OLLSCOIL NA hÉIREANN
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
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Semester 1 - Winter 2016

CS4616: Distributed Algorithms

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One and a half hour

Total marks: 80

Answer all Questions

Calculators allowed

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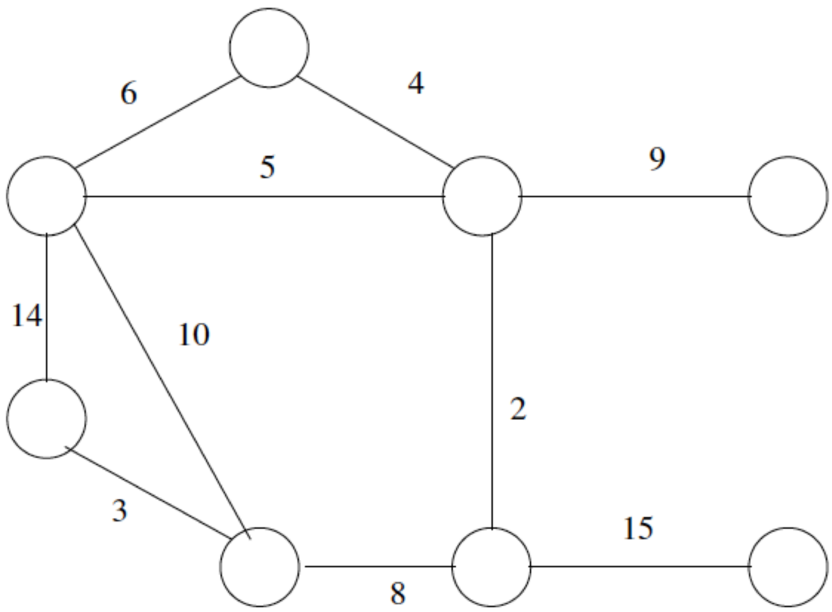
ENSURE THAT YOU HAVE THE CORRECT EXAM PAPER

Question 1 [20 marks]
LubyMIS is a distributed algorithm that aims to compute a maximal independent subset of vertices in an undirected graph. LubyMIS uses randomisation.

a) [4 marks] Give an example of a graph in which the selection (as specified in the code of LubyMIS) of an independent subset could yield an empty set. Explain your answer, i.e. for the example given, explain why the algorithm would construct an empty set.

b) [3 marks] The problem outlined in part a) of this question shows that it is possible for LubyMIS to exhibit non-termination. Why is the algorithm still useful in practice?

c) [6 marks] Consider the following undirected graph $G = (V, E)$, where all the edges have distinct weights. Draw the forest computed the SynchGHS algorithm during the computation that proceeds from level zero to level one (where level zero is the forest consisting of all the vertices in the graph, i.e. single-node trees). Clearly indicate the edges that are selected in this process. Also indicate which leaders are selected per component (i.e. for each tree in the forest). As no UID's are provided, there are two potential leaders per component. Indicate both potential choices for each component.



d) [4 marks] Explain how you can adapt SynchGHS so it can execute on graphs for which not all weights are distinct.

e) [3 marks] Give an example of how distributed computing can occur in nature.

Question 2 [20 marks]
The zero-one principle states that comparator networks that sort all *binary* sequences, must sort all sequences. The proof proceeds by contradiction, assuming that there is a sequence, say $L = (a_1, \ldots, a_n)$, that is not sorted by the network. A binary sequence then is constructed from the sequence L, which can be shown not to be sorted by the network. We refer to such a binary sequence as a "binary-witness" sequence.

Consider a list $L = (a_1, \ldots, a_n)$. For each list element a_i of this list we define a function f_{a_i} in the same spirit as the function used to produce binary-witness sequences:

$$\text{if } x \leq a_i \text{ then } f_{a_i}(x) = 0 \text{ otherwise } f_{a_i}(x) = 1$$

This function f_{a_i} can be applied to each element of L to produce the binary-witness sequence

$$(f_{a_i}(a_1), \ldots, f_{a_i}(a_n)).$$

In general you cannot reverse this process, i.e. it is not possible in general to recover the list $L = (a_1, \ldots, a_n)$ from a *single* binary-witness sequence $(f_{a_i}(a_1), \ldots, f_{a_i}(a_n))$. In other words, such a binary-witness sequence forms a "lossy encoding."

Consider the *list* of binary-witness sequences produced by the functions $f_{a_1}, f_{a_2}, \ldots, f_{a_n}$ when applied to the list $L = (a_1, \ldots, a_n)$:

$$(**)\ [(f_{a_1}(a_1), \ldots, f_{a_1}(a_n)), \ldots, (f_{a_n}(a_1), \ldots, f_{a_n}(a_n))]$$

Show that this list of binary-witness sequences together with the *set* $\{a_1, \ldots, a_n\}$ of all elements contained in the list L is sufficient to recover the original list $L = (a_1, \ldots, a_n)$. Specify a decoding that computes L from this list of binary-witness sequences and the set $\{a_1, \ldots, a_n\}$.

Hint for question 2: Consider a list $L' = (7, 1, 5, 3)$. Check how you can recover this list from the following *list* of binary-witness sequences and where you can make use of the set of values $\{7, 1, 5, 3\}$ to achieve the decoding. The list of binary-witness sequences in this case is:

$$[(f_7(7), f_7(1), f_7(5), f_7(3)),\ (f_1(7), f_1(1), f_1(5), f_1(3)),$$
$$(f_5(7), f_5(1), f_5(5), f_5(3)),\ (f_3(7), f_3(1), f_3(5), f_3(3))]$$

a) [4 marks] Compute the bit-values for each element of these binary-witness sequences.

b) [8 marks] Explain how you can decode the original list from the given list of four binary-witness sequences *and* from the set of values $\{7, 1, 5, 3\}$.

c) [8 marks] Generalise your approach obtained under part b) of this question to arbitrary lists L, i.e., explain how to decode such a list $L = (a_1, \ldots, a_n)$ from its set of values $\{a_1, \ldots, a_n\}$ and the *list* of binary-witness sequences displayed in $(**)$ above.

Question 3 [20 marks]
a) [10 marks] Hirschberg and Sinclair's HS algorithm elects a leader in an efficient way by selecting the process with maximum UID, allowing bi-directional communication in a ring network. Give the pseudo-code for the state transition function of this distributed algorithm.

b) [10 marks] For a ring-shaped network with four nodes, assume that the nodes are marked with a unique identifier as follows:

- a node in the northern position (top of the ring) has UID 10
- a node in the western position (left side of the ring) has UID 5
- a node in eastern position (right side of the ring) has UID 2
- a node in the southern position (bottom of the ring) has UID 6

Show the execution of the algorithm on the ring-shaped network, where you display the messages sent during each phase of the execution. Count the number of messages transmitted during the phases of the execution of the HS algorithm. Display the local leaders on a drawing, for each phase, as well as arrows for the messages passed during the execution of each phase.

Question 4 [20 marks]
a) [2 marks] What is the outcome of the coordinated attack algorithm when one of the inputs is zero?

b) [3 marks] What is the goal of the coordinated attack algorithm? Give an example in a distributed context of where this algorithm would be applied.

c) [5 marks] For the following set of triples, draw the graph depicting the corresponding good communication pattern. The set of triples for the good communication pattern is given by:

$$\{(1,2,1),(1,2,2),(2,1,2),(1,2,3),(1,2,4),(2,1,4),(2,1,5)\}.$$

d) [5 marks] Consider the coordinated attack algorithm. For a given adversary B, we know that Prob_B[some process decides 0 and some process decides 1] $\leq \frac{1}{r}$, where r is the number of rounds. Consider the good communication pattern depicted by the graph you drew under part b). Indicate the information levels for the graph.

e) [5 marks] Compute the exact value of Prob_B[some process decides 0 and some process decides 1] for the communication pattern given in b), and using the information levels you derived in c). You can assume that the initial values for the two processes all are 1. Give sufficient detail in your argument to show how you arrived at the answer.