MIT Political Science Math Prefresher
Pset 0

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Due August 24, 5 PM (Eastern time)

Please read the introductory notes we sent together with this pset beforehand. Once you are done, submit both your *write-up* and the *RScript* with your answers electronically to loffredo@mit.edu *and* liuchst@mit.edu. For the write-up, you can either print the pset, answer by hand, and send us a scanned version of your responses, or edit directly on the PDF. If you feel comfortable using LaTeX then you can also typeset it.

1 Basic Notation

1. Write the following in math notation:

   (a) Suppose that $z = [z_1, z_2, z_3, \ldots, z_n]$. Formally write the sum of the first five elements of the vector z using the summation notation $\sum$. $\sum_{i=1}^{5} z_i$

   (b) Formally write the product of all the elements of vector z, where $z = [z_1, z_2, z_3, \ldots, z_n]$, using the product notation $\prod$. $\prod_{i=1}^{n} z_i$

   (c) Expand and calculate the following double summation: $\sum_{i=2}^{5} \sum_{j=1}^{3} i^j$ $= \sum_{i=2}^{5} i + \sum_{i=2}^{5} i^2 + \sum_{i=2}^{5} i^3$
   2,3,4,5
   $14 + 54 + 224$
   292

   (d) Write the following expression in words:
   $\forall x \in \mathbb{R}^- \text{ if } y = \sqrt{x} \implies y \notin \mathbb{R}$
   For every element x, in a set of real numbers when y equals the square root of x, y is not a positive real number

2 Functions

1. Consider a function $f(x) = \sqrt{x}$. Formally write the natural domain (all the potential values that x can take on given the nature of the function and not considering imaginary numbers) and the range of the function. Hint: use the following format. f : domain $\rightarrow$ range. $f: 0 \rightarrow \infty$

2. What is the domain (note the values that x can take on given the nature of the function and its definition) of $f(x) = \sqrt{3x - 9} + 8, x < 5$. $f: 3 \rightarrow 4$

3. Find the inverse of the following function:

$$f(x) = y = \frac{-3}{x - 4}$$

$x = \frac{-3}{y-4}$

$x(y-4) = -3$

$y - 4 = -3/x$

$y = \frac{-3}{x} + 4$

$f^{-1}(x) = \frac{-3}{x} + 4$

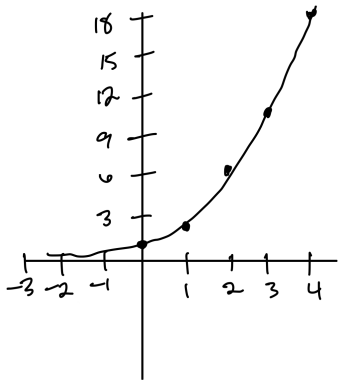
3 Graphing Functions

1. Represent the following functions graphically. Note that you will need to represent a two-dimensional space (a plane) to be able to present the functions in a coordinate system. You can graph the functions by hand, by drawing several pairs of points and joining them with a line to approximate their shape visually or you can use an statistical package or TikZ if you feel comfortable with them.

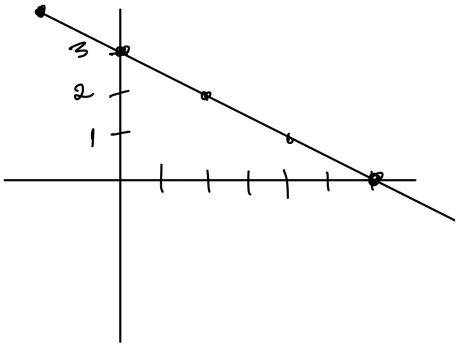
   (a) $f(x) = e^x$ * answered on next page

   (b) $f(x) = -\frac{x}{2} + 3$

3a. $f(x) = e^x$



3b. $f(x) = -\frac{1}{2}x + 3$



4 Important Functions and their Properties

1. **Exponents**. Simplify or compute the following:

   (a) $(2^3)^3$

   2^9

   $\boxed{512}$

   (b) $(a \times b)^2$

   ab^2

   (c) $\frac{a^4}{a^3}$

   a^{4-3}

   $\boxed{a}$

   (d) $\left(\frac{3}{4}\right)^2$

   $\boxed{\frac{9}{16}}$

   (e) $a^6 \times a^2$

   a^{6+2}

   $\boxed{a^8}$

   (f) a^0

   $\boxed{1}$

2. **Logarithms**. Simplify or compute the following:

   (a) $\log(x \times y)$

   $\log x + \log y$

(b) $\log(1) \times \log(4)$

$\log(1) = 0$

$\boxed{0}$

(c) $\log(1 \times 4)$

$\log 4$

$\log_{10} 4 = 1.386$

5 Differentiation Problems

1. Differentiate the following functions:

(a) $f(x) = 6x^3 - 9x + 4$

$f'(x) = 18x^2 - 9$

(b) $f(x) = 2x^4 - 10x^2 + 13x$

$f'(x) = 8x^3 - 20x + 13$

(c) $f(x) = (2x + 1)^3$

$f(x) = x^3$, $g(x) = 2x+1$, $F(x) = f(g(x))$

$F'(x) = f'(g(x)) \cdot g'(x)$

$F'(x) = 3(2x+1)^2 \cdot 2$

$F'(x) = 6(2x+1)^2$

6 Software Installation

To prepare for Math Camp and for completing assignments throughout the methods sequences, please complete the following steps to ensure you have installed and can use the necessary software.

1. Throughout the methods sequence, students will utilize the `R` programming language. `R` is an open-source statistical computing environment widely used in statistics and political science. During Math Camp, we will provide an introduction to the basic syntax, features, and packages of `R`. Please download and install `R` on your computer by for free from https://cran.r-project.org.

2. For those who code in `R`, the primary environment for writing and compiling code is `RStudio`. Please follow the instructions for installing `RStudio` at https://posit.co/products/open-source/rstudio/?sid=1. The "Open Source Edition" is all you need!

3. `RStudio` is great for write and running your code in scripts, but you can also create files that produce nice reports in LaTeX. When completing problems sets in the method sequence, you will take advantage of this feature. Please follow Step #1 at the following link to install `Quarto` on your computer: https://quarto.org/docs/get-started/. By creating and rendering `Quarto` documents, you will be able to integrate your `R` code into documents just like this—more on this during Math Camp!