OLLSCOIL NA hÉIREANN, CORCAIGH
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLAISTE NA hOLLSCOILE, CORCAIGH
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CS1113 Foundations of Computer Science II

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Answer all questions

Total Marks 80

90 minutes

The use of electronic calculators is permitted

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ENSURE THAT YOU HAVE THE CORRECT EXAM PAPER

1. Translate the following predicate logic formulae into English:

(i) $\forall x$ *alphanumeric(x),* where the domain is the set of all filenames, and *alphanumeric(x)* says x is made up of alpha-numeric characters only (i.e. only letters or digits)

(ii) $\exists y$ *coversLogic(y)*, where the domain is the set of all first year CS modules, and *coversLogic(x)* says that module x has an introduction to logic

(iii) $\forall x$ *(P(x)* $\leftrightarrow$*R(x)),* where the domain is the set of all files, and *P(y)* says y is a file containing private information, and *R(z)* says z is a file stored in the restricted area.

(iv) $\exists y$ *(U(y)* $\vee$ *C(y))* , where the domain is the set of all database tables, and *U(x)*says table x is being updated, while *C(z)* says that table z is in a consistent state.

(4 marks)

2. For the following situation,

U = {a,b,c}
A(a) is false, A(b) is true, A(c) is false
B(a) is true, B(b) is false, B(c) is false
C(a,a) is false, C(a,b) is true, C(a,c) is false, C(b,a) is true, C(b,b) is false, C(b,c) is true, C(c,a) is false, C(c,b) is false, C(c,c) is true

state whether the following formulae are true or false:

(a) $\exists z$ *A(z)*

(b) $\forall z$ *(A(z)* $\vee$ *B(z))*

(c) $\forall y \exists z$ *C(y,z)* .

(6 marks)

3. (i) Consider a software development exercise where teams consisting of 2 people must complete a task. Assuming the following sets and predicates,

P is a set of people
D is a set of databases
The universe of discourse $U = P \cup D$

team(x,y): person x and person y are in a team together
hasAccess(x,z): person x has access to database z
eq(x,y): object x and object y are the same

give a clear English language equivalent of the following logical statements:

(a) $\forall x \in P$ *hasAccess(x, 'DB1')*

(b) $\forall x \in P \forall y \in P \forall z \in D$*(team(x,y)* $\rightarrow$ *(hasAccess(x,z)* $\vee$ *hasAccess(y,z)))*

(6 marks)

(ii) and express the following statements in quantified logic:

(a) "at least one person has access to database 'DB3'"

(b) "Everybody has another person as a partner" (i.e. in a team together) .

(5 marks)

4. Consider a simple graph G = (V,E) with no edge loops. For each part of the question, you may assume the definitions from the previous parts.

(i) What does it mean if two vertices x and y are *adjacent*?

(2 marks)

(ii) Define the *degree* of a vertex x in G.

(2 marks)

(iii) Define a *path* between a pair of vertices x and z in G.

(3 marks)

(iv) Define a *circuit* in a graph.

(2 marks)

(v) Define an *Euler circuit* in a graph.

(2 marks)

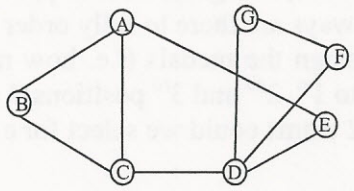
(vi) State a simple test for determining whether or not a graph G has an Euler circuit, without actually generating a circuit.

(2 marks)

(vii) Define a *Hamiltonian circuit* in a graph.

(2 marks)

(viii) Is there a Hamiltonian circuit in the following graph? If there is, sketch it clearly; if there is not, give a clear explanation of why not, for this particular graph.



(3 marks)

5. Consider the symmetric undirected weighted graph represented below as a table.

	A	B	C	D	E	F	G	H
A	0	2	3	0	0	0	0	0
B	2	0	0	1	0	3	5	0
C	3	0	0	1	7	0	0	0
D	0	1	1	0	0	3	0	0
E	0	0	7	0	0	2	0	4
F	0	3	0	3	2	0	0	0
G	0	5	0	0	0	0	0	2
H	0	0	0	0	4	0	2	0

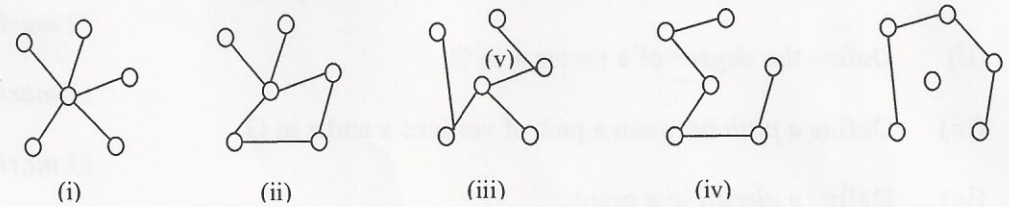
(i) Sketch the graph as vertices connected by weighted edges with no arrows.

(5 marks)

(ii) Apply Dijkstra's algorithm to find the cheapest path from C to H. In your answer, show all the partial paths and costs that were discovered and modified, either by doing another sketch of the graph with arrows and additional weights, or by showing the table you developed as you ran the algorithm. In either case, make sure you clearly mark any updates you make to costs or path edges as you run the algorithm.

(12 marks)

6. Which of the following graphs are trees? For those that are not, state why not.



(i) (ii) (iii) (iv)

(5 marks)

7. (i) For the sets A = {1,3,5,7,9} and B = {0,2,4,6,8},
   (a) how many pairs (a,b) are there, where a ∈ A and b ∈ B?
   (b) how many pairs (a,b) are there, where a ∈ A, b ∈ B, and a < b?
   (c) how many pairs (a,b) are there, where a ∈ A, b ∈ B, and a + b < 10?

(3 marks)

(ii) For a set of teams in an Olympic sports tournament
{Denmark, Egypt, France, Greece, Hungary, Ireland },
   (a) how many different possible ways are there to fully order the teams?
   (b) how many ways could we assign the medals (i.e. how many different ways could we assign teams to 1st, 2nd and 3rd positions)?
   (c) how many different pairs of 2 teams could we select for a playoff?

(5 marks)

8. Write out the steps taken during a search of the list <1,4,6,9,13,15,19,23> for the element 15, using first linear search, and then binary search (show the state of the list after each iteration of the outer loop).

(6 marks)

9. For the following algorithm, state a formula in terms of *n* (i.e. the length of the array *points*) for how many times we call the function `distance(x,y)`.

```
Algorithm: sumPairwiseDistance
Input: array points of n coordinates  (index starts at 1)
Output: integer equal to the sum over all pairs of points of
        the distance between them
1.    sum := 0
2.    for each i from 1 to n-1
3.       for each j from i+1 to n
4.          sum = sum + distance(points[i], points[j])
5.    return sum
```

(5 marks)

Total *80 marks*