# Sets (cont.)

## Cardinality

Indicated by | | surrounding the set. Is the number of elements in the set.

## Extended Example (student record system)

(Incomplete notes, see lecturer slides)

Each module is represented by the set of students taking that module e.g. CS101 = {Alice, Darragh, Oliver, Ronan}

This allows e.g.

> CS101 ≠ EC101 ⇔ The modules CS101 and EC101 do not have identical class lists

Using set notation we can now be completely unambiguous and clear, once we understand set notation fully.

Sets can contain sets as elements.

Also sets can contain a mix of types of elements.

For example, a set can contain {b} but not b, which is an important distinction (by definition) in set notation.

More practically, code will expect this distinction.

The *power set* of a set A is the set containing all possible subsets of A.

Example:

> A = {1,2,3}
> P(A) = { {}, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3} }

For any A;

> $|P(A)| = 2^{|A|}$

A *partition* of a set A is a division of all the members of A into non-empty, non-intersecting subsets.

Graphically (picture a venn diagram) this is dividing it into sections (it's a very simple image).

Union and intersection are commutative (you can put them in any order e.g. $A \cup B = B \cup A$).

In general set difference is not commutative, though.

Union and intersection are associative, e.g. $(A \cup B) \cup C = A \cup (B \cup C)$

Set difference isn't.

Intersection distributes over union, and union distributes over intersection.

De Morgan's Laws:

- $(A \cup B)' = (A' \cap B')$
- $(A \cap B)' = (A' \cup B')$

Note De Morgan's Laws very important in logical circuits and in programming.

# Ordered Pairs

An *ordered pair* is two objects listed inside round brackets and separated by commas.

Indicates a connection between two elements.

The order is important.

(1, 0) is a different ordered pair to (0, 1).

You can repeat elements in an ordered pair, and you can have sets as members of an ordered pair.

The definition extends to ordered n-tuples of any length.

The *Cartesian product (aka cross product)* is the set of all possible ordered pairs, where the first element comes from A, and the second element comes from B.

The *projection* of B onto $A_i$ is the subset of elements of $A_i$ that appear (in the right position) in one of the tuples of B.

I'm guessing that's the dot product. I don't understand it though.

All definitions will be in the notes which will be online.