Øving 1

Oppg 1

a)

$$\frac{dv}{dt} = -\left[\frac{2g}{L} \cdot z + \frac{f}{2D} v \cdot |v|\right]$$

$D = 0{,}5$, $f = 0{,}05$, $L = 392$, $g = 9{,}8$

- Gives:

$$\frac{dv}{dt} = -0{,}05 \cdot z \; - 0{,}05 v \cdot |v| = f_1(z, v)$$

- Eulers method gives

$$v_{i+1} = v_i + h f_1(z_i, v_i)$$

- needs to find an expression for z

$$v = \frac{dz}{dt} \quad \Rightarrow \quad \text{Gives us:}$$

$$z_{i+1} = z_i + h f_2(v_i)$$

- The systeme of equations we have is then.

$$z_{i+1} = z_i + h \cdot v_i$$

$$v_{i+1} = v_i - 0{,}05 h(z_i + v_i \cdot |v_i|)$$

- we have given:

$z_0 = -6$

$v_0 = 0$, $h = \Delta t = 1s$

- 1. iteration

$V_1 = 0 - 0{,}05\left(-6 + 0 \cdot |0|\right) = 0{,}3$

$Z_1 = -6 + 1 \cdot 0 \qquad = -6$

- 2. iteration

$V_2 = 0{,}3 - 0{,}05\left(-6 + 0{,}3^2\right) = 0{,}5955$

$Z_2 = -6 + 1 \cdot 0{,}3 \qquad = -5{,}7$

- 3. iteration

$V_3 = 0{,}5955 - 0{,}05\left(-5{,}7 + 0{,}5955^2\right) = 0{,}8628$

$Z_3 = -5{,}7 + 1 \cdot 0{,}5955 \qquad = -5{,}105$

b) Heun's method

- same as before exept we use a prediction

$V_{i+1}^{P} = V_i + h f_1(z_i, V_i)$ Prediction

$V_i = V_i + \frac{h}{2}\left(f(z_i, V_i) + f(z_{i+1}^{P}, V_{i+1}^{P})\right)$ correction

$Z_{i+1}^{P} = Z_i + h \cdot f_2(V_i)$ Prediction

$Z_{i+1} = Z_i + \frac{h}{2}\left(f_2(V_i) + f_2(V_{i+1}^{P})\right)$ Correction

1. iteration

$V_1^P = 0 - 0{,}05(-6 + 0 \cdot 101) = 0{,}3$

$Z_1^P = -6 + 0 \cdot 1 = -6$

$V_1 = 0 + \frac{1}{2}\left(-0{,}05(-6+0) - 0{,}05(-6 + 0{,}3^2)\right) = 0{,}298$

$Z_1 = -6 + \frac{1}{2}(0 + 0{,}3) = -5{,}85$

2. iteration

$V_2^P = 0{,}289 - 0{,}05(-5{,}85 + 0{,}289^2) = 0{,}577$

$Z_2^P = -5{,}85 + 0{,}298 = -5{,}55$

$V_2 = 0{,}289 + \frac{1}{2}\left(-0{,}05(-5{,}85 + 0{,}289^2) - 0{,}05(-5{,}55 + 0{,}577^2)\right)$

$V_2 = 0{,}563$

$Z_2 = -5{,}85 + \frac{1}{2}(0{,}298 + 0{,}577) = -5{,}417$

3. iteration

$V_3^P = 0{,}563 - 0{,}05(-5{,}417 + 0{,}563^2) = 0{,}818$

$Z_3^P = -5{,}417 + 0{,}563 = -4{,}854$

$V_3 = 0{,}563 + \frac{1}{2}\left(-0{,}05(-5{,}417 + 0{,}563^2 - 4{,}854 + 0{,}818^2\right)$

$V_3 = 0{,}795$

$Z_3 = -5{,}417 + \frac{1}{2}(0{,}818 + 0{,}563) = -4{,}74$

close enough

c)

$$\frac{dv}{dt} = -0{,}05\left(z + v \cdot |v|\right)$$

∘ series expansion around $t = 0$

$$z(t) = z(0) + z'(0) \cdot \Delta t + z''(0)\,\frac{\Delta t^2}{2} + z'''(0) \cdot \frac{\Delta t^3}{6} +$$
$$z''''(0) \cdot \frac{\Delta t^4}{24} + O(\Delta t^5)$$

∘ need to find the derivatives.

$$z(0) = -6$$

$$z'(0) = v(0) = 0$$

$$z''(0) = \left.\frac{dv}{dt}\right|_{t=0} = -0{,}05\left(z(0) + v(0)\,|v(0)|\right) = 0{,}3$$

$$z'''(0) = \left.\frac{d^2v}{dt^2}\right|_{t=0} = -0{,}05\left(z'(0) + v'(0) \cdot |v(0)| + v(0) \cdot \frac{d}{dt}|v(0)|\right)$$
$$= -0{,}05\left(z'(0) + v'(0)\,|v(0)| + v(0) \cdot |v'(0)|\right)$$
$$= 0 - 0{,}05\left(0{,}3 \cdot 0 + 0 \cdot 0{,}3\right) = 0$$

$$z''''(0) = \left.\frac{d^3v}{dt^3}\right|_{t=0} = -0{,}05\left(z''(0) + v''(0)\,|v(0)| + v'(0) \cdot |v'(0)|\right.$$
$$\left. + v'(0)\,|v'(0)| + v(0) \cdot |v''(0)|\right)$$
$$= -0{,}05\left(0{,}3 + 0 + 0{,}3^2 + 0{,}3^2 + 0\right)$$
$$= -\frac{3}{125}$$

$$z(\Delta t) = -6 + 0 \cdot \Delta t + 0{,}3 \cdot \frac{\Delta t^2}{2} + 0 \cdot \frac{\Delta t^3}{6} + \frac{3}{125} \cdot \frac{\Delta t^4}{24} + O(\Delta t^5)$$
$$= -6 + \frac{3}{20}\Delta t^2 - \frac{\Delta t^4}{1000} + O(\Delta t^5)$$