

AND as an electrical circuit

A circuit with a battery, two simple switches, and a light.

The light only lights if both switches are closed.

Open corresponds to false and closed corresponds to true.

AND as a water circuit

Water flowing through a pipe with two force-controlled switches. If there's force on both switches, the water flows at the output.

OR as an electrical circuit

Two parallel switches in circuit with a battery and a light.

The light lights if either switch is (or both are) closed.

Similar for a water circuit.

NOT as an electrical circuit

A switch and resistor pair in parallel with a light. By managing the resistance ratio of the resistor and the light, closing the switch can be made to turn the light off.

NOT as a water circuit

A switch that blocks the flow of water when force is applied.

AND, OR, and NOT are basic gates, meaning all functions can be made from combinations of those gates.

They form a fundamental set.

As well as these, there are other gates which are convenient to have. These are often pre-built using NOT, AND, and OR gates.

NAND

This is the result of applying NOT to the output of AND.

A	B	A NAND B
0	0	1
0	1	1
1	0	1
1	1	0

Algebraically: $\overline{A \cdot B}$ (with a bar over all of it)

Circuit symbol: $\text{=Do}\sim$

NOR

NOT(OR)

A	B	A NOR B
0	0	1
0	1	0
1	0	0
1	1	0

Algebraically: $\overline{A + B}$ (with a bar over all of it)

Circuit symbol: $\text{=})>0\sim$

There are two more useful gates:

Exclusive OR

A	B	A XOR B
0	0	0
0	1	1

1	0	1
1	1	0

Have seen the algebraic and circuit symbols for this already.

Exclusive NOR (NOT(XOR))

A	B	A XNOR B
0	0	1
0	1	0
1	0	0
1	1	1

Also called COIN for coincident.

Making XOR from AND, OR, and NOT

$$(A \cdot B') + (A' \cdot B)$$

A	B	A'	B'	A.B'	B.A'	OR
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

This truth table is an exhaustive proof of $(A \cdot B') + (A' \cdot B) = A \text{ XOR } B$

This tabulation method is called perfect induction because it lists all possibilities and all results.