OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

2016/2017

Semester 1 - Winter 2016

CS2502 Logic Design

Dr. Helen Purchase
Professor Cormac J. Sreenan
Dr Frank Böhme

90 minutes

Answer all questions.

Total marks: **80**

PLEASE DO NOT TURN THIS PAGE UNTIL INSTRUCTED TO DO SO.

PLEASE ENSURE THAT YOU HAVE THE CORRECT EXAM PAPER.

1. These questions deal with the *implication* function, i.e. $Y = f(A, B) = A \rightarrow B$.

20 marks

a) Recall that the AND and OR function posses the *commutative property*, e.g. $A+B = B+A$. Does the implication function also possess the commutative property?

Justify your answer by a proof. (*8 marks*)

b) Draw a circuit realization of the implication function using only NOR gates. (*6 marks*)

c) Draw a circuit realization of the implication function using only NAND gates. (*6 marks*)

2. Consider a combinational circuit with 4 inputs A, B, C, D and one output Y.

The output $Y = f(A, B, C, D)$ shall assume 1 if and only if the unsigned binary integer n composed by $ABCD$ can be divided by 5 without remainder. The input D stands for the least significant bit.

For example, if $ABCD = 0000$ then we have $n = 0$ and the output $Y = 1$ since $0/5 = 0$. If $ABCD = 1100$ then $n = 12$ and therefore $Y = 0$ since 12 is not a multiple of 5. **36 marks**

a) Use a Karnaugh-map to find an optimized expression for f. Your result should be a notation with a minimum number of literals while the form of the expression (such as SPOS or SSOP) is not relevant. (*11 marks*)

b) Rewrite your result from a) into an expression that contains only OR and NOT operations. The result should be an expression that can be realized with NOR gates only but you do not need to draw the circuit diagram. (*7 marks*)

c) Rewrite your result from a) into an expression that contains only AND and NOT operations. The result should be an expression that can be realized with NAND gates only but you do not need to draw the circuit diagram. (*7 marks*)

d) Obtain a realization of f that is made of an 8-to-1 demultiplexer and one inverter gate.

Draw the corresponding circuit diagram.

Make sure that all inputs are clearly labelled. (*11 marks*)

3. The questions below deal with a *sequential* circuit which is to be implemented as a *Moore-machine* with one binary input X and one binary output Y. Its input-output behaviour is described as follows: The output Y always depends on the three values of X which occurred at the *most recent three clock steps*. If and only if exactly 2 of these three recent input values are 1, then the output Y shall assume 1. Y shall assume 0 otherwise.

   For reasons of simplicity, we don't pay attention to the output at the very first two clock steps. The machine has exactly *8 states* that should correspond to the binary number composed by the input values of the most recent three clock steps.

   For example, if the most recent sequence of input values was $1, 1, 0$ (0 being the latest), then the machine is in state 6 (because the binary representation of 6 is 110) and the output is $Y = 1$ because there are exactly two occurrences of 1 within the most recent three input values. Suppose a new clock step occurs and X still remains 0. Then the machine enters state 4 (binary representation 100) and the output is now 0 because there is now only one occurence of 1 in the sequence of recent inputs. **24 marks**

   a) Determine and draw the *output table* of this Moore-machine, i.e. a table that shows how the output depends on the current state. (*6 marks*)

   b) Determine and draw the *state transition table* of this Moore-machine. (*6 marks*)

   c) Is it possible that the output value alternates forever, e.g. $Y = 0, 1, 0, 1, 0, 1, \ldots$? If so, give a corresponding sequence of inputs. (*6 marks*)

   d) Is it possible that the output value stays constantly at 1, e.g. $Y = 1, 1, 1, 1, 1, 1, \ldots$? If so, give a corresponding sequence of inputs. (*6 marks*)