Note that all definitions form the course will be included in a single pdf on the webpage.

Properties of functions

There are certain properties functions need to have in order to be reversible/inversible.

Image and Range

If f(a) = b then b is the *image* of a under f.

The *range* of f is the subset of B to which elements of A are mapped.

The range is the set of all images. Subset of the codomain that contains all images of A under f.

Surjective/Injective/Bijective

surjective/onto means range(f) = B.

So every element in the codomain is an image of a value in the domain.

Every element of B is mapped onto by something in A.

injective means no elements of A map to the same element.

For all elements a_1 and a_2, $f(a_1)$ /= $f(a_2)$

A function is *bijective* if it is both surjective and injective.

Surjective because each ID is owned by a student, and injective because no students share the same ID.

Functions have a direction

f(b) may not be defined. We do not know if b is an element of A. Even if b is an element of A, f(b) isn't necessarily = a.

Inverse functions

If f maps a to f(a), then f^{-1} maps f(a) to a.

If f is the inverse of g, then g must be the inverse of f.

A function f has an inverse function f^{-1} only if f is a bijection (it is bijective).

Composition of functions

If f:A->B and g:B->C, then:

g after f ($g \circ f$) = g(f(a))

But can we be certain that g is actually a function from A to C?

Yes, as long as it specifies a single image in C for each element of A.

He has a proof for this that's pretty straightforward:

- prove each element has an image
- prove it only has one image

(there's a lot of stating the obvious)

Some other properties of compositions

range($g \circ f$) $\subseteq$ range(g)

If f and g are injective, so is $g \circ f$.

If f and g are surjective, so is $g \circ f$.

If f and g are bijective, so is $g \circ f$.

Why do we care?

If we have a number of bijective functions, we can apply them one after another, and we can always get back the original data.

E.g. lossless file compression.

Functions between more than 2 sets

The cartesian product of n sets is still just a set. Can define a function between two cartesian products.

So we might have $f((a_1, a_2, \ldots)) = (b_1, b_2, \ldots)$.

Each element of the domain will be an ordered n-tuple from (A_1 x A_2 x … x A_n) and each element of the image will be an ordered m-tuple from (B_1 x B_2 x … x B_m).

Set Cardinality

We said it was the number of elements. That only applied to finite sets. We can now extend this definition.

Two sets A and B have the same cardinality if and only if it is possible to create a bijection from A to B.