Øving 6

a)

$$CFL = \alpha_0 \frac{\Delta t}{\Delta x} \Rightarrow \alpha_0 = CFL \frac{\Delta x}{\Delta t}$$

Figure 1.1

$$\frac{\Delta x}{\Delta t} = \frac{7-1}{1,5} = 4$$

1.2

$$\frac{\Delta x}{\Delta t} = \frac{6,25 - 0,25}{1,5} = 4$$

b) Diffusion happens when the numerical solution error is increasing with higher frequencies

$$\varepsilon_D = \frac{|G|}{|G_a|} \Rightarrow \frac{\text{numerical amplification factor}}{\text{analytical amplification factor}}$$

It related to the gain of the numerical scheme compared to the gain of the analytical solution.

Thus relates to when our numerical solution differs from the analytical

- From Figure 2 we see that Lax Wendroff has initially very low divergens of ε_D, meaning low diffusion.

Dissipation error occur when the phase shift of the numerical scheme differs from the analytical one.

$$\varepsilon_\phi = \frac{\phi}{\phi_a}$$

when the numerical propogation speed is greater than the physical one, This would result into numerical oscillation.

- From figure 2.4 we see that $\varepsilon_\phi > 1$ for FTBS, indicating $\phi > \phi_a$
- From figure 2.3 we see that the phase shift of Lax $|\phi_{Lax}| > |\phi_{FTBS}|$
- Summary: FTBS is highly dominated by diffusion which we see occurs for green.

  Lax Wendroff is highly dominated by dispersion which we see from the red.

c) Green = FTBS , Red = Lax

Fig 1.1

- Low diffusion for both $\varepsilon_\phi \approx 1$
- Low dissipation for bdth $\varepsilon_D \approx 1$

CFL = 0,7

Fig 1.2

- Low diffusion for Lax
- high diffusion for FTBS

$\varepsilon_{D,Lax} < \varepsilon_{D,FTBS}$

- Low dissipation for FTBS $\varepsilon_{\phi,FTBS} > 1$
- High dissipation for Lax $\varepsilon_{\phi,Lax} < 1$

CFL = 0,7

Fig 1.3

- Low diffusion for Lax
- medium diffusion for FTBS

$\varepsilon_{D,FTBS} > \varepsilon_{D,Lax}$

- Low dissipation for FTBS
- Low dissipation for Lax

$\frac{\phi}{\phi_a} \approx 1$

CFL = 0,2

Fig 1.4

- High diffusion of FTBS
- Low diffusion of Lax

$\left.\begin{array}{l}\\\\\end{array}\right\} \varepsilon_{D,Lax} < \varepsilon_{D,FTBS}$

- High dispersion of Lax
- Low dispersion of FTBS

$\varepsilon_{\phi,Lax} < 1$

$\varepsilon_{\phi,FTBS} \approx 1$

$CFL = 0,2$

a)

$$\alpha_0 = \frac{\Delta x}{\Delta t} \cdot CFL$$

Lax Wendroff

1. $CFL = 0,7$, $T = 4$

$$\alpha_0 = \frac{6,9974 - 1}{1,5} = 3,9983 \quad \& \quad \varepsilon_\phi = \frac{\alpha_0}{\alpha_a} = \frac{3,9983}{4} = 0,9996$$

2. $CFL = 0,7$, $T = 1$

$\varepsilon_\phi = \frac{x_{num}}{x_a}$ can also use

$$\alpha_0 = \frac{6,2156 - 0,25}{1,5} = 3,977 \quad \& \quad \varepsilon_\phi = \frac{6,2156}{6,25} = 0,994$$

3. $CFL = 0,2$, $T = 4$

$$\alpha_0 = \frac{6,9952 - 1}{1,5} = 3,9968 \quad \& \quad \varepsilon_\phi = \frac{6,9952}{7} = 0,9993$$

4. $CFL = 0,2$, $T = 1$

$$\alpha_0 = \frac{6,1927 - 0,25}{1,5} = 3,96 \quad \& \quad \varepsilon_\phi = \frac{6,1927}{6,25} = 0,991$$

FTBS

1. $CFL = 0,7$, $T = 4$

$$\alpha_0 = \frac{7,0005 - 1}{1,5} = 4,0003 \quad \& \quad \varepsilon_\phi = \frac{7,0005}{7} \approx 1,00$$

2. $CFL = 0,7$, $T = 1$

$$\alpha_0 = \frac{6,2524 - 0,25}{1,5} = 4,0016 \quad \& \quad \varepsilon_\phi = \frac{6,2524}{6,25} = 1,00$$

3. $CFL = 0,2$, $T = 4$

$$\alpha_0 = \frac{6,9983 - 1}{1,5} = 3,999 \quad \& \quad \varepsilon_\phi = \frac{6,9983}{7} = 0,9998$$

4. $CFL = 0,2$, $T = 1$

$$\alpha_0 = \frac{6,246 - 0,25}{1,5} = 3,997 \quad \& \quad \varepsilon_0 = \frac{6,246}{6,25} = 0,999$$