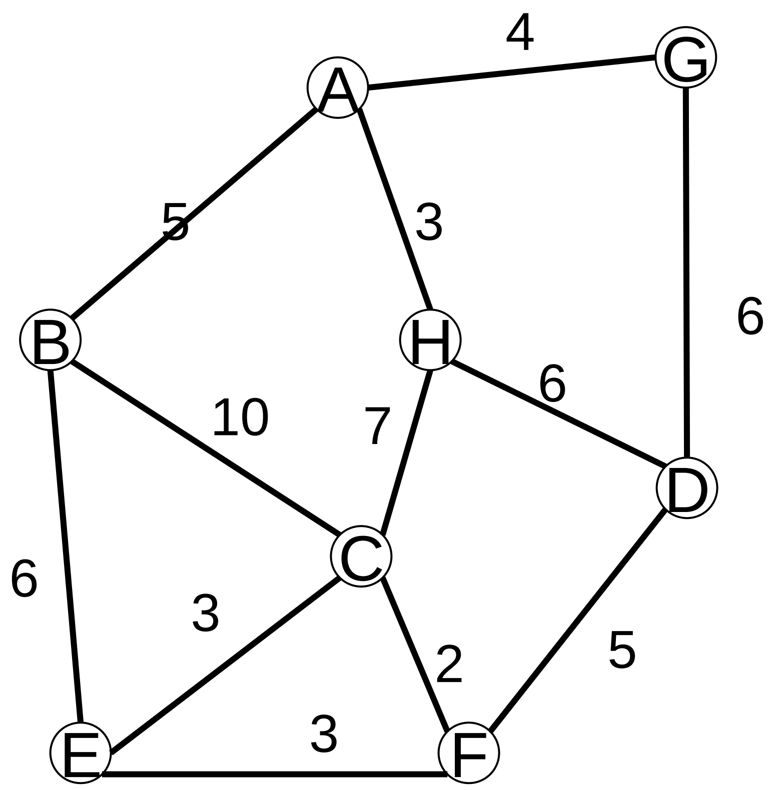
All-pairs shortest paths

	Dingle ***An Daingean***	Annascaul *Abhainn an Scáil*	Ballydavid *Baile na nGall*	Ballyferriter *Baile an Fheirtéaraigh*	Brandon *Cé Bhréanainn*	Camp *An Com*	Castlegregory *Caisleán Ghriaire*	Cloghane *An Clochán*	Conor Pass *An Conair*	Dunquin *Dún Chaoin*	Feohanagh *An Fheothanach*	Inch *An Inse*	Kinard *Ceann Áird*	Lispole *Lios Póil*	Minard *Minn Aird*	Ventry *Ceann Trá*
Dingle ***An Daingean***		17.4	11.5	11.7	20.0	32.5	25.8	14.9	8.3	20.9	12.2	23.7	9.1	8.3	13.9	7.5
Annascaul *Abhainn an Scáil*	10.8		28.8	29.0	42.2	15.1	24.2	37.1	25.7	38.2	29.6	7.1	13.6	9.1	8.4	24.9
Ballydavid *Baile na nGall*	7.1	17.9		7.2	31.4	43.9	37.3	26.4	19.8	15.8	3.9	35.1	20.7	19.8	25.3	11.4
Ballyferriter *Baile an Fheirtéaraigh*	7.3	18.0	4.5		31.6	44.2	37.5	26.6	20.0	8.6	10.3	35.3	20.9	20.0	25.5	5.9
Brandon *Cé Bhréanainn*	12.4	26.2	19.5	19.6		25.7	19.8	5.0	11.7	40.8	31.2	39.9	29.2	28.3	33.9	27.5
Camp *An Com*	20.2	9.4	27.3	27.4	16.0		9.1	22.0	23.4	52.5	42.9	14.2	28.7	24.2	23.5	40.0
Castlegregory *Caisleán Ghriaire*	16.1	15.0	23.2	23.3	12.3	5.7		16.2	17.5	46.6	37.1	23.3	37.7	33.3	32.6	33.3
Cloghane *An Clochán*	9.3	23.1	16.4	16.5	3.1	13.7	10.0		6.6	35.7	26.2	34.9	23.8	23.3	28.8	22.5
Conor Pass *An Conair*	5.2	16.0	12.3	12.4	7.3	14.5	10.9	4.1		29.1	19.5	32.0	24.4	16.6	22.2	15.8
Dunquin *Dún Chaoin*	13.0	23.7	9.8	5.3	25.3	32.6	29.0	22.2	18.1		18.9	44.5	29.9	29.2	34.7	13.4
Feohanagh *An Fheothanach*	7.6	18.4	2.4	6.4	19.4	26.7	23.0	16.3	12.1	11.7		35.9	21.3	20.5	26.1	17.1
Inch *An Inse*	14.7	4.4	21.8	22.0	24.8	8.8	14.5	21.7	19.9	27.7	22.3		20.1	15.4	14.7	31.2
Kinard *Ceann Áird*	5.6	8.4	12.9	13.0	18.2	17.8	23.4	14.8	15.2	18.6	13.2	12.5		4.4	8.4	16.6
Lispole *Lios Póil*	5.2	5.6	12.3	12.4	17.6	15.0	20.7	14.5	10.3	18.1	12.8	9.5	2.7		3.4	15.8
Minard *Minn Aird*	8.6	5.2	15.7	15.9	21.0	14.6	20.3	17.9	13.8	21.6	16.2	9.1	5.2	5.5		21.4
Ventry *Ceann Trá*	4.7	15.5	7.1	3.7	17.1	24.9	20.7	14.0	9.8	8.3	10.6	19.4	10.3	9.8	13.3	

KILOMETRES

MILES

Given a weighted graph, compute the length of the shortest paths between all pairs of vertices.



	A	B	C	D	E	F	G	H
A	0	5	10	9	11	12	4	3
B	5	0	9	14	6	9	9	8
C	10	9	0	7	3	2	13	7
D	9	14	7	0	8	5	6	6
E	11	6	3	8	0	3	14	10
F	12	9	2	5	3	0	11	9
G	4	9	13	6	14	11	0	7
H	3	8	7	6	10	9	7	0

How should we compute 'all pairs shortest paths'?

For each vertex v in the graph
    compute the shortest path from v to all vertices using Dijkstra?

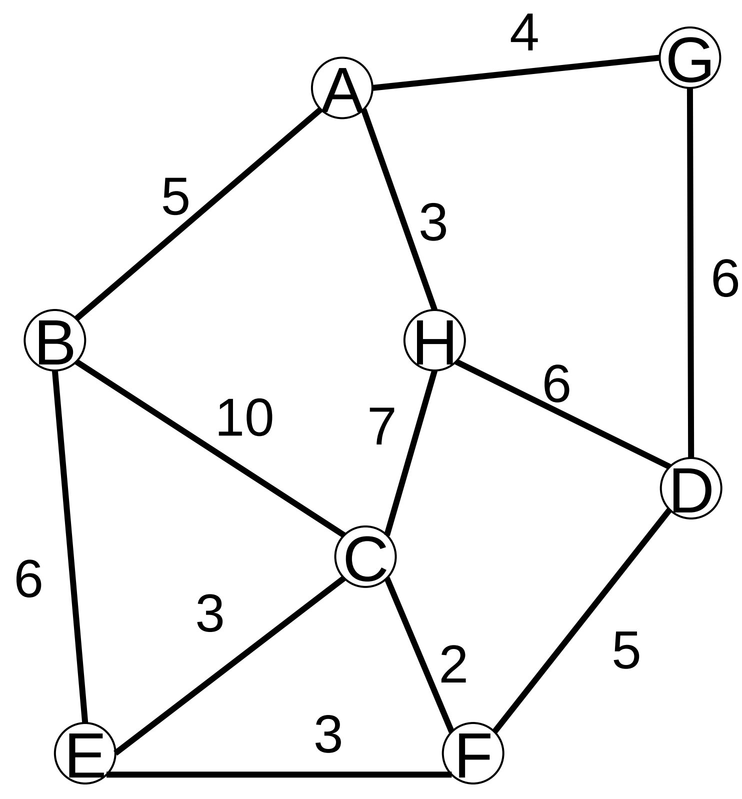
How should we compute 'all pairs shortest paths' in dense graphs?

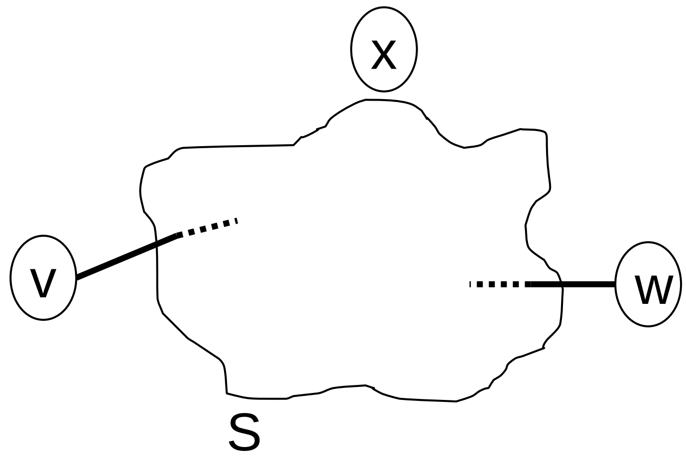
Adapt the Floyd Warshall algorithm for finding the transitive closure?

Find all-pairs shortest paths that only visit vertices in {A}

Find all pairs shortest paths that only visit vertices in {A,B}

etc





Let P be the shortest path from v to w which only visits vertices in S.

Now let Q be the shortest path from v to w which only visits vertices in $S \cup \{x\}$

Either Q = P, or Q is made up of the shortest path from v to x which only visits vertices in S, then the shortest path from x to w which only visits vertices in S.

So if we have sp(v,w,S), and sp(v,x,S) and sp(x,w,S), we can compute $sp(v,w,S \cup \{x\})$: minimum(sp(v,w,S),sp(v,x,S)+sp(x,w,S))

```
floydwarshall(): pseudocode
    initialise an nxn 2D structure with value ∞ for each entry
    for each v in graph
        assign   table[v][v] = 0     #cost of path from v to v = 0
    #start visitable set S = {}, so shortest paths visiting only S are just edge costs
    for each edge (x,y) in the graph
        table[x][y] = w((x,y))      #initial cost of path is the edge weight, if edge exists
        table[y][x] = w((x,y))
    for each v in the graph       #each time round the loop, add v to S
        for each w in the graph
            for each x in the graph
                if table[w][x] > table[w][v] + table[v][x]:     #if path via v is cheaper
                    table[w][x] = table[w][v] + table[v][x]     #record that as shortest path
    return table
```

```python
def floydwarshall(self):

    allpairs = {}                   #create a dictionary, vertices as keys
    for v in self._structure:
        allpairs[v] = {}            #each value is a dictionary
        for w in self._structure:
            allpairs[v][w] = float('inf')
        allpairs[v][v] = 0

    for e in self.edges():
        (v,w) = e.vertices()
        allpairs[v][w] = e.element()
        allpairs[w][v] = e.element()

    for v in self._structure:
        for w in self._structure:
            for x in self._structure:
                if allpairs[w][x] > allpairs[w][v] + allpairs[v][x]:
                    allpairs[w][x] = allpairs[w][v] + allpairs[v][x]

    return allpairs
```

Complexity:
- initialising the 2D dictionary is $O(n^2)$
- adding the edge costs is $O(m)$, which is $O(n^2)$
- triple loop, n times round each loop: $O(n^3)$

So complete algorithm is $O(n^3)$

If graph is dense, this is lower complexity than repeated Dijkstra-heap (and same as Dijkstra-list, but tends to be faster)

If graph is sparse, this is higher complexity than repeated Dijkstra-heap

Next lecture

Various algorithms in matching and text processing