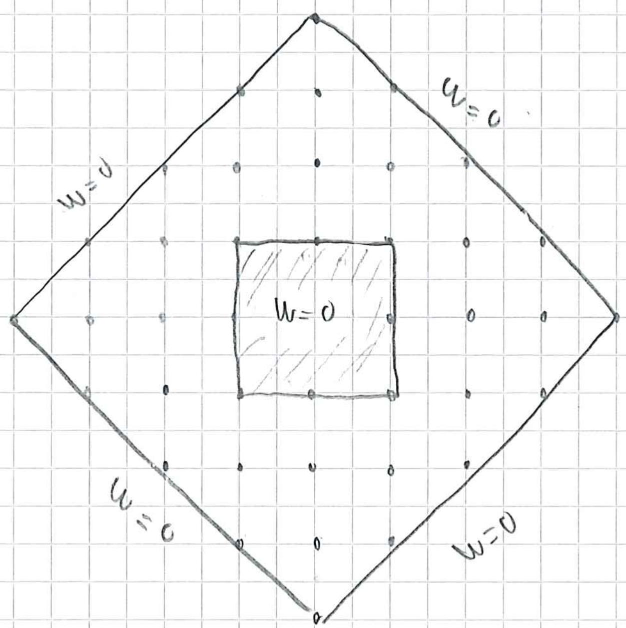
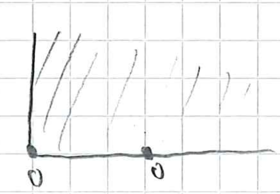
Øving 4

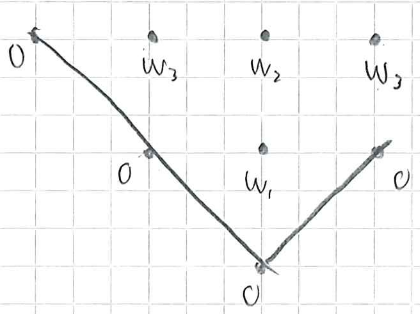
$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -1$$

Fig 1)



- Because of symetry, we only need to solve 3 values of w.





- Discretizing equation.

$$\frac{\partial^2 w}{\partial x^2} = \frac{w_{i+1}^n + w_{i-1}^n - 2w_i^n}{\Delta x^2} + O(\Delta x^3)$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{w_i^{n+1} + w_i^{n-1} - 2w_i^n}{\Delta y^2}$$

- Gives us:

$$\frac{w_{i+1}^n + w_{i-1}^n - 2w_i^n}{\Delta x^2} + \frac{w_i^{n+1} + w_i^{n-1} - 2w_i^n}{\Delta y^2} = -1$$

- collecting all terms

$$w_{i+1}^n + w_{i-1}^n + w_i^n\left(-2 - 2\frac{\Delta x^2}{\Delta y^2}\right) + w_i^{n+1}\frac{\Delta x^2}{\Delta y^2} + w_i^{n-1}\frac{\Delta x^2}{\Delta y^2} = -\Delta x^2$$

- $\Delta x = \Delta y = 0{,}5$

$$w_{i+1}^n + w_{i-1}^n - 4w_i^n + w_i^{n+1} + w_i^{n-1} = -0{,}25$$

$$w_i^n = \frac{1}{4}\left(0{,}25 + w_i^{n+1} + w_i^{n-1} + w_{i+1}^n + w_{i-1}^n\right)$$

- Set up the system of equations

$$w_1 = \frac{1}{4}(0{,}25 + w_2) \Rightarrow w_1 - \frac{1}{4}w_2 = \frac{1}{16}$$

$$w_2 = \frac{1}{4}(0{,}25 + 2w_3 + w_1) \Rightarrow w_2 - \frac{1}{2}w_3 - \frac{1}{4}w_1 = \frac{1}{16}$$

$$w_3 = \frac{1}{4}(0{,}25 + w_2) = w_1 \Rightarrow$$ so we remove the equation

- Set up the matrix

$$\begin{bmatrix} 1 & -\frac{1}{4} \\ -\frac{3}{4} & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{16} \\ \frac{1}{16} \end{bmatrix} \Rightarrow \quad Aw = d$$

$$A^{-1} = \begin{bmatrix} 1 & \frac{3}{4} \\ \frac{1}{4} & 1 \end{bmatrix} \cdot \det(A) = \begin{bmatrix} 0{,}813 & 0{,}609 \\ 0{,}203 & 0{,}813 \end{bmatrix}$$

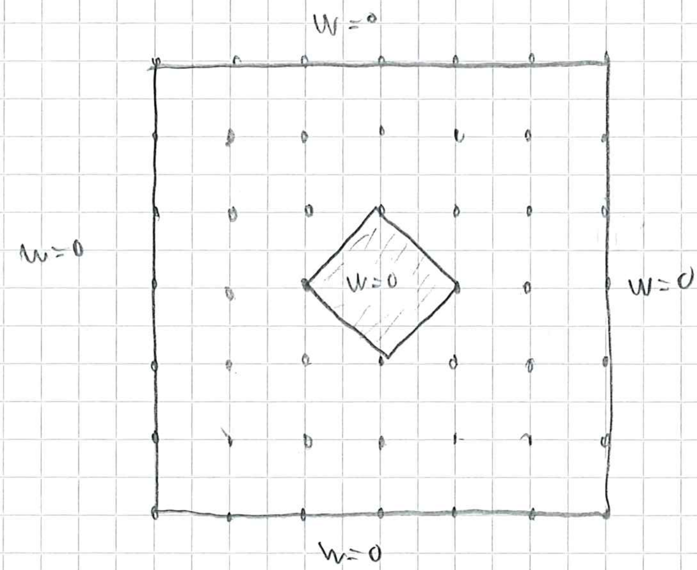
$$\underbrace{A^{-1} \cdot \begin{bmatrix} \frac{1}{16} \\ \frac{1}{16} \end{bmatrix}}_{A^{-1} \cdot d} = \begin{bmatrix} 0{,}089 \\ 0{,}069 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

close enough

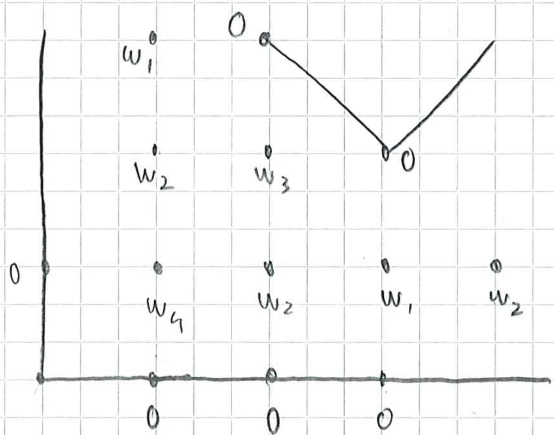
LF says $w_1 = 0{,}0962$

$w_2 = 0{,}1346$

Fig 2



- Again we see symetry



$$W_1 = \frac{1}{4}(0{,}25 + 2W_2)$$

$$W_2 = \frac{1}{4}(0{,}25 + W_1 + W_3 + W_4)$$

$$W_3 = \frac{1}{4}(0{,}25 + 2W_2)$$

$$W_4 = \frac{1}{4}(0{,}25 + 2W_2)$$

see that $W_1 = W_3 = W_4$

- Gives us the system of equations

$$W_1 = \frac{1}{4}(0{,}25 + 2W_2)$$

$$W_2 = \frac{1}{4}(0{,}25 + 3W_1)$$

- This time I just solve it.

$$W_1 = \frac{1}{4}\left(0{,}25 + 2\left(\frac{1}{4}(0{,}25 + 3W_1)\right)\right)$$

$$4W_1 = 0{,}25 + \frac{1}{2}\cdot 0{,}25 + \frac{3}{2}W_1$$

$$W_1\left(4 - \frac{3}{2}\right) = 0{,}25\left(1 + \frac{1}{2}\right)$$

$$W_1 = \frac{\frac{3}{8}}{\frac{5}{2}} = 0{,}15$$

$$W_2 = \frac{1}{4}(0{,}25 + 3\cdot 0{,}15) = 0{,}175$$