ELEC 4700

ASSIGNMENT 2 – FINITE DIFFERENCE METHOD

Submitted By: Emeka Peters

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Introduction

This experiment involved using finite difference method and Laplace's equation to solve basic electrostatic problems modelled as a mesh of voltages and resistances. The first part of the experiment involved modelling the voltage in a simple rectangular device with specific boundary conditions. The second part of the experiment involved modelling the voltage in a simple device with insulator bottlenecks added. This part of the experiment also involved calculation of current in the device and the behavior of the current as certain parameters are varied.

Question 1: Finite Difference Method with Boundary Conditions

a. In this case, the voltage increased linearly from 0 (at x = W) to 1V (at x = 0). Figure 1 below shows the modelled voltage:

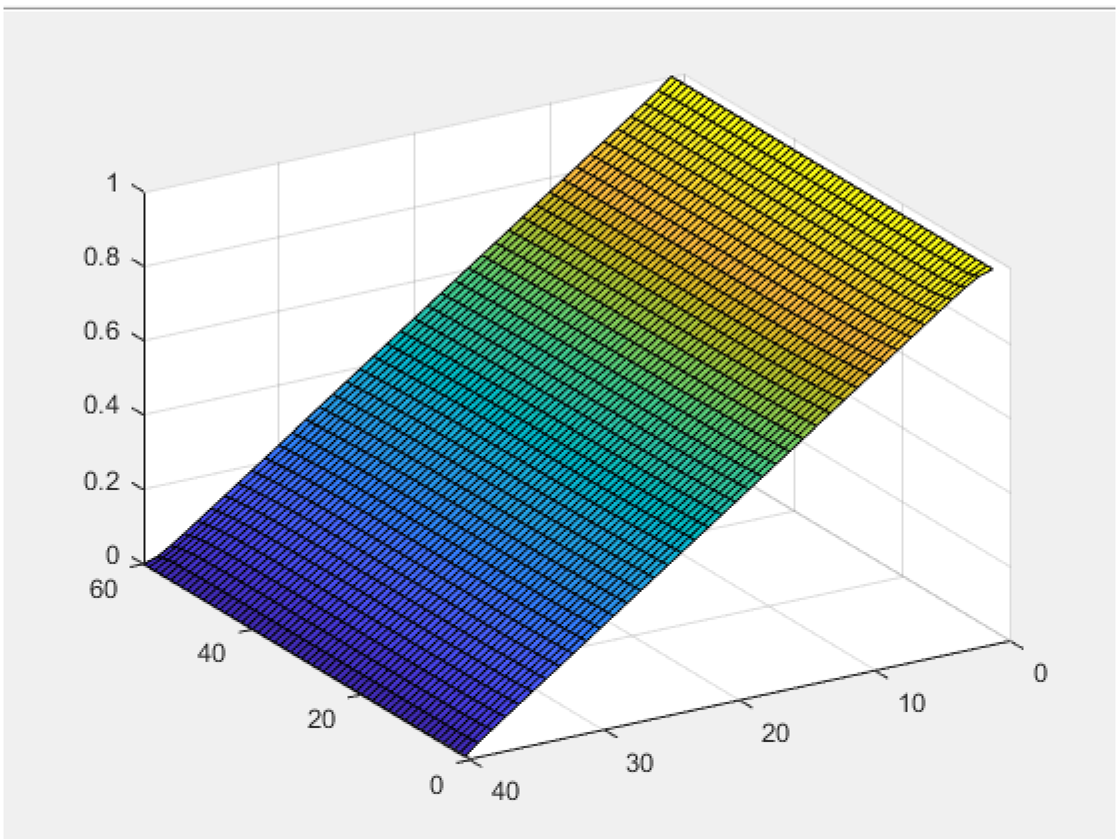

Figure 1: Voltage model for fixed BCs on two sides

b. This part of the experiment was done with two methods; meshing, and analysis. Figure 2 below shows the model that was derived with the meshing method:

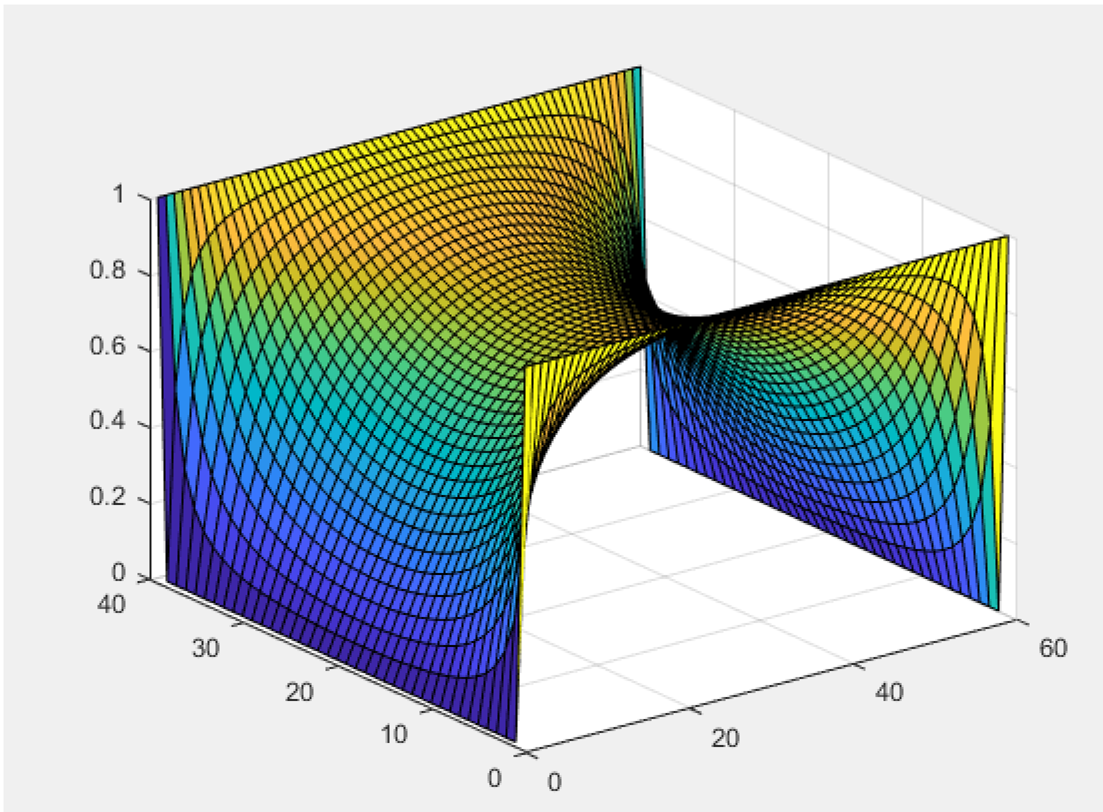

Figure 2: Voltage model for device with fixed BCs on four sides (numerical solution)

Figure 3 below shows the model that was derived with the analytical solution method:

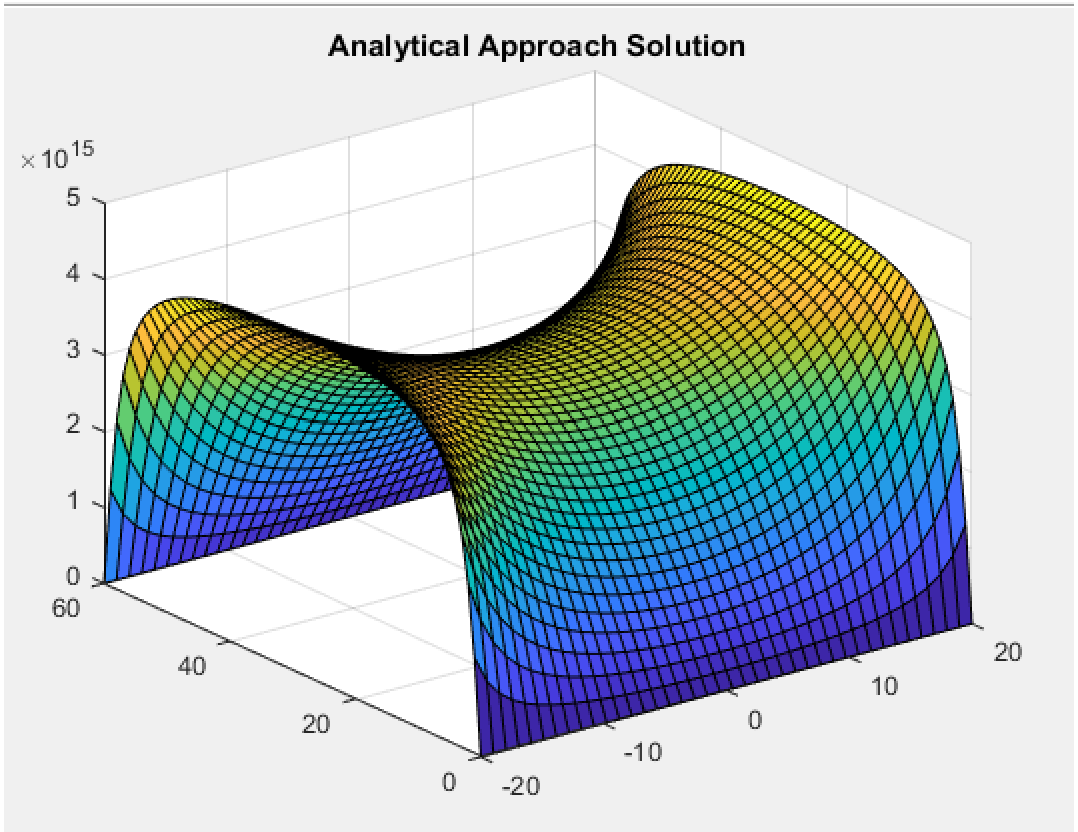

Figure 3: Voltage model for device with fixed BCs on four sides (Analytical Solution)

The solution using meshing seemed to be more correct than that which was obtained using the analytical method. The meshing method will take a lot of memory but gets computed faster than the analytical solution which has to be evaluated multiple times, which can be time consuming. The analytical method would be expected to have less error because the solution is computed multiple times and will converge closer to the actual solution than the meshing (numerical) method.

Question 2: Finite Difference Method with Boundary Conditions and Added Bottleneck

a. The current flow at the two contacts was measured as the sum of the magnitude of current along the line connecting the two contacts (sum of currents at L/2). First of all, the current matrix was obtained by multiplying the voltage matrix (map) with the conductivity matrix (map). This part was done with a mesh size of (L, W) = (120, 80) and the current value obtained was 2.72×10^{-6} A. Figure 4 below shows the conductivity map with the given default values.

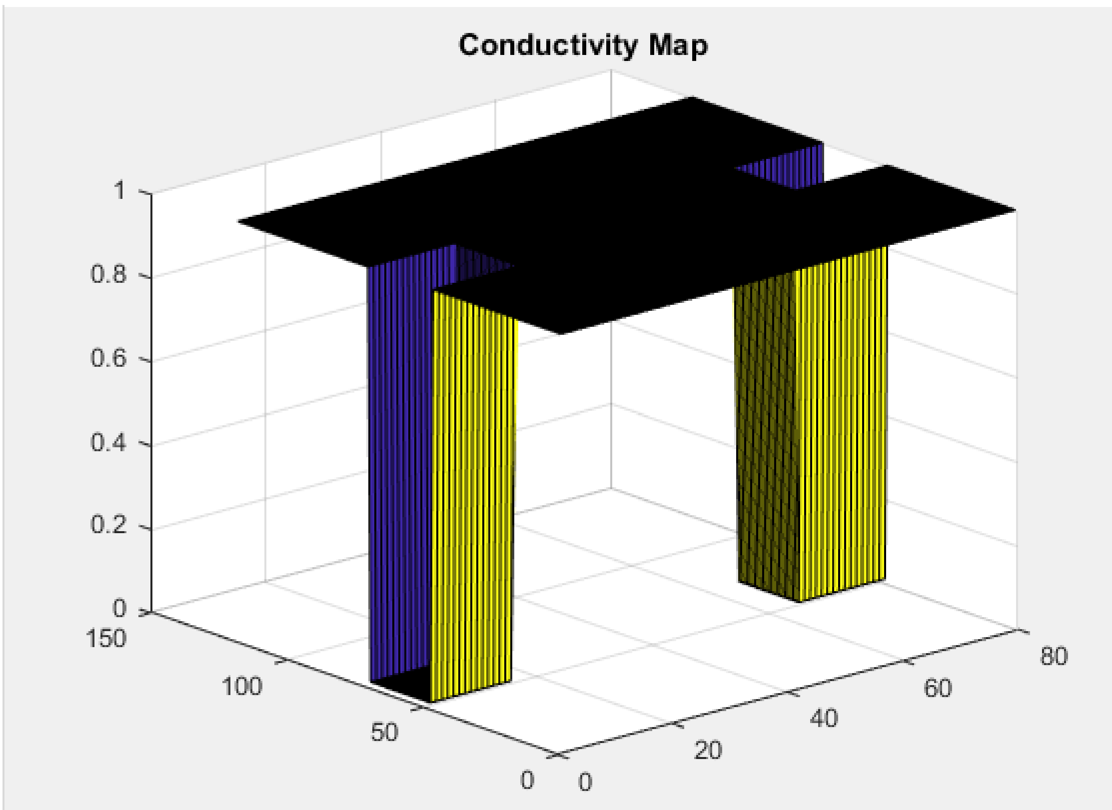

Figure 4: Conductivity map with (120, 80) mesh size

The voltage decreased linearly from 1V (at x = 0) to 0V at the first contact with the bottle neck. There is still some voltage left between the bottlenecks though but decreases to zero more quickly than the case examined in question 1. Figure 5 below shows the plot of the voltage map:

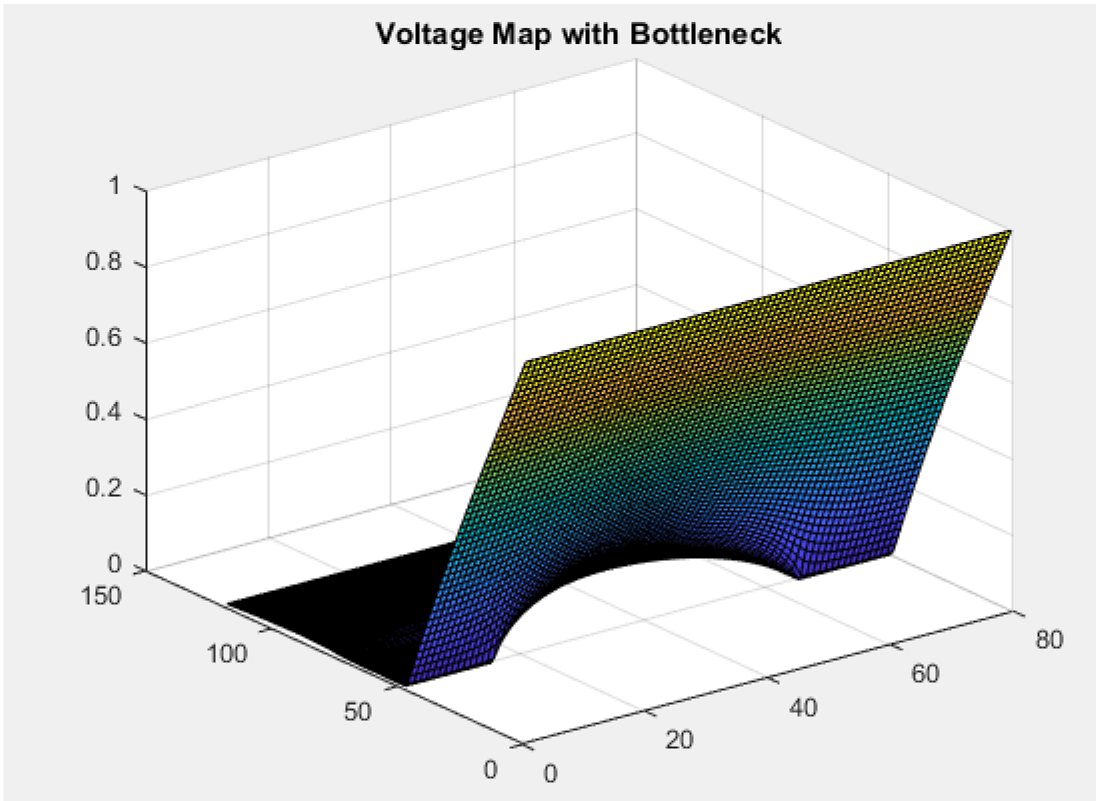

Figure 5: Voltage map with bottle necks added

The electric field was found by finding the gradient of the voltage map matrix and plotting with the MATLAB quiver function. Figures 6 and 7 below show the electric field in x and y directions respectively:

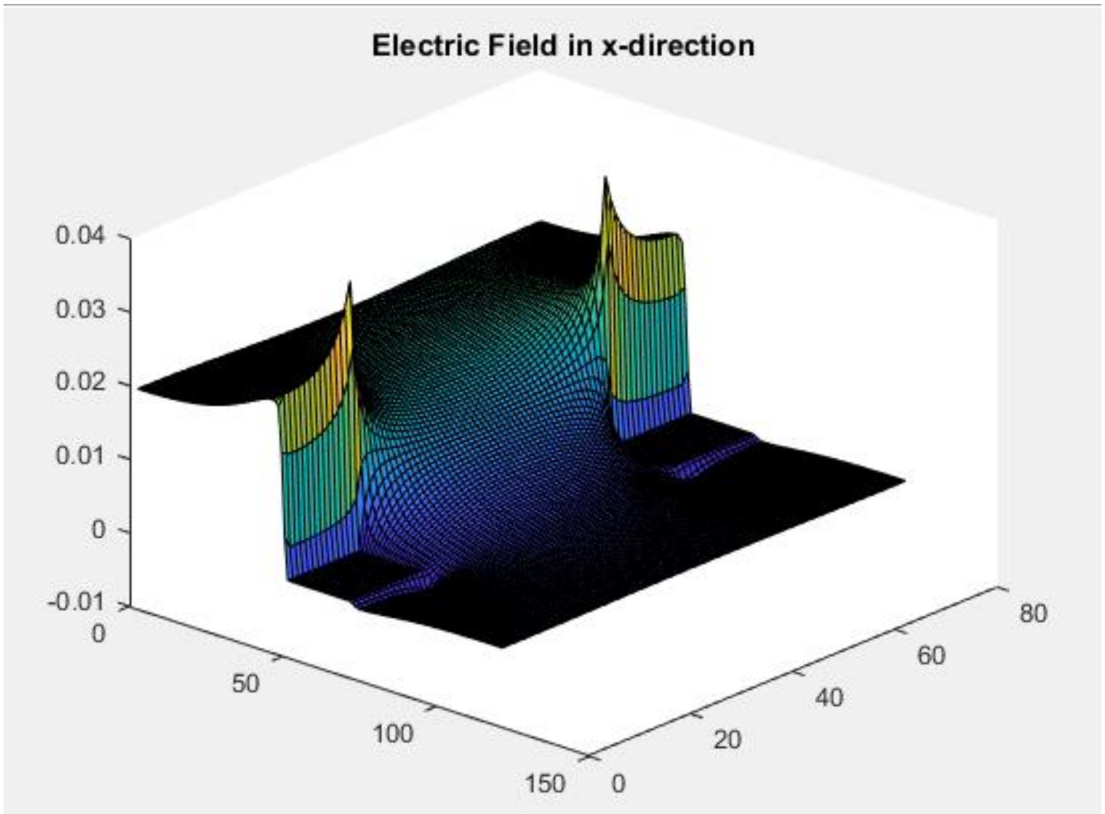

Figure 6: Electric field in x-direction

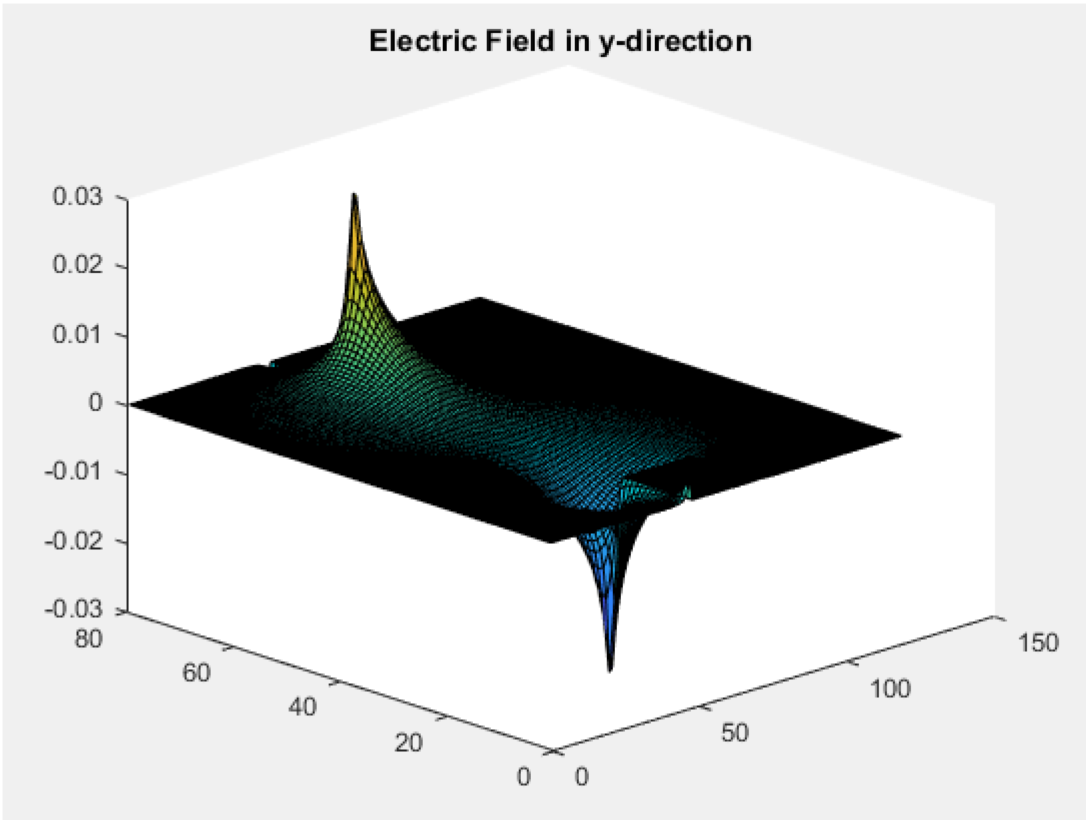

Figure 7: Electric field in y-direction

The current density was plotted using the MATLAB quiver function. Figure 8 below shows the quiver plot of the current density:

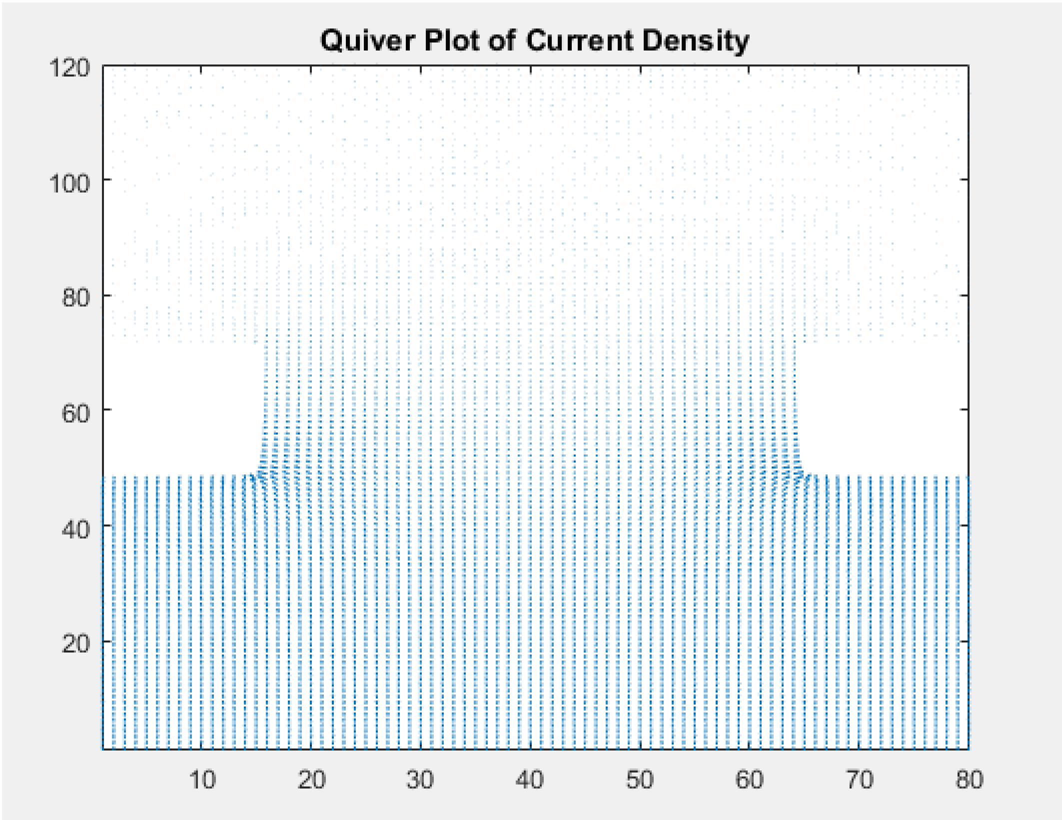

Figure 8: Quiver plot of current density

b. The current value was observed to be decreasing with mesh size. In this part of the experiment, the mesh size was started with (W, L) = 30, 20. The mesh size was increased in 10 loops by adding 30 to the width and 20 to the length to maintain the desired 3/2 ratio. Figure 9 below shows the graph of current vs mesh area:

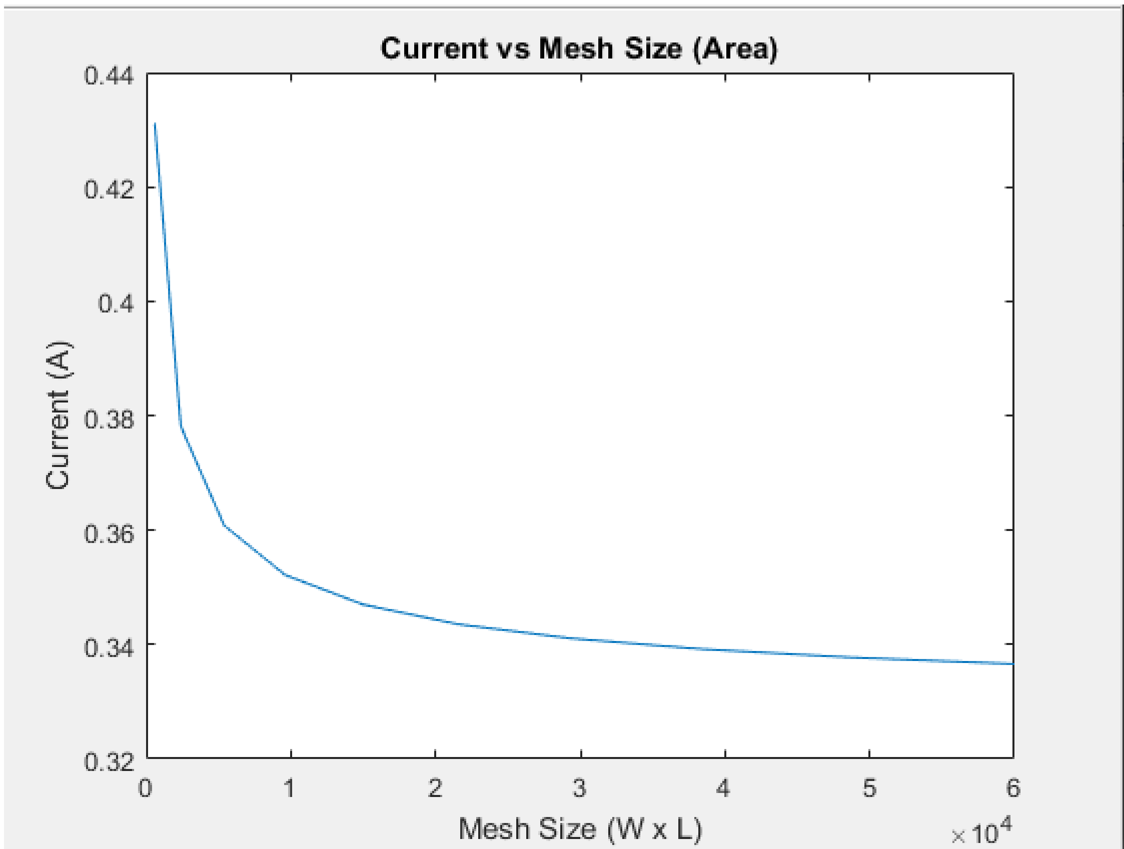

Figure 9: Current vs Mesh size (Area)

c. The current value increased with increased bottle-neck width. The bottleneck width is the space between the two boxes, so with increasing width of the box, the bottleneck width reduces. Figure 10 below shows the graph of current vs bottleneck width (This was done with (W, L) = (120, 80)):

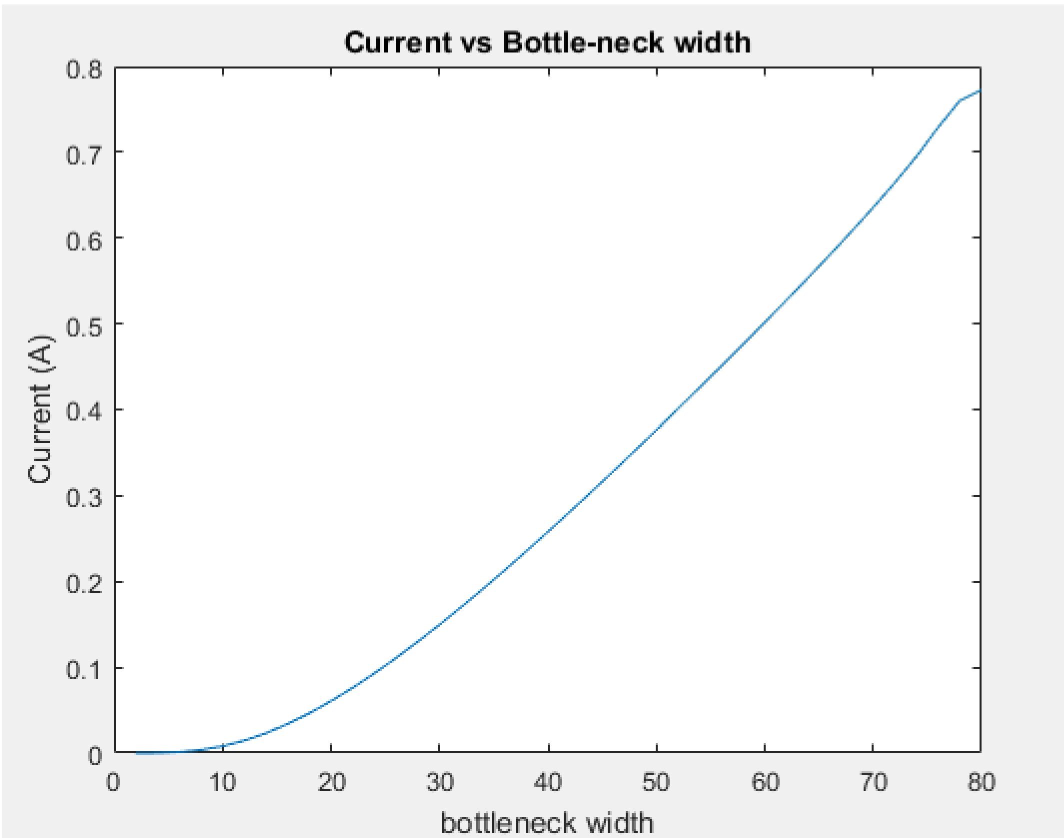

Figure 10: Current vs Bottle-neck width

d. The current increased linearly with conductivity inside the boxes. The starting conductivity was 0.01 and was increased by 0.01 for 10 iterations. Figure 11 below shows the plot of current versus conductivity.

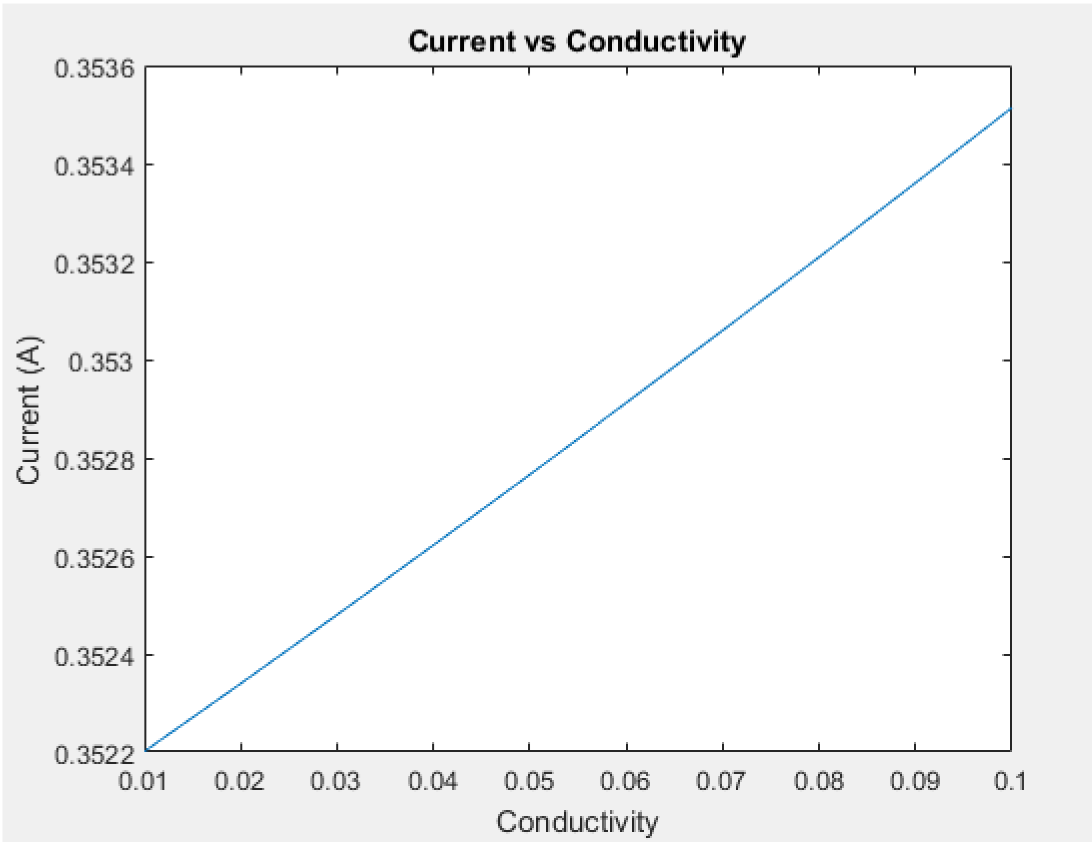

Figure 11: Current vs Conductivity

<u>Conclusion</u>

The voltage models in question 1, part-a, were obtained successfully. In the second part of question 1, the numerical solution model seemed accurate and was obtained successfully with reasonable voltage levels, but the analytical solution model had the right shape and somewhat unreasonable voltage values. Question 2 was done successfully and the results that were obtained were reasonable. The behavior of the model under the specified varying conditions were also reasonable and as expected. In conclusion, the experiment was very educative and successful.