ddot{q}_1\right) + \frac{7}{4}\ddot{q}_2 \\
\ddot{q}_1 & \ddot{q}_2 \\
\ddot{q}_1 & 0 \\
\dot{q}_1 & 0 \\
\mathrm{sign}(\dot{q}_1) & 0 \\
0 & \dot{q}_2 \\
0 & \mathrm{sign}(\dot{q}_2)
\end{bmatrix}^T \\
s_{12} &\equiv \sin(q_1 + q_2) \qquad c_{12} \equiv \cos(q_1 + q_2)
\end{aligned}$$

this equation is nonlinear in $(q_1, q_2, \dot{q}_1, \dot{q}_2, \ddot{q}_1, \ddot{q}_2)$
(but linear in the parameter vector $\mathbf{p}$)

NONLINEAR ROBOT CONTROL



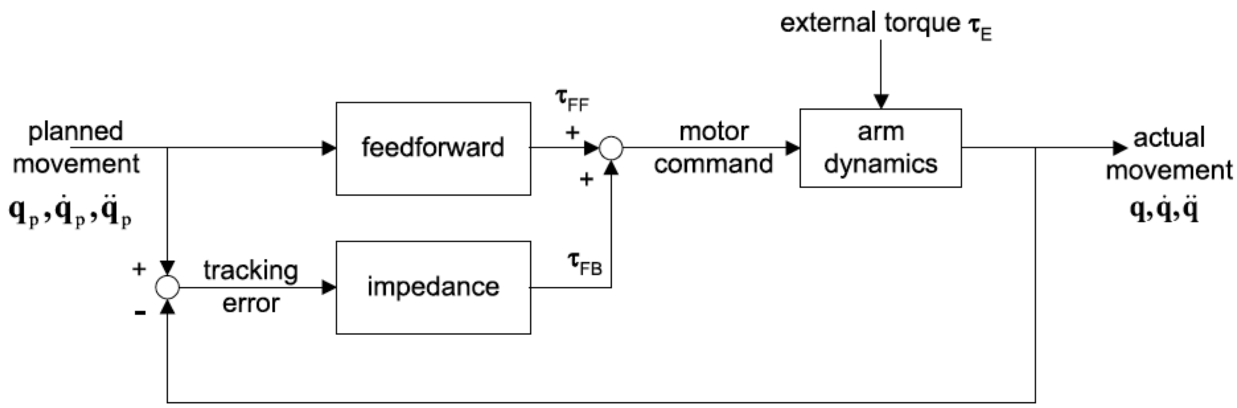
nonlinear control along desired trajectory $\mathbf{q}_u$:

$$\boldsymbol{\tau} = \boldsymbol{\Psi}(\mathbf{q}_u, \dot{\mathbf{q}}_u, \ddot{\mathbf{q}}_u)\,\mathbf{p} + \mathbf{K}\boldsymbol{\varepsilon}$$

- motion dynamics is provided by the **feed-forward** term $\boldsymbol{\Psi}\mathbf{p}$

- **feedback** $\mathbf{K}\boldsymbol{\varepsilon} \equiv \mathbf{e} + \kappa\dot{\mathbf{e}}\,,\ \mathbf{e} \equiv \mathbf{q}_u - \mathbf{q}$ provides stability and robustness to disturbances and to error of the feedforward model

FF CONTROL MODEL (1)



$$
\begin{aligned}
\boldsymbol{\tau} &= \boldsymbol{\tau}_{FF} + \boldsymbol{\tau}_{FB} \\
\boldsymbol{\tau}_{FF} &= \boldsymbol{\Psi}(\mathbf{q}_u, \dot{\mathbf{q}}_u, \ddot{\mathbf{q}}_u) - \mathbf{J}(\mathbf{q})^T \mathbf{F}_E \\
\boldsymbol{\tau}_{FB} &= \mathbf{K}(|\boldsymbol{\tau}_{FF}|)\,\boldsymbol{\varepsilon}\,,\ \boldsymbol{\varepsilon} \equiv \mathbf{e} + \kappa\dot{\mathbf{e}}\,,\ \mathbf{e} \equiv \mathbf{q}_u - \mathbf{q}
\end{aligned}
$$

- effect of impedance and neural feedback lumped

- $\mathbf{K}$ is a linear function of FF torque magnitude

- the task dynamics depend on the position, velocity and acceleration, but the feedforward torque approximating it depends on the muscle variables $(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u})$

FF CONTROL MODEL (2)

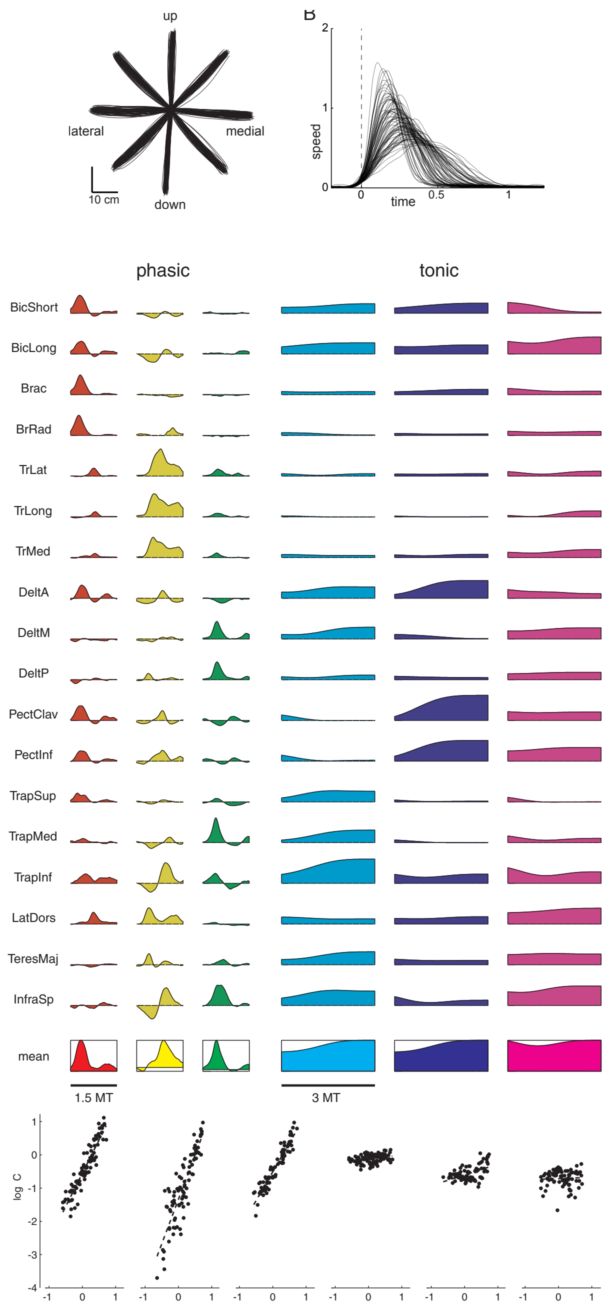
- the dynamics of the rigid-body arm model $\boldsymbol{\tau}_B = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{G}(\mathbf{q}) = \boldsymbol{\Psi}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\,\mathbf{p}$ are complex, so it is not clear how the CNS could produce the torques to simultaneously move multiple body segments

- human movements have regular patterns, so the CNS may only have to produce a limited set of **primitive movements**

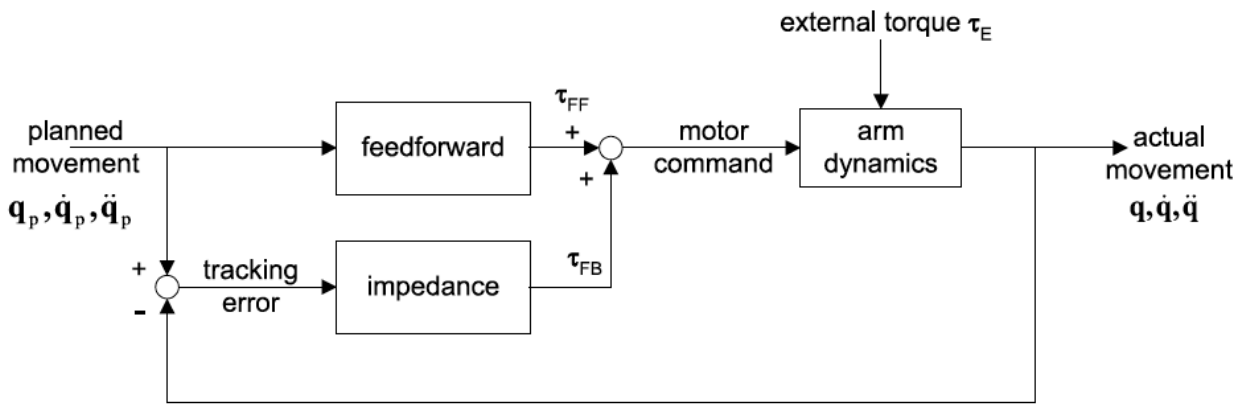
- the CNS may just scale torque corresponding to inertial terms quadratically with time:

$$
\begin{aligned}
t \;\rightarrow\; \xi t\,,\; \dot{x}(t) \;&\rightarrow\; \dot{x}(t)\xi\,,\; \ddot{x}(t) \;\rightarrow\; \ddot{x}(t)\xi^2 \\
\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} \;&\rightarrow\; [\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}]\,\xi^2 \\
\mathbf{G}(\mathbf{q}) \;&\rightarrow\; \mathbf{G}(\mathbf{q})
\end{aligned}
$$

MUSCLE SYNERGIES



MODEL OF FORCE AND IMPEDANCE IN MOTION



- torque to move the limbs similar to inverse dynamics
- external force compensated by the joint torques
- restoring force to perturbation from impedance
- stiffness as a linear function of the joint torque magnitude (posture independent)
- damping proportional to the stiffness matrix and inversely proportional to the speed

SIMULATION OF PLANAR ARM MOVEMENTS

- undisturbed trajectory: straight line movement with bell shaped velocity profile

- data to simulate planar movements:

	mass [kg]	length [m]	center of mass from proximal joint [m]	mass moment of inertia [kg m^2]
upper arm	1.93	0.31	0.165	0.0141
forearm	1.52	0.34	0.19	0.0188

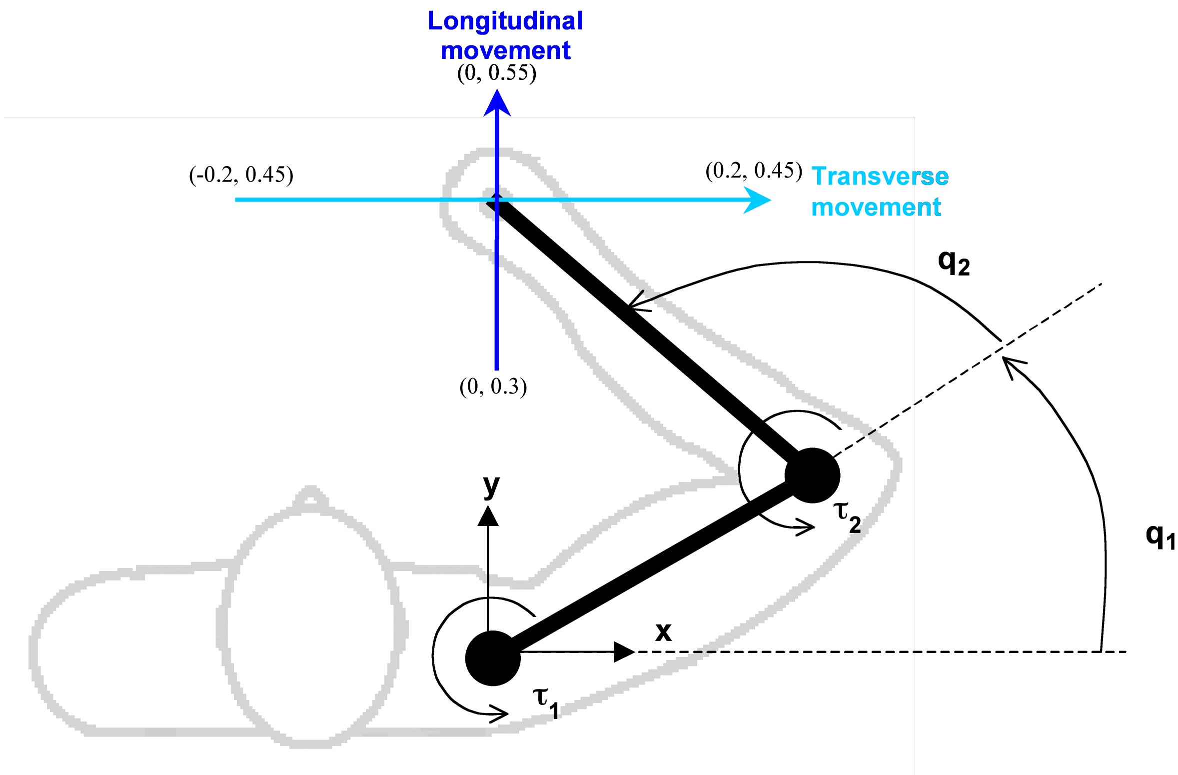
- stiffness:

$$\mathbf{K} = \begin{pmatrix} 10.8 + 3.18|\tau_s| & 2.83 + 2.15|\tau_e| \\ 2.51 + 2.34|\tau_e| & 8.67 + 6.18|\tau_e| \end{pmatrix} \frac{Nm}{rad}$$

- viscosity:

$$\mathbf{D} = \frac{0.42}{\sqrt{1 + \dot{\mathbf{q}}^T \dot{\mathbf{q}}}} \mathbf{K}$$

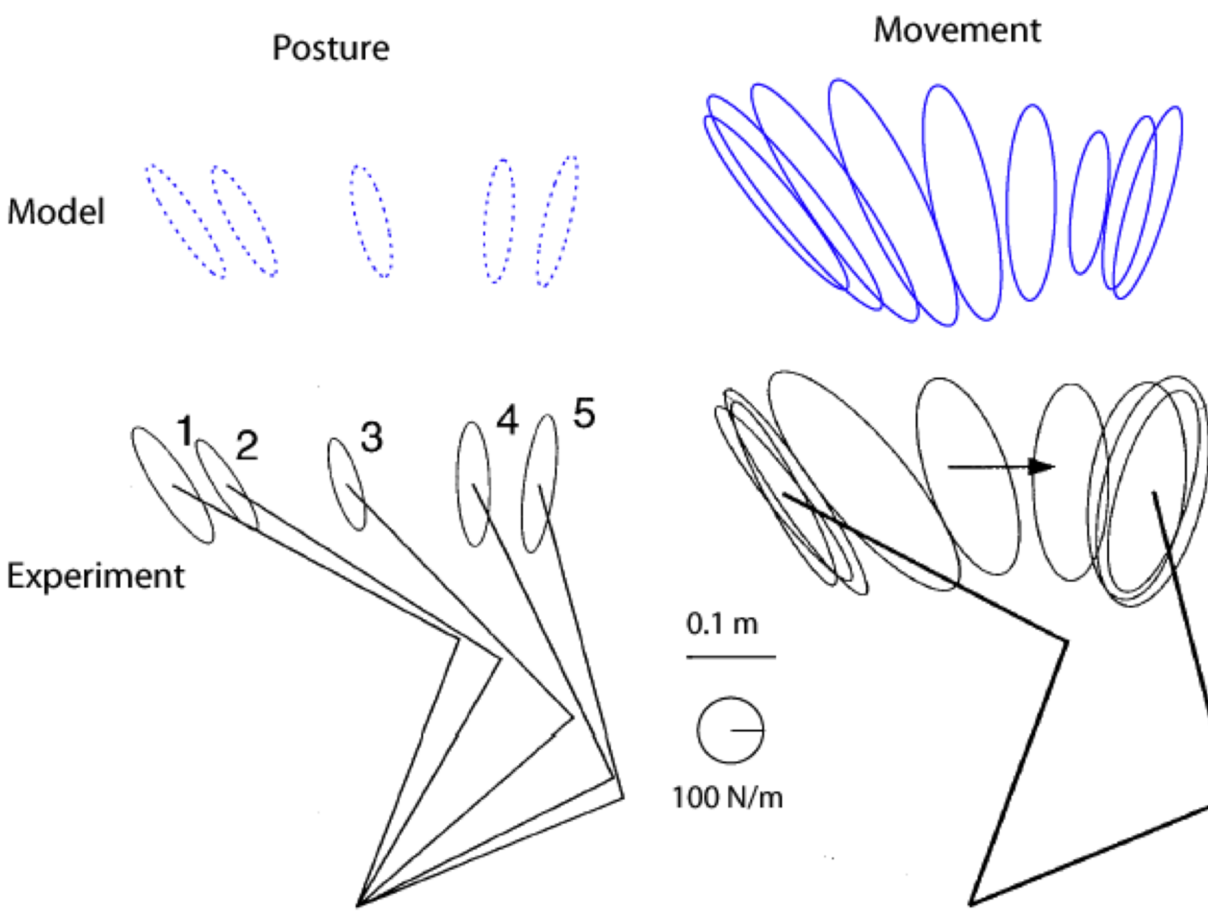
STIFFNESS GEOMETRY DURING MOVEMENT



- horizontal movements at shoulder height with the hand supported (such that the influence of gravity could be neglected)

- the rigid body dynamics are computed along the minimal jerk planed trajectory

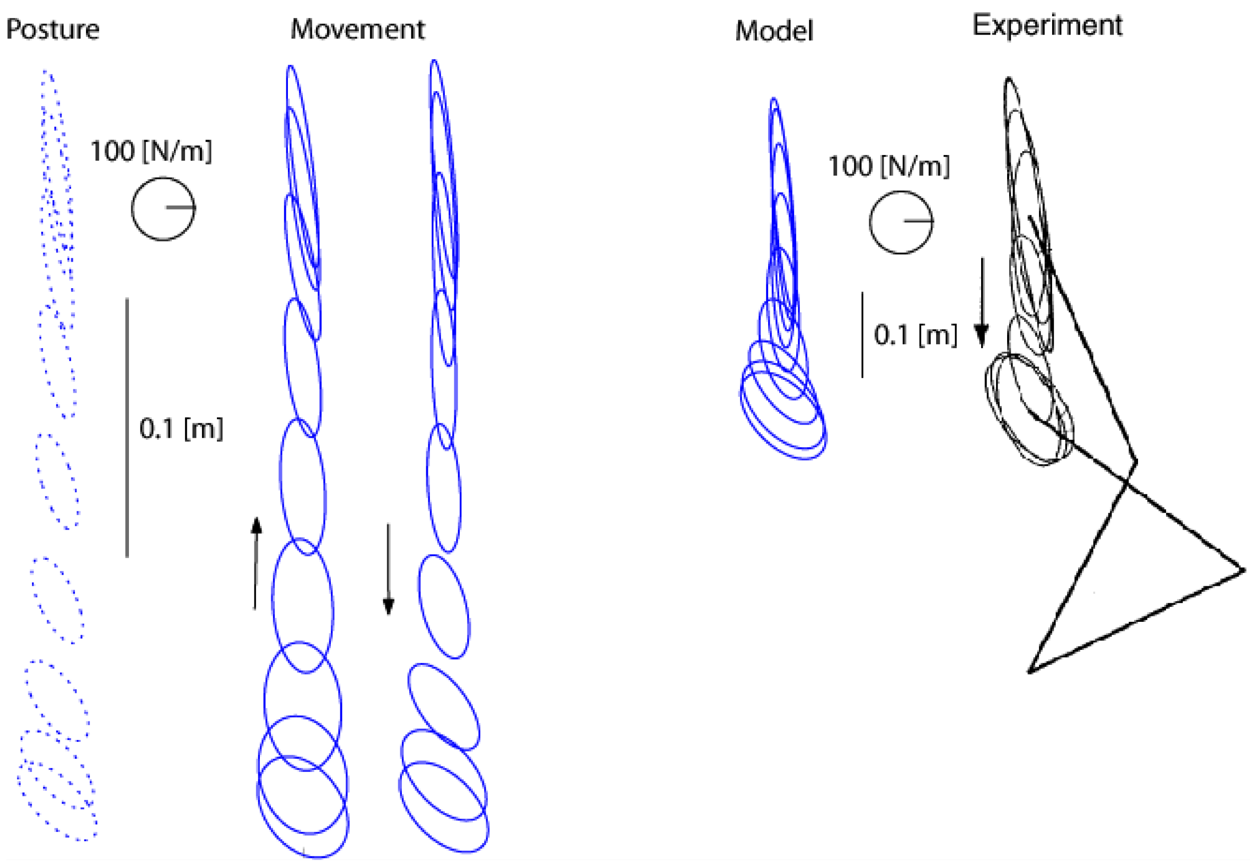
TRANSVERSE MOVEMENT



- snapshots of the stiffness ellipses every 100*ms*

- dynamic stiffness is higher as compared to postural stiffness since joint torque is larger in magnitude

LONGITUDINAL MOVEMENT

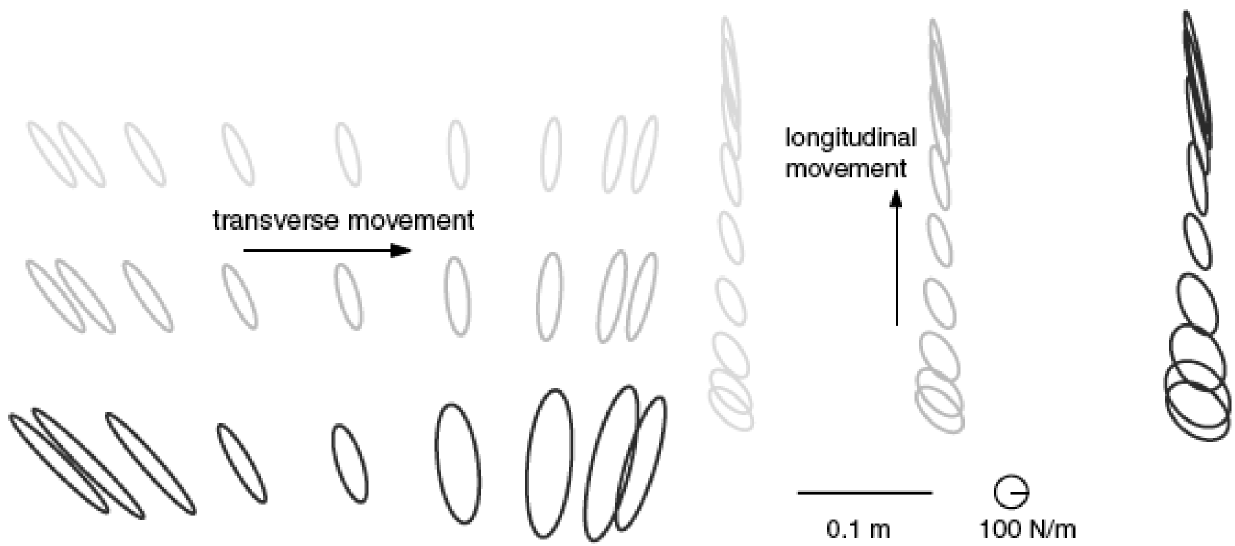


- snapshots of the stiffness ellipses every 100*ms*

- dynamic stiffness is higher as compared to postural stiffness since joint torque is larger in magnitude

- good match with the experimental data

VELOCITY SCALING

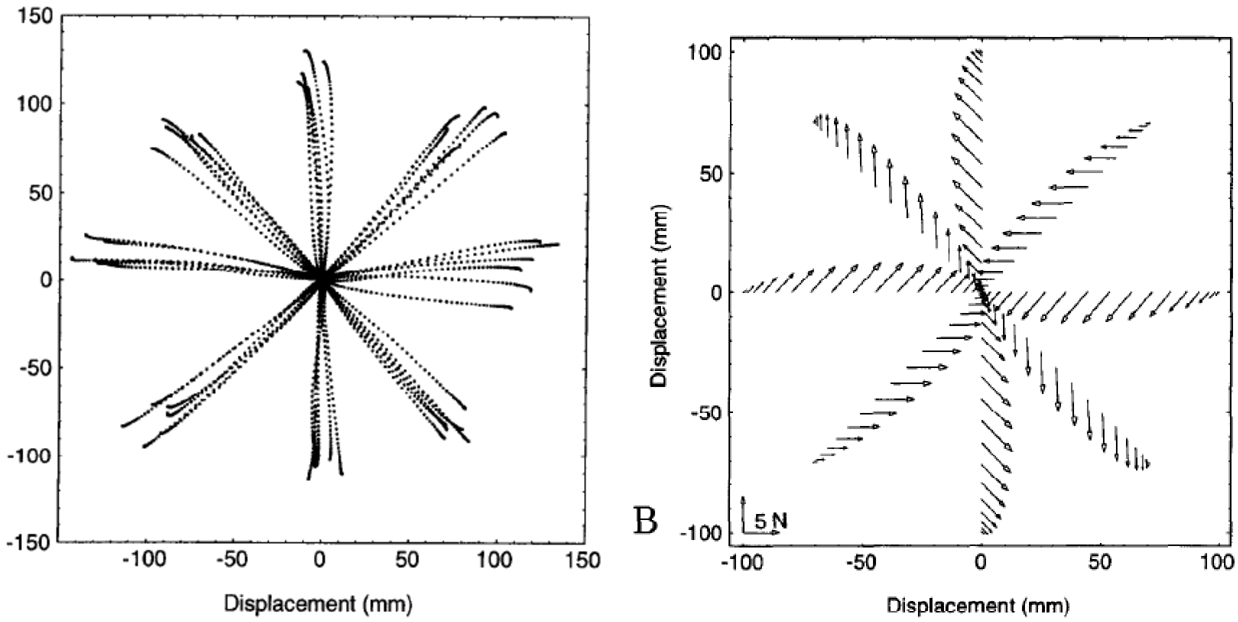


- prediction of movements with durations of 0.5, 1, and 2*s*

- increase of stiffness ellipse size in the acceleration and deceleration phases

- no change in the shape and orientation of stiffness among different movement speeds

REACHING MOVEMENTS IN NOVEL DYNAMICS (1)



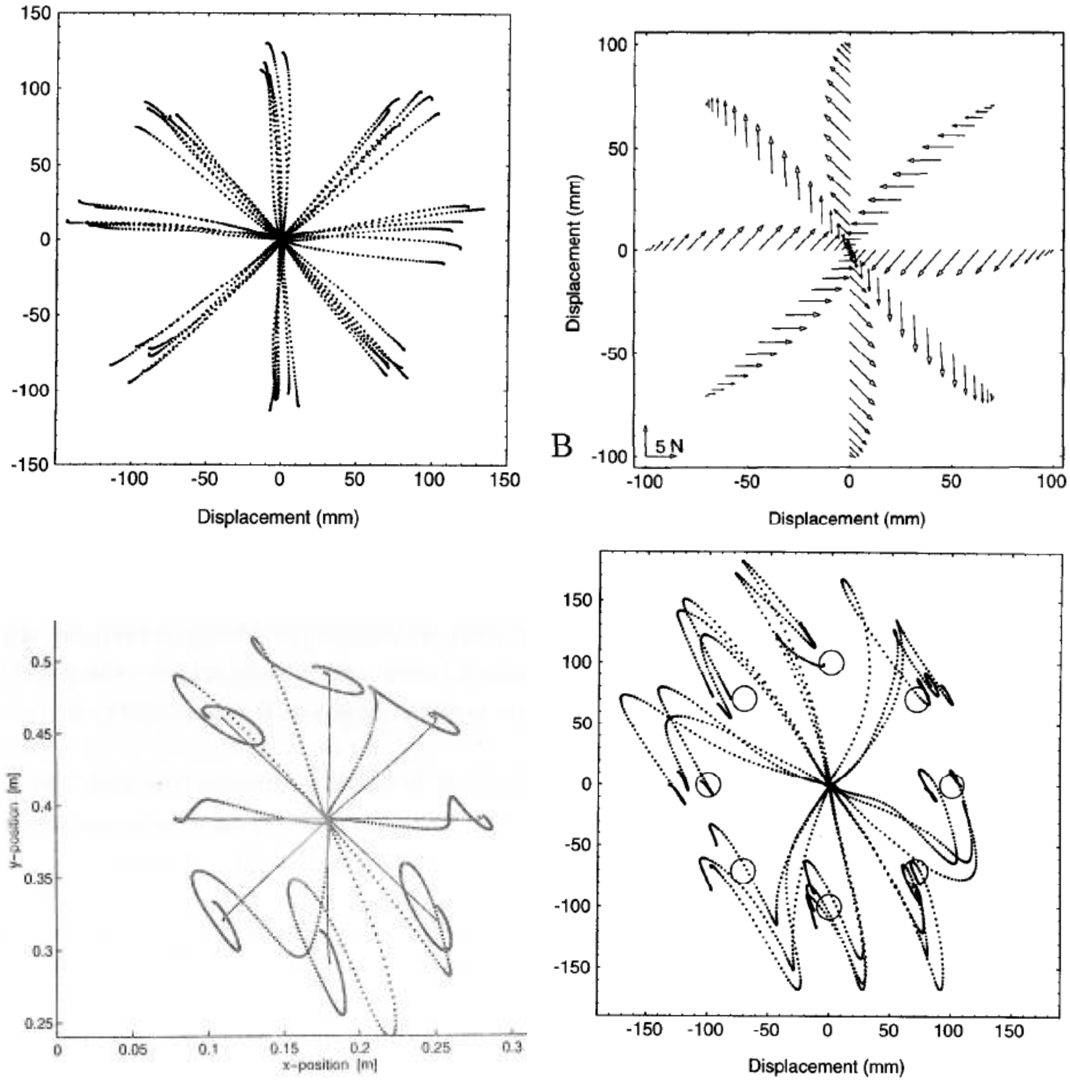
- horizontal arm reaching movements at shoulder height performed in 8 directions

- haptic interface exercises a force

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} -10.1 & -11.2 \\ -11.2 & 11.1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

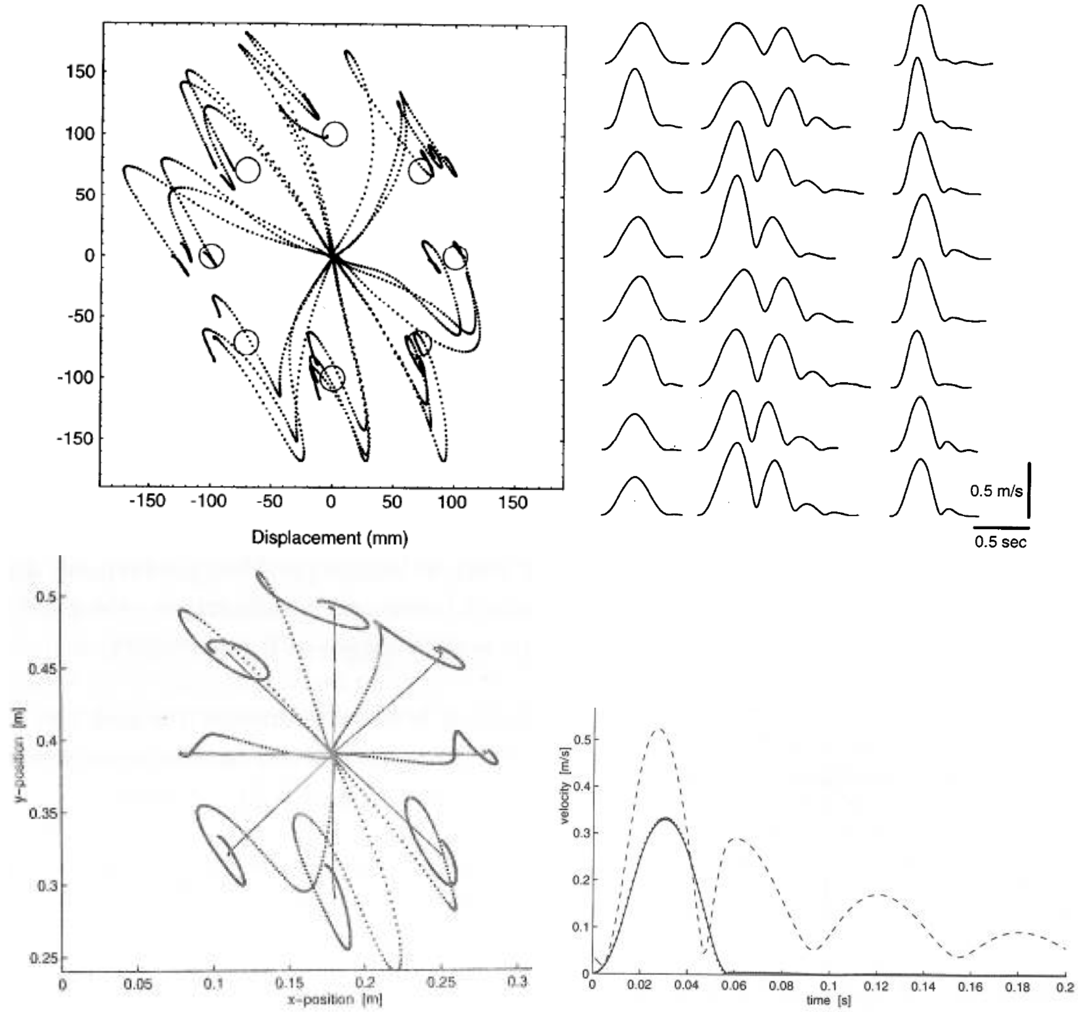
on the hand during movement (force in *N*, velocity in *m/s*)

REACHING MOVEMENTS IN NOVEL DYNAMICS (2)



the trajectories deformations in novel dynamics are well predicted by our model

REACHING MOVEMENTS IN NOVEL DYNAMICS (3)

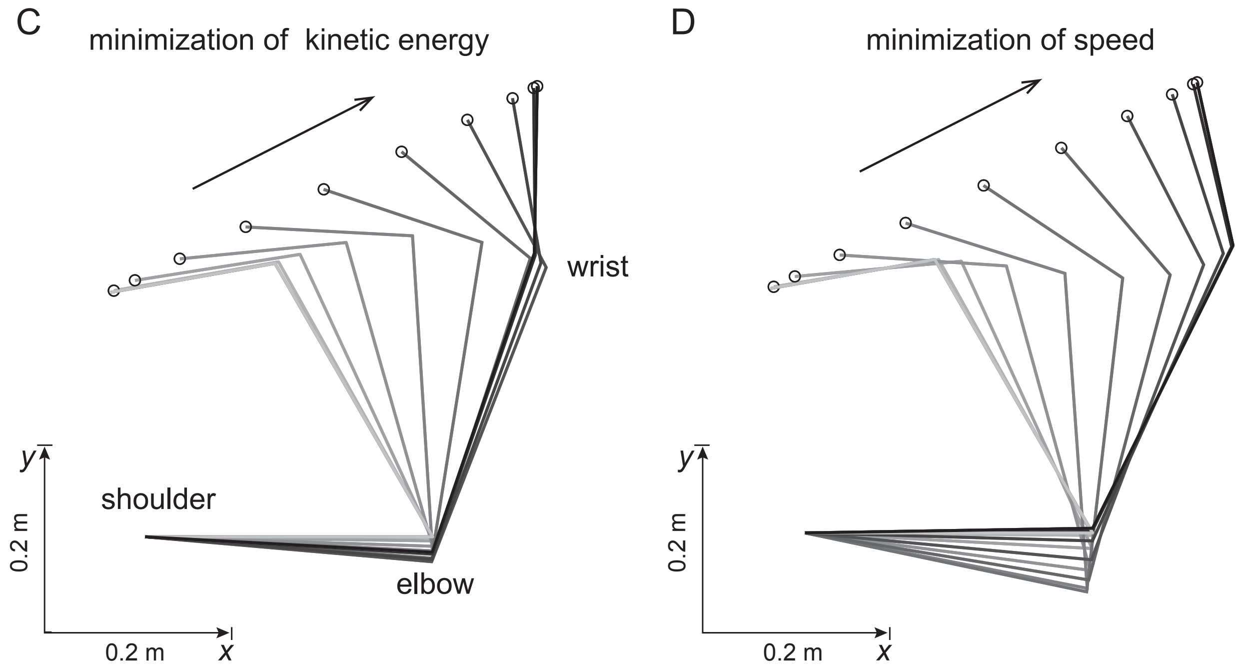


the trajectories deformations in novel dynamics are well predicted by our model

DYNAMIC REDUNDANCY

how to best use redundancy in a mechanism with more joints than required by the task?

DYNAMIC REDUNDANCY (2)



to minimise the kinetic energy

$$E \equiv \frac{1}{2}\dot{\mathbf{q}}^T \mathbf{H}\dot{\mathbf{q}}$$

($\mathbf{H}(\mathbf{q})$ mass matrix) under $\dot{\mathbf{x}} = \mathbf{J}\dot{\mathbf{q}}$ yields

$$\dot{\mathbf{q}}^* = \mathbf{J}_H^\dagger\,\dot{\mathbf{x}}\,, \quad \mathbf{J}_H^\dagger \equiv \mathbf{H}^{-1}\mathbf{J}^T\left(\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^T\right)^{-1}$$

$\mathbf{J}^\dagger(\mathbf{q})$: **weighted pseudo-inverse**

DYNAMIC REDUNDANCY (3)

- additional constraints (e.g. gravitation, joint boundaries) can be implemented by reducing suitable potentials, using their gradient

- this enables generation of human-like motion for tasks given in Cartesian space, which can be implemented on (humanoid) robots and on animats in computer graphics

DYNAMIC REDUNDANCY (4)

- it is also possible to minimise muscle activation

$$V(\mathbf{u}) \equiv \mathbf{u}^2 = \sum_{i=1}^{n} u_i^2$$

with constraints

$$\mathbf{J}^T \mathbf{F} = \boldsymbol{\tau} = \mathbf{J}_\mu^T \, \boldsymbol{\mu}(\mathbf{u})$$

- muscle tension $\boldsymbol{\mu}(\boldsymbol{\lambda}, \dot{\boldsymbol{\lambda}}, \mathbf{u})$ is a nonlinear function of the activation $\mathbf{u}$

- the solution can be computed numerically

- this may enable one to "understand" human postures during various tasks

SUMMARY (1)

- we have introduced techniques to model and simulate the dynamics of the multi-joint arm, as well as muscle or kinematic redundancy

- the concept of a real-time controller is particularly important as it can be used to elucidate the stable and smooth control of human arm motion, as will be used in the next parts to interpret human motor control

SUMMARY (2)

- a simple model to predict endpoint force and impedance during movement postulates that the CNS compensates for the interaction dynamics, and stiffness depends linearly on the load to move the limb and interact with the environment

- the good predictions of this model suggests that impedance of a limb exerting a force on the environment does not depend on whether the force is produced to move the arm or to interact with the environment

SUMMARY (3)

- using measurement of stiffness in static interactions with various levels of force, it becomes possible to predict stiffness during arbitrary movements adapted to a known dynamic environment

- the model is valid for learned movements of the (possibly redundant) arm or leg in three dimensional space but requires knowledge of the trajectory

- this model does not consider accurate muscle properties, but can be used to simulate animat or implement "naturally looking" motion on humanoid robots