OLLSCOIL NA hÉIREANN
THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH
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CS3509: Theory of Computing

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One and a half hour

Question1 [10 marks]
A real-time company develops software that is guaranteed to be "time-able". More precisely, the company developed a programming language for which sofware can be timed in the following way: ahead of executing the software, the worst-case running time of this software can be determined by analyzing the code. The language developed by the company is not as powerful as Java. It can only compute a subset of the problems that Java can compute. Can the company create a language that is as powerful as Java for their safety-critical applications such that this language still has the same property, i.e. the worst-case time of all software can be determined? Justify your answer.

Question 2 [20 marks]
Construct the decision tree for the algorithm described by the following pseudo-code:

For i = 1 to n - 1 do
  If L[i] > L[i+1] then swap(L[i],L[i+1])
For i = n-1 downto 2 do
  If L[i] < L[i-1] then swap(L[i],L[i-1])

Note that this code does not determine a sorting algorithm. However, on lists of size 3, the code does sort these lists. Produce the decision tree of this algorithm for lists of size 3.

Question3 [10 marks]
a) (5 marks) State a semi-modularity inequality that holds for the best-case time for the sequential composition of programs P and Q.

b) (5 marks) When does equality hold? State a condition on P or Q that guarantees equality to hold for the inquality you stated under a). Justify your answer.

Question 4 [20 marks]
We have seen that the independent set problem in a graph, denoted by IND, is NP complete. Recall that in a graph $G = (V, E)$ a set of vertices $S \subseteq V$ is independent if and only if no two vertices in S are connected by an edge. The independent set problem in a graph concerns finding the largest independent set in a the graph.

Below we describe the clique problem, denoted by CLIQUE. A clique in a graph $G = (V, E)$ is a subset of vertices $C \subseteq V$ such that for every two vertices in C there *is* an edge that connects them. The clique problem in a graph concerns finding the largest clique.

Show that the clique problem in a graph is NP complete by using the fact that the independent set problem in a graph is NP complete. In other words, show: IND $\leq_P$ CLIQUE.

Question5 [20 marks]
a) (5 marks) Construct a heap using the procedure heapify, for the inputs given by the list:

$$(10, 2, 9, 16, 8, 6, 1, 3, 12).$$

b) (5 marks) A company requests that their software developer designs an algorithm that uses only comparisons to sort its data. The algorithm should run in $O(\frac{n}{2})$ *worst-case* time. How should the sofware developer respond?

c) (5 marks) A company requests that their software developer designs an algorithm that uses only comparisons to sort its data. The algorithm should run in $O(\frac{n}{2})$ *average-case* time. How should the sofware developer respond?

d) (5 marks) If a comparison based sorting algorithm A is optimal in worst-case time, i.e. $T_A^W(n) \in O(nlogn)$, then what can you conclude about its average-case time? Justify your answer.