

1. Section 1

Consider one auto company that receives parts from three suppliers; assume 50% of the parts from supplier 1, 30% from supplier 2, and 20% from supplier 3. The quality of the parts could be summarized in the following table based on historically data.

	Percentage Good Parts	Percentage Bad parts
Supplier 1	98	2
Supplier 2	95	5
Supplier 3	92	8

Question: A bad part broke one of the machines (observed), what is the probability the part came from supplier 1?

ANSWER:

Let A_1 denote Supplier 1, A_2 denote Supplier 2, and A_3 denote Supplier 3

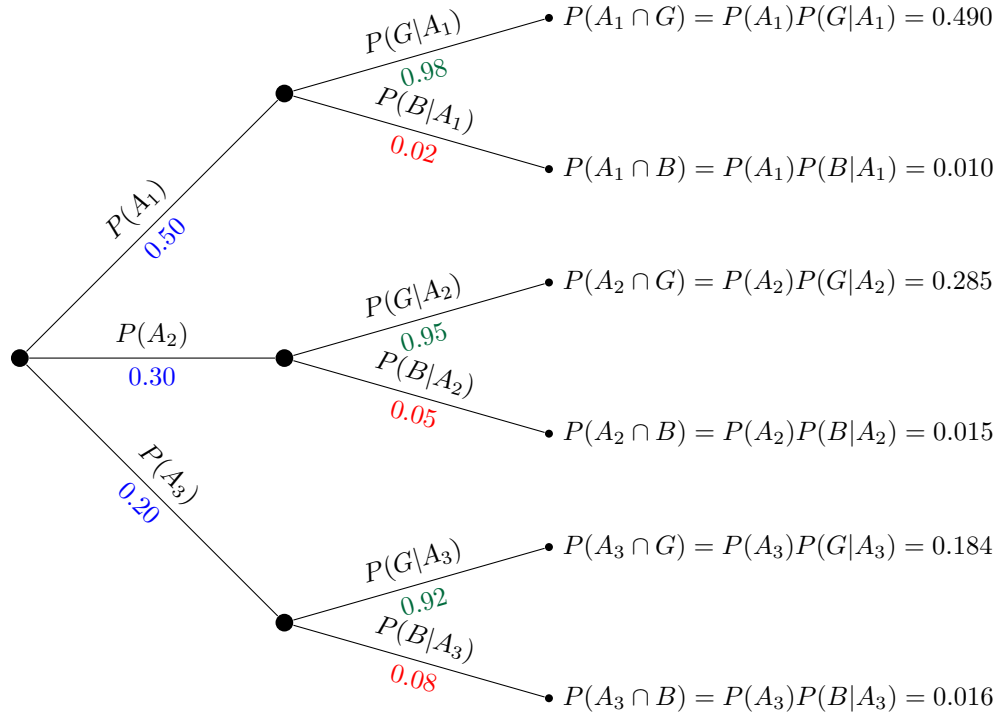
$$\begin{aligned}P(A_1) &= 0.50 \\P(A_2) &= 0.30 \\P(A_3) &= 0.20\end{aligned}\tag{1.1}$$

Let G denote that a part is good and B denote that a part is bad.

$$\begin{aligned}P(G|A_1) &= 0.98 \\P(G|A_2) &= 0.95 \\P(G|A_3) &= 0.92\end{aligned}\tag{1.2}$$

$$\begin{aligned}P(B|A_1) &= 0.02 \\P(B|A_2) &= 0.05 \\P(B|A_3) &= 0.08\end{aligned}\tag{1.3}$$

Here is the Probability Tree for Three-Suppliers



Law of Conditional Probability gives us the following equation

$$P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)} \quad (1.4)$$

We know from are probability tree that

$$P(A_1 \cap B) = P(A_1)P(B|A_1) = 0.010 \quad (1.5)$$

We also know that

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3) \quad (1.6)$$

Combining are equations together we obtain Bayes' Theorem

$$1 \leq k \leq n$$

$$P(B_k|A) = \frac{P(A|B_k)P(B_k)}{\sum_{i=1}^n P(A|B_i)P(B_i)} \quad (1.7)$$

$$\begin{aligned} P(A_1|B) &= \frac{P(A_1)P(B|A_1)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)} \\ P(A_2|B) &= \frac{P(A_2)P(B|A_2)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)} \\ P(A_3|B) &= \frac{P(A_3)P(B|A_3)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)} \end{aligned} \quad (1.8)$$

Now all we have to do is plug in the numbers and solve the equations

$$\begin{aligned}P(A_1|B) &= \frac{(0.50)(0.02)}{(0.50)(0.02) + (0.30)(0.05) + (0.20)(0.08)} = \frac{0.010}{0.041} = 0.2439024390 \\P(A_2|B) &= \frac{(0.30)(0.05)}{(0.50)(0.02) + (0.30)(0.05) + (0.20)(0.08)} = \frac{0.015}{0.041} = 0.3658536585 \\P(A_3|B) &= \frac{(0.20)(0.08)}{(0.50)(0.02) + (0.30)(0.05) + (0.20)(0.08)} = \frac{0.016}{0.041} = 0.3902439024\end{aligned}\tag{1.9}$$

Therefore the probability the part came from supplier 1: 0.243902439

2. Section 2

For the play tennis data set shown below:

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No