If we're asked to prove something logically we'll be given all proofs we need.

Equivalent formulae

Can prove things exhaustively. If two statements have the same truth table, they are equivalent statements.

This gets long quickly for complicated statements, so you can also prove things by building on things you already know.

Conditional vs. disjunction

p implies q is equivalent to not(p) or q

Distributive law 1

Too fast to take notes. OR distributes over AND.

Logic notation and set notation

AND corresponds to intersection, OR corresponds to union.

F corresponds to {}.

Tautologies and Contradictions

A propositional formula that is always true, no matter what truth values are given to the atomic propositions, is a tautology.

A propositional formula that is always false is a contradiction.

Example of a tautology

$(p \wedge q) \rightarrow (p \vee q)$

The above is true for any values of p and q. You can prove that with a truth table or with equivalences and laws etc.

Example: Is $\neg(p \wedge s) \vee q \leftrightarrow (p \rightarrow q) \vee (\neg(\neg q \wedge s))$

Note: We will have proofs like this in the exam.

Satisfiability

A propositional formula where T is output for at least one set of inputs.

Entails - |=

Means the formula on the right is true whenever the statements on the left are true.

E.g. $\neg p \models \neg p \vee q$

Boolean satisfiability

What?