

1 Simulation Scenario

We simulate data from this model

$$\begin{aligned} Y_s &\sim N(X_s\beta_s, \sigma_s^2) \\ \beta_s &\sim N(\mu, \Sigma) \\ \sigma_s^2 &\sim IG(\alpha/2, \alpha\lambda/2) \end{aligned} \tag{1}$$

We use 70 groups in the simulation, so $s = 1, \dots, 70$, the explicative variable mimic the centered "year" we've used in the real data, $\mathbf{x}=-10:10$, in all cases $X_s\beta_s = \beta_{1s} + x\beta_{2s} + x^2\beta_{3s}$.

We need to set values for μ , Σ , α and λ , we've set those values with similar values to those find in bird forest data. We start focusing on one single parameter ρ_{23} , the correlation between β_{s2} and β_{s3} , so we repeat the simulation exercise for $\rho_{23} = (-.8, -.3, 0, .3, .8)$.

The rest parameter are set as $\mu = (0, 0, 0)$, $\alpha = 2$, $\lambda = .1$ and

$$\Sigma = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0.5 & \rho_{23}0.5^2 \\ 0 & \rho_{23}0.5^2 & 0.5 \end{pmatrix}$$

The structure for Σ is actually more extreme in bird data, where the variances for the 2nd and 3rd coefficient are even lower. The problem is the covariance between coef 2 and 3 get too close to zero and is not possible for the models distinguish from it.

Once we have data simulated from the model (1) we estimate a bayesian model on those data. To do this, we need to specify priors distributions for the parameters we've fixed, we used $\lambda_s \sim unif(0, 1000)$ and $\alpha_s \sim unif(0, 1000)$

Table 1: Priors for μ and Σ

IW	SIW
	$\xi_i = Unif(0, 100)$
$\mu_i \sim N(0, 1000)$	$\mu_i = \xi_i \mu_i^*$
	$\mu_i^* \sim N(0, 1000)$
$\Sigma \sim IW(4, I_3)$	$Q \sim IW(4, I_3)$
	$\sigma_i^2 = \xi_i^2 Q_{ii}$
	$\sigma_{ij} = \xi_i \xi_j Q_{ij}$

Finally, for each value of ρ_{23} we simulate 5 data sets (25 data sets in total) and for each data set we fit both models, using IW and SIW priors.

2 Simulation Results

```
ddply(bet, .(r, model), function(x) cor(x$b2, x$b3))
```

```
##           r model           V1
## 1  -0.8      iw -0.356061
## 2  -0.8     siw -0.355637
## 3  -0.3      iw -0.193537
## 4  -0.3     siw -0.193284
## 5   0.0      iw  0.007975
## 6   0.0     siw  0.007956
## 7   0.3      iw  0.147667
## 8   0.3     siw  0.147513
## 9   0.8      iw  0.416734
## 10  0.8     siw  0.416194
```

3 R code

```
# inverse wishart
mtext.hh.iw <- "
model {
  for (i in 1:n) {
    y[i] ~ dnorm(eff[i]+slop[i]+quad[i] , eta.e[abbrev[i]] )
    eff[i] <- beta[1, abbrev[i]]
    slop[i] <- beta[2, abbrev[i]]*year[i]
    quad[i] <- beta[3, abbrev[i]]*year[i]^2
  }

  # Priors.
  for (j in 1:ns) {
    beta[1:3,j] ~ dmnorm(mu,prec.be )

    # use scale chi param
    eta.e[j] ~ dgamma(alpha/2, alpha*lamba/2)
    sigma.e[j] <- 1/sqrt(eta.e[j])
  }

  # hyperpriors
  prec.be ~ dwish(R, df)
  sigma.be <- inverse(prec.be)
  for (i in 1:3) { mu[i] ~ dnorm(0,0.001) }

  df <- 4
  alpha ~ dunif(0, 1000)
  lambda ~ dunif(0, 1000)
  rho23 <- sigma.be[3,2]/sqrt(sigma.be[3,3]*sigma.be[2,2])
}
"

# Scaled Inverse Wishart prior
mtext.hh.siw = "
model {
  for (i in 1:n) {
    y[i] ~ dnorm(eff[i]+slop[i]+quad[i] , eta.e[abbrev[i]] )
    eff[i] <- beta[1, abbrev[i]]
    slop[i] <- beta[2, abbrev[i]]*year[i]
    quad[i] <- beta[3, abbrev[i]]*year[i]^2
  }

  # Priors.
  for (j in 1:ns) {
```

```

beta[1,j] <- xi[1]*beta.raw[1,j]
beta[2,j] <- xi[2]*beta.raw[2,j]
beta[3,j] <- xi[3]*beta.raw[3,j]
beta.raw[1:3,j] ~ dmnorm(mu.raw, tau.raw)

# use scale chi param
eta.e[j] ~ dgamma(alpha/2, alpha*lambda/2)
sigma.e[j] <- 1/sqrt(eta.e[j])
}

# hyperpriors
tau.raw ~ dwish(R, df)
sigma.raw <- inverse(tau.raw)

for (i in 1:3) {
mu.raw[i] ~ dnorm(0,0.001)
xi[i] ~ dunif(0, 100)
mu[i] <- xi[i]*mu.raw[i]
sigma.be[i,i] <- (xi[i])^2*sigma.raw[i,i]
}
sigma.be[1,2] <- xi[1]*xi[2]*sigma.raw[1,2]
sigma.be[1,3] <- xi[1]*xi[3]*sigma.raw[1,3]
sigma.be[2,3] <- xi[3]*xi[2]*sigma.raw[2,3]
sigma.be[2,1] <- sigma.be[1,2]
sigma.be[3,1] <- sigma.be[1,3]
sigma.be[3,2] <- sigma.be[2,3]
rho23 <- sigma.be[2,3]/sqrt(sigma.be[3,3]*sigma.be[2,2])

df <- 4
alpha ~ dunif(0, 1000)
lambda ~ dunif(0, 1000)
}
"

#-----
library(plyr)
library(reshape2)
library(ggplot2)
library(rjags)
library(gridExtra)
library(mnormt)

# function to run the jags models with simulated data
runjags.sim <- function(d, mod) {
  # d: are the simulated data set
  # mod: is the 'alias' for the model character object

```

```

if (mod=='siw') model = mtext.hh.siw
if (mod=='iw') model = mtext.hh.iw
dat = list(y = d$y ,
           abbrev = as.numeric(d$group) ,
           year= d$x,
           n = nrow(d),
           ns = nlevels(factor(d$group)),
           R = diag(3))
m = jags.model(textConnection(model), dat, n.chains=3, n.adapt=100)
update(m, 500)
coda.samples(m, c('alpha', 'lambda', 'rho23', 'mu', "sigma.be", 'beta', 'sigma.e'), 4000)
#codas.samples(m, 'rho23', 3000)
}
#-----
# function to simulated a data set with a similar structure of the
# bird data. The only 'free' parameter is the correlation between
# slope and quad term.
simdat <- function(r) {
  # r: is the rho23 value

  # set parameter values: mostly based on the estimations from data
  rho23 <- r
  s <- diag(c(2, .5, .5)); s[6] <- rho23*.5^2 ; s[8] <- rho23*.5^2
  mu <- rep(0,3)
  alpha <- 2
  lambda <- 0.1
  G <- 70

  # get simulated data
  beta <- rmnorm(G, varcov=s)
  sigma.e <- 1/sqrt(rgamma(G,shape=alpha/2, rate=alpha*lambda/2))
  simpar <- data.frame(group=1:G,beta, sigma.e)
  colnames(simpar) <- c('group','b1', 'b2', 'b3', 'sigma.e')
  simy_f <- function(d) {
    x <- -10:10
    eps <- rnorm(length(x), mean=0, sd=sqrt(d$sigma.e))
    y <- with(d, b1+b2*x+b3*x^2+eps)
    data.frame(y,x)
  }
  ddply(simpar, .(group), simy_f)
}
#-----
# this function simulate a set of data and fit all models in ms list to it
simres <- function(r, ms) {
  sdat <- simdat(r)

```

```

fitmod <- function(mod) {
  res <- runjags.sim(sdat, mod)
  prnam <- attributes(res[[1]])$dimnames[[2]]
  aux1 <- summary(res[, c('alpha', 'lambda', prnam[grepl('mu', prnam)], prnam[grepl('sigma.be
  out <- as.data.frame(aux1)
  colnames(out) <- paste('Q', c(2.5, 25, 50, 75, 97.5), sep='')
  s <- grepl('sigma.be', prnam)[c(4, 6, 7)]
  aux2 <- gelman.diag(res[, prnam[-s] ])
  out$gd=aux2$mpsrfr
  out$param <- rownames(out)
  out$model <- mod
  return(out)
}
ldply(ms, fitmod)
}

#-----
# 5 values of rho, 2 types of sigma prior, N reps of each
setwd('~\\GitHub\\mnfb_yearly\\R')
source('codemodels.R')
N <- 5
rhos <- data.frame(r=c(-.8, -.3, 0, .3, .8))
ptm <- proc.time()
sim.res <- rdply(.n=N, mdply(rhos, simres, ms=list('iw', 'siw')) )
proc.time() - ptm

setwd('~\\GitHub\\mnfb_yearly')
save(sim.res, file='simulations.Rdata')

```

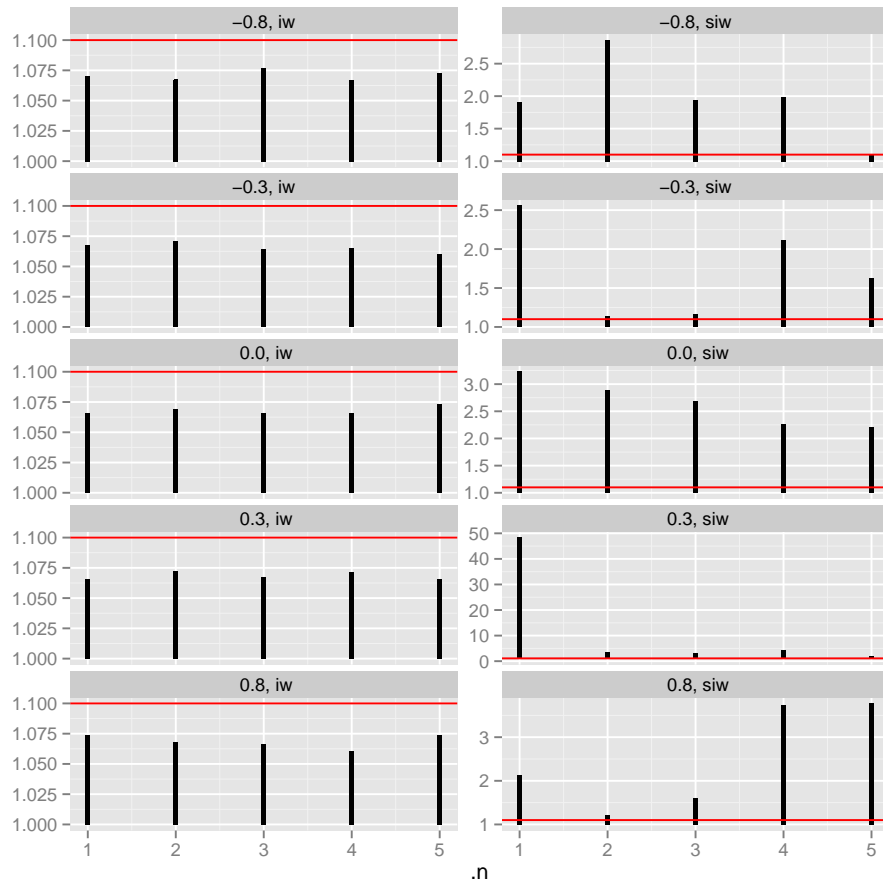


Figure 1: Multivariate gelman diagnostic Statistic. *IW* model presents no problems, but in most cases the *SIW* have not converged yet. The statistic summarize convergence from all the parameters in the model (about 150), when I've checked some individuals models I've found out almost every parameter has presents good gd statistic except for the variance of β_3 or sometimes a individual covariance. For those is like the chains are shifted, as there were 2 modes for instances. This is sensitive to the ξ distribution, which Gelman propose use uniform, and I saw on the web using a gamma and normal

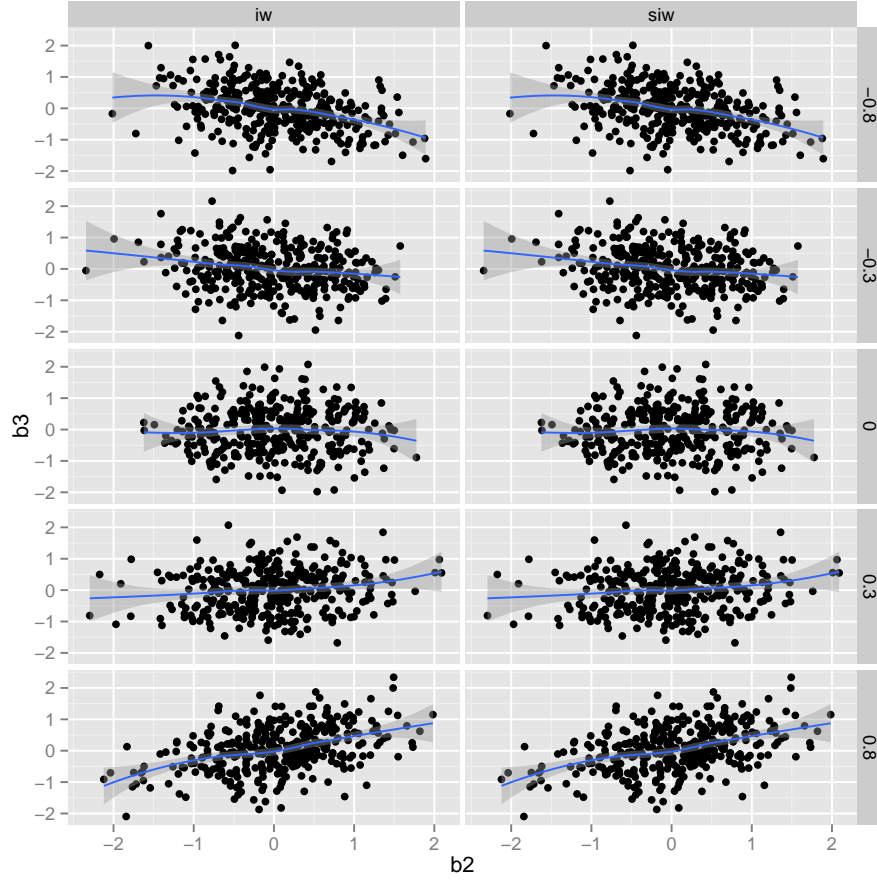


Figure 2: Relationship among β_{2s} and β_{3s} medians. The sign of the relationship is correct, however the true correlation is more extreme than the one implies in the plot, I've computed the "sample" correlation coefficient among medians and we can see the same effect, for instance the true $\rho = 0.8$ have generated a correlation among medians about 0.4

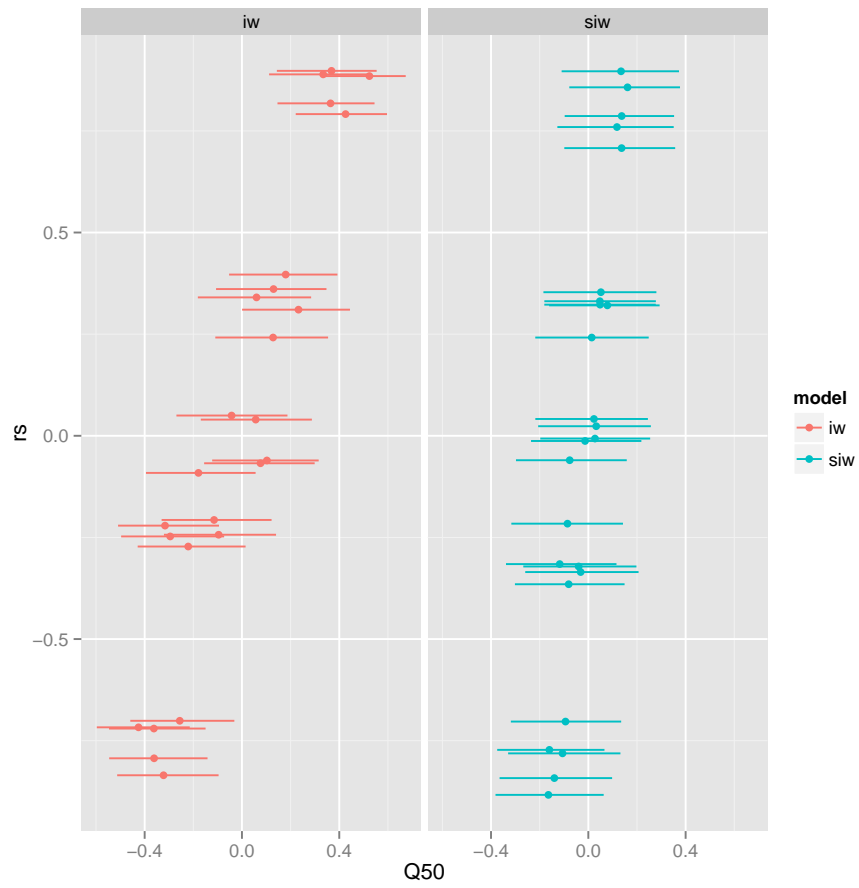


Figure 3: Credible intervals for ρ_{23} .

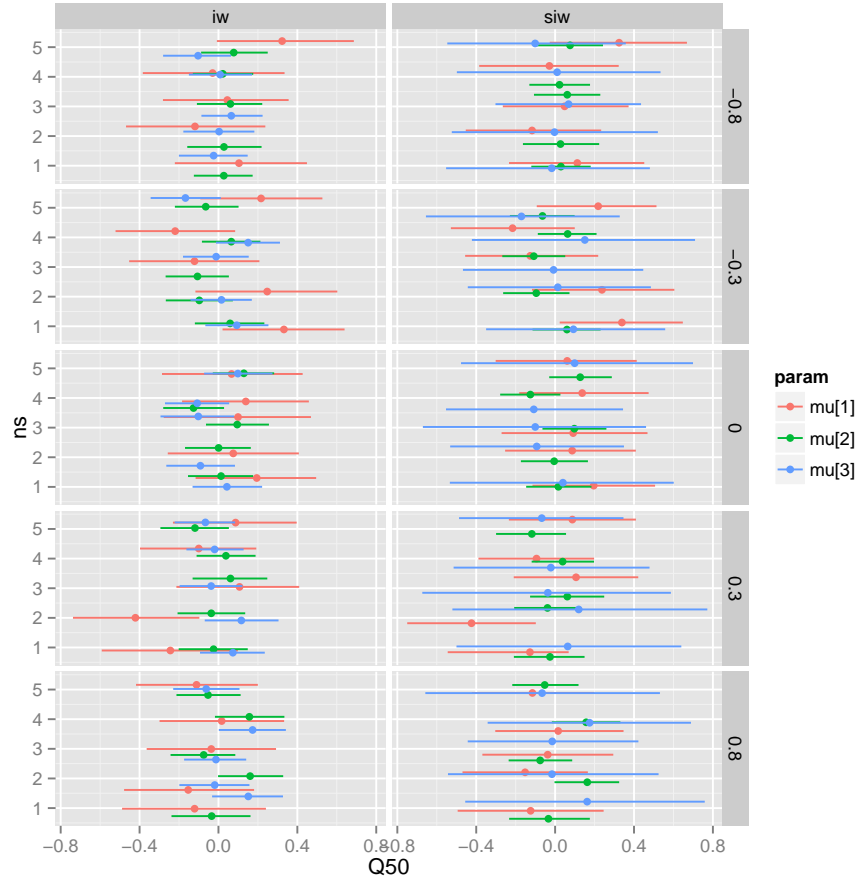


Figure 4: Credible interval for μ , this results are not changing with the covariance matrix distribution and most of the cases are around the true value

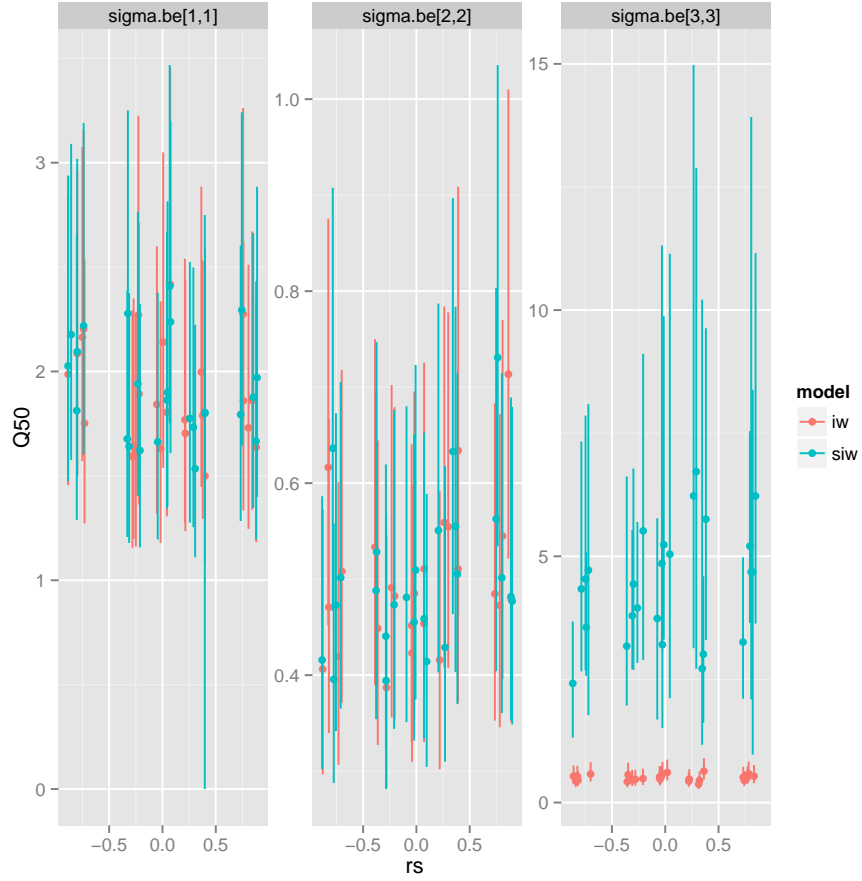


Figure 5: Credible intervals for Σ variance parameters. The true values for the parameter were 2, 0.5 , 0.5 respectively. There is a problem with the 3rd variance in the SIW model is much higher than it should be, this is one of the parameters with bad gelman diagnostic statistic so I hope that is the problem.

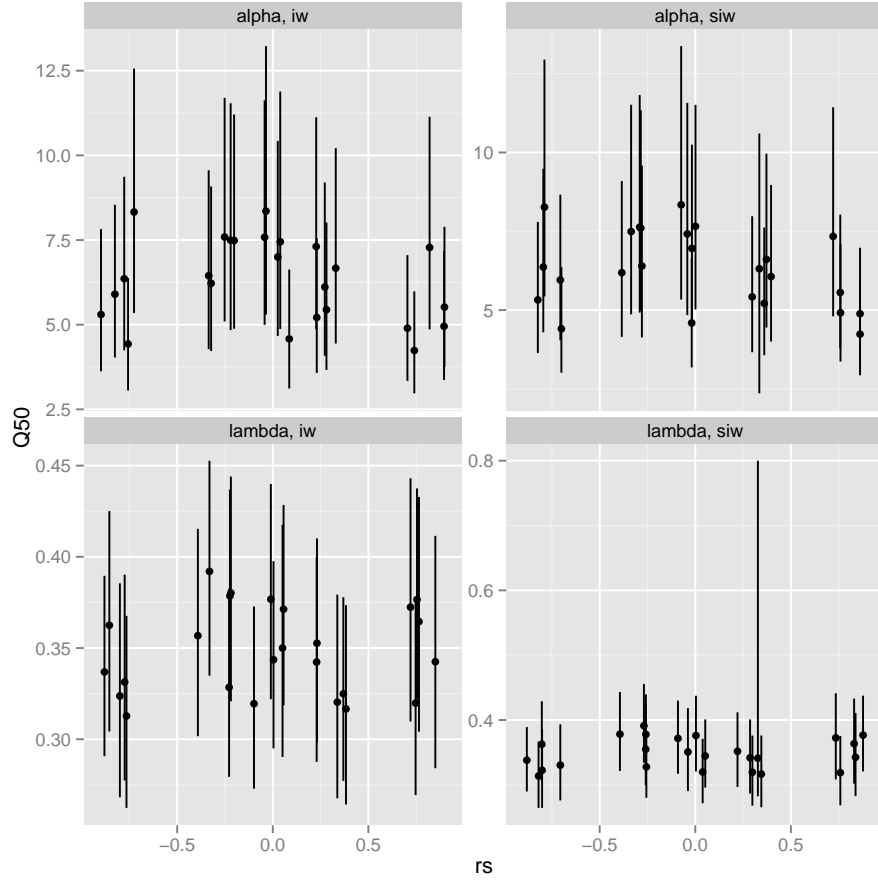


Figure 6: Credible intervals for α and λ . There seems to be problems in both parameters, for α both models tend to get higher values than the true, actually $\alpha = 2$ does not lie in any of the credible intervals in the plot. For λ it seems that SIW model is doing a better job.