Øving 3

Oppg 1)

$$x\frac{d^2\theta}{dx^2} + 2\frac{d\theta}{dx} - \beta^2\theta = 0$$

a) Discretizing by central method

$$\frac{d^2\theta}{dx^2} = \frac{\theta_{i+1} + \theta_{i-1} - 2\theta_i}{\Delta x^2} + O(\Delta x^3)$$

$$\frac{d\theta}{dx} = \frac{\theta_{i+1} - \theta_{i-1}}{2\Delta x} + O(\Delta x^2)$$

- we remove the higher order terms and continuous with the approximation

$$x_i\frac{\theta_{i+1} + \theta_{i-1} - 2\theta_i}{\Delta x^2} + \cancel{2}\left(\frac{\theta_{i+1} - \theta_{i-1}}{\cancel{2}\Delta x}\right) - \beta^2\theta_i = 0$$

- collect all variables

$$\theta_{i+1}(x_i + \Delta x) + \theta_{i-1}(x_i - \Delta x) + \theta_i(-2x_i - \beta^2\cdot\Delta x^2))$$

- set up the diagonal matrix

Main diag : $\{-2x_0 - \beta^2\Delta x, -2x_1 - \beta^2\Delta x, \dots, -2x_N - \beta^2\Delta x\} \in \mathbb{R}^N$
sub diag : $\{x_1 - \Delta x, x_2 - \Delta x, \dots, x_N - \Delta x\} \in \mathbb{R}^{N-1}$
superdiag : $\{x_0 + \Delta x, x_1 + \Delta x, \dots, x_{N-1} + \Delta x\} \in \mathbb{R}^{N-1}$

N = number of iterations

• Gives us the matrix

$$
\begin{bmatrix}
-2x_1-\beta^2\Delta x & x_1+\Delta x & & & \\
x_2-\Delta x & -2x_2-\beta^2\Delta x & x_1+\Delta x & & \\
 & \ddots & \ddots & \ddots & \\
 & & x_{N-1}-\Delta x & -2x_{N-1}-\beta^2\Delta x & x_{N-1}+\Delta x \\
 & & & x_N-\Delta x & -2x_N-\beta^2\Delta x
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_{N-2} \\ \theta_{N-1}
\end{bmatrix}
=
\begin{bmatrix}
\theta_0\,(x-\Delta x) \\ 0 \\ 0 \\ \vdots \\ 0 \\ -\theta_N(x_N+\Delta x)
\end{bmatrix}
$$

↑ assumed known.

b)

$$\theta(x) \approx \theta_0 + x\cdot\theta_0' + \frac{x^2}{2}\theta_0''$$

• Have the equation:

$$x\cdot\frac{d^2\theta}{dx} + 2\frac{d\theta}{dx} - \beta^2\cdot\theta = 0 \qquad (2)$$

at $x=0$, $\theta=\theta_0$, $x_0 = 0$

$$\cancel{0\cdot\frac{d^2\theta_0}{dx}} + 2\cdot\frac{d\theta_0}{dx} - \beta^2\cdot\theta_0 = 0$$

$$\theta_0' = \frac{\beta^2\cdot\theta_0}{2}$$

• want to find θ_0'', so derivate (2)

$$\theta_0'' + \cancel{x_0\cdot\theta_0'''} + \theta_0''\cdot 2 - \beta^2\cdot\theta_0' = 0$$

$$\theta_0''(1+2) = \beta^2\cdot\theta_0'$$

$$\theta_0'' = \frac{1}{3}\left(\beta^2\cdot\left(\frac{\beta^2\cdot\theta_0}{2}\right)\right) = \frac{\beta^4\cdot\theta_0}{6}$$

$$\theta(x) \approx \theta_0 + x \cdot \frac{\beta^2 \cdot \theta_0}{2} + \frac{x^2}{2}\left(\frac{\beta^4 \cdot \theta_0}{6}\right)$$

$$\approx \theta_0\left(1 + x \cdot \frac{\beta^2}{2} + \frac{x^2 \cdot \beta^4}{12}\right)$$

- we set $x = h$ to find $\theta_0(\theta_1)$

$$\theta_0 = \theta_1 \cdot \left(1 + h \cdot \frac{\beta^2}{2} + \frac{h^2 \cdot \beta^4}{12}\right)^{-1}$$

c)

$\beta^2 = 4$ & $h = 0{,}25$

$$\theta_1 = \frac{\cancel{T_L} - \cancel{T_\infty}}{\cancel{T_L} - \cancel{T_\infty}} = 1$$

- Gives us:

$$\theta_0 = 1 \cdot \left(1 + 1 \cdot \frac{4}{2} + \frac{1 \cdot 4^2}{12}\right)^{-1} = 0{,}23$$

- Gives us the matrix.

$$A = \begin{bmatrix} -1{,}5 & 0{,}5 & & \\ 0{,}25 & -2 & 0{,}25 & \\ & 0{,}5 & -2{,}5 & 1 \\ & & 0{,}75 & -3 \end{bmatrix}, \quad d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1{,}25 \end{bmatrix}$$

$$A \cdot \theta = d \Rightarrow \theta = A^{-1} \cdot d$$ would give us the solution

d) Nei, det gidder jeg ikke.