

AND as an electrical circuit

A circuit with a battery, two simple switches, and a light.

The light only lights if both switches are closed.

Open corresponds to false and closed corresponds to true.

AND as a water circuit

Water flowing through a pipe with two force-controlled switches. If there's force on both switches, the water flows at the output.

OR as an electrical circuit

Two parallel switches in circuit with a battery and a light.

The light lights if either switch is (or both are) closed.

Similar for a water circuit.

NOT as an electrical circuit

A switch and resistor pair in parallel with a light. By managing the resistance ratio of the resistor and the light, closing the switch can be made to turn the light off.

NOT as a water circuit

A switch that blocks the flow of water when force is applied.

AND, OR, and NOT are *basic* gates, meaning all functions can be made from combinations of those gates.

They form a *fundamental set*.

As well as these, there are other gates which are convenient to have. These are often pre-built using NOT, AND, and OR gates.

NAND

This is the result of applying NOT to the output of AND.

A	B	A NAND B
0	0	1
0	1	1
1	0	1
1	1	0

Algebraically: $\overline{A \cdot B}$ (with a bar over all of it)

Circuit symbol: 

NOR

NOT(OR)

A	B	A NOR B
0	0	1
0	1	0
1	0	0
1	1	0

Algebraically: $\overline{A + B}$ (with a bar over all of it)

Circuit symbol: 

There are two more useful gates:

Exclusive OR

A	B	A XOR B
0	0	0
0	1	1
1	0	1

1	1	0
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Have seen the algebraic and circuit symbols for this already.

Exclusive NOR (NOT(XOR))

A	B	A XNOR B
0	0	1
0	1	0
1	0	0
1	1	1

Also called COIN for coincident.

Making XOR from AND, OR, and NOT

$$(A.B')+(A'.B)$$

A	B	A'	B'	A.B'	B.A'	OR
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

This truth table is an exhaustive proof of $(A.B')+(A'.B) = AXORB$

This tabulation method is called perfect induction because it lists all possibilities and all results.