

# 1. Section 1

## 1.1 a.

The vertebral column data was first read from the ARFF file, then split into classes for processing.

```
library(foreign)
vert <- read.arff("column_2C_weka.arff")
vert_split <- split(vert, vert[, "class"])

sapply(vert_split$Abnormal[0:6], mean)
sapply(vert_split$Abnormal[0:6], median)
sapply(vert_split$Abnormal[0:6], sd)
sapply(vert_split$Normal[0:6], mean)
sapply(vert_split$Normal[0:6], median)
sapply(vert_split$Normal[0:6], sd)
```

### 1.1.1 Abnormal Data

mean			
pelvic_incidence	pelvic_tilt	lumbar_lordosis_angle	
64.69256	19.79111	55.92537	
sacral_slope	pelvic_radius	degree_spondylolisthesis	
44.90145	115.07771	37.77771	
standard deviation			
pelvic_incidence	pelvic_tilt	lumbar_lordosis_angle	
65.27489	18.79890	56.15000	
sacral_slope	pelvic_radius	degree_spondylolisthesis	
44.63960	115.65032	31.94652	
median			
pelvic_incidence	pelvic_tilt	lumbar_lordosis_angle	
17.66213	10.51587	19.66947	
sacral_slope	pelvic_radius	degree_spondylolisthesis	
14.51556	14.09060	40.69674	

### 1.1.2 Normal Data

mean			
pelvic_incidence	pelvic_tilt	lumbar_lordosis_angle	
51.685244	12.821414	43.542605	
sacral_slope	pelvic_radius	degree_spondylolisthesis	
38.863830	123.890834	2.186572	
standard deviation			
pelvic_incidence	pelvic_tilt	lumbar_lordosis_angle	
50.12312	13.48243	42.63892	
sacral_slope	pelvic_radius	degree_spondylolisthesis	
37.05969	123.87433	1.15271	
median			
pelvic_incidence	pelvic_tilt	lumbar_lordosis_angle	
12.368161	6.778503	12.361388	
sacral_slope	pelvic_radius	degree_spondylolisthesis	
9.624004	9.014246	6.307483	

## 1.2 b.

```
library(foreign)
vert <- read.arff("column_2C_weka.arff")

pairs(vert[0:6], pch = 21, bg = c("green", "blue")[unclass(vert$class)])
```

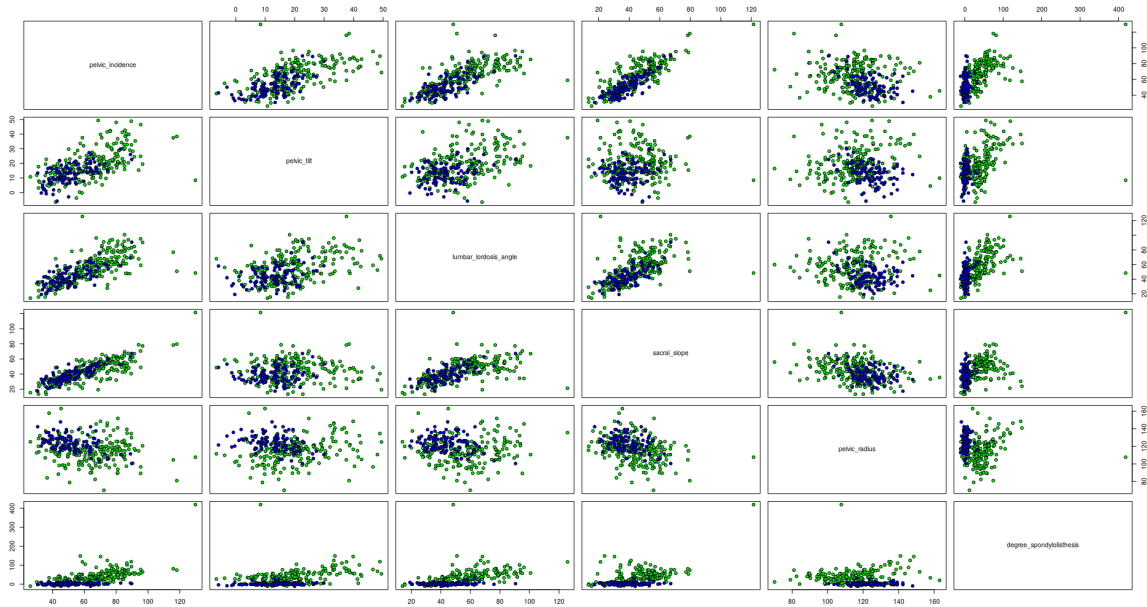


Figure 1.1: Feature Scatter Plot

## 1.3 c.

Given the values from section a and the scatter plot from section b we can see that the two classes are separated well by their values. For example if we pick two classes, pelvic\_radius and degree\_spondylolisthesis, we can compare the values and see how well they are separated. If we also take into account the scatter plot from Figure 1.1 we can see that abnormal classes have a larger value with respect to degree\_spondylolisthesis than the normal class.

## 2. Section 2

### 2.1 a.

Generating 100 3-dimensional vectors from a normal distribution with a mean vector as  $[1 \ 2 \ 1]$  and a 3x3 covariance matrix as  $[4 \ 0.8 \ -0.3; 0.8 \ 2 \ 0.6; -0.3 \ 0.6 \ 5]$

```
mean <- c(1,2,1)
cov <- matrix(c(4, 0.8, -0.3, 0.8, 2, 0.6, -0.3, 0.6, 5), 3,3)
mvnd <- MASS\dotsmvnrnorm(n = 100, mean, cov)
```

$$mean = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

$$cov = \begin{bmatrix} 4 & 0.8 & -0.3 \\ 0.8 & 2 & 0.6 \\ -0.3 & 0.6 & 5 \end{bmatrix}$$

### 2.2 b.

### 2.3 c.

### 3. Section 3

```
records <- read.table("five-dimensional-records.txt")
mean <- colMeans(records)
cov <- cov(records)
```

$$\text{mean} < - \begin{bmatrix} 6241.66667 & 11.44167 & 2333.33333 & 120.83333 & 17000.00000 \end{bmatrix}$$

$$\text{cov} < - \begin{bmatrix} 1.183356e+07 & 59.924242 & 4152121.2121 & 173507.5758 & 490909.091 \\ 5.992424e+01 & 3.191742 & 342.1212 & 141.9621 & 9818.182 \\ 4.152121e+06 & 342.121212 & 1540606.0606 & 73424.2424 & 963636.364 \\ 1.735076e+05 & 141.962121 & 73424.2424 & 13208.3333 & 569090.909 \\ 4.909091e+05 & 9818.181818 & 963636.3636 & 569090.9091 & 40545454.545 \end{bmatrix}$$

```
Eigenvalues <- eigen(cov)$values
Eigenvectors <- eigen(cov)$vectors
```

$$\text{Eigenvalues} < - \begin{bmatrix} 4.058981e+07 & 1.327940e+07 & 6.078551e+04 & 2.835137e+03 & 5.672175e-01 \end{bmatrix}$$

$$\text{Eigenvectors} < - \begin{bmatrix} 0.0210326211 & 9.430084e-01 & 0.332053840 & 0.0057119337 & -0.0006728422 \\ 0.0002420299 & -8.480421e-06 & -0.002006136 & -0.0006633593 & -0.9999977384 \\ 0.0269236336 & 3.312548e-01 & -0.942852072 & 0.0239126004 & 0.0018793378 \\ 0.0141541619 & 1.292481e-02 & -0.020405538 & -0.9996077844 & 0.0007073532 \\ 0.9993159400 & -2.895527e-02 & 0.018703144 & 0.0133939822 & 0.0001957042 \end{bmatrix}$$

```
PCA <- as.data.frame(prcomp(records)$x)[1:2]
```

$$\text{PCA} < - \begin{bmatrix} 7989.734 & -685.3013 \\ -7153.694 & -5315.8562 \\ -8091.763 & -2891.1789 \\ 7926.393 & -2743.7012 \\ 7927.907 & -2588.2250 \\ -4949.073 & 2079.0494 \\ -1158.977 & -5367.2371 \\ -2912.664 & 3101.7248 \\ 1105.817 & 3774.9869 \\ 8103.076 & 3358.3627 \\ -4900.497 & 3631.3980 \\ -3886.258 & 3645.9779 \end{bmatrix}$$

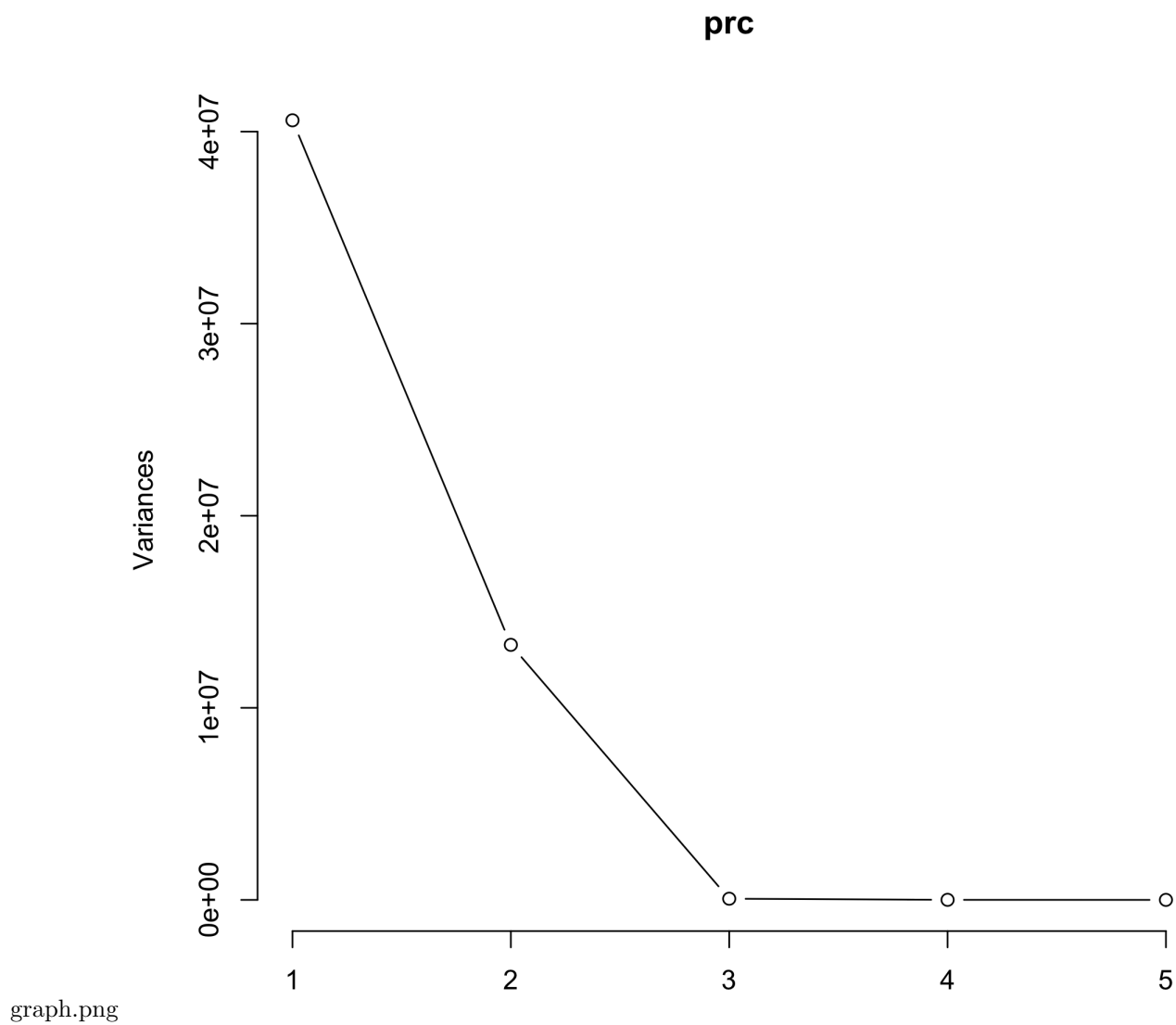


Figure 3.1: Eigenvectors line graph

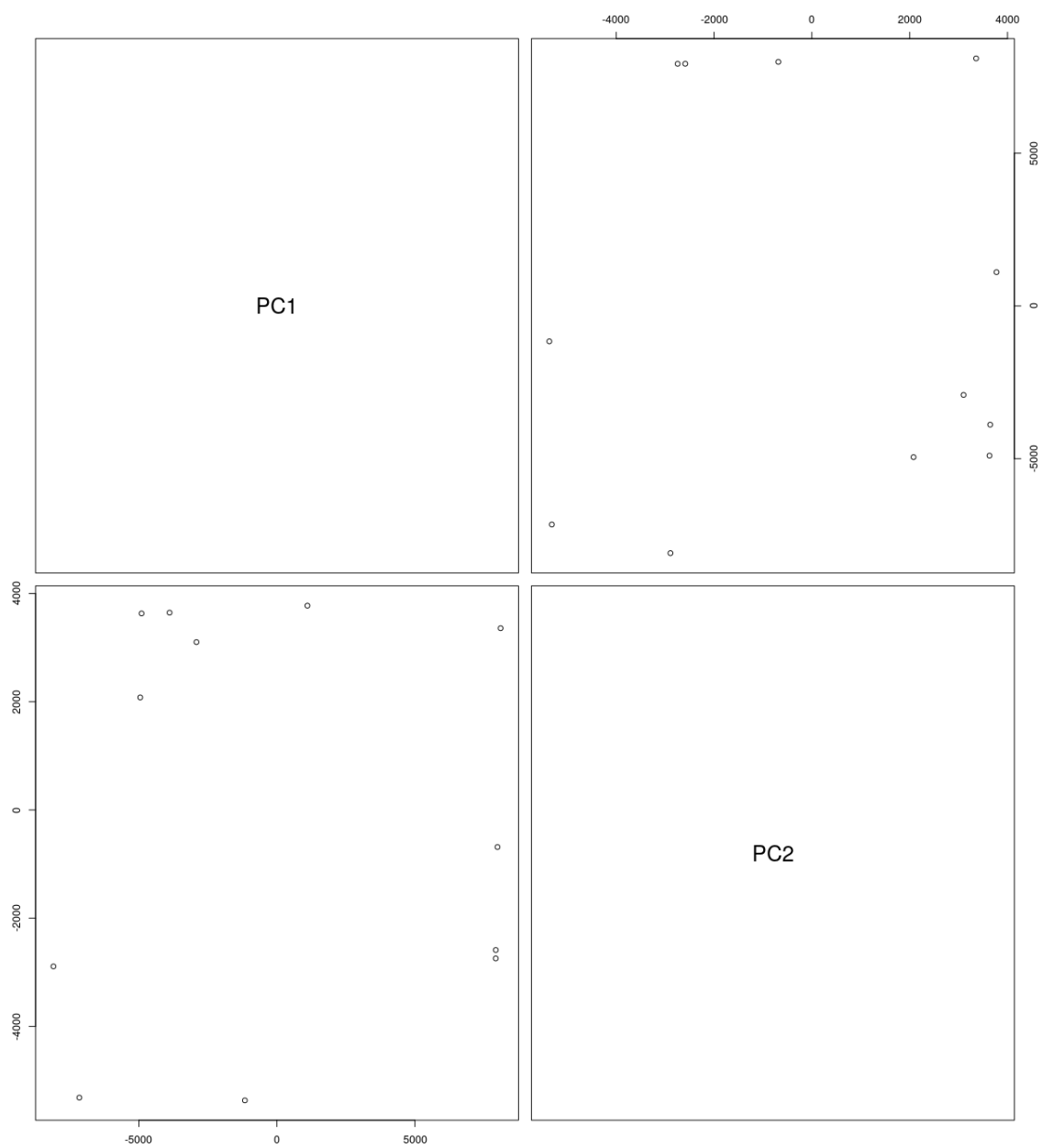


Figure 3.2: PCA Scatter Plot

## 4. Section 4

```
PCA <- prcomp(vert[0:6])
Eigenvalues <- PCA$sdev^2
Eigenvectors <- PCA$rotation
reduced <- as.data.frame(PCA$x)[1:2]
pairs(reduced, pch = 21, bg = c('green', 'blue')[unclass(vert$class)])
```

$Eigenvalues < - [1.780994e + 03 \quad 3.453271e + 02 \quad 1.887770e + 02 \quad 1.060179e + 02 \quad 8.861407e + 01 \quad 7.207841e - 18]$

$Eigenvectors < - \begin{bmatrix} -0.32364565 & 0.47663485 & -0.001544813 & 0.37367725 & -0.44170387 & -5.773503e - 01 \\ -0.11319229 & 0.09856328 & -0.264657410 & 0.75411376 & 0.07354147 & 5.773503e - 01 \\ -0.30367474 & 0.53278398 & -0.496541893 & -0.33941176 & 0.51202411 & 1.089295e - 11 \\ -0.21045336 & 0.37807157 & 0.263112598 & -0.38043651 & -0.51524534 & 5.773503e - 01 \\ 0.02995983 & -0.32180920 & -0.774612852 & -0.17510604 & -0.51463973 & 3.590517e - 12 \\ -0.86315378 & -0.48243804 & 0.118940778 & -0.03291431 & 0.08359925 & -3.067324e - 12 \end{bmatrix}$

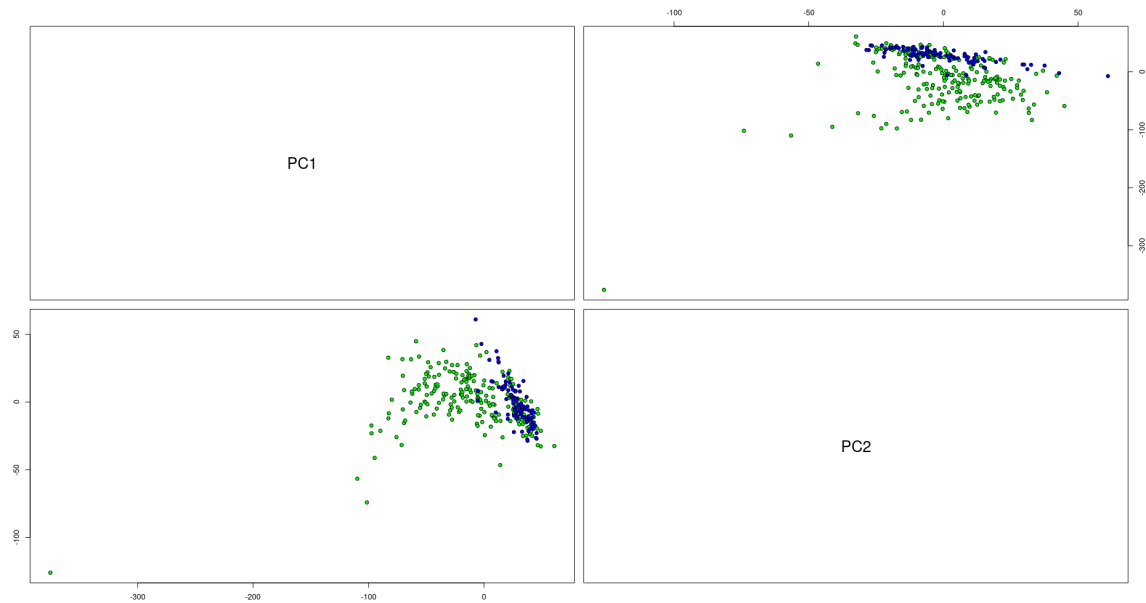


Figure 4.1: PCA Vertebral Column Data Set

## 5. Section 5