Background

Revision of Functions

If a function maps from a set A to a set B, then A is the domain and B is the codomain. The codomain is a set that contains all outputs of a function (but may contain other values). The range of a function is the set which contains only the output values of that function.

Input-Output Function of a Program

If a program describes a computation that transforms inputs to outputs, then for every program there is a corresponding input-output function.

If you take sorting algorithms, the input-output function for any sorting algorithm is the same – it takes a list of integers as input and gives the sorted list as output.

The input-output function can be described as a specification of the problem, or of programs to compute the function.

For programs with the same input-output function, we say they're solving the same problem.

Finite Sequences

The input for a sorting program is the set of all finite sequence.

Bijections, Injections, and Surjections

If a function f: A -> B is a bijection, then |A| and |B| are the same (sets A and B contain the same number of elements). We're gonna use this to measure the sizes of infinite sets.

Countability

Binary Sequences

The set of binary sequences of size n encodes the possible subsets of $\{0, \ldots\ n\}$.

Countability and Enumeration

If an infinite set is countable, it can be represented as a sequence, because you can form a bijection between it and the natural numbers, giving the set an order.