

Lab Sessions:

I am in Monday 3-5 lab group for this module. The other group is Monday 10-12. Both starting next week. In WGB.G20.

Labs will consist of exercises on paper.

No mark for attendance, but it will be noted.

Tomorrow's lecture will be given by Ken Brown.

Sets & Collections

Often things you want to calculate are sets.

A *set* is a collection of things. These things can be anything, physical or abstract:

- students registered for CS1105 in 2008
- ID numbers of students registered for CS1105 in 2008
- the objects in my suitcase when I go on holiday

There are many ways you can refer to the same thing (guy in seat x , guy with jumper colour y , where x and y are unique), it doesn't matter.

You can either list things out ($\{\text{banana, apple, orange}\}$, which is extensional definition) or describe them using constraints.

Extensional: between $\{\}$ and separated by commas.

A set is different from a list because the number of times one element occurs in a set doesn't matter. $\{\text{Cork, Cork, Kerry}\} = \{\text{Cork, Kerry}\}$

Also the order doesn't matter.

Note that sets don't have to be finite in length, they can be infinite.

So we have intensional set definition:

- $\{y \mid y \text{ is a library book with "databases" in the title}\}$
- $\{z \mid z \text{ is divisible by 7, } z > 0, z \text{ is an integer}\}$

Specify a template and then a rule, such that everything that matches the template and obeys the rule is in the set, and nothing else is in the set.

Set Membership

To say x is a *member* of the set A : $x \in A$

To say x is not a member of the set A : $x \notin A$

Note that $\{CS1112\} \notin \{CS1106, CS1112\}$ while $CS1112 \in \{CS1106, CS1112\}$.

The set $\{CS1112\}$ is a subset, not an element, of the set $\{CS1106, CS1112\}$.

$\{CS1112\} \subseteq \{CS1106, CS1112\}$, though.

The two most important qualities of a set:

- The order you write the elements in does not matter.
- Repeated elements are ignored, there is only one copy of each element in a set.

Equal Sets

Two sets are equal if they have exactly the same members

Two sets are not equal if one of them has a member that the other doesn't.

$$\{1,2,3\} = \{3,1,1,2\}$$

$$\{1,2,3\} \neq \{1,2\}$$

Some Important Sets

The *empty set* is the set with no elements, and is written $\{\}$.

For every possible thing x , $x \notin \{\}$.

\mathbb{Z} is the set of integers.

\mathbb{Q} is the set of rational numbers (the numbers that can be expressed as fractions).

\mathbb{R} is the set of real numbers.

Subsets

A set A is a subset of B if every member of A is also a member of B .

Written as $A \subseteq B$.

If A is not a subset of B, there must be at least one member of A that is not a member of B. $A \not\subseteq B$.

Note that the empty set is a subset of all sets. It contains no elements that aren't in any other set. Any set is a subset of itself for the same reason.

You can redefine equality:

$$A \subseteq B \text{ and } B \subseteq A \Leftrightarrow A = B$$

This condition \Leftrightarrow (if and only if), is also sometimes written iff.

A is a *proper subset* of B if A is a subset of B but not equal to B. Written $A \subset B$.

Limiting the Scope of Our Collections

Sometimes we only want to talk about collections of things taken from some clearly defined larger collection.

We call this larger collection the *universal set* and write it as U.

E.g. when talking about collections of students within UCC, the universal set is (the collection of all students? I missed it).

The union of two sets is a new set consisting of every member of the two sets. Written $A \cup B$.

Intersection is a new set consisting of the members that appear in both sets. Written $A \cap B$.

The *set difference* of two sets is a new set consisting of each element of the first that is not also an element of the second.

$A \setminus B$ or $A - B$.

For any element x of U, $x \in A \setminus B$ if and only if $x \in A$ and $x \notin B$.

Symmetric set difference is $(A - B) \cup (B - A)$.