

OLLSCOIL NA hÉIREANN
THE NATIONAL UNIVERSITY OF IRELAND, CORK
COLÁISTE NA hOLLSCOILE, CORCAIGH
UNIVERSITY COLLEGE, CORK

2017/2018

Semester 1 - Winter 2017

CS4616 Distributed Algorithms

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Professor Cormac Sreenan
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1.5 hours

Calculators Allowed

Total marks: 80

Answer all Questions

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THE CORRECT EXAM PAPER**

Question 1 [20 marks]

- a) [12 marks] Show that the following result holds: if a comparator network sorts all binary bitonic sequences (i.e. bitonic sequences formed only from zeros and ones) then it sorts all bitonic sequences. This is a version of the zero-one principle. However, the fact that the zero-one principle holds does not imply the statement on bitonic sequences. Adapt the proof of the zero-one principle to verify that the above statement holds true.
- b) [8 marks] Draw the Bitonic-Sorter network for the case of 8 wires, where the recursion has been unrolled.

Question 2 [20 marks]

- a) [2 marks] What is the outcome of the random attack algorithm when one of the inputs is zero?
- b) [2 marks] What is the goal of the random attack algorithm?
- c) [5 marks] For the following set of triples, draw the graph depicting the corresponding good communication pattern. The set of triples for the good communication pattern is given by:
- $$(1, 2, 1), (2, 1, 2), (1, 2, 3), (1, 2, 4), (2, 1, 5), (1, 2, 5)$$
- d) [5 marks] Consider the good communication pattern depicted by the graph you drew under part c). Indicate the information levels for the graph.
- e) [6 marks] Compute the exact value of $\text{ProbB}[\text{some process decides 0 and some process decides 1}]$ for the communication pattern drawn in c), and using the information levels you derived in d). You can assume that the initial values for the two processes all are 1. Give sufficient detail in your argument to show how you arrived at the answer.

Question 3 [20 marks]

- a) [5 marks] What is the time complexity for convergecast in case of a directed graph? Justify your answer.
- b) [5 marks] What is the communication complexity for convergecast in case of a directed graph? Justify your answer.
- c) [5 marks] The LubyMIS algorithm makes random choices of Unique Identifiers (UID's) from the set $1 \dots n^4$, for each of the n processes. Give an upper bound for the probability that two such UID's coincide and justify your answer.
- d) [5 marks] Describe how the LubyMIS algorithm picks the initial independent set in the graph, and why this choice yields an independent set.

Question 4 [20 marks]

HS is a distributed algorithm that computes a leader in a bidirectional ring. Give the pseudo-code for the state transition function of HS.