Software Development (cs2500)

Lectures 48 & 49: The Collections Framework

M. R. C. van Dongen

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Outline

Collections

Linked Lists

Sets

Maps

Queues

Hashing

For Wednesday

Acknowledgements

About this Document

Outline

- Study the `Java` Collection Framework.
- Finalise our in-depth study of linked lists.
- Explore the `Set` interface.
- Use the `Map` interface to implement lookup tables.

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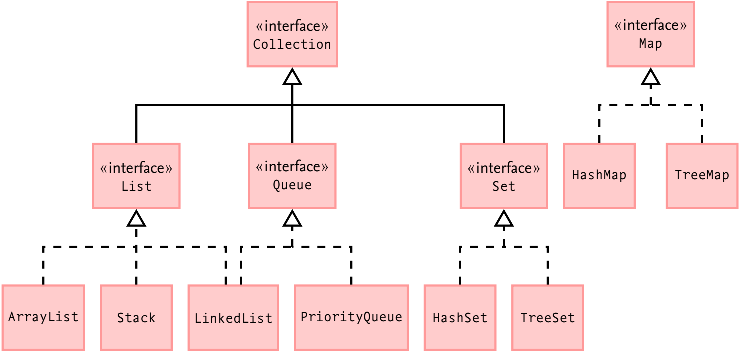
Java Collections Framework

- Framework for storing items, querying, and traversing.
- All interfaces and classes are generic.
- At the top are the `Collection` and `Map` interfaces.
  - A `Collection<T>` is for storing/retrieving `T`s.
  - A `Map<K,V>` is for looking up value `V` using key `K`.
  - In some sense, a `Collection<T>` is a `Map<T,T>`.

Collections Hierarchy

All Interfaces/Classes are Generic



`Collection<T>` API

`size( )`	Determine the size of the collection.
`add( T item )`	Add a member to the collection.
`toString( )`	Compute `String` of what's in the collection.
`remove( T item )`	Tentatively remove `item` from collection. - Returns `true` if and only if successful.
`contains( T item )`	Detemine whether `item` is in the collection. - Returns `true` if and only `item` is in it.
`iterator( )`	Returns an `Iterator<T>`. - Defined in `Iterable<T>` interface. - Used to traverse the collection. - If a class implements `Iterable<T>`, you can use enhanced `for` notation.

Implementing `remove( )`

Java

```
public class MyList<T> {
    private Link<T> nodes;

    ...

    private static class Link<T> {
        private T head;
        private Link<T> tail;

        ...
    }
}
```

Implementing `remove( )` (Continued)

Worst-Case Performance Proportional to Length of List

Java

```java
public boolean remove( T item ) {
    final boolean result;

    if (nodes == null) {
        result = false;
    } else if (nodes.head.equals( item )) {
        result = true;
        nodes = nodes.tail;
    } else {
        Link<T> link = nodes;
        while ((link.tail != null) && (!link.tail.head.equals( item ))) {
            link = link.tail;
        }
        result = link.tail != null;
        if (result) {
            link.tail = link.tail.tail;
        }
    }

    return result;
}
```

`Iterator<T>` and `Iterable<T>`

`Iterator` Used to traverse collection.
Implementing classes must override:

- `hasNext( )` Determine whether there's another member.
- `next( )` Get the next member.
- `remove( )` Remove most recent `next( )` value.
  - Implementing this method is optional.

`Iterable` Implementing classes must override `iterator( )`:

- `public Iterator<T> iterator( )`
- Traverse `Iterable` classes with enhanced `for`:
  - ```
    for (T item : collection) {
       /* use item */
    }
    ```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    ```

Implementing `Iterable`

Java

```
  @Override
  public Iterator<T> iterator( ) {
      return new Iterator<T>( ) {
          private Link<T> current = nodes;

          @Override
          public boolean hasNext( ) {
              return current != null;
          }

          @Override
          public T next( ) {
              final T next = current.head;
              current = current.tail;
              return next;
          }

          @Override
          public void remove( ) {
          }
      };
}
```


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Singly Linked Lists

- So far we've implemented singly linked lists.
- At the top level a `Link` instance, `nodes`, represents the members.
- Inserting a new `Link` depends on two cases:

`nodes == null` No need to locate insertion position:

1. Create the new `Link`.
2. Assign `Link` to `nodes`.

`nodes != null` We must locate insertion position:

1. Create the new `Link`, `insertee`.
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3. Assign `position.tail` to `insertee.tail`.
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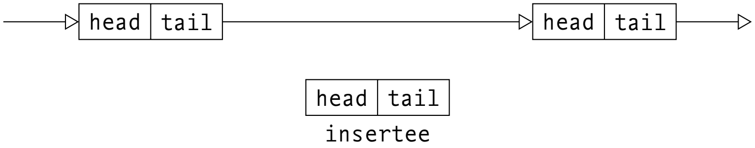

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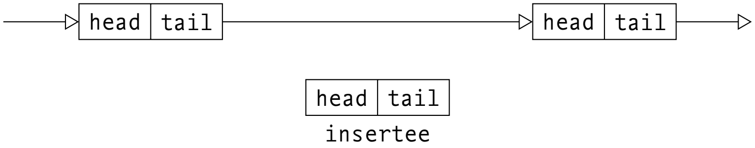

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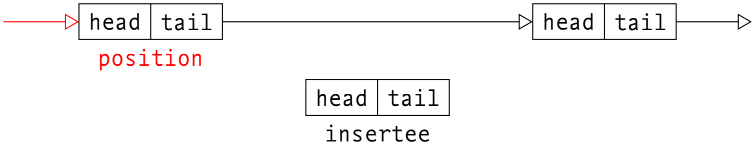

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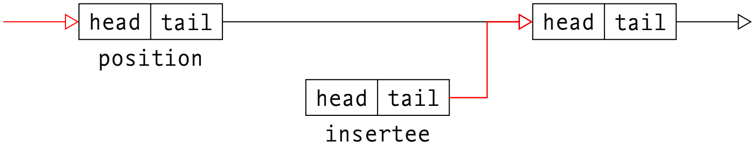

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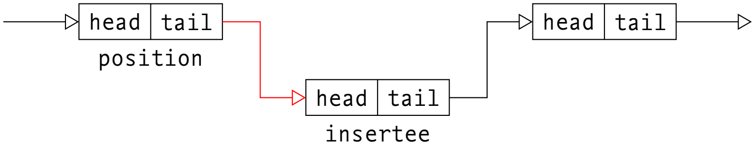

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Singly Linked Lists

Analysis

- Modern CPUs try to predict next instruction.
  - With branching, this prediction becomes difficult.
  - Inserting an item needs branching, so it's not ideal.
- Inserting at start of list is constant time operation.
  - Inserting at end requires linear time (in length of list).

Enters the Doubly Linked List

- A *doubly linked list* overcomes the singly linked list's problems.
- It requires no branching when inserting a new `Link`.
  - (Provided the insertion position is known.)
- Also supports constant time opration for appending items.
- Allows removal of `Link` in constant time.
  - (Provided a reference to `Link` is known.)
- Can be implemented with `addAfter( )` and `addBefore( )`.

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The Data Structure

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Java

```
public class DList<T> {
    private DLink<T> sentinel;

    public DList( ) {
        sentinel = new DLink( );
    }

    ...

    private static class DLink<T> {
        private T data;
        private DLink<T> prev;
        private DLink<T> next;

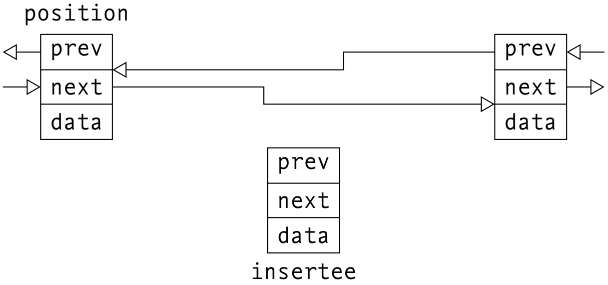
        private DLink( ) {
            prev = next = this;
        }
    }
}
```

The addAfter( ) Operation

AddBefore( ) is Similar

Java

```java
private static void addAfter( final DLink<T> position, final DLink<T> insertee ) {
    insertee.next = position.next;
    insertee.prev = position;
    position.next.prev = insertee;
    position.next = insertee;
}
```



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    position.next = insertee;
}
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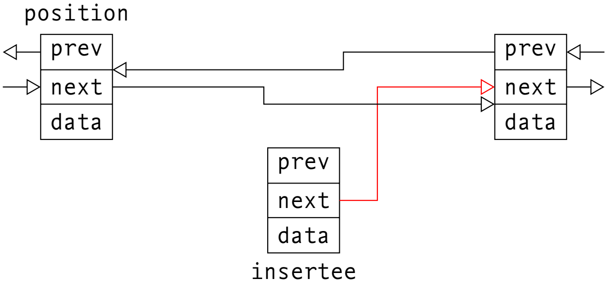

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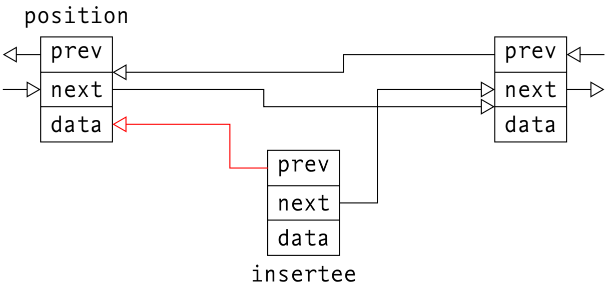

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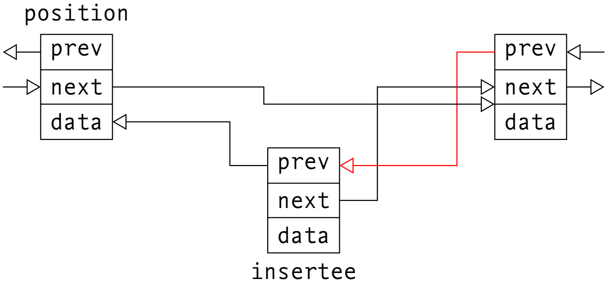

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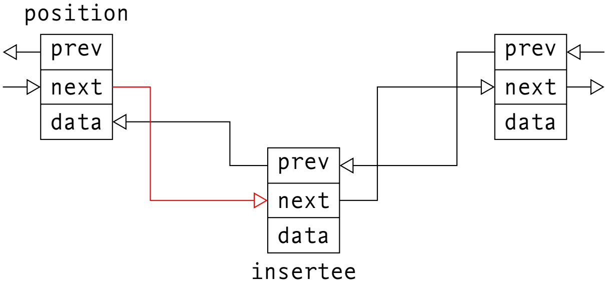

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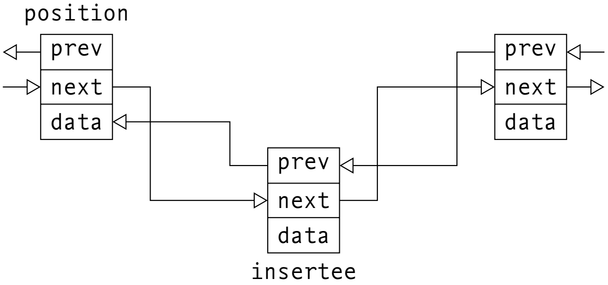

Introduction

- The `Set` interface is for collections without repetitions.
- The order of traversal depends on the implementing class.

`HashSet` No “real” order.

  - Uses `item.hashCode( )` for `add( item )` and `remove( item )`.
  - Good average performance for `add( )` and `remove( )` if there are few `hashCode( )` collisions.

`TreeSet` Set with order.

  - Uses tree to represent the set.
  - Order determined by `compareTo( )`.
  - Usually poorer performance for `add( )` & `remove( )`.

Introduction to `HashSets`

- `HashSet` usually provides good average performance.
- Uses `hashCode( )` to partition members in to subcollections.
- Members with “similar” `hashCode( )` go to same subcollection.
  - Equality in subcollections is decided with `equals( )`.
  - Classes usually override `equals( )`.
- Good performance guaranteed if subcollections don’t “overflow.”
- Diversifying the `hashCode( )` values makes “overflow” less likely.

Contract

- Deciding deep equality is an expensive operation.
- The `HashSet` class uses `equals( )` to decide equality.
- If you override `equals( )` you must override `hashCode( )`.
- All `Java` classes make assumptions about `hashCode( )`:
  - The same object must always return the same `hashCode( )`.
  - Two deeply equal objects must have the same `hashCode( )`.
  - (Different objects may have the same `hashCode( )`.)
- With these assumptions, we can decide deep equality as follows:
  - If `a.hashCode( ) != b.hashCode( )` then `!a.equals( b );`
  - Else use `a.equals( b )`.
- This is usually very efficient, provided
  - `hashCode( )` is fast; and
  - `hashCode( )` depends on as many relevant attributes as possible.

Computing Hash Codes

- By default different objects have different `hashCode( )` values.
- This is usually not useful, so we must override it.
- Take as much information into account as possible.
- “Information” are the attributes that determine `equals( )`.
- This generally reduces collisions.

Computing Hash Codes (Continued)

Overall Method

1. Initialise `result` with some nonzero value;
2. For each relevant attribute:
   1. Compute an `int`, `intCode`, for the attribute;
   2. Combine the the current value of `result` and `intCode`:
      - `result = result * 31 + intCode`.
3. Return `result`.

Computing Hash Codes (Continued)

Computing `intCode` of Attribute, `attr`

- If attribute is `boolean` return `attr ? 1 : 0;`
- If it's `byte`, `char`, or `short`, return `(int)attr;`
- If it's an `int` return `attr;`
- If it's a `long` return `attr ^ (attr >>> 32);`
- If it's a `float`, return `Float.floatToIntBits( attr );`
- If it's a `double`, return `Double.doubleToIntBits( attr );`
- If it's `null` return 0;
- If it's an array:
  - If all values are relevant, return `Arrays.hashCode( attr )`.
  - Recursively compute a hash code for the relevant values.
- Otherwise, return `attr.hashCode( )`.

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Example

Java

```
public class Person {
    final static int INITIAL_HASH_CODE = 17;
    final static int HASH_MULTIPLIER = 31;
    private final String name;
    private final boolean isMale;
    private final int age;

    ...

    @Override
    public boolean equals( Object that ) {
        final Person p = (Person)that;
        return (name.equals( p.name )) && (age == p.age);
    }

    @Override
    public int hashCode( ) {
        int result = INITIAL_HASH_CODE;
        result = result * HASH_MULTIPLIER + age;
        result = result * HASH_MULTIPLIER + (isMale ? 1 : 0);
        result = result * HASH_MULTIPLIER + name.hashCode( );
        return result;
    }
}
```

Using the Person Class

Java

```
// Always use an interface variable to reference the set.
final Set<Person> set = new HashSet<Person>( );
set.add( new Person( "Joe", true, 20 ) );
set.add( new Person( "Jane", false, 20 ) );
...
```

When Computations get Expensive

Don't Try This at Home

```
public class Casher {
    ...

    public Casher( ... ) {
        // initialise attribute values.
    }

    @Override
    public int hashCode( ) {
        return motherOfAllHashCodeComputations( );
    }
}
```


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To Avoid Recomputing, you can Cache the Value

Java

```
public class Casher {
    private static final hashCode;
    ...

    public Casher( ... ) {
        // first initialise attribute values.
        ...
        hashCode = motherOfAllHashCodeComputations( );
    }

    @Override
    public int hashCode( ) {
        return hashCode( );
    }
}
```

Lazy Computation

Code is Correct for Single-Threaded Classes

- Initialising many hash code values may take too long.
- When too many hash codes are initialised at the same time, this may cause unacceptable delays.
- You can postpone the expensive initialisation.
  - Makes sense if you don't need the hash code values.
- This is called *lazy initialisation*.
  - Intelligent classes such as `String` use lazy initialisation.

Java

```
public class Lazy {
    private Integer hashCode;

    ...

    @Override
    public int hashCode( ) {
        if (hashCode == null) {
            hashCode = motherOfAllInitialHashCodeComputations( );
        }
        return hashCode;
    }
}
```


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Maps

- A `Map` is like a mathematical function.
- It maps a *key* to a *value*.
- The `Map` interface is generic: `Map<K,V>`.
- A given key can only have no more than one value.
- Some keys have no values: their values are `null`.
- There are several `Map` implementations:
  - `HashMap`.
  - `TreeMap`.

Instance Methods

`put( key, value )`	Add `value` for `key`. - Returns current value if present; - Otherwise `null`.
`get( key )`	Get value of `key`; `null` if not present.
`keySet( )`	Get a set representation of the keys.

Example

Java

```
final Map<Color,String> map = HashMap<Color,String>( );
map.put( Color.RED, "Bad" );
map.put( Color.Green, "Good" );
map.put( Color.Blue, "Cold" );
```


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Introduction

- The `Queue` class implements a first-in-first-out (FIFO) collection.
- The `PriorityQueue` class implements a *priority queue*.
    - The elements in the collection are ordered.
    - Order depends on `compareTo( )`.
    - Least significant instances have higher *priority*.
    - The `remove( )` method removes instance with highest priority.

Example

Java

```java
public class MyProcess implements Comparable<MyProcess> {
    private int priority;
    private final Task task;

    public static void main( String[] args ) {
        final PriorityQueue<MyProcess> tasks = new PriorityQueue( );
        tasks.add( ... );
        ...
        while (!tasks.isEmpty( )) {
            final Task task = queue.remove( );
            task.run( );
        }
    }

    ...
    @Override
    public int compareTo( MyProcess that ) {
        return (this.priority < that.priority) ? -1 :
               (this.priority > that.priority) ?  1 : 0;
    }
}
```

Outline

- Hashing maps keys to positions in arrays.
- *Perfect hashing* maps different keys to different positions.
- Perfect hashing is rarely possible:
  - In general *collisions* will occur.
  - Should they occur, these collisions should be *resolved*.
- We shall study two classes of collision resolution strategies.
  - Open addressing: here we look for other free cells.
  - Separate chaining: stores colliding keys in a special data structure.

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Random Hash Functions

Collision Resolution

Running Example

Linear Probing

Double Hashing

Limitation of Open Addressing

Separate Chaining

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Implementing a `Table` Class

- Arrays are the basic data structure for many applications.
- However, storing and retrieving by index isn't always ideal.
- E.g., let's assume we want to implement a `Table` (`Map`) class.
- Each item in the `Table` has a *key*.
- Different items have different keys.
- The `Table` supports the following methods.

`Table( int size ):`	Create an *empty* `Table` object.
`isFull( ):`	Determine if the `Table` is full.
`add( Object o ):`	Add a new item to the `Table`.
`remove( Object o ):`	Remove existing item from the `Table`.
`contains( Object o ):`	Decide if the `Table` contains `o`.

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`add( Object o ):`	Add a new item to the `Table`.
`remove( Object o ):`	Remove existing item from the `Table`.
`contains( Object o ):`	Decide if the `Table` contains `o`.

- All operations should be fast.
- We shall use an array to store each object in the `Table`.
- We use the keys to determine the object's positions.
- How do we map the keys to index positions in the array?

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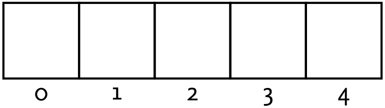
First Attempt: Use hashCode( )

Don't Try This at Home

```
public class Table {
    private final Object[] members;

    public Table( int size ) {
        members = new Object[ size ];
    }

    private int hashFunction( Object o ) {
        return Math.abs( o.hashCode( ) ) % members.length;
    }

    public void add( Object o ) {
        members[ hashFunction( o ) ] = o;
    }

    public void remove( Object o ) {
        members[ hashFunction( o ) ] = null;
    }

    public boolean contains( Object o ) {
        return (o != null)
                  && (o == members[ hashFunction( o ) ]);
    }
}
```

Let's See



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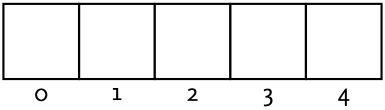
Let's See

Add Object with HashCode 123



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Add Object with HashCode 123

0	1	2	3	4
			123	

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Add Object with HashCode 20

			123	
0	1	2	3	4

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Add Object with HashCode 20

20			123	
0	1	2	3	4

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Add Object with HashCode 666

20			123	
0	1	2	3	4

Let’s See

Add Object with HashCode 666

20	666		123	
0	1	2	3	4

Let's See

Add Object with HashCode 33

20	666		123	
0	1	2	3	4

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Add Object with HashCode 33

0	1	2	3	4
20	666		123 33	

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Add Object with HashCode 33

20	666		123 33	
0	1	2	3	4

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20	666		123 33	

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Hash Functions

Definition (Hash Function)

- A *hash function* is a function that maps its argument to an index position of an array.

Java

```
public class IntTable {
    final int[] members;

    private int hashFunction( int key ) {
        return Math.abs( key ) % members.length;
    }
    ...
}
```


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Ideal Hash Function Properties

- Good hash functions should be correct:
  - The result should be an allowed index.
- They should be fast.
- They should minimise collisions.
- Random keys should result in random index positions.

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More Complications

- Let's assume we have three objects.
- The hash codes of the objects are 0, 1, and 5.
- Let's also assume we have two `Table` objects.
- `Table` 1 has a capacity of 3: its hash function is $h_3(c) = c \,\%\, 3$.
- `Table` 2 has a capacity of 4: its hash function is $h_4(c) = c \,\%\, 4$.
- With the first hash function we get no collisions:
  - $h_3(0) = 0$, $h_3(1) = 1$, and $h_3(5) = 2$.
- With the second hash function we get a collision:
  - $h_4(0) = 0$, $h_4(1) = 1$, and $h_4(5) = 1$.
- So, how do we fix this problem?

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Different Hash Functions

- Possible solution:
  - Let each `Table` object *compute* its own hash function.
- They start with a random hash function.
- They use it until a collision arises.
- When the collision arises:
  - The `Table` computes a new random hash function.
  - The new hash function should resolve the colission.

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A Family of 2-Universal Hash Functions

- Assume the hash codes are in the domain $U = \{0, \ldots, m-1\}$.
- Furthermore, assume our array has length $n \leq m$.
- Let $p \geq m$ be a random prime.
- Finally, let $0 < a < p$ and $0 \leq b \leq p$ be random integers.
- Consider the following hash function:

$$h_{a,b}(x) = ((ax + b) \,\%\, p) \,\%\, n \,.$$

- You can prove that if x and y are random members from U, then

$$\mathbb{P}\left(h_{a,b}(x) = h_{a,b}(y)\right) \leq 1/n \,. \tag{1}$$

- Mitzenmacher, and Upfal [2005, Lemma 13.6] prove (1).

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Improving our Hash Functions

Java

```java
public class IntTable {
    final Random rand;
    int[] array;
    int a, b, m, p;

    public IntTable( int size, int m ) {
        array = new int[ size ];
        rand  = new Random( );
        this.m = m;
        computeNewHashFunctionConstants( );
    }

    private void computeNewHashFunctionConstants( ) {
        p = randomPrime( m );
        a = 1 + rand.nextInt( p - 2 );
        b = rand.nextInt( p + 1 );
    }

    // ASSUMPTION 0 <= x < m.
    private int hashFunction( int x ) {
        final int n = array.length;
        return ((a * x + b) % p) % n;
    }
    ...
}
```


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Collision Resolution

- A *perfect hash function* maps each key to a unique index.
- Non-perfect hash function may result in collisions.
  - They cannot be avoided.
- If a collision occurs when adding a key,
  - We must *resolve the collision*.
- There are two techniques:

  Open addressing: Use a different, free index.

  Buckets: Allow multiple keys per index.

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0	1	2	3	4	5	6

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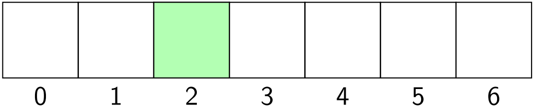

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Letters Left: B_2, J_{10}, S_{19}

		B_2				
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0	1	2	3	4	5	6
		B_2				

Running Example (Start)

Letters Left: J_{10}, S_{19}

0	1	2	3	4	5	6
		B_2				

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Letters Left: J_{10}, S_{19}

0	1	2	3	4	5	6
		B_2	J_{10}			

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0	1	2	3	4	5	6

Open Addressing with Linear Probing

- Let $n > 2$ be a prime and let's assume we have an n-sized array that isn't full.
- Furthermore, let's assume we want to insert a key, k.
- However, this time a collision occurs.
- To resolve the collision we need to find a free cell.
- We start at $h(k)$, next visit $(h(k) - 1) \% n$, visit $(h(k) - 2) \% n$, ...
- Eventually, we should find some free cell.
- This collision resolution policy is called *linear probing*.

Example

Letters Left: N_{14}, X_{24}, W_{23}

0	1	2	3	4	5	6
		B_2	J_{10}		S_{19}	

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Letters Left: N_{14}, X_{24}, W_{23}

		B_2	J_{10}		S_{19}	
0	1	2	3	4	5	6

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Letters Left: X_{24}, W_{23}

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0	1	2	3	4	5	6

Probe Sequences

- Linear probing visits a sequence of occupied and free indices.
- The sequence of occupied cells is called the *probe sequence*.
- The key, k, and the hash function determine the sequence's first index: $h(k)$.
- The remaining indices are determined by the collision resolution policy.
- The ith next index is $(h(k) - i \times p(k)) \,\%\, n$,
  - Where $p(\cdot)$ is the *probe decrement function*.
- For linear probing the probe decrement function is given by $p(k) = 1$.

Removing an Item:

- When removing a value we shouldn't break a probe sequence.
- E.g. let's assume we remove B_2 by making Cell 2 empty.
- If make it empty we'll have problems locating X_{24}:
  - A free cell in the sequence should indicate that B_2 isn't in the table.

N_{14}	X_{24}	B_2	J_{10}		S_{19}	W_{23}
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Removing an Item: Remove B_2

- When removing a value we shouldn't break a probe sequence.
- E.g. let's assume we remove B_2 by making Cell 2 empty.
- If make it empty we'll have problems locating X_{24}:
  - A free cell in the sequence should indicate that B_2 isn't in the table.

N_{14}	X_{24}	B_2	J_{10}		S_{19}	W_{23}
0	1	2	3	4	5	6

Removing an Item: Remove B_2

- When removing a value we shouldn't break a probe sequence.
- E.g. let's assume we remove B_2 by making Cell 2 empty.
- If make it empty we'll have problems locating X_{24}:
  - A free cell in the sequence should indicate that B_2 isn't in the table.

N_{14}	X_{24}		J_{10}		S_{19}	W_{23}
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Removing an Item: Locate X_{24}

- When removing a value we shouldn't break a probe sequence.
- E.g. let's assume we remove B_2 by making Cell 2 empty.
- If make it empty we'll have problems locating X_{24}:
  - A free cell in the sequence should indicate that B_2 isn't in the table.

N_{14}	X_{24}		J_{10}		S_{19}	W_{23}
0	1	2	3	4	5	6

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Removing an Item: Locate X_{24}

- When removing a value we shouldn't break a probe sequence.
- E.g. let's assume we remove B_2 by making Cell 2 empty.
- If make it empty we'll have problems locating X_{24}:
  - A free cell in the sequence should indicate that B_2 isn't in the table.

N_{14}	X_{24}		J_{10}		S_{19}	W_{23}
0	1	2	3	4	5	6

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Removing an Item: Locate X_{24}

- When removing a value we shouldn't break a probe sequence.
- E.g. let's assume we remove B_2 by making Cell 2 empty.
- If make it empty we'll have problems locating X_{24}:
  - A free cell in the sequence should indicate that B_2 isn't in the table.

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Removing an Item:

- Instead of making B_2's cell available, we mark it $\perp$.
  - When looking for items we treat $\perp$ as a used cell.
  - When looking for free cells we treat $\perp$ as a free cell.

N_{14}	X_{24}	B_2	J_{10}		S_{19}	W_{23}
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- Instead of making B_2's cell available, we mark it $\perp$.
  - When looking for items we treat $\perp$ as a used cell.
  - When looking for free cells we treat $\perp$ as a free cell.

N_{14}	X_{24}	B_2	J_{10}		S_{19}	W_{23}
0	1	2	3	4	5	6

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Removing an Item: Remove B_2

- Instead of making B_2's cell available, we mark it $\perp$.
  - When looking for items we treat $\perp$ as a used cell.
  - When looking for free cells we treat $\perp$ as a free cell.

N_{14}	X_{24}	$\perp$	J_{10}		S_{19}	W_{23}
0	1	2	3	4	5	6

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Removing an Item: Locate X_{24}

- Instead of making B_2's cell available, we mark it $\perp$.
  - When looking for items we treat $\perp$ as a used cell.
  - When looking for free cells we treat $\perp$ as a free cell.

N_{14}	X_{24}	$\perp$	J_{10}		S_{19}	W_{23}
0	1	2	3	4	5	6

Removing an Item: Locate X_{24}

- Instead of making B_2's cell available, we mark it $\perp$.
  - When looking for items we treat $\perp$ as a used cell.
  - When looking for free cells we treat $\perp$ as a free cell.

N_{14}	X_{24}	$\perp$	J_{10}		S_{19}	W_{23}
0	1	2	3	4	5	6

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Removing an Item: Locate X_{24}

- Instead of making B_2's cell available, we mark it $\perp$.
  - When looking for items we treat $\perp$ as a used cell.
  - When looking for free cells we treat $\perp$ as a free cell.

N_{14}	X_{24}	$\perp$	J_{10}		S_{19}	W_{23}
0	1	2	3	4	5	6

Removing an Item: Locate X_{24}

- Instead of making B_2's cell available, we mark it $\perp$.
  - When looking for items we treat $\perp$ as a used cell.
  - When looking for free cells we treat $\perp$ as a free cell.

N_{14}	X_{24}	$\perp$	J_{10}		S_{19}	W_{23}
0	1	2	3	4	5	6

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Linear Probing Revisited

- Linear probing is not a good collision resolution policy.
- It tends to lead to clusters, which in their turn lead to bigger clusters, ...
- With large clusters you get long probe sequences.
- The larger the clusters get the faster they grow.
- In general it is better to have *different* probe decrements for different keys.
- This is called *double hashing*.

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Double Hashing

- With double hashing we have two hash functions.
  - $h_1(k)$ should be random index.
  - $h_2(k)$ should be a random probe decrement as a function of k.
- We use the two to compute the probe sequence (modulo n):

$$h_1(k), h_1(k) - h_2(k), h_1(k) - 2h_2(k), h_1(k) - 3h_2(k), \ldots .$$

- Square hopping should still work with $h_2(\cdot)$.
  - Therefore $h_2(k)$ should be *relative prime* to n.
- Usually, n is a power of two.
- If n is a power of two ands $h_2(k)$ is odd,
  - Then $h_2(k)$ and n are relative prime [Cormen et al. 2001].

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Limitations of Open Addressing

- Frequent additions cause clustering.
- Frequent additions will eventually fill the table.
- Both problems may be overcome by resizing the table.
- `Java` has many classes based on hashing.
- Many re-size themselves so as to ensure good performance.

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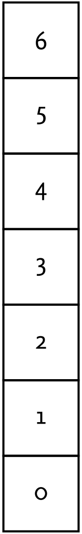
Separate Chaining

- Another collision resolution strategy is *separate chaining*.
- It changes the structure of the hash table.
- Table locations now store *multiple* values.
- Each non-empty table index now has a *bucket*.
- Initially all buckets are empty.
- When a key is mapped to a location,
  - We add it to the location's bucket.
- A possible implementation for the bucket is a linked list.
- Separate chaining provides a simple way to resolve collisions.
- However, it requires more memory than open addressing.

Example

Letters Left: B_2,J_{10}, S_{19},N_{14},X_{24},W_{23}



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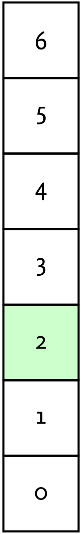

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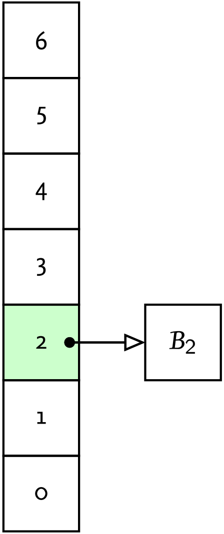
Letters Left: B_2,J_{10}, S_{19},N_{14},X_{24},W_{23}



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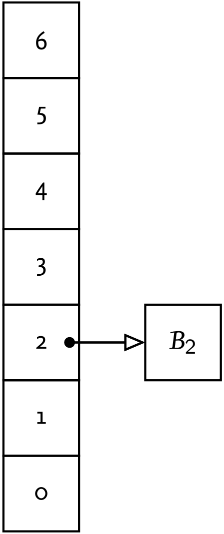

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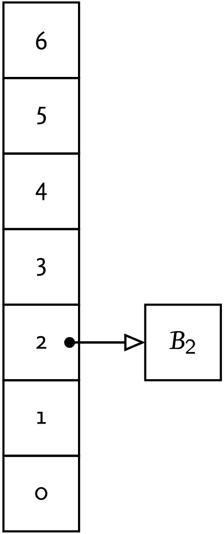

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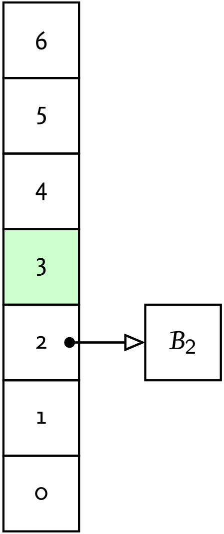

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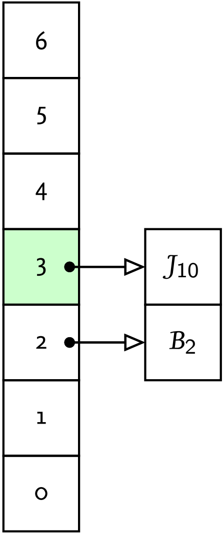

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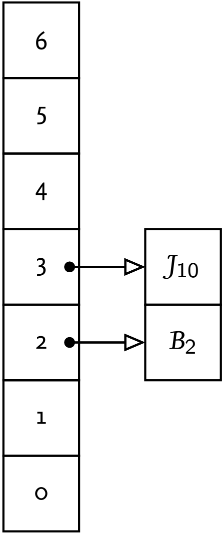

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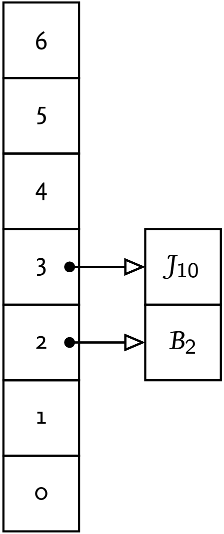
Letters Left: $S_{19}, N_{14}, X_{24}, W_{23}$



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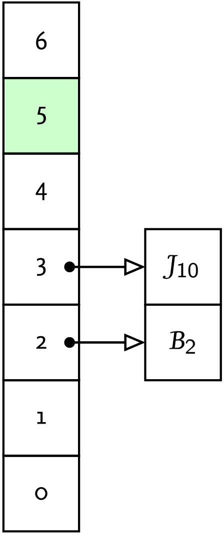

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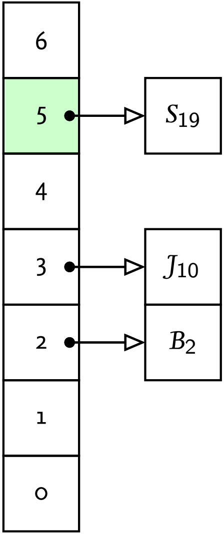

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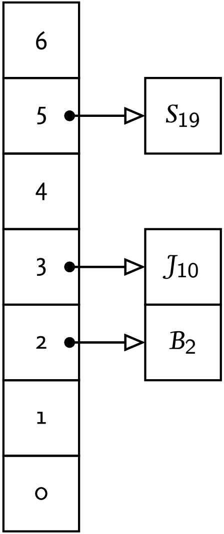
Letters Left: $S_{19}, N_{14}, X_{24}, W_{23}$



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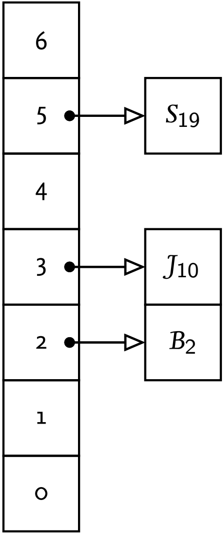

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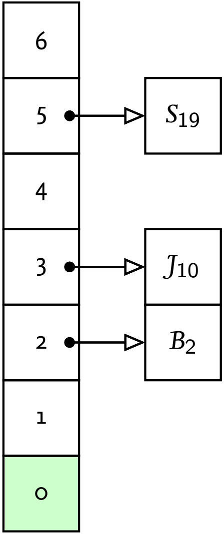

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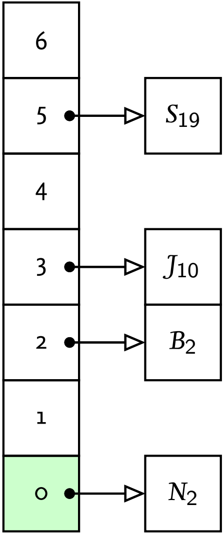

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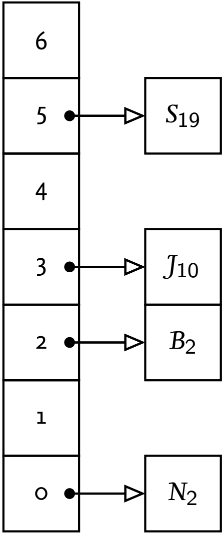

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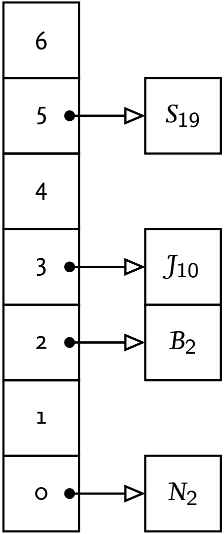

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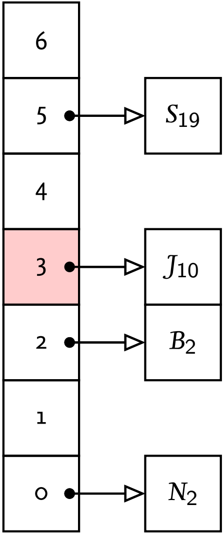

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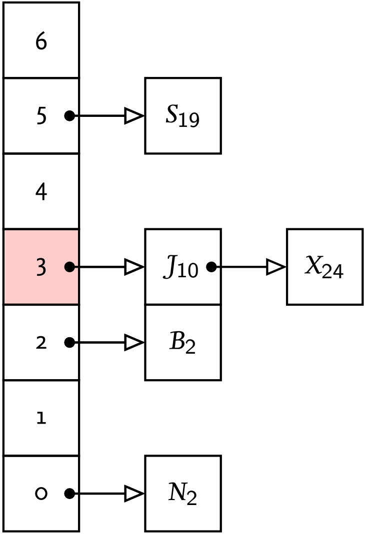

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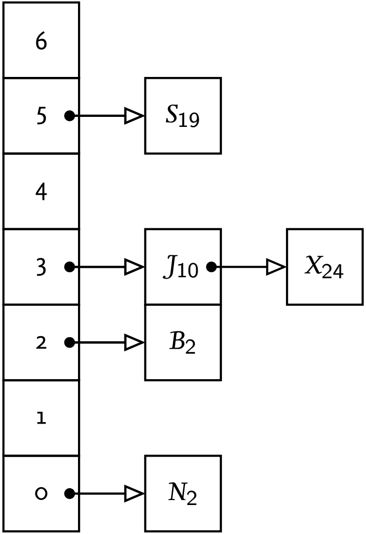

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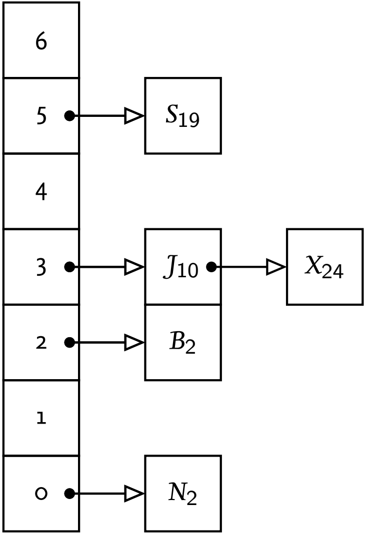

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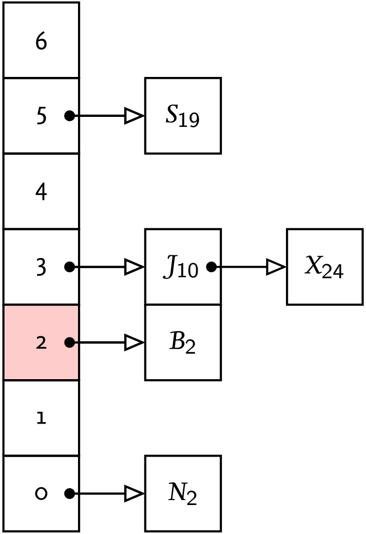

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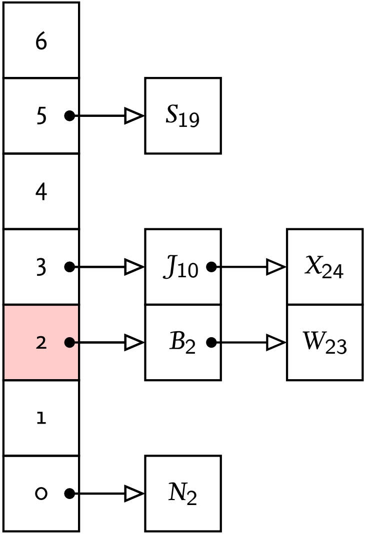

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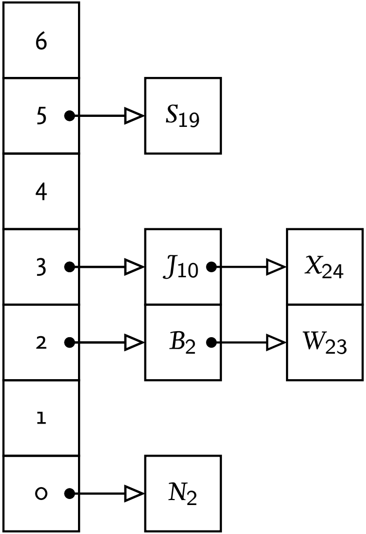

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- Study [Horstmann 2013, Chapter 14.1–14.4].
- Read [Horstmann 2013, Chapter 15.4].

Acknowledgements

- Parts of this lecture correspond to [Horstmann 2013, Chapter 14.1–14.4].
- Overriding `hashCode( )` is based on [Bloch 2008, Item 9].
- The part about random hashing is based on [Mitzenmacher, and Upfal 2005].
- The running linear probing example is based on [Standish 1994, Chapter 11].

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- This document was created with `pdflatex`.
- The LaTeX document class is `beamer`.