Quantifying the Sensitivity of Quantitative Trade Models*

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Abstract

A modern revolution in spatial economic modeling aims to answer quantitative counterfactual questions by using models that feature micro-level heterogeneity. This heterogeneity is then often assumed to come from particular parametric families—such as Frechet in Eaton and Kortum's (2002) Ricardian model. While these parametric choices greatly enhance the tractability of model simulations, it is unknown how sensitive the answers to counterfactual questions are to these assumptions of convenience because there are infinitely many alternative distributions of heterogeneity to be evaluated. We overcome this challenge by building a general trade model that leverages recent advances in the robustness literature. Our method calculates sharp bounds on the values of model counterfactuals that could obtain—while still exactly matching all aggregate trade data points, a gravity-like moment condition, and satisfying equilibrium constraints—under all possible distributions of underlying heterogeneity that lie within a given divergence from a chosen reference distribution. Applying this method to the Eaton and Kortum (2002) model, we find that the gains from trade in these models could be several times larger or smaller than they appear to be under standard benchmark distributions, even if heterogeneity is drawn from a distribution that is at least as similar to Frechet as are the types of parametric alternatives that are commonly explored in sensitivity analysis.

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1 Introduction

A fundamental question in the field of international economics concerns the size of the gains from trade. In the canonical Ricardian model, these gains hinge on countries' relative productivity levels across all the goods in the economy. When relative productivities are the same for all goods there are no gains to be had from trading, but when relative productivities are highly dispersed across goods then two countries can mutually gain from specializing in the goods for which their productivity is relatively largest. However, because these gains hinge precisely on relative productivity comparisons among goods that countries don't produce, they are not observable in conventional datasets.

To fill in this missing data gap, the seminal work of Eaton and Kortum (2002) posits that countries' productivity levels across goods are drawn independently form an extreme-value (Frechet) distribution. Doing so results in a remarkably tractable model, and one in which the gains from trading for any country depend on just two characteristics (the extent to which the country is currently open to trade and the dispersion parameter of the Frechet distribution). However, it remains an open question how sensitive are the gains from trade in this canonical Ricardian model as one departs from Eaton and Kortum's (2002) functional form choice.

In this paper we develop a procedure for quantifying the sensitivity of existing estimates of the gains from trade—among other counterfactual questions that quantitative models are used for—in Ricardian models. In particular, we draw on recent advances in the econometrics literature (Christensen and Connault, 2023) in order to consider every distribution of productivities around the world that is within a given measure of "divergence" (a standard measure of the similarity of any two probability distributions) of that used by Eaton and Kortum (2002). We then calculate the maximum and minimum values of the gains from trade (the welfare cost of shutting down trade altogether) that can occur in a market equilibrium for this set of distributions. In addition, a core component of our approach is that, while considering every productivity distribution within a given divergence of Frechet, we ensure that the resulting Ricardian model associated with each distribution can always generate bilateral trade flows that exactly match those in the data. This ensures that gains from trade, and other counterfactual questions, are grounded in the realities of how much countries actually trade, along with other key empirical facts about trade flows such as the fit of the gravity equation or the value of how trade flows respond to trade costs (the so-called "trade elasticity").

Our main finding is that the gains from trade (for any country considered) are extremely sensitive to small departures from Eaton and Kortum's (2002) chosen distribution of productivities around the world. To interpret this, consider the world distribution of productivities that is used in Eaton and Kortum (2002)—one in which there are 17 countries, each with productivities drawn independently from a Frechet distribution with dispersion parameter θ and with a location parameter chosen to match trade flows at baseline. A commonly used value is $\theta = 6$. For this distribution the gains from trade for France, a country with typical openness, is 4.4% of GDP. A common form of sensitivity analysis in the literature on Ricardian models is to consider an alternative distribution that remains Frechet but with a different dispersion parameter. For example, lowering this parameter from $\theta = 6$ to $\theta = 4.97$ increases France's gains from trade to 5.1%. The divergence, denoted δ, between these two (Frechet) distributions is $\delta = 0.5$. Our calculations repeat an exercise like this for *all* distributions that are within a given divergence δ from that in Eaton and Kortum (2002). Doing so, we find that the minimum amount by which France gains from trade, across all distributions with $\delta \leq 0.5$, is 0.05% and the maximum possible gain is 9.2%. That is, despite matching the value of every bilateral trade flow at baseline, there exist ways to make relatively small (i.e. divergence within $\delta = 0.5$) changes to the distribution that yield strikingly different gains from trade.

Our second finding repeats this exercise at larger values of δ. Consider $\delta = 10$. One example of a global productivity distribution that has divergence $\delta = 10$ from that in Eaton and Kortum (2002) is one that changes θ to 3.6, where French gains from trade are 7.0% of GDP. We find that, even with the considerable added flexibility of a high δ, the maximum and minimum gains from trade for France are 12.3% and 0.03%, respectively. These values are relatively close to the $\delta = 0.5$ bounds we obtain. Our findings therefore imply that Ricardian gains from trade in Eaton and Kortum (2002) are quite sensitive to small changes to the productivity distribution but the range of possible gains is relatively stable across larger changes in the distribution.

We believe these results are surprising for two reasons. First, one might conjecture that once key facts about trade flows are matched—facts such as the extent of openness, the trade elasticity, and the impressive fit of the gravity equation—then there may be little room to maneuver in search of alternative models that generate the same data, feature similar productivity distributions, and yet display considerably different gains from trade. Yet our findings imply that this conjecture is wrong. Second, one might conjecture that, because Eaton and Kortum (2002) assumed an extreme value distribution, their results might not be sensitive to this functional form assumption, at least for the case of a large number of countries. This conjecture follows from the extremal types theorem—analogous to the central limit theorem, but for extremal statistics rather than means—which in this context implies that if consumers are always consuming the cheapest version sold by many suppliers (who themselves sell competitively and have independent productivity distributions) then the prices of what consumers actually buy would be extreme-value distributed even if the suppliers' underlying productivity distributions were not. Our findings imply that the relevant rate of convergence in the extremal types theorem in our context must be "slow" in the economic sense that it does not appear to provide much robustness for the problem that interests us here (quantifying the gains from trade in the case of 17 countries).

2 Theory

In this section we describe the core economic environment of a multi-country Ricardian trade model with many goods and arbitrary distributions of productivities (across goods and countries) around the world. We then describe how one can conduct counterfactual exercises (such as computing the gains from trade for any country) in this model. Finally, we describe the version of this model proposed by Eaton and Kortum (2002) in which the global productivity distribution takes a particular form.

2.1 Economic environment

We begin with a standard Ricardian model of international trade.

Setup. There are D countries, potentially trading any of the goods along a continuum indexed by $\omega \in [0, 1]$. Each country produces goods using an immobile (across countries) factor of production that is owned by a representative household there.

Demand. Preferences in each country d take the CES form across the continuum of goods:

$$U_d = \left(\int_0^1 q_d(\omega)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

with σ capturing the elasticity of substitution and $q_d(\omega)$ denoting the quantity of good ω consumed in country d. As is standard, we work with the case in which goods are substitutes (i.e. $\sigma > 1$) so that the gains from trade are finite.

We let $p_{od}(\omega)$ denote the price of good ω in country d, if it were to be shipped from country o to country d. The determination of these prices involves the supply side of the economy, introduced below. But regardless of the determination, utility-maximization implies that the expenditure in d on good ω from country o will be

$$X_{od}(\omega) = Y_d p_{od}(\omega)^{1-\sigma}\lambda_d \tag{1}$$

where Y_d is the income (and total expenditure) in d and $\lambda_d \equiv \left(\sum_o \int_0^1 p_{od}(\omega)^{1-\sigma} d\omega\right)^{-1}$.

Supply. The core of the Ricardian model lies on the supply side. We assume that producers in country o are endowed with an "idea" $U_o(\omega)$ about how to produce each good ω. Ideas are useful because they enhance production. To describe this, we first refer to a "latent" price, $p_{od}^{\text{latent}}(\omega)$, which is what the perfectly competitive producers of good ω in o would charge in destination d *if they were* to sell this good in that market. These latent prices are given by

$$p_{od}^{\text{latent}}(\omega) \equiv \frac{w_o c_{od}}{[U_o(\omega)]^{1/\theta}} \tag{2}$$

where w_o is the cost of a unit of the factor of production in o, and c_{od} denotes the per-unit cost of trading the good (e.g. due to tariffs and transport costs) from o to d. In this expression, ideas $U_o(\omega)$ shift downwards the marginal cost of producing ω in o, and the extent to which they do so is governed by the parameter $\theta > 0$. In particular, when θ is low, marginal costs, and hence prices, are relatively more sensitive to the quality of ideas.

Because the good ω is homogeneous and there is perfect competition, the actual price of the version of good ω that hails from o and is sold in d must satisfy

$$p_{od}(\omega) = \begin{cases} p_{od}^{\text{latent}}(\omega) & \text{if } o = \arg\min_{o' \in D} p_{o'd}^{\text{latent}}(\omega), \\ \infty & \text{otherwise,} \end{cases} \tag{3}$$

with the convention that $X_{od}(\omega) = 0$ whenever $p_{od}(\omega) = \infty$. That is, country d only buys from the minimum-price supplier of any good.

Finally, we specify trade costs c_{od} as a function of ad-valorem tariffs (denoted t_{od}) and transport costs (A_{od}^{-1}), namely

$$c_{od} = (1 + t_{od})A_{od}^{-1}. \tag{4}$$

In this sense, A_{od} can be thought of as a measure of o's productivity in shipping to d. Tariffs do not apply on domestic trade, so $t_{dd} = 0$ for all d.

Equilibrium. We let L_o denote the (exogenous) supply of the factor of production in o. A competitive, balanced-trade, global equilibrium in this Ricardian model is then one in which the factor market clears in all countries. This requires that the vector of wages w_o

satisfies

$$Y_o \equiv R_o + w_o L_o = \sum_d \int_0^1 X_{od}(\omega) d\omega, \tag{5}$$

where R_o denotes the tariff revenue collected by country o.

2.2 Stochastic representation

The key ingredient of this Ricardian model is the distribution of each country's ideas, $U_o(\omega)$. For example, if ideas were to be distributed in such a way that they are perfectly correlated across countries (i.e. $U_o(\omega) = \alpha U_{o'}(\omega)$ for some $\alpha > 0$ for any pair of countries o and o') then no country would have a comparative advantage in any good and there would be no trade (or gains from trade). On the other hand, if countries' distributions are relatively uncorrelated, or even negatively correlated, then a rationale for international specialization and hence trade emerges in the competitive equilibrium.

To emphasize such distributions we, like Eaton and Kortum (2002), use notation that invokes a stochastic representation (even though, applying a law-of-large-numbers convention to the continuum of goods, this is merely notational). Letting F denote the *arbitrary* cumulative distribution function (CDF) of the global distribution of $U_o(\omega)$—across all countries o and all goods ω—we can write the key equations for λ_d and equilibrium (5) in this stochastic notation as

$$\lambda_d^{-1} = \mathbb{E}_F \left[\sum_o p_{od}(\omega)^{1-\sigma} \right] \tag{6}$$

$$R_o + w_o L_o = \sum_d \mathbb{E}_F \left[X_{od}(\omega) \right], \tag{7}$$

where $\mathbb{E}_F [\cdot]$ denotes an expectation over the ideas distributed according to the CDF, F.

2.3 Counterfactuals

Our goal is to study how economic conditions in any given country d respond to counterfactual changes in the exogenous primitives of this model. In particular, we imagine a generic change from the set of tariffs $\{t\}$ to the set $\{t'\}$, or from the set of factor endowments $\{L\}$ to the set $\{L'\}$. One central counterfactual exercise of interest concerns the gains from trade for country d, an object that is defined as the inverse of the welfare cost to country d from changing its import tariffs from $\{t_d\}$ to the tariffs that would imply autarky (i.e. $t_{od} = \infty$ for all $o \neq d$).

A counterfactual equilibrium is one in which all endogenous variables have changed correspondingly (e.g. the wage in country o has changed from w_o to w'_o) so as to satisfy the counterfactual equilibrium values that solve:

$$
\begin{aligned}
(\lambda'_d)^{-1} &= \mathbb{E}_F\left[p'_{od}(\omega)^{1-\sigma}\right], && (8)\\
Y'_d &= R'_d + w'_d L'_d, && (9)\\
X'_{od}(\omega) &= Y'_d p'_{od}(\omega)^{1-\sigma}\lambda'_d, && (10)\\
Y'_o &= \sum_d \mathbb{E}_F\left[X'_{od}(\omega)\right], && (11)
\end{aligned}
$$

and where the prices p'_{od} follow from (3) but evaluated at latent prices now given by $w'_o(1+t'_{od})A^{-1}_{od}[U_o(\omega)]^{-1/\theta}$.

After determining the values of all endogenous variables in this counterfactual equilibrium, we then solve for a particular function of those values that is of interest. For example, we can define (as standard) the gains from trade for country d as (one minus) the proportional change in the real income of factor owners in d when that country's trade moves from its current level to autarky. Then the counterfactual calculation of interest is given by

$$
\kappa \equiv 1 - \frac{Y'_d}{Y_d}\left(\frac{\lambda'_d}{\lambda_d}\right)^{\frac{1}{1-\sigma}} \qquad (12)
$$

when the values of (Y_d, λ_d) and (Y'_d, λ'_d) are calculated as their corresponding equilibrium values in the economy with tariffs $\{t\}$ set to their factual values and $\{t'\}$ set such that country d is in autarky, respectively. Additional sets of counterfactual calculations are just as feasible.[1]

2.4 The Eaton and Kortum (2002) reference case

The seminal model of Eaton and Kortum (2002) is a special case of the framework introduced above. In particular, Eaton and Kortum (2002) proposed the enormously tractable assumption that the global distribution of ideas F takes a particular form—which we denote by F^*—in which each country o draws its idea $U_o(\omega)$ for each good ω independently and identically from an exponential distribution with location parameter equal to one, ie $Exp(1)$. It is worth emphasizing that this involves three separate types of restrictions relative to the unrestricted F: (i) countries' distributions are independent from one another, so the joint global distribution is the product of country-specific marginal distributions;

[1] In particular, for any known function $k(\cdot)$, the methods described below can be applied to any counterfactual calculation that takes the form $\kappa = \mathbb{E}_F[k(\{U\}\{w,w'\},\{\lambda,\lambda'\},\{t,t'\},\{L,L'\}\{A\},\theta,\sigma)]$.

(ii) countries all share a common marginal distribution; and (iii) the common marginal distribution that countries share takes the particular form of $Exp(1)$.

This presentation differs from that in Eaton and Kortum (2002) in two respects. The first difference is merely notational. We follow Alvarez and Lucas (2007) in referring to ideas that are distributed $Exp(1)$, and where ideas enter production in a way that is shifted multiplicatively by the cost-shifters A_{od} in (4) and scaled by the technology parameter θ in (3); by contrast, in the Eaton and Kortum (2002) presentation of technology, productivities are drawn from a Frechet distribution that has a separate location parameter for each country and a common scale parameter. Standard properties of the Exponential and Frechet distributions imply the equivalence between these two conventions. The second difference offers slightly more generality: Eaton and Kortum (2002) has country-specific shifters (isomorphic to A_o rather than A_{od}). However, this added generality has become conventional in more recent work (such as Dekle et al. (2008)).

3 Estimation

Answering the counterfactual questions posed above is impossible without estimates of the model's exogenous features. These include both observed inputs (tariffs $\{t\}$ and endowments $\{L\}$) and unobserved parameters (the demand elasticity σ, the technology stretcher θ, and the set of technology shifters $\{A\}$). And perhaps most fundamentally of all, this includes the global distribution of ideas F that governs comparative advantage and trade possibilities.

3.1 Estimation in Eaton and Kortum (2002)

Eaton and Kortum (2002) pioneered the estimation of the unobserved inputs necessary to do counterfactuals in their version of the continuum of goods Ricardian model. In particular, they noted that all of the necessary inputs could be estimated from aggregate (country-to-country) trade data, without the need to observe micro-data on production, productivity, or specialization.[2]

[2]This use of aggregate data offers three advantages beyond mere convenience. First, it avoids the need to take a stand on how the products ω map into product categories in any available dataset, let alone one that is globally comparable. Second, as with any competitive model with constant returns technologies, the Ricardian model does not make a prediction about which firms will produce which products within a given country; so even if appropriate micro-data were available it could not be easily mapped to the model. Finally, the essence of the Ricardian model is complete specialization—a country gains from trade because of the goods that it does not produce in the trading equilibrium—so an unavoidable selection problem confronts attempts to measure productivities for non-produced goods.

In terms of the model above, we let the aggregate (across all goods ω) value of trade exported from country o to country d be denoted $X_{od} \equiv \int_0^1 X_{od}(\omega)d\omega$. And we let $\widetilde{X}_{od}$ denote the corresponding equivalent in an available dataset from the global cross-section of interest. Then, as Eaton and Kortum (2002) showed, in the case of $F = F^*$, the parameter θ can be estimated as the slope coefficient β in a bilateral gravity regression of the form

$$\ln \widetilde{X}_{od} = \alpha_o + \alpha_d + \beta \ln(1 + t_{od}) + \varepsilon_{od}, \tag{13}$$

where α_o and α_d denote exporter and importer fixed effects, respectively, and ε_{od} is a residual. In particular, if ε_{od} is mean independent of $\ln(1 + t_{od})$, then the OLS estimate of β will be an unbiased estimator of $-\theta$. Further, up to a normalization, the fitted values of the residual will satisfy $\widehat{\varepsilon}_{od} = \widehat{\theta} \ln A_{od}$, allowing estimation of each A_{od} as well. Having estimated all of the model parameters in this way the estimated Eaton and Kortum (2002) model perfectly matches all of the D^2 bilateral values of aggregate trade, $\widetilde{X}_{od}$.

3.2 Estimation beyond Eaton and Kortum (2002)

Having seen how estimation works in the Eaton and Kortum (2002) case, when $F = F^*$, our goal now is to replicate these steps for the case in which F is unrestricted except that it lies within the same "neighborhood" as F^*. We begin by imposing the same data restrictions as were imposed imposed by the Eaton and Kortum (2002) procedure, but in the setting of a more general F. In particular, the above gravity estimation equation (13) can be equivalently written as

$$\sum_{o,d} \left(\Delta_{o_1}\Delta_{d_1}\mathbb{E}_{F^*}\left[\ln X_{od}(\omega)\right]\right)\left(\Delta_{o_1}\Delta_{d_1}\ln(1 + t_{od})\right) = \theta \left(\sum_{o,d}\Delta_{o_1}\Delta_{d_1}\ln(1 + t_{od})\right)^2, \tag{14}$$

where the notation $\Delta_{o1} y_o$ denotes differencing with respect to a reference origin country, i.e. $\Delta_{o_1} y_o \equiv y_o - y_{o_1}$, etc. It follows that the analogous moment condition for the case of general F is simply

$$\sum_{o,d} \left(\Delta_{o_1}\Delta_{d_1}\mathbb{E}_{F}\left[\ln X_{od}(\omega)\right]\right)\left(\Delta_{o_1}\Delta_{d_1}\ln(1 + t_{od})\right) = \theta \left(\sum_{o,d}\Delta_{o_1}\Delta_{d_1}\ln(1 + t_{od})\right)^2. \tag{15}$$

Likewise, the Dekle et al. (2008) tradition of ensuring that each of the D^2 bilateral values of aggregate trade, $\widetilde{X}_{od}$, can be perfectly matched by the estimated model amounts to ensuring that the moment

$$E_F\left[X_{od}(\omega)\right] = \widetilde{X}_{od}, \tag{16}$$

holds for every pair of countries, o and d.

It may seem tempting to use the moments in (15) and (16) to estimate both the model's parameters (θ and all A_{od} values) and the distribution F. But this is impossible—indeed, the model's parameters were only just identified even in the Eaton and Kortum (2002) case in which F was known to be F^*.

Therefore, rather than seeking to estimate all of the model's unknowns (θ, $\{A\}$, and F), we instead seek to construct the largest and smallest values of κ that are consistent with the data moments (15) and (16) and feature an F that is within a given neighborhood of the Eaton and Kortum (2002) reference distribution, F^*. In particular, we define the neighborhood $\mathcal{N}_\delta$ of F^* as all distributions $F \in \mathcal{N}_\delta$ that lie within a statistical divergence from F^*, denoted $D(F, F^*)$, that is less than δ. For example, a "hybrid" measure of divergence that is well suited to applications in which F^* is fat-tailed (like it is for the case of the exponential distribution in Eaton and Kortum (2002)) is the following "hybrid" divergence function

$$D^H(F||F^*) \equiv \int \phi\left(\frac{f(x)}{f^*(x)}\right) f^*(x)dx, \tag{17}$$

with $f(\cdot)$ denoting the PDF of $F(\cdot)$, etc., and the function $\phi(x) \equiv x\log x - x + 1$ for all $x \leq e$ and $\phi(x) \equiv (x-e)^2/(2e) + (x-e) + 1$ for all $x > e$. This means that $D^H(F||F^*)$ corresponds to the Kullback-Leibler divergence at low values and the χ^2-divergence at higher values.

Summarizing the discussion so far, when seeking the largest value of gains from trade κ that are possible for any distribution of global technologies that are "close" to the Frechet distribution in Eaton and Kortum (2002), denoted $\overline{\kappa}_\delta$, we seek to solve for:

$$\begin{aligned} \overline{\kappa}_\delta \quad &\equiv \quad \max_{\theta,\{A\},F} \kappa \\ \text{subject to} \quad &\quad D^H(F||F^*) \leq \delta, \end{aligned} \tag{18}$$

and subject to the constraints (1)-(7) (equilibrium holds in the factual economy), (8)-(11) (equilibrium holds in the counterfactual economy), (15) (gravity), and (16) (all bilateral trade flows in the factual equilibrium match the data). That is, we seek to estimate the largest value for the gains from trade (for any given country d) that are consistent with the Ricardian model whose global technology distribution F is within divergence delta of the distribution F^* used by Eaton and Kortum (2002), and yet which is capable of matching all bilateral values of trade flow data $\widetilde{X}_{od}$ as well as the orthogonality restriction (15) that is at the heart of the gravity equation. We will also estimate the lower bound on the value of gains from trade, denoted $\underline{\kappa}$; this is analogous to $\overline{\kappa}$ but with the max operator in (18) replaced with the min operator.

While the goal of the program in (18) is well-defined, solving for the optimum required is far from trivial. At any value of $\delta > 0$, this is an infinite-dimensional optimization problem that requires a search over every possible distribution F within $\mathcal{N}_\delta$ and then, at each candidate distribution, find the values (if any) of the parameters that allow the model to match the required moments. Conducting such a search in the space of distributions F is clearly infeasible. Fortunately, however, Christensen and Connault (2023) show that, for any given value of the parameters $\psi \equiv (\theta, \{A\})$, the optimization program that is dual to (18) is feasible—it is concave, and its dimensionality is given only by the number of constraints. This opens up the possibility of a feasible two-step procedure in which one first, for given ψ, uses the Christensen and Connault (2023) algorithm to find $\overline{\kappa}_\delta(\psi)$, and then, second, one repeats this procedure over all ψ.

3.3 Adding Restrictions on the Productivity Distribution

The global productivity distribution F^* used by Eaton and Kortum (2002) features three key elements: (i) each country takes independent draws (that is, there is no cross-country dependence or correlation across their draws), (ii) every country draws from the same marginal distribution, and (iii) that common marginal distribution takes the particular form of $Exp(1)$. So far, our analysis has relaxed all three of these features by allowing for any distribution F that lies within the neighborhood $\mathcal{N}_\delta$ of the distribution F^* used by Eaton and Kortum (2002) and is capable of generating a Ricardian model that can match all trade data. But it is natural to restrict this search to a subset of distributions F that satisfy, in addition to these same data and economic model constraints, maintain some (but not all) of the three restrictions in Eaton and Kortum (2002). Doing so is the goal of this sub-section.

Common marginal distributions. We begin by retaining the restriction that countries all have the same marginal distribution, but drop the restriction that this common marginal needs to be $Exp(1)$. Doing so is natural if one seeks a parsimonious Ricardian model, where countries aren't allowed to differ in arbitrary respects.

We encode this restriction by ensuring that the marginal distributions of idea draws U_o are the same at a large number L of quantiles l of their marginal CDFs; as L grows this should provide an increasingly accurate approximation to the desired restriction that the marginal CDFs are identical. To do this, we subdivide the support of U into L points indexed by l and denoted by $\overline{U}_l$.[3] We then impose

$$\mathbb{E}_F[\mathbb{1}\{U_o < \overline{U}_l\}] = \mathbb{E}_F[\mathbb{1}\{U_{o'} < \overline{U}_l\}] \text{ for all } l \leq L \text{ and } o' \neq o. \tag{19}$$

While this method works arbitrarily well for large L, in principle, one challenge in practice occurs in regards to the likelihood of the highest-productivity idea draws, which are particularly important for generating gains from trade. In particular, since gains from trade depend on the price index, which is a transformation of the ηth moment of the price draws that consumers face (with $\eta \equiv (1-\sigma)/\theta$), we mitigate against the risks of inadequate upper-tail approximations by further imposing the additional constraints

$$\mathbb{E}_F[U_o^\eta \mathbb{1}\{U_o < \overline{U}_l\}] = \mathbb{E}_F[U_{o'}^\eta \mathbb{1}\{U_{o'} < \overline{U}_l\}] \text{ for all } l \leq L \text{ and } o' \neq o. \tag{20}$$

This ensures that the draws within each quantile have the same ηth moment.

Frechet marginal distributions. The previous restriction is less restrictive than Eaton and Kortum (2002) in two respects: it allows countries' productivity draws to be correlated and it allows for each country's marginal distribution (while common) to take any shape (i.e. not necessarily Frechet). We can move closer to the Eaton and Kortum (2002) case by restricting the common marginal distribution to be Frechet (i.e. $Exp(1)$ in our modeling convention) while still retaining the possibility that countries take correlated (i.e. dependent) draws. Indeed, this gets closer to the case of Lind and Ramondo (2023), who studied Ricardian models in which countries' marginal distributions were common, and Frechet distributed in particular, but their correlations were parameterized in various respects. The restriction we study here is in the same spirit except that we allow for countries to be taking draws from *any* joint process (formally, any copula) with Frechet marginals (that leads to a Ricardian model that matches all data), not any particular parameterized correlation structure.

To see how this can be done, let $H(\cdot)$ denote the CDF of the inverse of the distribution $Exp(1)$. Divide the unit-interval support of $H(\cdot)$ into $\overline{H}$ discrete and evenly-spaced points indexed by h. Finally, let $\overline{H}_h$ denote the value of the hth quantile of $H(\cdot)$. Then we can ensure that the marginal of the distribution of ideas U_o in any country o has the same

[3] In practice we choose the points $\overline{U}_l$ as the L quantiles of the $Exp(1)$ distribution. This has the advantage of using quantiles that are spaced out in the same proportion to their distribution under F^*. We also set $L = 50$.

marginal distribution as $Exp(1)$ by matching each of these $\overline{H}$ quantile values, as long as $\overline{H}$ is large enough for this approximation to be accurate. Formally, this amounts to imposing the constraint

$$\mathbb{E}_F[\mathbb{1}\{U_o < \overline{H}_h\}] = H(\overline{H}_h) \quad \text{for all } o \text{ and all } h \leq \overline{H}. \tag{21}$$

In addition, for the same reasons as above, we ensure that the ηth moment of each quantile of every country's distribution is the same as that under $Exp(1)$:

$$\mathbb{E}_F[U_o^\eta \mathbb{1}\{U_o < \overline{H}_h\}] = \mathbb{E}_{F^*}[U_o^\eta \mathbb{1}\{U_o < \overline{H}_h\}] \quad \text{for all } o \text{ and all } h \leq \overline{H}. \tag{22}$$

Independent draws. Finally, we can explore relaxations of Eaton and Kortum (2002) in the opposite direction. Rather than keeping the marginals identical (or even identical and of the Frechet form) and being flexible with cross-country productivity draw dependence, we can instead maintain independence and be flexible with countries' marginals. Recall that two jointly distributed random variables are independent if and only if it is the case that their joint CDF is equal to the product of their two marginal CDFs. Like the restriction of common marginals above, we can again encode the independence restriction by using quantiles. But doing so is considerably harder in this case because we require it to hold not only for every country pair but also for every pair of quantiles. That is, for independence we now impose

$$\mathbb{E}_F[\mathbb{1}\{U_o < \overline{U}_l\}\mathbb{1}\{U_{o'} < \overline{U}_{l'}\}] = \mathbb{E}_F[\mathbb{1}\{U_o < \overline{U}_l\}]\mathbb{E}_F[\mathbb{1}\{U_{o'} < \overline{U}_{l'}\}], \tag{23}$$

for all pairs of countries $o \neq o'$ and all pairs of points of support $l \neq l'$.[4] We note that the weaker condition of no correlation can be obtained by imposing the far simpler version based on first moments alone:

$$\mathbb{E}_F[U_o U_{o'}] = \mathbb{E}_F[U_o]\mathbb{E}_F[U_{o'}] \quad \text{for all } o \neq o'. \tag{24}$$

3.4 Data

Our analysis draws on the data used in Eaton and Kortum (2002). In particular, we use their same set of 17 countries and their estimated measure of trade costs (as our "tariff" t_{od}). But we update the bilateral trade flow matrix $\widetilde{X}_{od}$ from that in the World Input-Output Database for 2015.

[4]In practice, we again choose the points $\overline{U}_l$ as the L quantiles of the $Exp(1)$ distribution. But the large number of pairwise combinations requires a lower value of L than before to be feasible, so we set $L = 5$.

4 Results

4.1 Interpreting measures of divergence δ

Before turning to estimates of $\overline{\kappa}_\delta$ and $\underline{\kappa}_\delta$ for various values of δ, we pause to anticipate the natural question: how can one interpret the divergence implied by any given value of δ?

To provide one benchmark, consider the following exercise in sensitivity analysis which we see as representative of common approaches in the field. Begin with the reference global productivity distribution given by that in Eaton and Kortum (2002): 17 countries, each drawing independently from a Frechet distribution whose shape parameter is 6. This corresponds to our case in which F^* is $Exp(1)$ and the technology parameter θ is set to $\theta = 6$. Now consider changing the global distribution to one that remains (independently drawn from) Frechet marginals but with a different shape parameter.

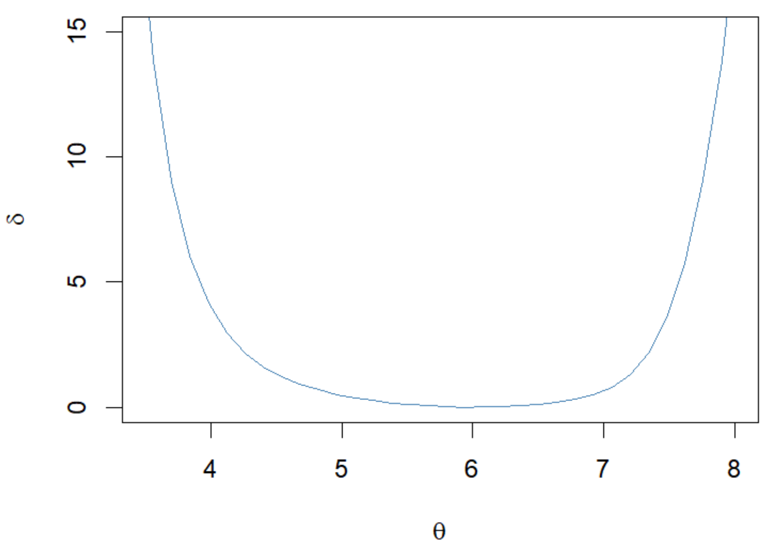

Figure 1: Divergence between Frechet distributions

Figure 1 presents the value of the divergence, as defined in equation (17), for a range of such shape parameters. This figure allows us to compare common sensitivity analysis directed towards the dispersion parameter θ to their implied divergences δ. For example, reducing θ from 6 to 5 corresponds to $\delta = 0.5$, whereas reducing it from 6 to 3.6 corresponds to $\delta = 10$. The blue line in Figure 1 is not exactly symmetric, but it is approximately so. This means that values of $\delta = 0.5$ and $\delta = 10$ can also be found by changing θ (from 6) by approximately the same amounts in the positive direction, i.e. to $\theta = 7$ and

$\theta = 7.7$, respectively. We will return to these benchmarks when discussing bounds on the gains from trade in the next section.

4.2 Baseline results

We now turn to the main goal of our paper, which is to estimate bounds on the gains from trade. We focus on the gains for France, whose extent of trade openness is in the middle of the range for those in our sample.

Recall, our procedure solves for the maximum possible gains from trade $\overline{\kappa}_\delta$, by searching over all global productivity distributions that are within divergence level δ of the Eaton and Kortum (2002) benchmark, subject to the constraint that any such candidate distribution can generate a Ricardian trade model with equilibrium trade flows that match those in the data. This is equivalent to solving the program in (18). We then repeat this exercise for the minimum possible gains, $\underline{\kappa}_\delta$ using the analog of (18) in which the minimum κ is found.

While this procedure is feasible, it is computationally costly, despite the advances developed by Christensen and Connault (2023). In particular, while those authors establish that the optimization problem in (18) is concave (and hence typically fast) for any given value of the parameters $\psi \equiv (\theta, \{A\})$, the outer loop that searches over all $D(D-1)+1$ such parameters has no guarantees (and in practice, for our problem, is typically slow, suffering from a natural curse of dimensionality as D grows). We therefore compute bounds ($\overline{\kappa}_\delta$ and $\underline{\kappa}_\delta$) that are constrained by the fact that we hold the parameters $(\theta, \{A\})$ constant at the values one would obtain when matching the trade flow and gravity equation constraints under F^*. Of course, this makes our results conservative, since expanding the parameter search over $(\theta, \{A\})$ can only widen the bounds that we report. However, an alternative interpretation of these results is that they take the reference distribution to be that in Eaton and Kortum (2002), and very literally so. That is, the reference distribution used below is not only $Exp(1)$ with location and scale parameters in each country (i.e. $(\theta, \{A\})$) taken to be free technological parameters (as in the model described above) but the particular Frechet distribution used in Eaton and Kortum (2002): a Frechet distribution with dispersion parameter θ^* and location parameters in each country pair given by the values of $\{A^*\}$.

Figure 2 reports estimated values of $\underline{\kappa}_\delta$ and $\overline{\kappa}_\delta$ (for France's gains from trade) for values of δ less than or equal to 10. These bounds are shown in blue. For reference, we compare them to the point estimate (shown in orange) of the same counterfactual object (French gains from trade) obtained when using the Eaton and Kortum (2002) productivity distribution, F^*. We denote this orange-line value by κ^*. Of course, at $\delta = 0$ the blue and orange lines coincide, but for $\delta > 0$ the blue bounds separate from the orange line, since allowing for distributions F that differ from F^* (i.e. a positive divergence δ from F^*) permits larger or smaller gains from trade than those under Eaton and Kortum (2002).

The question of interest here is how large this separation between blue and orange is for any given δ. Two results stand out. First, even small divergences result in economically meaningful differences in the gains from trade. For example, at $\delta = 0.5$ we find that the gains for France could be as low as effectively zero (i.e. $\underline{\kappa}_\delta = 0.05\%$) or as high as a value that is more than twice as large as what Eaton and Kortum (2002) would calculate (i.e. $\overline{\kappa}_\delta = 9.2\%$, whereas $\kappa^* = 4.4\%$). Recall from Figure 1 that an exercise of exploring sensitivity within the set of Frechet distributions that are diverge from F^* by less tahn $\delta = 0.5$ can be done by changing the Frechet distribution's shape parameter from $\theta = 6$ to $\theta = 5$ or $\theta = 7$. Such changes have a relatively small impact on gains from trade in the Eaton and Kortum (2002) model, moving them from 4.4% (at $\theta = 6$) to 5.0% and 3.8% (at $\theta = 5$ and $\theta = 7$, respectively). The fact that the $\overline{\kappa}_\delta$ and $\underline{\kappa}_\delta$ bounds are so much wider than this implies that the conventional way of doing robustness to Eaton and Kortum (2002) is not exploring very much of the space of distributions that are equally dissimilar from F^* and are equally successful at matching all data.

The second striking feature of Figure 2 is that the bounds at $\delta = 10$ are not that much wider than those at $\delta = 0.5$. This implies that relatively small divergence offers many ways to create larger or smaller gains from trade while still matching all data, but allowing greater divergence doesn't afford as much flexibility when subject to those same data constraints.

4.3 Restricting the Global Productivity Distribution

So far we have seen how large or small the gains from trade can be for a given country (France) when the global productivity distribution F is unrestricted (other than lying within divergence δ from F^* and generating a Ricardian model that matches all trade data). Yet one may be willing to restrict the set of admissible distributions F in some economically motivated respects. We now apply the methods described in Section 3.3 to do so.

We begin with the case in which F is restricted such that all countries have the same marginal distribution (though this common marginal is unrestricted, as is the dependence of the draws that countries take). Figure 3 reports (in red) the resulting values of $\overline{\kappa}_\delta$ and $\underline{\kappa}_\delta$, at various values of δ, when this restriction is imposed. This can be compared with

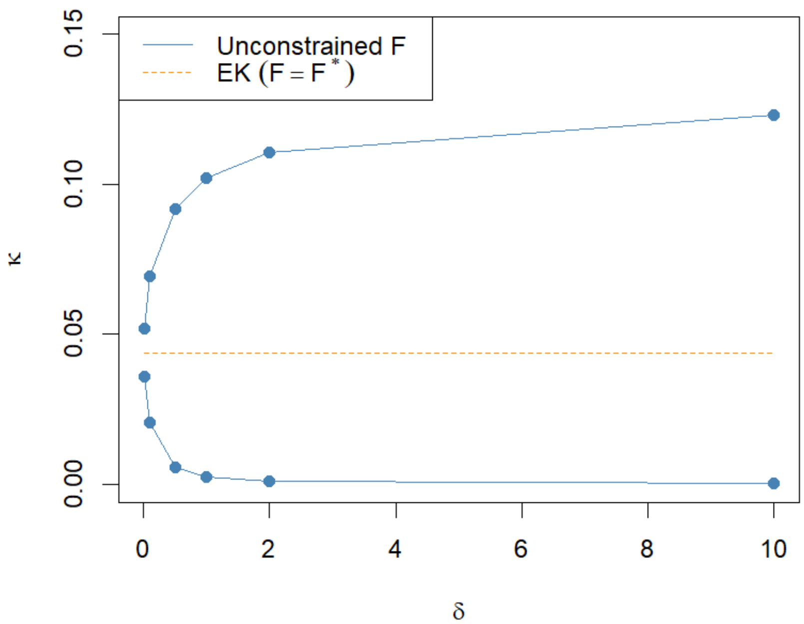

Figure 2: Bounds on Gains from Trade (for France) Across Alternative Productivity Distributions (within Divergence δ from that in Eaton and Kortum (2002))

the values (in blue) seen earlier in Figure 2, but repeated here for ease of comparison, in which F is unrestricted. Evidently, the restriction to common marginals has effectively no impact on the lower-bound gains from trade, $\underline{\kappa}_\delta$. But it does have a modest impact on the upper-bound, $\overline{\kappa}_\delta$, which (at the value of $\delta = 0.5$ for example) falls from 9.2% when F is unrestricted to 8.0% when F is restricted to have common marginals around the world. These relatively modest impacts of what may seem like a drastic restriction in global productivity heterogeneity—the underpinning of gains from trade in the Ricardian model—are indicative of the complexities of thinking about the difference between Ricardian models in general, and those that are restricted by the data in particular.

5 Concluding Remarks

A modern revolution in spatial economic modeling aims to answer quantitative counterfactual questions by using models that feature micro-level heterogeneity. This heterogeneity is then often assumed to come from particular parametric families—such as Frechet in the Eaton and Kortum (2002) model of Ricardian trade. While these parametric

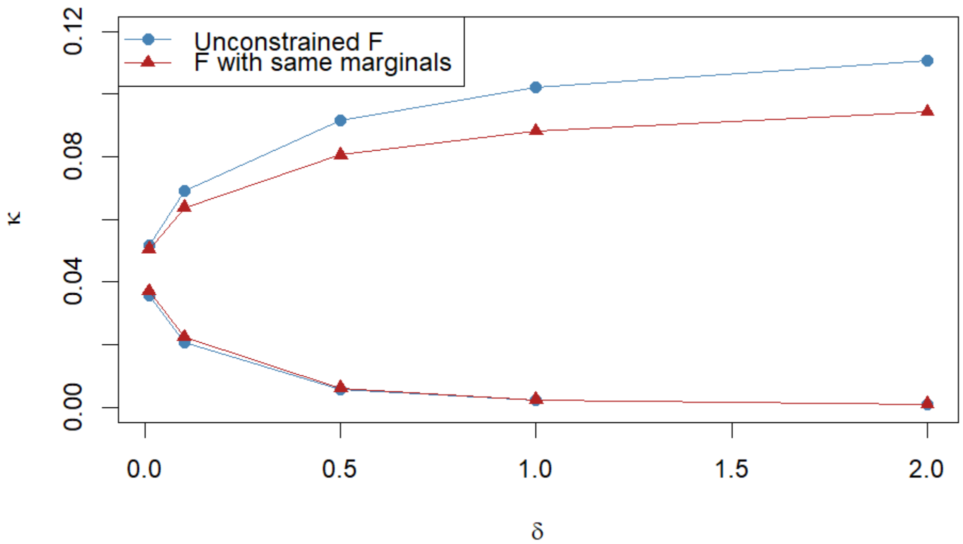

Figure 3: Bounds on Gains from Trade (for France) When Countries Have the Same Marginal Distribution of Productivities

choices greatly enhance the tractability of model simulations, it is unknown how sensitive the answers to counterfactual questions are to these assumptions of convenience because there are infinitely many alternative distributions of heterogeneity to be evaluated.

We overcome this challenge by building a general trade model that leverages recent advances in the robustness literature. Our method calculates sharp bounds on the values of model counterfactuals that could obtain—while still exactly matching all aggregate trade data points, a gravity-like moment condition, and satisfying equilibrium constraints—under all possible distributions of underlying heterogeneity that lie within a given divergence from a chosen reference distribution.

Applying this method to the Eaton and Kortum (2002) model, we find that the gains from trade in these models could be several times larger or smaller than they appear to be under standard benchmark distributions, even if heterogeneity is drawn from a distribution that is at least as similar to Frechet as are the types of parametric alternatives that are commonly explored in sensitivity analysis. Interestingly, the bounds that we obtain are not very sensitive to the imposition of restrictions on the global productivity distribution, such as that all countries share the same (unrestricted) marginal distribution.

References

Alvarez, Fernando and Robert E. Lucas, "General Equilibrium Analysis of the Eaton-Kortum Model of International Trade," *Journal of Monetary Economics*, 2007, *54* (6), 1726–1768.

Christensen, Timothy and Benjamin Connault, "Counterfactual sensitivity and robustness," *Econometrica*, 2023, *91* (1), 263–298.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum, "Global Rebalancing with Gravity: Measuring the Burden of Adjustment," *IMF Staff Papers*, 2008, *55* (3), 511–540.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography and Trade," *Econometrica*, 2002, *70* (5), 1741–1779.

Lind, Nelson and Natalia Ramondo, "Trade with correlation," *American Economic Review*, 2023, *113* (2), 317–353.