Øving 2

OPPG 1

$$u''(x) + 2x \cdot u'(x) = 0$$

$$u(0) = 0 \quad , u(1) = 1$$

a) shooting method:

set: $u'(x) = g(x)$ & $g(x_0) = s$, unknown parameter

- This gives us the equation:

$$g'(x) + 2x \cdot g(x) = 0$$

- The system of equations is:

$$u'(x) = g(x) = f_1(x)$$
$$g'(x) = -2x \cdot g(x) = f_2(x, g(x))$$

$h = \frac{1}{4}$ for eulers method

$$u_{i+1} = u_i + h \cdot f_1(x_i)$$
$$g_{i+1} = g_i + h \cdot f_2(x_i, g(x_i))$$

, set $g(x_0) = s$

1. iteration at $x = 0,25$

$$u_1 = u_0 + \frac{1}{4} \cdot g(x_0)$$
$$u_1 = 0 + \frac{1}{4} \cdot s = \frac{s}{4}$$
$$g_1 = g_0 + \frac{1}{4} \cdot (-2 \cdot x_0 \cdot g(x_0))$$
$$= s + \frac{1}{4}(-2 \cdot 0 \cdot s) = s$$

$$u_1 = \frac{s}{4}$$
$$g_1 = s$$

2. iteration at $x = 0{,}5$

$$u_2 = u_1 + \frac{1}{4}(g_1) = \frac{s}{4} + \frac{1}{4} \cdot s = \frac{s}{2}$$

$$g_2 = g_1 + \frac{1}{4}(-2 \cdot x_1 \cdot g_1) = s + \frac{1}{4}\left(-2 \cdot \frac{1}{4} \cdot s\right)$$
$$= \frac{7}{8} \cdot s$$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \begin{matrix} u_2 = \frac{s}{2} \\ g_2 = \frac{7}{8} s \end{matrix}$$

3. iteration at $x = 0{,}75$

$$u_3 = u_2 + \frac{1}{4} \cdot g_2 = \frac{s}{2} + \frac{1}{4} \cdot \frac{7}{8} \cdot s$$
$$= \frac{23}{32} s$$

$$g_3 = g_2 + \frac{1}{4}(-2 \cdot x_2 \cdot g_2) = \frac{7}{8} \cdot s + \frac{1}{4}\left(-2 \cdot \frac{2}{4} \cdot \frac{7}{8} s\right)$$
$$= \frac{21}{32} s$$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \begin{matrix} u_3 = \frac{23}{32} s \\ g_3 = \frac{21}{32} s \end{matrix}$$

4. iteration at $x = 1$

$$u_4 = u_3 + \frac{1}{4} \cdot g_3 = \frac{23}{32} s + \frac{1}{4} \cdot \frac{21}{32} s = \underline{\underline{\frac{113}{128} s}}$$

$$g_4 = g_3 + \frac{1}{4} \cdot (-2 \cdot x_3 \, g_3) = \frac{21}{32} s + \frac{1}{4} \cdot \left(-2 \cdot \frac{3}{4} \cdot \frac{21}{32} s\right)$$
$$= \underline{\underline{\frac{105}{256} s}}$$

b)

we know that at $x = 1$

$u(x=1) = 1$ from $u(x) = \dfrac{\operatorname{erf}(x)}{\operatorname{erf}(x=1)}$

$$1 = \frac{113}{128} s \quad \Rightarrow \quad s = \underline{\underline{\frac{128}{113}}}$$

b)

- Since we know that

$$u(1) = A\cdot\sinh(H\cdot(y=1)) + B\cosh(H\cdot y=1) + \frac{P}{H^2}$$

- we calculate $u(1)$

$$A = \frac{1 + \frac{P}{H^2}\left[\cosh(H) - 1\right]}{\sinh(H)}, \quad B = -\frac{P}{H^2}$$

$$A = \frac{1 + \frac{3}{18^2}\left[\cosh(18) - 1\right]}{\sinh(18)} = 9{,}26\cdot10^{-3}$$

$$B = -\frac{3}{18^2} = -\frac{1}{108}$$

$$u(1) = 9{,}26\cdot10^{-3}\cdot\sinh(18) - \frac{1}{108}\cosh(18) + \frac{1}{108}$$

$$= 1$$

- We can now correct our solution

$$1 = 13s - 1 \Rightarrow s^* = \frac{2}{13}$$

Didn't bother to check the other values

- $P = 3$ & $H = 18$
- $y_0 = s$ & $u_0 = 0$

<u>1. iteration</u> to $y = \frac{1}{3}$

$$y_1 = s + \frac{1}{3}(18^2 \cdot 0 - 3) = s - 1$$
$$u_1 = 0 + \frac{1}{3} \cdot s = \frac{s}{3}$$

$y_1 = s - 1$

$u_1 = \frac{s}{3}$

<u>2. iteration</u> to $y = \frac{2}{3}$

$$y_2 = y_1 + \frac{1}{3}(18^2 \cdot u_1 - 3)$$
$$y_2 = s - 1 + \frac{1}{3}\left(18^2 \cdot \frac{s}{3} - 3\right)$$
$$= s - 1 + 36s - 1$$
$$= 37s - 2$$
$$u_2 = u_1 + \frac{1}{3} \cdot y_1$$
$$= \frac{s}{3} + \frac{1}{3}(s - 1)$$
$$= \frac{s}{3} + \frac{s}{3} - \frac{1}{3}$$
$$= \frac{2}{3}s - \frac{1}{3}$$

$y_2 = 37s - 2$

$u_2 = \frac{2}{3}s - \frac{1}{3}$

<u>3. iteration</u> to $y = 1$

$$y_3 = y_2 + \frac{1}{3}(18^2 \cdot u_2 - 3)$$
$$u_3 = u_2 + \frac{1}{3}(y_2)$$
$$y_3 = 37s - 2 + \frac{1}{3}\left(18^2 \cdot \left(\frac{2}{3}s - \frac{1}{3}\right) - 3\right)$$
$$y_3 = 109s - 39$$
$$u_3 = \frac{2}{3}s - \frac{1}{3} + \frac{1}{3} \cdot (37s - 2)$$
$$= 13s - 1$$

$y_3 = 109s - 39$

$u_3 = 13s - 1$

Oppg 2

$$\frac{d^2u}{dy^2} - H^2 \cdot u = -P$$

$u(y=0) = 0$

$u(y=1) = 1$

a) shooting method

set $\frac{du}{dy} = g$ & $g(y=0) = s$

- Gives us:

$$\frac{dg}{dy} - H^2 \cdot u = -P$$

- The system of equations:

$$g'(y) = H^2 \cdot u(y) - P \quad = f_1(u(y))$$
$$u'(y) = g(y) \quad = f_2(g(y))$$

- use euler to iterate forward

$$g_{i+1} = g_i + h \cdot f_1(u_i)$$
$$u_{i+1} = u_i + h \cdot f_2(g_i)$$

- set $h = \frac{1}{3}$ & start iteration at $y=0$

$$g_1 = g_0 + \frac{1}{3} \cdot (H^2 \cdot u_0 - P)$$
$$u_1 = u_0 + \frac{1}{3}(g_0)$$

I didn't bother check the othe values at $x = \frac{1}{4}, \frac{2}{4}, \frac{3}{4}$

c)

$$s^* = \frac{\phi^1 \cdot s^0 - s^1 \cdot \phi^0}{\phi^1 - \phi^0}$$, secant method for correction of s

$$\phi^1 = u(1; s^1) - 1 = \frac{113}{128} \cdot s^1 - 1$$

$$\phi^0 = u(1; s^0) - 1 = \frac{113}{128} \cdot s^0 - 1$$

$$s^* = \frac{\left(\frac{113}{128} \cdot s^1 - 1\right) \cdot s^0 - s^1\left(\frac{113}{128} \cdot s^0 - 1\right)}{\frac{113}{128} \cdot s^1 - 1 - \frac{113}{128} \cdot s^0 + 1}$$

$$= \frac{(\cancel{113} \cdot s^1 - 128) \cdot s^0 - s^1(\cancel{113} \cdot s^0 - 128)}{113 \cdot s^1 - \cancel{128} - 113 s^0 + \cancel{128}}$$

$$= \frac{128(\cancel{s^1 - s^0})}{113(\cancel{s^1 - s^0})} = \frac{128}{113} \qquad \square$$